SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

Content Area	Fifth Grade Mathematics
Recommended Days of Instruction	**Second Nine Weeks**

Standards/Indicators Addressed:

Standard 5-3: The student will demonstrate through the mathematical processes an understanding of the use of patterns, relations, functions, models, structures, and algebraic symbols to represent quantitative relationships and will analyze change in various contexts.

5-3.1* Represent numeric, algebraic, and geometric patterns in words, symbols, algebraic expressions, and algebraic equations. (B2)
5-3.2* Analyze patterns and functions with words, tables, and graphs. (B4)
5-3.3* Match tables, graphs, expressions, equations, and verbal descriptions of the same problem situation. (B2)
5-3.4* Identify applications of commutative, associative, and distributive properties with whole numbers. (A1)
5-3.5* Analyze situations that show change over time. (B4)

Standard 5-4: The student will demonstrate through the mathematical processes an understanding of congruency, spatial relationships, and relationships among the properties of quadrilaterals.

5-4.1* Apply the relationships of quadrilaterals to make logical arguments about their properties. (B3)
5-4.2* Compare the angles, side lengths, and perimeters of congruent shapes.
5-4.3* Classify shapes as congruent. (B2)
5-4.4* Translate between two-dimensional representations and three-dimensional objects. (B2)

Standard 5-5: The student will demonstrate through the mathematical processes an understanding of the units and systems of measurement and the application of tools and formulas to determine measurements.

5-5.2* Use a protractor to measure angles from 0 to 180 degrees. (C3)

* These indicators are covered in the following 5 Modules for this Nine Weeks Period.
Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.

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Module 2-1 Patterns, Relationships and Functions			
Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines
Module 2-1 Lesson A 5-3.2 Analyze patterns and functions with words, tables, and graphs. (B4)	**STANDARD SUPPORT DOCUMENT** http://www.ed.sc.gov/agancy/standard-and-learning/academic standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and	See Instructional Planning Guide Module 2-1 Introductory Lesson A	See Instructional Planning Guide Module 2-1 Lesson A Assessing the Lesson

	Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4		

Module 2-2 - Representations, Properties, and Proportional Reasoning			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-2 Lesson A 5-3.1 Represent numeric, algebraic, and geometric patterns in words, symbols, algebraic expressions and algebraic equations. (B2)	**STANDARD SUPPORT DOCUMENT** http://www.ed.sc.gov/agancy/standard-and-learning/academic standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th	See Instructional Planning Guide Module 2-2 Introductory Lesson A	See Instructional Planning Guide Module 2-2 Lesson A Assessing the Lesson
Module 2-2 Lesson B 5-3.3 Match tables, graphs, expressions, equations and verbal descriptions of the same problem situation. (B2)		See Instructional Planning Guide Module 2-2 Introductory Lesson B	See Instructional Planning Guide Module 2-2 Lesson B Assessing the Lesson

Module 2-2 Lesson C 5-3.4 Identify applications of the commutative, associative, and distributive properties with whole numbers. (A1)	Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4	See Instructional Planning Guide Module 2-2 Introductory Lesson C	See Instructional Planning Guide Module 2-2 Lesson C Assessing the Lesson

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Module 2-3 Change in Various Contexts			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-3 Lesson A 5-3.5 Analyze situations that show change over time. (B4)	**STANDARD SUPPORT DOCUMENT** http://www.ed.sc.gov/agancy/standard-and-learning/academic standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School	See Instructional Planning Guide Module 2-3 Introductory Lesson A	See Instructional Planning Guide Module 2-3 Lesson A Assessing the Lesson

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	<u>Mathematics</u> (PSSM) NCTM, <u>Mathematics Assessment Sampler</u>: Grades 3-5 ETA Cuisenaire, <u>Hands-On Standards</u>: Grades 3-4		

Module 2-4 Patterns, Relationships and Functions			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**

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Module 2-4 Lesson A 5-5.2 Use a protractor to measure angles from 0 to 180 degrees. (C3)	**STANDARD SUPPORT DOCUMENT** http://www.ed.sc.gov/agancy/standard-and-learning/academic standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler:	See Instructional Planning Guide Module 2-4 Introductory Lesson A	See Instructional Planning Guide Module 2-4 Lesson A Assessing the Lesson

	Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4		

Module 2-5 - Dimensionality			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-5 Lesson A 5-4.1 Apply the relationships of quadrilaterals to make logical arguments about their properties. (B3)	**STANDARD SUPPORT DOCUMENT** http://www.ed.sc.gov/agancy/standard-and-learning/academic standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th	See Instructional Planning Guide Module 2-5 Introductory Lesson A	See Instructional Planning Guide Module 2-5 Lesson A Assessing the Lesson
Module 2-5 Lesson B 5-4.2 Compare the angles, side lengths, and perimeters of congruent shapes. (B2) 5-4.3 Classify shapes as congruent. (B2)		See Instructional Planning Guide Module 2-5 Introductory Lesson B	See Instructional Planning Guide Module 2-5 Lesson B Assessing the Lesson

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Module 2-5 Lesson C 5-4.4 Translate between two-dimensional representations and three-dimensional objects. (B2)	Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4	See Instructional Planning Guide Module 2-5 Introductory Lesson C	See Instructional Planning Guide Module 2-5 Lesson C Assessing the Lesson

MODULE

2-1

Patterns, Relationships, and Functions

This module addresses the following indicators:

5-3.2 Analyze patterns and functions with words, tables, and graphs. (B4)

This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY.** Lessons in S^3 begin to build the conceptual foundation students need. **ADDITONAL LESSONS will be required** to fully develop the concepts.

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I. Planning the Module

- **Continuum of Knowledge**

In fourth grade, students analyzed numeric, nonnumeric, and repeating patterns involving all operations and decimal patterns through hundredths (4-3.1). They generalized a rule for numeric, nonnumeric and repeating patterns involving all operations (4-3.2).

In fifth grade, students represent numeric, algebraic, and geometric patterns using words, symbols, algebraic expressions and algebraic equations (5-3.1) and analyze patterns and functions with words, tables and graphs (5-3.2).

In sixth grade, students will analyze numeric and algebraic patterns and pattern relationships (6-3.1).

- **Key Concepts/ Key Terms**

These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use and conversation for students.

Teacher Note: This is the first time students are exposed to the word "function".

Pattern
*Function
Rule
*Input/output machines

*Table
*Graph
*Output
*Input
*Function machine

II. Teaching the Lesson: Analyze Patterns and Functions

I. Teaching the Lesson

Fourth grade students focused on analyzing numeric, nonnumeric, and repeating patterns involving all operations and decimal patterns through hundredths. Fifth grade students are analyzing patterns and functions with words, tables, and graphs. Students should recognize the connections between these three representations which will lay the foundation for the understanding of functions in middle school. When students analyze functions in fifth grade they should look for patterns within the function. For example: Teachers could use an input/output table, function machine, etc. This is the first time that students are required to use function rules to make generalizations.
Emphasis should be placed on student's understanding of the relationship between related quantities that result in a function and how changing one quantity impacts the other related quantity.

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For this indicator, it is **essential** for students to:

- Identify patterns
- Represent patterns using a concrete models, where appropriate
- Describe patterns in words, symbols, algebraic expressions, and algebraic equations
- Translate between different representations (words, symbols and algebraic)

For this indicator, it is **not essential** for students to:

- Solve algebraic equations

a. Indicators with Taxonomy

Indicator → 5-3.2 Analyze patterns and functions with words, tables, and graphs. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual

b. Introductory Lesson

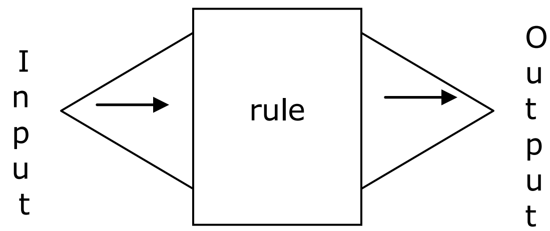
Materials Needed

Function "machine" – drawn or built

Lesson

(Adapted From Teaching Student Centered Mathematics Grades 3-5, Van de Walle, John A. and Lovin, LouAnn, 2006, page 316.)

