SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

Content Area	Fourth Grade Mathematics
Recommended Days of Instruction	**First Nine Weeks**

Standards/Indicators Addressed:

Standard 4-2: The student will demonstrate through the mathematical processes an understanding of decimal notation as an extension of the place-value system; the relationship between fractions and decimals; the multiplication of whole numbers; and accurate, efficient, and generalizable methods of dividing whole numbers, adding decimals, and subtracting decimals.

4-2.1* Recognize the period in the place-value structure of whole numbers: units, thousands, millions, and billions. (A1)
4-2.3* Apply an algorithm to multiply whole numbers fluently. (C3)
4-2.4* Explain the effect on the product when one of the factors is changed. (B2)
4-2.6* Analyze the magnitude of digits through hundredths on the basis of their place value. (B4)
4-2.7* Compare decimals through hundredths by using the terms *is less than, is greater than,* and *is equal to* and the symbols <, >, and =. (B2)
4-2.8* Apply strategies and procedures to find equivalent forms of fractions. (C3)
4-2.9* Compare the relative size of fractions to the benchmarks 0, $\frac{1}{2}$, and 1. (B2)

4-2.10* Identify the common fraction/decimal equivalents $\frac{1}{2}$, =.5, $\frac{1}{4}$=.25, $\frac{3}{4}$ =.75, $\frac{1}{3}$ ≈ .33, $\frac{2}{3}$ ≈ .67, multiples of $\frac{1}{10}$, and multiples of $\frac{1}{100}$. (A1)

4-2.11* Represent improper fractions, mixed numbers, and decimals. (B2)

4-2.12* Generate strategies to add and subtract decimals through hundredths. (B6)

* These indicators are covered in the following 6 Modules for this Nine Weeks Period.
Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.

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Module 1-1 Number Structure and Relationships - Whole Numbers			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 1-1 Lesson A 4-2.1 Recognize the period in the place-value structure of whole numbers: units, thousands, millions, and billions. (A1)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle	See Instructional Planning Guide Module 1-1 Introductory Lesson A	See Instructional Planning Guide Module 1-1 Lesson A Assessing the Lesson

	Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4 ETA Cuisenaire, Mathematics with Manipulatives: Base Ten Blocks Video, Marilyn Burns		

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Module 1-2 Number Structure and Relationships - Fractions			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 1-2 Lesson A 4-2.9 Compare the relative size of fractions to the benchmarks 0, $\frac{1}{2}$, and 1. (B2)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle	See Instructional Planning Guide Module 1-2 Introductory Lesson A See Instructional Planning Guide Module 1-2, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 1-2 Lesson A Assessing the Lesson
Module 1-2 Lesson B 4-2.8 Apply strategies and procedures to find equivalent forms of fractions. (C3)		See Instructional Planning Guide Module 1-2 Introductory Lesson B See Instructional Planning Guide Module 1-2, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 1-2 Lesson B Assessing the Lesson

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	Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4		

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Module 1-3 Number Structure and Relationships – Decimals			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 1-3 Lesson A 4-2.6 Analyze the magnitude of digits through hundredths on the basis of their place value. (B4)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries	See Instructional Planning Guide Module 1-3 Introductory Lesson A See Instructional Planning Guide Module 1-3, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 1-3 Lesson A Assessing the Lesson
Module 1-3 Lesson B 4-2.7 Compare decimals through hundredths by using the terms *is less than, is greater than,* and *is equal to* and the symbols <, >, and =. (B2)		See Instructional Planning Guide Module 1-3 Introductory Lesson B	See Instructional Planning Guide Module 1-3 Lesson B Assessing the Lesson

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	NCTM's <u>Principals and Standards for School Mathematics</u> (PSSM) NCTM, <u>Mathematics Assessment Sampler</u>: Grades 3-5 ETA Cuisenaire, <u>Hands-On Standards</u>: Grades 3-4		

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Module 1-4 Number Structure and Relationships - Fractions and Decimals			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 1-4 Lesson A 4-2.10 Identify the common fraction/decimal equivalents $\frac{1}{2}$, =.5, $\frac{1}{4}$ =.25, $\frac{3}{4}$ =.75, $\frac{1}{3}$ ≈.33, $\frac{2}{3}$ ≈ .67, multiples of $\frac{1}{10}$, and multiples of $\frac{1}{100}$. (A1) 4-2.11 Represent improper fractions, mixed numbers, and decimals. (B2)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle	See Instructional Planning Guide Module 1-4 Introductory Lesson A See Instructional Planning Guide Module 1-4, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 1-4 Lesson A Assessing the Lesson

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Module 1-5 Operations - Addition and Subtraction			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 1.5 Lesson A 4-2.12 Generate strategies to add and subtract decimals through hundredths. (B6)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries	See Instructional Planning Guide Module 1-5 Introductory Lesson A See Instructional Planning Guide Module 1-5, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 1-5 Lesson A Assessing the Lesson

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Module 1-6 Operations – Multiplication			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 1-6 Lesson A 4-2.3 Apply an algorithm to multiply whole numbers fluently. (C3)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle	See Instructional Planning Guide Module 1-6 Introductory Lesson A See Instructional Planning Guide Module 1-6, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 1-6 Lesson A Assessing the Lesson

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Module 1-6 Lesson B 4-2.4 Explain the effect on the product when one of the factors is changed. (B2)	Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4	See Instructional Planning Guide Module 1-6 Introductory Lesson B	See Instructional Planning Guide Module 1-6 Lesson B Assessing the Lesson

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MODULE

1-1

Number Structure and Relationships – Whole Numbers

This module addresses the following indicator:

4-2.1 Recognize the period in the place-value structure of whole numbers: units, thousands, millions, and billions. (A1)

* This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

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I. Planning the Module

- **Continuum of Knowledge**

  Students have explored place value structures in second grade (2-2.4) and third grade (3-2.1) as they compared whole number quantities. Students also used their knowledge of place value to generate strategies for addition, subtraction and multiplication.

  In fourth grade, students Recognize the period in the place-value structure of whole numbers: units, thousands, millions, and billions (4-2.1).

  In fifth grade, students analyze the magnitude of a digit on the basis of its place value, using whole numbers and decimal numbers through thousandths.

- **Key Concepts/Key Terms**
  *These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge, and use in conversation with students.

  * period
  * millions
  * billions
  * units Submitted from A5
  * place value
  * thousands
  * hundred thousands

II. Teaching the Lesson

1. Teaching Lesson A: *What Comes Next*

Place value concepts are complex and are developed slowly over a long period of time. In grades K-3 students have had the opportunity to explore place value structure by exploring patterns within each period (<u>ten</u>, hundred, thousand then <u>ten</u> thousand, hundred thousand, etc.).

In fourth grade the emphasis should be on exploring the progression of the periods (ones, thousands, millions, billions) and the relationship among the periods using manipulatives such as base ten blocks, Cuisenaire rods, place value mats, and number cards. Lessons in place value at the fourth grade level must begin with a re-emphasis on the structure of place value. Students must understand that digits have different values depending on their position and that changing the position or place of a digit will affect its value. When moving to the

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left, from one period to the next will increase the value of the number by multiples of ten. Moving to the right, from one period to the other decreases the value of the number by multiples of ten. As with previous grades, the use of concrete experiences is the best method to introduce this concept before moving on to abstract representations.

For this indicator, it is **essential** for students to:

- Understand place value
- Identify the place value as units, thousands, billions, millions
- Read whole numbers using appropriate periods
- Write whole numbers using the appropriate period

For this indicator, it is **not essential** for students to:

None noted

- For clarification: units is the ones period

- The emphasis should be on exploring the progression of the periods (ones, thousands, millions, billions) and the relationship among the periods.

a. Indicators with Taxonomy

4-2.1 Recognize the period in the place-value structure of whole numbers: units, thousands, millions, and billions. → (A1)

Cognitive Process Dimension: Remember
Knowledge Dimension: Factual

b. Introductory Lesson A

Materials Needed:

Base-ten materials
Butcher paper/markers or pavement/sidewalk chalk
Linear measuring devices

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. Teaching Student Centered Mathematics: Grades 3-5, pages 48-51.

Engage students in a “What Comes Next?” discussion with the use of base-ten strips and squares. The unit or ones place is a 1-centimeter square. The tens place is a 10 x 1 strip. The hundreds place is a square, 10 cm x 10 cm. What is next? Ten

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hundreds is called a thousand. What shape? (10 hundreds squares) Have students tape 10 hundreds together. What is next? (Reinforce the idea of "10 makes 1" that has progressed to this point.) Ten one-thousands strips would make a square measuring 1 meter on each side. Once the class has figured out the shape of the thousand piece, the problem-based task is "What comes next?" Let small groups work on the dimensions of a ten-thousand piece.

Have students measure and draw the big pieces for "What Comes Next?" on butcher paper or on the pavement using sidewalk chalk. Ten ten-thousand squares (100,000) go together to make a huge strip. Have students draw this strip and mark off the 10 squares that make it up. How far to extend this activity is up to the teacher. (1 million = 10m x 10, 10 million = 100 m x 10 m strip, etc.)

You can also try the "What Comes Next?" discussion with three dimensional base-ten materials. The first three shapes are distinct: a cube, a long, and a flat. What comes next? Stack 10 flats. What comes next? Ten thousands cubes make another long. What comes next? Ten big longs make a big flat. etc.

Teacher Note: It is imperative to remember the intent of this indicator: the emphasis should be on exploring the progression of the periods (ones, thousands, millions, billions) and the relationship among the periods. A debriefing will be necessary after the activity to accomplish the intent of the indicator.

Possible Literature Connections

A Million Fish…More or Less (McKissack)
How Much is Million? (Schwartz)
If You Made a Million (Schwartz)
Millions of Cats (Wanda Gag)

c. Misconceptions/Common Errors

When reading a number that has more than one digit in the period, students may have difficulty understanding that they need to read both digits in the period. For example: 94,100,000 may be more difficult for students to understand than 9,100,000 where there is only one digit in the millions place.

It is important for students to realize that the place value system is not arbitrary and does have a logical structure.

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d. Additional Instructional Strategies/Differentiation

Count on Math: Making Your First Million
http://illuminations.nctm.org/LessonDetail.aspx?ID=L367
In this activity, students attempt to identify the concept of a million by working with smaller numerical units, such as blocks of 10 or 100, and then expanding the idea by multiplication or repeated addition until a million is reached. Additionally, they use critical thinking to analyze situations and to identify mathematical patterns that will enable them to develop the concept of very large numbers. This activity can be extended to billions.

Use base ten blocks to build numbers.

Write numbers in a teacher-made chart, which identifies the value of each digit.
Write numbers in word form underlining period names, for example: four hundred twenty-five million, two hundred three thousand, two

A place value table is useful to visualize how large a number is.

The following is an example of a place value table.

Billions			Millions			Thousands			Ones		
Hundred billion	Ten billion	billion	Hundred million	Ten million	million	Hundred thousands	Ten thousand	thousand	Hundreds	Tens	Ones

- 3 digits make a period
- These periods represent breaks in large numbers and are always separated by a comma.
- At every comma you name the period these numbers are in.

Millions			Thousands			Ones		
	7	1	4	2	2	3	3	6

Draw student's attention to the repeated pattern in each period (hundreds, tens, ones.)

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e. *Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

In addition to the questions asked during the discussion of the "What Comes Next?" activity, the student drawings, and the debriefing of the activity, it is suggested that the teacher use an exit ticket approach to formatively assessing the introductory lesson. A possible question might be: What affect does the period have on a digit's value? What evidence can you give to prove your explanation?

III. Assessing the Module

The objective of this indicator is to recognize which is in the "remember factual" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is on remembering factual knowledge related to the position of the period in the place value structure, students need a variety of experiences to support retention. The learning progression to **remember** requires students to recall the overall place value structure. Students use concrete models to explore the progression of periods. They analyze these relationships (4-1.1) and generalize connections (4-1.6) between and among periods. Students use correct, clear and complete oral and written language to communicate their understanding (4-1.5). When given the number, students recognize the words form. When given the word form, students recognize the number form.

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

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The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Which of these is the number 5,005,014?

   a. A five million, five hundred, fourteen
   b. B five million, five thousand, fourteen
   c. C five thousand, five hundred, fourteen
   d. D five billion, five million, fourteen

2. The estimated cost to build a new baseball stadium is ninety-four million dollars. What is this number in standard form?

   a. A $90,400
   b. B $94,000
   c. C $90,400,000
   d. D $94,000,000

3. Give the number: 40, 376, 572, 018

   i. Underline the millions period in Red.
   ii. Underline the thousands period in Blue.
   iii. Underline the billions period in Green.
   iv. Underline the units in Yellow.

4. What period is the underlined digit in?

   201,7<u>8</u>4,602

   A. ones period
   B. thousands period
   C. millions period
   D. billions period

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MODULE

1-2

Number Structure and Relationships - Fractions

This module addresses the following indicators:

4-2.8 Apply strategies and procedures to find equivalent forms of fractions. (C3)

4-2.9 Compare the relative size of fractions to the benchmarks 0, $\frac{1}{2}$, and 1. (B2)

* This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

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I. *Planning the Module*

- ***Continuum of Knowledge***

4-2.8
In third grade, students developed an understanding of fraction as parts of a whole (3-2.5) and represent fractions that are greater than or equal to 1 (3-2.6).

In fourth grade, students apply strategies and procedures to fine equivalent forms of fractions (4-2.8), compare the relative size of fractions to the benchmarks 0, $\frac{1}{2}$,and 1(4-2.9) and identify the common fraction/decimal equivalents $\frac{1}{2}$ =.5, $\frac{1}{4}$ =.25, $\frac{3}{4}$ =.75, $\frac{1}{3}$ ≈ .33, $\frac{2}{3}$ ≈ .67, multiples of $\frac{1}{10}$, and multiples of $\frac{1}{100}$(4-2.10). They also represent improper fractions, mixed numbers, and decimals (4-2.11).

In fifth grade, students compare whole numbers, decimals, and fractions by using the symbols <, > and = (5-2.4) and generate strategies to add and subtract fractions with like and unlike denominators (5-2.8).

4-2.9
In third grade, students developed an understanding of fraction as parts of a whole (3-2.5) and represent fractions that are greater than or equal to 1 (3-2.6).

In fourth grade, students compare the relative size of fractions to the benchmarks 0, $\frac{1}{2}$, and 1(4-2.9), apply strategies and procedures to find equivalent forms of fractions (4-2.8) and identify the common fraction/decimal equivalents $\frac{1}{2}$ =.5, $\frac{1}{4}$ =.25, $\frac{3}{4}$ =.75, $\frac{1}{3}$ ≈ .33, $\frac{2}{3}$ ≈ .67, multiples of $\frac{1}{10}$, and multiples of $\frac{1}{100}$(4-2.10). They also represent improper fractions, mixed numbers, and decimals (4-2.11).

In fifth grade, students compare whole numbers, decimals, and fractions by using the symbols <, > and = (5-2.4) and generate

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strategies to add and subtract fractions with like and unlike denominators (5-2.8).

- **Key Vocabulary/ Key Concepts**

  *These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

  *equivalent fractions
  *numerator
  *denominator
  *benchmark
  hundredths submitted from A5
  tenths

II. Teaching the Lesson(s)

1. *Teaching Lesson A*: *Compare size of fractions to benchmarks.*

In fourth grade students will move to understanding benchmark fractions and developing equivalent fractions. The most important reference points for fractions are 0, $\frac{1}{2}$, and 1. Therefore, students should have many concrete experiences with manipulating fractional parts in reference to the benchmark fractions of 0, $\frac{1}{2}$, and 1. The goal is for students to begin to develop a sense of fractions.

Fourth grade is the first year students are introduced to the concept of equivalent forms of fractions. The indicator begins with the phrase "Apply strategies and procedures" which means that students should progress from developing strategies to finding equivalent fractions to applying a procedure to find equivalent fractions. To help students understand equivalent fractions, they should have many experiences using concrete and pictorial models to find different names for a fraction. "When students understand that fractions can have different names, they should be challenged to develop a method for finding equivalent names. It might also be argued that students who are experienced at looking for patterns and developing schemes for doing things can invent an algorithm for equivalent fractions without further assistance." (Van de Walle, p. 155)

Therefore, by providing students with experiences that require that they find equivalent fractions and discover how equivalent fractions

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can be generated, students will have sufficient understanding to become fluent in doing so by the end of 4th grade.

For this indicator, 4-2.8, it is **essential** for students to:
- Understand the meaning of numerator, denominator and fraction
- Recall basic multiplication facts
- Understand that the fractions have the same value even though they look different
- Understand that simplifying a fraction does not change the value of the fraction
- Develop and apply a strategy for finding equivalent fractions
- Use concrete and/or pictorial models to find equivalent fractions

For this indicator, 4-2.8, it is **not essential** for students to:
- Multiply by a fractional form of one to find the equivalent fractions

For this indicator, 4-2.9, it is **essential** for students to:
- Recognize the benchmark fraction
- Locate or place fractions on the number line
- Use concrete or pictorial models to represent the comparison
- Recognize fractions that are greater than or equal to one
- Determine if two fractions are equivalent
- Recognize a fractional form of one
- Explain their reasoning
- Understand the concept of equivalency

For this indicator, 4-2.9, it is **not essential** for students to:
- Use a traditional algorithm to compare fractions

a. Indicators with Taxonomy

4-2.9 Compare the relative size of fractions to the benchmarks $0, \frac{1}{2}$, and 1. → B2

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

b. Introductory Lesson

Materials Needed:

- Fraction Cards (made with index cards and grids)
- Number Line labeled from 0 to 1 (drawn on the board or using sentence strips)

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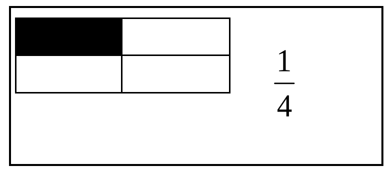
Introductory Lesson A

To refresh students' memories with fractions, have the students work in teams of four to create fraction cards for various fractions (such as: $\frac{1}{8}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{4}{5}$, $\frac{2}{3}$, $\frac{5}{8}$, $\frac{3}{9}$, $\frac{1}{10}$, $\frac{4}{12}$, $\frac{7}{10}$…) for a total of about 20 cards (making sure that there are some equivalent fractions in the set. These cards can be made in many ways, but it is important that the fraction shape is the same (a square is easiest).

Once these cards are made, they can be used for many different activities.



One activity is to locate fractions in relation to the benchmark 0, $\frac{1}{2}$, and 1. Have the students make a fraction card showing 0 and 1. Using the previously made fraction cards, have the students find their fraction card for $\frac{1}{2}$, and then place the 0, $\frac{1}{2}$, and 1 cards across their desk/table. (Have the same benchmark cards taped to and evenly spaced across the white/black board. Using the fraction card for $\frac{4}{5}$, for example, have the students compare the amount shaded in to the benchmarks 0, $\frac{1}{2}$, and 1. Which benchmark is it closest to? Place the $\frac{4}{5}$ fraction card near the benchmark it is closest too.) After modeling with one or two other fraction cards, have the groups sort the rest of their previously made fraction cards, placing them next to the benchmark they are closest to. (For modeling purposes, use cards that are very clear as to which fraction they are closest to. Save the ones like $\frac{1}{4}$, for example, that could be closest to 0 or $\frac{1}{2}$. This will cause students to self reflect as they sort their cards and generate good discussion when answers are shared. If students ask where a fraction card should go when they are sorting their fraction cards, simply tell them it is more important that they make a decision and be able to explain their thinking.)

Once all the groups have finished, lead a group discussion, having different groups share by which benchmark fraction they placed their fraction cards. Discuss any fractions that were not unanimous. Allow

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different students to explain why they placed the fraction card near that benchmark. Many students will notice equivalent fractions, too since this was introduced in 3rd grade. This is good for future activities. Through proper discussion as fraction cards are placed near the benchmarks, students will begin to see how they can look at the written fraction instead of the picture to determine its closest benchmark. Once the students seem to understand this, write several fractions on the board ($\frac{9}{12}$, $\frac{2}{8}$, and $\frac{5}{7}$). Have the students decide where these fractions belong. As you check these, encourage discussions about how they decided on their answers.
Another activity is to locate fractions on a number line. Once the students understand how to relate fractions to the benchmarks 0, $\frac{1}{2}$, and 1, tell them that the next task is to place the fractions on a number line. Simply tape a piece of string across the top of the benchmark fraction cards already on the white/black board or draw a line. Have the students make a similar number line at their desks. Working in their groups, have the students place the fractions where they think they belong on the number line (using the pictures to help them). When all the groups have finished, again share with the whole class, modeling on the number line on the board. Discuss any differences in the placing of fractions and encourage explanations. (For example, *Why did you choose to put four-twelfths and one-third in the same place?*) This activity leads to great discussions about fractions. (Encourage students to notice how fractions relate to one another, for example, one-eighth is smaller than one-seventh. *Why*?) As with the previous activity, once the students are finished, give them two or three new fractions (without illustrations) to place on the number line. Let the groups decide where they belong and have them explain their reasoning as they share.

c. Misconceptions/Common Errors

Student sometimes place fraction between the numerator value and denominator value. For example, ½ is between one and two. To emphasis that the focus is on the benchmark fraction, the number line should initially only show those fractions until they encounter a fraction greater than one.

d. Additional Instructional Strategies/Differentiation

The focus of the indicator is to build conceptual knowledge. Students are gaining a deeper understanding of the magnitude (size) of other fractions as they relate to the benchmark fractions. This

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understanding is not limited to mere procedural knowledge focused on a traditional algorithm that has not conceptual basis.

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. Teaching Student Centered Mathematics: Grades 3-5, pages 144-145

Fraction Model Sort
Have students sort fraction bars, fraction circles, etc. into three groups: those close to 0, those close to ½, and those close to 1.

Zero, One-Half or One
Write a collection of 10 to 15 fractions. A few of the fractions should be fractions greater than one (written as improper fractions) with the others ranging from 0 to 1. Let the students sort the fractions into 3 groups: those close to 0, those close to ½ and those close to 1. Students should draw pictorial models to justify their answers. The difficulty in this task depends on the fractions. This activity can be repeated over several days with fractions becoming more difficult on subsequent days.

Close Fractions
Have students name a fraction that is close to 1 but not more than 1. Next time have them name another fraction that is even closer to 1. For the second response, students should explain why they believe the fraction is closer to 1than the previous fraction using models to justify their answer. Repeat several times in the same manner. Similarly, try close to 0 or close to ½. Focus discussions on the relative size of fractions.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

This is a suggested resource:
http://illuminations.nctm.org/LessonDetail.aspx?ID=L80

f. Assessing the Lesson

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

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In addition to the questions asked during the lesson, it is suggested that the teacher use Red, Yellow and Green Cups. See Appendix A.

2. Teaching Lesson B: Equivalent Fractions

a. Indicators with Taxonomy

4-2.8 Apply strategies and procedures to find equivalent forms of fractions.

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural

b. Introductory Lesson B

Materials Needed:

- 12 Egg- Size Egg Cartons – one per pair of students
- Counters – about 30 per student
- Yarn – Cut in 8 inch lengths – 5 per student pair
- A Chart on the Board or on large Chart Paper – One column labeled Eggs Needed, one column labeled fractional part
- Transparency drawn to resemble the slots in the egg carton
- Overhead projector or interactive whiteboard

Before you begin, ask students to individually label a sheet of paper like your chart and ask them to record the responses like you are doing on your chart.

Ask students how many eggs the carton will hold. (12) Tell the students that you only need six eggs and that your grocer is willing to cut the carton and let you buy only the eggs you need. Ask the students to place a piece of yarn to show how the carton could be cut. The way the yarn is placed will vary and that is OK. Next ask:

- What fraction of the total number of eggs do I want? If you get $\frac{1}{2}$ and $\frac{6}{12}$, ask
- How can they both be correct?
- If students don't respond with "$\frac{6}{12}$", ask is there another way to say what fraction 6 out of 12 eggs represents?
- Record the number of eggs (6) and the fractions $\frac{1}{2}$ and $\frac{6}{12}$, on the chart.
- Ask students to use the transparency on the overhead or interactive whiteboard to share how they placed their yarn. It is important for students to see that one-half or six twelfths is the

same regardless of where the yarn is placed, provided it evenly divides the carton.

Repeat the above process asking the students to use yarn to divided the carton into groups of 2 eggs (which yields the fractions $\frac{2}{12}$ or $\frac{1}{6}$), 3 eggs (which yields the fractions $\frac{3}{12}$ or $\frac{1}{4}$), 4 eggs (which yields the fractions $\frac{4}{12}$ or $\frac{1}{3}$). Each time record the information on your chart and have students do the same with their individual charts.

After the activity is complete, ask students if an equal sign can be put between the two fractions that represent the same number of eggs. Then group students in pairs. Ask student pairs to determine how the fractions can be equal when they are written with different numbers. (The point is to get students to recognize that the area represented by $\frac{4}{12}$ is equal to the area represented by $\frac{1}{3}$. Thus the fractions are equivalent.) After giving students an opportunity to share their reasoning, challenge the students to generate more fractions equivalent to the ones listed on their chart and to draw pictures of each. The point here is to move students toward recognizing how equivalent fractions are generated.

c. Misconceptions/Common Errors

Using the phrase *reducing fractions* confuses students because it implies making a fraction smaller; therefore, the appropriate mathematical term *simplest form* should be used.

d. Additional Instructional Strategies/Differentiation

To help students understand equivalent fractions, they should have many experiences using concrete and pictorial models to find different names for a fraction. "When students understand that fractions can have different names, they should be challenged to develop a method for finding equivalent names.

Using concrete models and pictorial models will help students better visualize equivalent fractional relationships.

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Students should be provided with experiences that enable them to reach the conclusion that equivalent fractions can be generated by multiplying a fraction by a fractional form of one. Students can think about it like "what can I do to the fraction that I have to get this new equivalent fraction?" Student may need to guided through the process of recognize that they can multiply the numerator and denominator by the same number and get their equivalent fraction. Since students have not generated strategies for multiplication of fractions, they will not think of it as multiplying the fraction by a fractional form of one.

The following is a challenge problem for students who have a solid understanding of equivalent fractions:

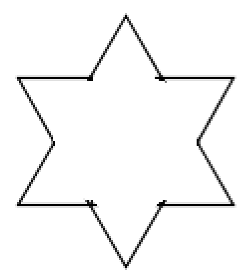
Materials Needed:

Triangular Graph Paper
Colored Pencils
Pattern Blocks

Problem Solving Application with the Pattern Blocks.

The following activity will challenge the students to use problem solving and higher level thinking skills to determine the equivalence. Many will begin to develop their own rules for finding equivalent fractions.

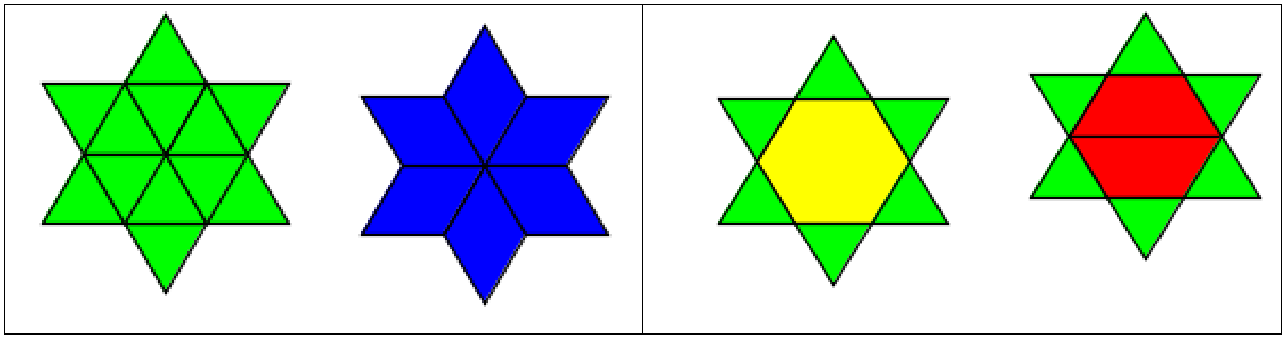
1. Construct a shape similar to the star below. If the star represents 1, use the pattern blocks to complete the equivalents below.



2. Ask the students what the relationship or ratio is of the trapezoid to the star. This relationship can be shown as trapezoid : star or $\frac{trapezoid}{star}$

3. Tell the students that you are going to find the equivalent ratio of this relationship using the fractional form of $\frac{trapezoid}{star} = \frac{?}{12} = \frac{1}{?}$

This becomes a challenge for the students, but allows them to really show an understanding of equivalence. Students may begin building this shape with the trapezoid, because it is located in the numerator. They will quickly realize that this particular shape cannot be constructed using just the trapezoid pieces. Teachers should encourage students to try all possible ways to construct the figure. They may then move to the parallelograms or the triangles. Do NOT discourage students from trying all variations of constructing the shape. Possible representations may be:



4. To determine the equivalency of the first fraction, students should conclude that they must use the green triangles, since it would take 12 green triangles to construct the star shape. Once this has been determined, guide them to place the trapezoid within the star and see how many triangles the trapezoid covers. Students should discover that is would take 3 triangles to cover the same area as one trapezoid. If it takes 12 triangles to cover the star, then each triangle would be worth $\frac{1}{12}$. If the trapezoid covers three of the triangles, then the trapezoid would be worth $\frac{3}{12}$. Therefore, students should conclude that $\frac{trapezoid}{star} = \frac{3}{12}$.

5. The challenge for the students now is to determine what the denominator would be if the numerator is 1. The given equation states that the trapezoid is what is represented in the numerator and the denominator represents the complete star shape. Looking at the given equation, students must determine what the denominator would be, given that the numerator represents the trapezoid and is represented by 1.

6. Students should have already determined that they cannot make the star shape with the trapezoid. However, they have just discovered that they can make the trapezoid with the triangles, and they can also construct the star with the triangles. With experimentation (<u>let the students explore this</u>), students should discover that they can take the twelve triangles and see how many trapezoids they can make by placing the 12 triangles on trapezoids until they run out of triangles. They should discover that they can construct 4 trapezoids, which would mean that 4 trapezoids have the same area as 12 triangles, which means that it takes 4 trapezoids to have the same equivalent area as the star shape. This conclusion is

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based on the equivalency of the triangles and the trapezoid because the students cannot construct the star with the trapezoids. Their solution to the equivalencies using the trapezoid would be

$$\frac{trapezoid}{star} = \frac{3}{12} = \frac{1}{4}$$

6. Have the students to solve the following problems in the same way, using the Pattern Blocks. Ask students to record their work on triangular graph paper and to write down the reasoning they used to find each equivalent fraction.

- Exercises

$$\frac{2\ blue\ parallelograms}{star} = \frac{?}{6} = \frac{1}{?} = \frac{?}{12}$$

$$\frac{hexagon}{star} = \frac{6}{?} = \frac{?}{6} = \frac{1}{?}$$

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- http://illuminations.nctm.org/LessonDetail.aspx?ID=L543
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L338
- http://www.northcanton.sparcc.org/~elem/kidspiration/bricker/EquivalentFractions.htm moved from comparing indicator

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

In addition to the questions asked during the activity, it is suggested that the teacher use Red, Yellow and Green Cups. See Appendix A.

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III. Assessing the Module

4-2.9

Assessment Guidelines

The objective of this indicator is to compare which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning therefore, students should are not just learning procedural strategies for comparing fractions but they are also building number sense related to fractions. The learning progression to **compare** requires students to recall the benchmark fractions and their value. Students then use their understanding of the various forms fractions (equivalent, improper, etc..) to create concrete and/or pictorial models to represent the relationship the benchmark fraction and the given fraction. Students analyze these representations (4-1.1) and construct arguments (4-1.2) to explain those relationships. Students recognize mathematical symbols <, >, and = and their meanings and select an appropriate symbol or word. They explain and justify their mathematical ideas (4-1.3) to classmates and their teacher using correct, complete and clearly written and oral mathematical language to communicate their reasoning (4-1.5).

4-2.8

The objective of this indicator is to apply which is in the "apply procedural" cell of the Revised Taxonomy. To apply means to carry out a procedure in familiar and unfamiliar situations; therefore, students should develop then apply their procedure to a variety of examples. The learning progression to **apply** requires students to recall and understand the meaning of numerator, denominator and fraction. Students explore concrete and pictorial models to investigate and visualize equivalent fractional relationships. As students analyze information from these experiences, they generalize connections (4-1.6) between equivalent fractions and explain and justify their reasoning (4-1.3) to their classmates and their teacher. Students use correct, complete and clearly written and oral mathematical language to communicate their ideas (4-1.5). Student should then generate mathematical strategies (4-1.4) and apply those strategies to find equivalent forms of fractions.

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

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The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Which fraction is closest to a whole?

A $\frac{1}{6}$ B $\frac{1}{4}$ C $\frac{1}{3}$ D $\frac{1}{2}$

2. Which set contains only equivalent fractions?

A $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}$

B $\frac{2}{4}, \frac{2}{5}, \frac{2}{6}$

C $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}$

D $\frac{1}{2}, \frac{2}{4}, \frac{4}{6}$

Item #3 is Adapted From Mathematics Assessment Sampler, Grades 3-5, NCTM, 2005, page 11.

3. Students in Mrs. Johnson's class were asked to tell why $\frac{4}{5}$ is greater than $\frac{2}{3}$. Whose reason is correct? Explain your answer.

A Kelly said, "Because 4 is greater than 2."
B Keri said, "Because 5 is larger than 3."
C Kim said, "Because $\frac{4}{5}$ is closer to 1 than $\frac{2}{3}$."
D Kevin said, "Because 4 + 5 is more than 2 + 3."

4. These two fractions are equivalent. Give <u>one</u> more fraction that is equivalent to these.

$\frac{2}{3}$ and $\frac{8}{12}$

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MODULE

1-3

Number Structure and Relationships - Decimals

This module addresses the following indicators:

4-2.6 Analyze the magnitude of digits through hundredths on the basis of their place value. (B4)
4-2.7 Compare decimals through hundredths by using the terms *is less than, is greater than,* and *is equal to* and the symbols <, >, and =. (B2)

* This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

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I. Planning the Module

- **Continuum of Knowledge**

4-2.6
Fourth grade is the first time students are introduced to the concept of decimals. Students analyze the magnitude of digits through hundredths on the basis of their place value (4-2.6) and compare decimals through hundredths by using the terms is less than, is greater than and is equal to and the symbols < ,> or = (4-2.7).

In fifth grade, students compare whole number, decimals, and fractions by using the symbols <, > and = (5-2.4) and apply an algorithm to add and subtract decimals through thousandths (5-2.5). In sixth grade, students generate strategies to multiply and divide fractions and decimals (6-2.5).

4-2.7
Fourth grade is the first time students are introduced to the concept of decimals. Students compare decimals through hundredths by using the terms is less than, is greater than and is equal to and the symbols < ,> or = (4-2.7).

In fifth grade, students compare whole number, decimals, and fractions by using the symbols <, > and = (5-2.4) and apply an algorithm to add and subtract decimals through thousandths (5-2.5). In sixth grade, students generate strategies to multiply and divide fractions and decimals (6-2.5).

- **Key Concepts/Key Terms**
  *These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

  * decimal
  * decimal point
  * hundredths
  * tenths
  compare from A5

II. Teaching the Lesson(s)

1. Teaching Lesson A: **Decimal Place Value**

4-2.6

For this indicator, it is **essential** for students to:

- Understand that the role of the decimal point is to designate the unit position.
- Use concrete and pictorial models to identify decimals through hundredths
- Understand that the pattern with tens is true on both sides of the decimal point

For this indicator, it is **not essential** for students to:
None noted

4-2.7

For this indicator, it is **essential** for students to:

- Understand place value
- Understand the role of the decimal point
- Model place value relationships i.e. what does tenths look like, what does hundredths look like, etc…
- Understand the difference between when zero is in the tenths like (0.05) and when zero is in the hundredths place (0.5)

For this indicator, it is **not essential** for students to:
Compare decimals beyond the hundredths place

Fourth grade is the first time students are introduced to the concept of decimals. Therefore, they should begin their work using concrete and pictorial models to identify decimals through hundredths. It is very important that students have a firm understanding of decimals and the place value system in order to make a shift from concrete to symbolic and to later work with fraction – decimal equivalencies. This will also get students ready to generate strategies to add and subtract decimals as part of the fourth grade standards.

When working with decimals through hundredths, the emphasis for students is conceptual development. Students should have enough experiences with concrete and pictorial models to form visual images of decimals like 0.5 versus 0.05 when they see or hear the number. Also, when developing the concept through pictorial and concrete models, the decimal relationship to the whole should be stressed. Therefore, when students use symbols or words to make a value comparison, the comparison is made based on an in-depth

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understanding of the relative size of each decimal, rather than using a comparison process or mnemonic strategy.

Using concrete and pictorial models, students should be able to identify place value, and read, and compare decimals through hundredths. When comparing decimals, students should be comfortable using both comparison words *is less than, is greater than,* and *is equal to* and their respective symbols(<, >, =).

a. Indicators with Taxonomy

4-2.6 Analyze the magnitude of digits through hundredths on the basis of their place value. →B4

Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual

b. Introductory Lesson A

Materials Needed:

- Base Ten Blocks
- Overhead Base Ten Blocks
- Centimeter Graph Paper
- Write place values *hundreds* through *hundredths* across the top of the board, including the decimal point

(Adapted from Teaching Student Centered Mathematics: Grades 3-5, Van de Walle, John A. & Lovin, LouAnn H., 2006. pages 183 – 190.)

Before introducing decimals to students, it is advisable to review some ideas of whole number place value (the 10 to 1 relationship between the value of any two adjacent positions. See Module 1-1, Grade 4).

Having established the progression to larger pieces, focus on the idea that each piece to the right gets smaller by 1/10. The critical question is "Is there ever a smallest piece?" Couldn't any piece be divided into 10 smaller pieces? There is no smallest strip or smallest square. The goal of this discussion is to help students see that a 10 to 1 relationship can extend indefinitely in two directions.

Important Points to Remember:
- ✓ Any unit can be chosen as the ones unit.
- ✓ The role of the decimal point is to designate the unit position.

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Allow students to work in pairs. Distribute base ten blocks to each pair.

Inform students that the hundreds block represent “one”. Ask students pairs to discuss how they would describe the ten strip and cube in relationship to the whole. Allow groups to share. Because of their work with fractions, they may report in fraction form. After groups have shared, inform students there is another form of fractions called *decimals* and demonstrate how to write the decimal notation, lining the numbers up with the place value name written across the top of the board. Do not stress the fraction decimal relationship at this time.

Ask student pairs to use their ten strip and cubes to form another decimal. Allow student groups to share, demonstrate on the overhead and write on the board, again lining up with the place values previously listed on the board. Keep a horizontal list of examples on the board.

Next, tell students to look closely at the place value chart and list of examples on the board. Ask students to share what they notice about the place value chart – this may include their knowledge of whole number place value as well. The object here is for students to see the relationship between the whole number and decimal place values. Even though not required, ask students to predict what they think the next decimal place value will be based on their knowledge of whole number place value. Again, the purpose is for them to recognize the pattern and relationships.

Next, make the connection to picture form. Give student groups centimeter grid paper. Ask them to build a decimal with the base ten blocks and challenge their partner to label and represent the decimal on the grid paper. Have students save their work for use in the lesson below that deals with comparison of decimals.

A game that can be played to reinforce this requires only one die or any random number generator and base ten blocks (or the paper model.) After the player rolls the die he/she chooses the appropriate number of hundredths cubes. Play goes to the next player who repeats the process with a new roll of the die. When play returns to the first player, he/she rolls and chooses again. When the player has enough hundredths cubes to trade for a tenth, he/she does so. The object of the game is to be the first player to gain the ones block. This game can also be played using pennies, dimes, and a dollar once the children are comfortable with the proportional relationship.

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After students have had ample experience discussing and representing decimals through hundredths, have students work in pairs and challenge them to prepare a brief presentation about the relationship of money to decimals. Allow student groups to share.

c. *Misconceptions/Common Errors*

4-2.6 Student may have difficulty transferring their understanding of one, tens, hundreds to tenths and hundredth. They may mistakenly believe that they should be ones place to parallel the whole number place value system.

d. *Additional Instructional Strategies/Differentiation*

4-2.6 When working with decimals through hundredths, the emphasis for students is conceptual development. Students should have enough experiences with concrete and pictorial models to form visual images of decimals like 0.5 versus 0.05 when they see or hear the number. Also, when developing the concept through pictorial and concrete models, the decimal relationship to the whole should be stressed.

Students need to see decimal representations in formats other than square base ten blocks. Therefore, another manipulative for demonstrating decimal portions is a hundredths disk (Teaching Student Centered Mathematics: Grades 3-5, Van de Walle, John A. & Lovin, LouAnn H., 2006. blackline master 17.) The disk is created by copying it twice for each model, preferably on two different colored sheets of paper. Cut each disk on one of the solid lines to the center point and slide one into the other. With this you can easily model and have children create decimal regions. This manipulative is excellent for understanding decimals as part of a whole. It becomes an excellent model for comparing decimals when used to show such decimals as 0.34 and 0.4.

An excellent length model is a meter stick. Decimeters mark out tenths and centimeters mark out the hundredths.

e. *Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

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f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

In addition to the questions asked during the lesson, it is suggested that the teacher use Red, Yellow and Green Cups. See Appendix A.

2. Teaching Lesson B: Comparing Decimals

a. Indicators with Taxonomy

4-2.7 Compare decimals through hundredths by using the terms *is less than, is greater than,* and *is equal to* and the symbols <, >, and =. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

b. Introductory Lesson B

Materials Needed:

Previous centimeter grid work
Overhead base ten blocks

Return to students' work they did on centimeter grid paper from the above lesson when they were introduced to decimals. Allowing students to use their previous work is a good way to build on prior knowledge – the work is something they created and with which they are familiar.

Group the students in pairs by some random manner. Challenge student pairs to compare with their partner their centimeter grid work and be prepared to share with the class whether or not the decimal representations are equal, greater than, or less than their partner and how they know. Allow pairs to use overhead base ten blocks to show their representations on the overhead and include comparison symbols.

Have students record their work in their notebooks.

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c. Misconceptions/Common Errors

4-2.7 Student may not realize that if a digit is not present in the hundredths place that a zero can be used as place holder.

d. Additional Instructional Strategies/Differentiation

4-2.7 Student should explore a variety of representation to deepen their conceptual understanding of decimals. These experiences will build a sense of the magnitude (size) of the decimals so that when students use symbols or words to make a value comparison, the comparison is made based on an in-depth understanding of the relative size of each decimal, rather than using a comparison process or mnemonic strategy.

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Compare Decimals:
http://www.aaastudy.com/dec52_x2.htm

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

In addition to the questions asked during the lesson, it is suggested that the teacher use Red, Yellow and Green Cups. See Appendix A.

III. Assessing the Module

4-2.6
The objective of this indicator is to analyze which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To analyze

means to determine relevant features and relationships. The learning progression to **analyze** requires students to understand place value and be able to locate the correct place value. Students represent the place value using concrete and/or pictorial models and generalize the connections (4-1.6) between place values and the multiple of tenths. They use these connections to generate statements (4-1.4) about the magnitude of numbers. Students explain and justify their answers (4-1.3) and use correct, complete and clearly written and oral language to communicate their ideas (4-1.5).

4-2.7
The objective of this indicator is to compare which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning therefore, students should are not just learning procedural strategies for comparing decimals but they are also building number sense related to decimals. The learning progression to compare requires students to recognize the place value of digits through the hundredths, compare the place value of digits using concrete models to support understanding where appropriate. Students recognize mathematical symbols <, >, and = and their meanings. As students analyze place value patterns (4-1.1), they construct arguments and explain and justify their mathematical ideas to classmates about which symbol is appropriate (4-1.3), they should use correct, complete and clearly written and oral mathematical language to communicate their reasoning (4-1.5).

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Dana is weighing rocks in science class. The first rock weighs 0.62 pounds. The second weighs 0.8 pounds. Write a number sentence that compares the two numbers using <, >, or =.

2. Compare the numbers using the symbols <, >, and =.

   0.8 _____ 0.59

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3. Tenley finally got her Karaoke Master 5000 in the mail. It's a machine that automatically scores people's Karaoke performances. To test the machine out, Lois and Pete had a singing contest. Lois sang and got a score of 8.41. Pete sang and got a score of 8.14. Compare Lois and Pete's scores using the words *greater than, less than,* or *equals*.

4. In the number 123.45,
   a. What is the value of the 5?
   b. Which digit has the greatest value?
   c. What is the value of the 2?

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MODULE

1-4

Number Structure and Relationships - Fractions and Decimals

This module addresses the following indicators:

4-2.10 Identify the common fraction/decimal equivalents $\frac{1}{2}$, =.5, $\frac{1}{4}$ =.25, $\frac{3}{4}$ =.75, $\frac{1}{3}$ ≈.33, $\frac{2}{3}$ ≈ .67, multiples of $\frac{1}{10}$, and multiples of $\frac{1}{100}$. (A1)

4-2.11 Represent improper fractions, mixed numbers, and decimals. (B2)

* This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

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I. Planning the Module

- **Continuum of Knowledge**

4-2.10 In third grade, students developed an understanding of fraction as parts of a whole (3-2.5) and represent fractions that are greater than or equal to 1 (3-2.6).

In fourth grade, students compare the relative size of fractions to the benchmarks 0, $\frac{1}{2}$, and 1(4-2.9), apply strategies and procedures to find equivalent forms of fractions (4-2.8)and identify common the fraction/decimal equivalents $\frac{1}{2}$ =.5, $\frac{1}{4}$ =.25, $\frac{3}{4}$ =.75, $\frac{1}{3}$ $\approx$ .33, $\frac{2}{3}$ $\approx$ .67, multiples of $\frac{1}{10}$, and multiples of $\frac{1}{100}$(4-2.10).
They also represent improper fractions, mixed numbers, and decimals (4-2.11).

In fifth grade, students compare whole numbers, decimals, and fractions by using the symbols <, > and = (5-2.4) and generate strategies to add and subtract fractions with like and unlike denominators (5-2.8).

4-2.11 In third grade, students developed an understanding of fraction as parts of a whole (3-2.5) and represent fractions that are greater than or equal to 1 (3-2.6).

In fourth grade, students compare the relative size of fractions to the benchmarks 0, $\frac{1}{2}$, and 1(4-2.9), apply strategies and procedures to find equivalent forms of fractions (4-2.8)and identify common the fraction/decimal equivalents $\frac{1}{2}$ =.5, $\frac{1}{4}$ =.25, $\frac{3}{4}$ =.75, $\frac{1}{3}$ $\approx$ .33, $\frac{2}{3}$ $\approx$ .67, multiples of $\frac{1}{10}$, and multiples of $\frac{1}{100}$(4-2.10).
They also represent improper fractions, mixed numbers, and decimals (4-2.11).

In fifth grade, students compare whole numbers, decimals, and fractions by using the symbols <, > and = (5-2.4) and generate strategies to add and subtract fractions with like and unlike denominators (5-2.8).

In third grade, students experiences with decimals were limited to using the fewest possible number of coins when making change less than and greater than a $1.00 and even then the concept of decimals was not

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formally discussed. It is difficult for a child, whose main identity with decimals is money, to form the flexibility needed to work with decimals such as 47.8 and 6.123. Therefore, it is wise not to initiate teaching of decimals with money. Money is an application of decimals, and should be taught after conceptual development of the ten to one relationship is in place.

In third grade students represented fractions equal to or greater than one and left the fraction in improper form.

- **Key Concepts/Key Terms**

  *These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

  * common fraction
  * decimal equivalent
  * mixed number
  * improper fraction
  * decimals
  * represent
  * translate

II. Teaching the Lesson(s)

1.Teaching Lesson A: Fraction/Decimal Match

4-2.10

For this indicator, it is **essential** for students to:

- Recall the relationships outlined in the indicator
- Understand that a fraction and a decimal are two ways to write the same number
- Find a concrete and or real world representation of these equivalencies to support retention of these facts
- Understand the concept of equivalency
- Understand that not all fractions have exact decimal equivalent
- Understand the meaning of approximation (≈)

For this indicator, it is **not essential** for students to:

- Recall relationships not listed
- Divide fractions to find equivalent decimals

4-2.11

For this indicator, it is **essential** for students to:

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- Go back and forth between the three representations i.e. improper to decimal, mixed number to improper, decimal to mixed number, etc…
- Recall equivalent fractions
- Recall common benchmark fractions
- Recognize that a decimal has a whole number part and a decimal part
- Understand the role of the decimal is the designate the unit position
- Understand that a mixed number has a whole number part and a fractional part

For this indicator, it is **not essential** for students to:

- Use traditional algorithms to convert from improper to mixed number
- Divide fractions to convert from fraction to decimal

In fourth grade Indicators 4-2.6 and 4-2.7 (Module 1-3), students were introduced to the concept of decimals through hundredths. Now, students pull together their knowledge of fractions [began in third grade and extended in fourth grade indicators 4-2.8 and 4-2.9 (Module 1-2)] and decimals. Therefore, they will need adequate concrete and pictorial experiences in order to develop a conceptual understanding of relative size.

Fourth grade students should identify and represent the common fraction-decimal equivalents $\frac{1}{2}$, =.5, $\frac{1}{4}$ =.25, $\frac{3}{4}$ =.75, multiples of $\frac{1}{10}$, and multiples of $\frac{1}{100}$. They should also identify the approximate fraction-decimal equivalents $\frac{1}{3} \approx .33$ and $\frac{2}{3} \approx .67$. It is important that students understand that a fraction and a decimal are two different ways of writing the same number. Students must have a good understanding of fractions and decimals, so it is easiest to begin with fractions of base ten (tenths and hundredths). By using tenths and hundredths grids (or another concrete model), students can see that shading $\frac{1}{10}$ as a fraction or a decimal is the same. By having the students read both the fraction and decimal correctly, they can see that they are read with the same words.

Once students have a conceptual understanding of tenths and hundredths, they can begin to look at other common fraction-decimal equivalents (i.e. $\frac{1}{2}$ is the same as 0.5, $\frac{1}{4}$ is the same as 0.25, and $\frac{3}{4}$ is the same as 0.75). The students may need to use their skills with equivalent fractions to help see the common decimal equivalent (for example $\frac{1}{4} = \frac{25}{100}$, so the decimal is 0.25). It is not the intent in fourth grade for students to memorize how to divide a fraction to get a decimal. Fourth graders need lessons (using concrete examples where possible) to become familiar with these common equivalents. It is more important for students to be able to relate fraction-decimal equivalents and approximate equivalents with everyday concepts with which they are familiar. For example, students can connect $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$ to amounts of money (for example, if you ask students how much money is half a dollar, they know it is fifty cents and can be written 0.5 or 0.50. The same is true for a fourth of a dollar= 0.25 and three fourths of a dollar = 0.75. Although, student experiences with fraction/decimal equivalents should not be limited to the concept of money.

It is important that students have a clear understanding that not all fractions have exact decimal equivalents. Therefore, asking students to use manipulatives to show fraction/decimal relationships such as $\frac{1}{3} \approx .33$ and $\frac{2}{3} \approx .67$ will generate interesting conversation and serve as the foundation for work with decimals in later grades. It is important that students understand that .33 and .67 are approximate equivalents and thus, the use of the *approximately equal to* sign(≈). Again, a simple teaching strategy might be to ask students to determine one-third and two-thirds of a dollar. The discovery by students can then be linked to the concept of rounding.

In fourth grade students will simplify those fractions greater than one. When simplifying, students should be able to make the connection between concrete and pictorial models, improper fractions, and simplified fractions. For example, if you asked students to make a model using paper and crayon of the improper fraction $\frac{9}{4}$, students should be able to see that the denominator is 4 and thus they are working with fourths. They should then draw nine parts with a value of one-fourth each. They should use their knowledge that $\frac{4}{4} = 1$ and

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group the fourths to make wholes. Thus resulting in $\frac{9}{4}$ = 2 $\frac{1}{4}$ = 2.25 using this kind of model.

a. Indicators with Taxonomy

4-2.10 Identify the common fraction/decimal equivalents $\frac{1}{2}$, =.5, $\frac{1}{4}$ =.25, $\frac{3}{4}$ =.75, $\frac{1}{3}$ ≈.33, $\frac{2}{3}$ ≈ .67, multiples of $\frac{1}{10}$, and multiples of $\frac{1}{100}$. → A1
Cognitive Process Dimension: Remember
Knowledge Dimension: Factual

4-2.11 Represent improper fractions, mixed numbers, and decimals. → B2
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

b. Introductory Lesson
Materials Needed:

- Index cards with fractions on some and decimals on others. Make certain the cards are pairs. For example, if $\frac{5}{10}$ is written on one card, .5 should be written on another.
- 10 x 10 grid paper
- Transparency of 10 x 10 grid paper
- Overhead markers

As previously stated, since students have had experiences with fraction models and with decimal models, the goal here is to combine the two.

Give each student a sheet of 10 x 10 grid paper and either a fraction or decimal index card. Then ask students to shade their grid paper to represent the fraction or decimal on their index card. Next, ask students to find their partner – fraction/decimal partner. Call on a couple of student pairs to show on the overhead their fraction/decimal numbers and to explain how they know they are partners.

Collect all cards, mix them up, redistribute and repeat the find your fraction/decimal partner activity.

Ask students to select a fraction and to write the equivalent decimal and to explain how they know they are equivalent. Record findings in their notebooks.

c. Misconceptions/Common Errors

When exploring multiples of tenths and hundredths, students may incorrect assume that the denominator should increase. For example, they may think the progression for multiples of tenths is 1/10, 1/20, 1/30, instead of 1/10, 2/10, 3/10, etc….

d. Additional Instructional Strategies/Differentiation

Write each of the fractions and decimals $\frac{1}{2}$ =.5, $\frac{1}{4}$ =.25, $\frac{3}{4}$ =.75, $\frac{1}{3}$ ≈.33, $\frac{2}{3}$ ≈ .67, multiples of $\frac{1}{10}$, and multiples of $\frac{1}{100}$ on post-it notes and place them on the board. Have students match the decimals and fractions.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Here is a suggestion for a resource:

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=11

f. Assessing the <u>Lesson</u>

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

See Appendix A. Use green, yellow, and red cups during the activities. An exit ticket may also be used. You may use

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information gathered in student explanations for formative assessment.

III. Assessing the Module

4-2.10
The objective of this indicator is to identify which is in the "remember factual" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is to recall which is to retrieve from long term memory learning experiences should integrate both memorization and concept building strategies to support retention. The learning progression to **identify** requires student to explore a variety of representations (money, hundreds chart, etc.) of these equivalent relationships. They analyze these examples and generate descriptions (5-1.4) of what they observe using correct, complete and clearly written and oral language (5-1.5) to communicate their understanding. Students translate these descriptions into mathematical statements and connect these statements to the fraction-decimal equivalents outlined in indicator. They use correct, complete and clearly written and oral language (3-1.5) to communicate their understanding of these equivalent relationships. Students should develop meaningful and personal strategies that enable them to recall these relationships.

4-2.11
The objective of this indicator is to represent which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand means to construct meaning; therefore, the students' focus is on building conceptual knowledge of the relationships between the forms as opposed to gaining computational fluency in converting between forms. The learning progression to **represent** requires students to recall basic fraction-decimal equivalents. Students demonstrate flexibility in the use of mathematical representations (4-1.7) to represent improper fractions, mixed numbers and decimals using concrete and pictorial models. Students explore these representations with their classmates and generate mathematical statement summarizing their mathematical processes (4-1.4). They use correct, complete and clearly written and oral language to communicate their ideas (4-1.5).

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

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1. What is the decimal 0.7 written as a fraction?

A $\frac{1}{7}$ B $\frac{3}{4}$ C $\frac{3}{7}$ D $\frac{7}{10}$

The following item #2 is adapted from NCTM, 2005. Mathematics Assessment Sampler: Grades 3-5, page 13

2. Consider the following fraction: $\frac{23}{?}$

If this fraction is just a little more than 1, what would go in place of the "?"

A. 22 B. 23 C. 24

3. Maria ate $\frac{1}{4}$ of her sandwich at lunch. What decimal shows the part of the sandwich she ate?

A 0.2 B 0.25 C 0.4 D 0.5

4. Miss Kim asked each student to bring in 0.20 of a dollar for a field trip. Dana brought $\frac{1}{5}$ of a dollar. Margaret brought $\frac{1}{10}$ of a dollar. Justin brought in $\frac{1}{2}$ of a dollar. Who brought exactly the right amount?

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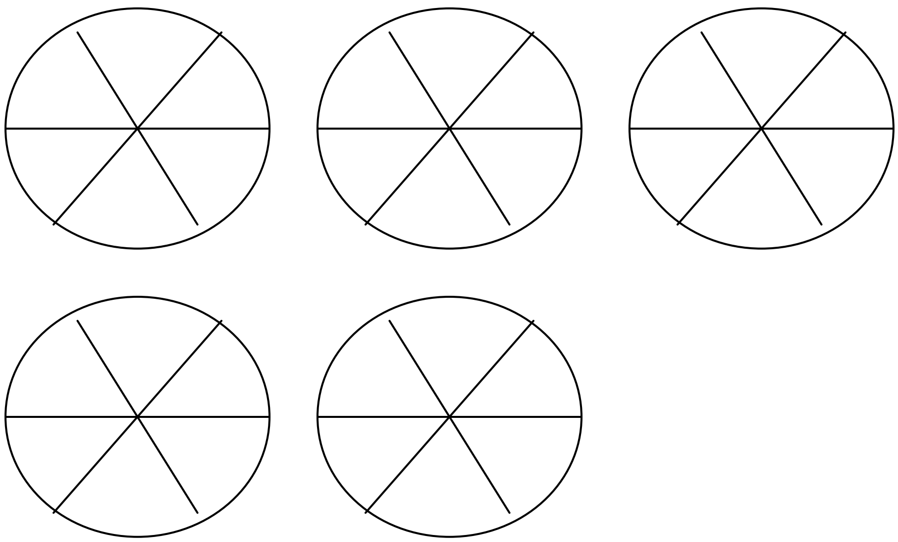
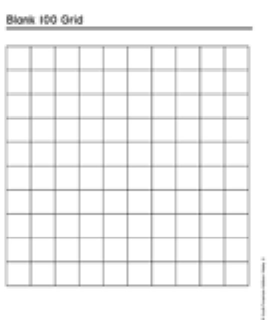
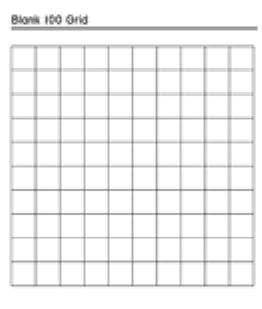
5. Brittany's class had a pizza party. There were 5 pizzas and each pizza was cut into 6 equal pieces. After the party, there were 13 slices left. What fraction of a pizza is left over? *Use the model to represent your solution.*



6. Model 0.30 and 0.03. Complete the ◯ with <, >, or =.





0.30 ◯ 0.03

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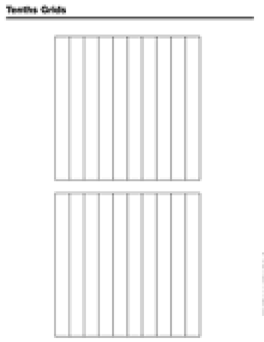
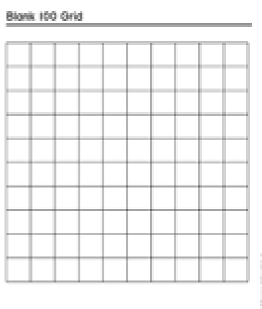
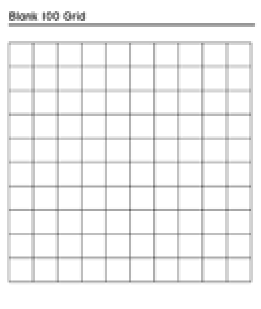
7. Model 0.50 and $\frac{1}{2}$. Complete the ◯ with <, >, or =.

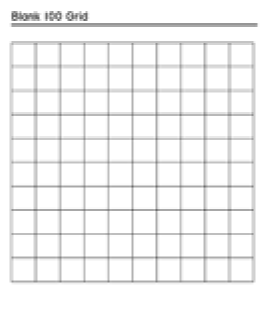


0.50 ◯ $\frac{1}{2}$

8. Model $2\frac{1}{4}$ using the grids below….







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MODULE

1-5

Operations – Addition and Subtraction

This module addresses the following indicators:

4-2.12 Generate strategies to add and subtract decimals through hundredths. (B6)

* This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

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I. *Planning the Module*

- **Continuum of Knowledge**

  Fourth grade is the first time students are introduced to the concept of decimals. Students compare decimals through hundredths by using the terms is less than, is greater than and is equal to and the symbols < ,> or = (4-2.7).

  In fifth grade, students compare whole number, decimals, and fractions by using the symbols <, > and = (5-2.4) and apply an algorithm to add and subtract decimals through thousandths (5-2.5). In sixth grade, students generate strategies to multiply and divide fractions and decimals (6-2.5).

- **Key Concepts/Key Terms**
  ***These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.**

  *Addends
  *Sum
  *Hundredths

II. Teaching the Lesson(s)

1. ***Teaching Lesson A: Using Concrete and Pictorial Models to develop Problem Soling Strategies with Decimals***

   For this indicator, it is **essential** for students to:
   - Understand place value
   - Name decimals through the hundredths
   - Generate strategies from story problems (in context)
   - Use concrete and or pictorial models to represent these operations
   - Use an estimation strategies to approximate the answer

   For this indicator, it is **not essential** for students to:
   - Apply a traditional algorithm for adding and subtracting decimals

   Students should move to generating strategies for adding and subtracting decimals through hundredths.

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Students should be given opportunities that foster conceptual understanding of adding and subtracting decimals through hundredths. This means, just as with whole numbers in earlier grades, a problem in context should be posed to students. Then using their knowledge of decimal models and manipulatives students should solve the problem using their own strategies. By sharing strategies with the whole class, students will be given the opportunities to test for accuracy and justify their methods. Fourth grade students should NOT progress to nor be taught an algorithm for adding and subtracting decimals. The emphasis in fourth grade is on conceptual understanding. Therefore, students need multiple opportunities to use concrete materials to solve problems that require addition and subtraction of decimals through hundredths. As with whole numbers, it is important that the aspects of mathematics related to addition and subtraction of decimals be addressed in classroom experiences. For example, if the goal is to determine if students understand the importance of combining/adding like decimal place values, then a problem like the following might be posed: Anna Lee ran the first part of the race in seven tenths of an hour. She ran the second part of the race in twenty-two hundredths of an hour. What was the total amount of time needed to run both parts of the race? To solve the student should draw a model of ten squares with seven shaded and a model of 100 squares with 22 shaded. The final model should show 100 squares with 92 shaded. (An explanation should accompany the models). This demonstrates an understanding of the need to combine like decimal place values (line up the decimal when adding symbolically). If on the other hand the final model is 100 with 29 shaded it demonstrates the student's lack of understanding of the need to combine like decimal place values.

So, the point here is to consider all the types of problems that might be posed when adding symbolically (numbers only) and translate that into classroom experiences with concrete and pictorial models.

a. Indicators with Taxonomy

4-2.12 Generate strategies to add and subtract decimals through hundredths. (B6)

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual

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b. Introductory Lesson

Materials Needed:

- Various manipulatives such as grid paper, two colored counters, base ten blocks, unifix cubes, etc.

A significant method of developing meaning for addition and subtraction of decimals is to have students solve contextual problems or story problems. However, there is more to think about than simply giving students word problems to solve. Consider the following problem: In P.E., Max and Moe timed each other in the quarter mile run. Max ran the quarter in 74.5 seconds. Moe's time was 81.34. How many seconds faster was Max than Moe? "Contextual problems are connected as closely as possible to children's lives. They are designed to anticipate and to develop children's mathematical modeling of the real world." Contextual problems might derive from recent experiences in the classroom (a field trip, a discussion, or from children's literature). Students are more likely to exhibit their most spontaneous and meaningful approaches when solving contextual problems because they have a connection to it.

Good lessons built around contextual problems will involve more than just students solving problems but also using words, pictures, and manipulatives to explain how they went about solving the problem and justifying their answers. Students should be able to use whatever physical materials they feel they need to help them, or they can simply draw pictures. A complete lesson will often revolve around one or two problems and the related discussion.

A good place to begin decimal computation is with estimation. A good time to begin computation with decimals is as soon as a conceptual background in decimal numeration has been developed. For more information, please refer to Teaching Student Centered Mathematics: Grades 3-5 by John A. Van de Walle and LouAnn H. Lovin, Pearson Learning, 2006.

Contextual problems involving addition and subtraction should be posed to students. Students should be given the opportunity to solve the problems using their own strategies and the manipulative of their choice. After solving the problems, students should be permitted to share their thinking with the class. An example of a problem might be: Mrs. Coleman, the art teacher, asked the

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students to cut two different lengths of ribbon to use in an art project. Jillian cut one piece of ribbon that was 0.7 meters long and another that was 0.5 meters long. How much yarn did Jillian cut altogether?

c. Misconceptions/Common Errors

Watch out for students who express a sum greater than 1 in tenths or hundredths. Use manipulatives to reinforce the idea of a sum with 10 or more tenths must be regrouped into a whole number. For example, 0.5 + 0.7 = 1.2 not 0.12.

Memorizing specific rules for decimal computation are not necessary if computation is built on a firm understanding of place value and a connection between decimals and fractions.

Avoid the key word strategy! In contrast to common practice, researchers and mathematics educators have long cautioned against the strategy for key words for the following reasons:

- Key words are misleading. Often the key word or phrase in a problem suggests an operation that is incorrect.
- Many problems have not key words.
- The key word strategy sends a terribly wrong message about doing mathematics. The most important approach to solving any contextual problem is to analyze its structure—to make sense of it. The key word approach encourages students to ignore the meaning and structure of the problem and look for an easy way out. Mathematics is about reasoning and making sense of situations. A sense-making strategy will always work.

d. Additional Instructional Strategies/Differentiation

For additional learning strategies, please refer to Teaching Student Centered Mathematics: Grades 3-5 by John A. Van de Walle and LouAnn H. Lovin.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

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There is no specific technology recommended for this lesson at this time.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Students sharing their strategies should serve as formative assessment.

III. Assessing the Module

The objective of this indicator is to generate which is in the "create conceptual" knowledge cell of the Revised Taxonomy. The create means to put ideas together into a new structure; therefore, students use prior knowledge to generate new strategies. The learning progression to **generate** requires students to recall the place value structure and understand place value. Using concrete and/or pictorial models, students apply their understanding of decimals to determine how to break down and solve problems. As students analyze information (4-1.1) from these experiences, they generate conjectures and mathematical statements (4-1.4) about the relationships they observe then explain and justify their strategies (4-1.3) to their classmates and their teachers. Students recognize the limitations of various strategies and representations (4-1.8) and use correct, complete and clearly written and oral language to communicate their ideas (4-1.5).

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At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

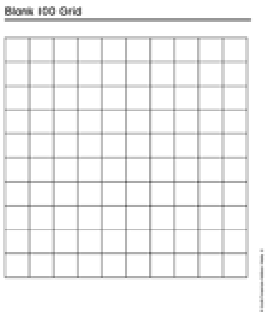
The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

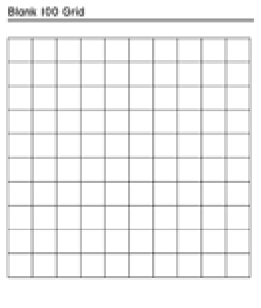
The following item #1 is adapted from NCTM, 2005. Mathematics Assessment Sampler: Grades 3-5, page 36.

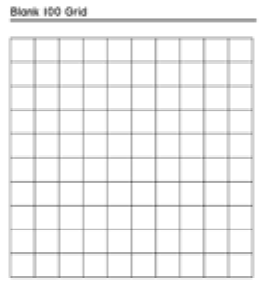
1. Chase bought a model airplane and some glue. How much did Chase spend? Show your thinking using models/drawings and words.

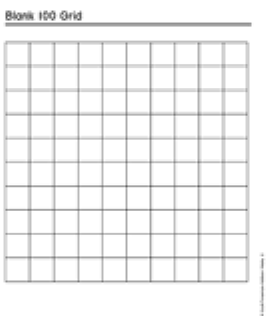
$1.29 **$0.45**

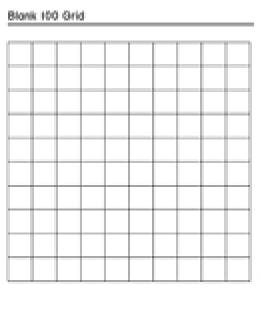
2. How much change would Chase receive from a $5 bill? Show your thinking using drawings and words. You may choose to use the hundreds grids below.











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MODULE

1-6

Operations – Multiplication

This module addresses the following indicators:

4-2.3 Apply an algorithm to multiply whole numbers fluently. (C3)

4-2.4 Explain the effect on the product when one of the factors is changed. (B2)

* This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

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1. *Planning the Module*

Continuum of Knowledge

4-2.3

In third grade, students generated strategies to multiply whole numbers by using one single-digit factor and one multi-digit factor (3-2.10) and recalled basic multiplications facts through 12 x 12 and the corresponding division facts (3-2.7)

In fourth grade, students apply an algorithm to multiply whole number fluently (4-2.3) and explain the effect on the product when one of the factor is changed (4-2.4).

4-2.4

In third grade, students generated strategies to multiply whole numbers by using one single-digit factor and one multi-digit factor (3-2.10) and recalled basic multiplications facts through 12 x 12 and the corresponding division facts (3-2.7)

In fourth grade, students apply an algorithm to multiply whole number fluently (4-2.3) and explain the effect on the product when one of the factor is changed (4-2.4).

In third grade, students generated strategies to multiply whole numbers by using one single-digit factor and one multi-digit factor.

- **Key Concepts/Key Terms**
  *These are vocabulary terms that are a reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.
  * factors
  * product
  * effect
  * outcome

The indicator requires for students to say more than “Oh, the product gets bigger or smaller.” Students should use their understanding of multiplication as grouping to explain the effect on the product. For example, changing the second factor in 22 x 7 to 22 x 6 and posing the question, “What happens to the product when the second factor is changed from a 7 to a 6?” Students should be able to explain that the product is smaller because instead of having 7 groups of 22, there are 6 groups of 22. Or the product of 22 x 7 = 154 so to find one less group of 22 you could subtract 154 - 22 = 132 so 22 x 6 is 132.

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Another example would be given 3 x 44, what would happen if one of the factors was doubled? Students may say that the product doubles to 264 from 132 because I have double the number of sets.

As the verb "Apply" implies from indicator 4-2.3, students should already have the conceptual understanding from third grade, and the goal must now be fluency. Review of strategies for multiplication should be emphasized at the beginning of fourth grade and experiences should enable students to link those prior concrete/pictorial experiences to the symbolic. The goal is that fourth grade students will be fluent with whole number multiplication. If the student understands the concept of multiplication, then the size of the numbers used when multiplying should not be an issue.

As with any operation, students should be able to estimate and determine the reasonableness of the product of whole numbers. They should be able to refine their estimates using terms such as closer to, between, and a little more than.

II. Teaching the Lesson(s)

1. ***Teaching Lesson*** A: Apply an Algorithm to multiply whole numbers

For this indicator, it is **essential** for students to:

- Recall basic multiplication fact
- Multiply numbers with fluently
- Understand that multiplication is creating equal grouping such as 4 x 5 is creating four sets of five

For this indicator, it is **not essential** for students to:
None noted

a. Indicators with Taxonomy

4-2.3 Apply an algorithm to multiply whole numbers fluently. →C3

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural

b. Introductory Lesson

(Adapted from: Hands on Standards: Grades 3-4, ETA Cuisenaire, 2006, p. 30-31.)

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Review of strategies for multiplication should be emphasized at the beginning of fourth grade and experiences should enable students to link those prior concrete/pictorial experiences to the symbolic.

To review multiplication strategies from third grade, pose contextual problems and provide various manipulatives for student use. For example, the following problem could be introduced: Kitty's class is having a bake sale. Each student in Kitty's class will bring 12 treats to sell. There are 25 students in Kitty's class. How many treats will the class bring in altogether?

After students solve the problem, they should be allowed to share their strategies with the class.

Problems and strategies used to review multiplication strategies can also be used to introduce applying procedures to multiply whole numbers.

Materials Needed

none

Divide the class into groups of 2 or 3. Have each group work on a problem such as, "There are 6 boxes of baseballs with 24 baseballs in each box. How many baseballs are there total?" (As with any operation, students should always begin by estimating an answer. Students can be asked to explain their estimated answer using the terms *closer to, between,* and/or *a little more than*.) Each group needs to solve the problem in their own way. Then ask them to illustrate how they solved it on paper so they can explain their answer to the class.

As students begin their study of multiplication, both the students and the teacher should explore a variety of inventive strategies. This usually involves both the associative and distributive properties. Those strategies will vary from class to class – student to student. As students create their own approaches, IF the idea of partial products is not seen or recorded in a consistent manner, then the following <u>may</u> be an approach the teacher wishes to demonstrate:

Ex.
$$\begin{array}{r} 85 \\ \underline{\text{x } 4} \\ 20 \\ \underline{320} \\ 340 \end{array} \qquad\qquad \begin{array}{r} 322 \\ \underline{\text{x } 6} \\ 1{,}800 \\ 120 \\ \underline{12} \\ 1{,}932 \end{array}$$

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Or with a double-digit multiplier and multiplicand, one could consistently record the partial products in this manner:

```
Ex.    57
     x 23
    1,000  (20 x 50)
      140  (20 x 7)
      150  (50 x 3)
       21  (7 x 3)
    1,311
```

This lesson is not an attempt to introduce OR force the "traditional algorithm" onto the students. Partial products should be seen and understood first before the traditional algorithm is ever shown.

The focus of this introductory lesson should be on developing understanding and developing the ability to explain one's thinking about multiplication.

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

Refer to Navigating through Number and Operations in Grades 3-5, NCTM, 2007.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Here is a suggestion for a resource:
http://www.learningplanet.com/articles/mathmachine/index.asp#list

f. Assessing the Lesson

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Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Use students solving problems and using words, pictures, and manipulatives to explain how they went about solving the problem and justifying their answers as formative assessment.

2. ***Teaching Lesson B:*** ***Explain the effect on the product when one of the factors is changed***

a. Indicators with Taxonomy

4-2.4 Explain the effect on the product when one of the factors is changed. → B2

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

b. Introductory Lesson B:

Materials Needed:

small blank pieces of paper
brown paper bags
paper and pencils

Give an example of a basic fact on the board (3 x 8). Have a dialogue about how 3 x 8 helps us solve 3 x 80, 3 x 800, and 3 x 8,000.

Have students work in pairs and write five basic multiplication facts on small pieces of paper. Place the pieces of paper in their paper bag and ask them to exchange bags with other pairs. Partners work together solving one problem at a time – first the basic fact, then creating and solving three more problems by making one factor a multiple of 10, 100, or 1,000. Students record their work.

The teacher leads a conversation about how the product is changed because one of the factors has been changed. A distinction should be made between the multiplier and the multiplicand. Though 80 x 3 and 30 x 8 produce the same product, in a contextual situation, there is a difference in the meaning (ex. You order 3 boxes of 80 candles vs. 8 boxes of 30 candles. If you are paying for shipping on each box, the number of boxes ordered does make a difference.). Students need to understand what each represents.

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Apply this same thinking to other multiplication problems. For example, write the following on the board: "50 x 3 = 150" and. "50 x 4 = 200" Note that the product is different, but ask the students "Why is the product different?" Students should understand the multiplier has increased by 1, thus increasing the number of sets (multiplicand)

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

There is no specific technology recommended for this lesson at this time.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Use students solving problems and using words, pictures, and manipulatives to explain how they went about solving the problem and justifying their answers as formative assessment.

III. Assessing the Module

4-2.3

The objective of this indicator is to apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. To apply means to carry out a procedure in familiar and unfamiliar situations; therefore, students should be able to multiply numbers regardless of their size. The learning progression to **apply** requires students to recall basic multiplication facts. Students connect experiences with

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concrete and pictorial models from third grade to symbolic procedures. Students use these models to generalize connections (5-1.6) between their models, their generated strategies and the symbolic procedure. As students exchange mathematical ideas with their classmates/teachers and explain and justify their answers (5-1.3), they are supporting conceptual understanding and building computational fluency. Student use estimation strategies to determine the reasonableness of their answers and explore these procedures in context to further deepen both procedural and conceptual knowledge.

4-2.4
The objective of this indicator is to explain which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To explain means to construct a cause and effect statement; therefore, students should not only tell the "what" but the "why". The learning progression to **explain** requires students to recall basic multiplication facts and understand that multiplication is creating equal grouping. Students explore a variety of problems and construct arguments about what they are observing. As student analyze information (4-1.1) from these problems, they explain and justify their answers to their classmates (4-1.3) and their teacher using correct, complete and clearly written and oral mathematical language (4-1.5).

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

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The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

The following item #1 is adapted from NCTM, 2005. Mathematics Assessment Sampler: Grades 3-5, page 32.

1. A store sells 168 DVDs each week. How many DVDs does the store sell in 24 weeks? Show your work.

2. SC Transport has been hired to deliver new seats to Death Valley stadium. The company will use 41 trucks to move the seats. If each truck holds 1025 seats, how many seats will be delivered to the stadium?

A 41,825 B 41,925 C 42,025 D 42,125

3a. There are 58 cases of soda in a warehouse. If there are 24 cans of soda in each case, how many cans of soda are in the warehouse?

A 1392 B 1362 C 1292 D 1262

3b. If there were 57 cases in problem 3a, what would be the result on the product? How could you determine the change in number of cans of soda? Explain in words or pictures.

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