Draw a function machine on the board, overhead or interactive white board. (any variation will do, but an example is below):



The function machine will take a number in (input) and give another number out (output). What happens to the input is called the rule. Once the rule is applied, the output is the resulting number. For example, if the rule is *2n* or *2 times n* where *n* is the number inputted, then the table would look like this.

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Input (*n*)	Output
1	2
2	4
3	6
4	8
5	10

The teacher should show a function machine… students will give numbers to put into the machine and the teacher (who knows the rule) will tell the output. Students should complete a table of values like the one above to show the different inputs and outputs. When a student thinks he/she has discovered the rule, a guess can be made.

Some rules that the teacher may want to use are:

- double the number and add 1 each time
- add three to the number each time
- divide the number by 2 each time
- double the number and subtract 2 each time
- subtract 4 each time
- adding 1.2 each time

Have students create their own rules and operate the function machine.

c. *Misconceptions/Common Errors*

Students may believe that the representations are unrelated.

d. *Additional Instructional Strategies/Differentiation*

Students should be asked to create their own patterns or function tables and allow others to solve it.

Adapted from Anderson 5
Write these numbers on the board: 1, 3, 7, 15, 31, 63, 127, 255. Ask students to find the pattern by having them ask themselves how we go from one to three. Test the answer to the other numbers. For example, if a student says, "We add two." Then ask, "Does that rule hold true for getting from three to seven?" "No, because three plus two is five not seven." Continue to ask the question of how we got from one number to another until the answer fits all the numbers and can be used to find the missing numbers.

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e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

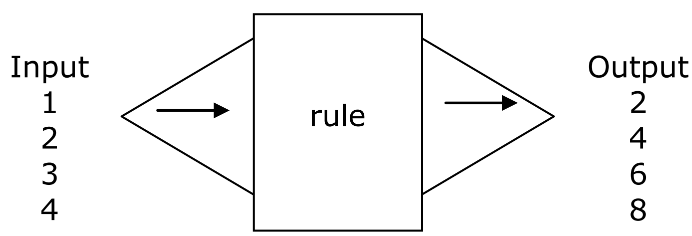
There is no specific technology recommended for this lesson at this time.

f. Assessing the Lesson

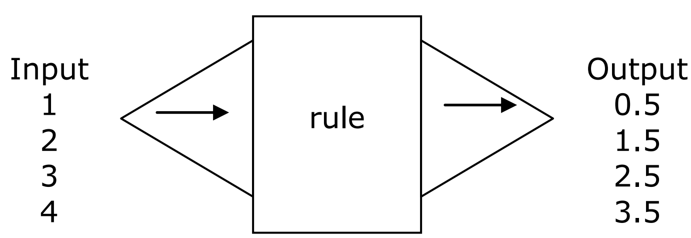
Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Write a rule for each of the following A and B. Explain how you determined your rule. Students should write answers on exit slips to be reviewed by the teacher.

A.



B.



III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

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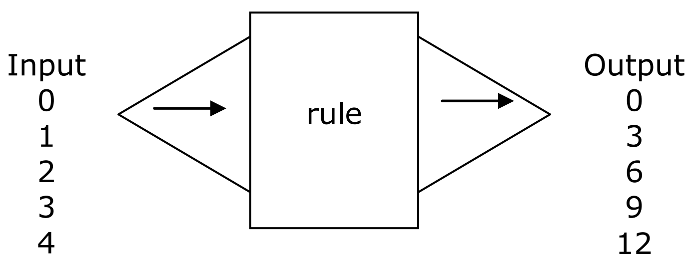
5-3.2
The objective of this indicator is to analyze, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge of these multiple representations (words, table and graph) should be explored using a variety of examples. The learning progression to **analyze** requires students to recall the structure of a function table and understand the purpose of function machines. Students evaluate function rules and patterns and translate their understanding to words, tables or graphs. They generalize connections (5-1.6) among the multiple representations and generate descriptions and mathematical statements about these connections (5-1.4) using correct, complete and clearly written and oral mathematical language (5 – 1.5). Students explain and justify answers (5 – 1.3) and place an emphasis on the similar meaning that is conveyed by each representation.

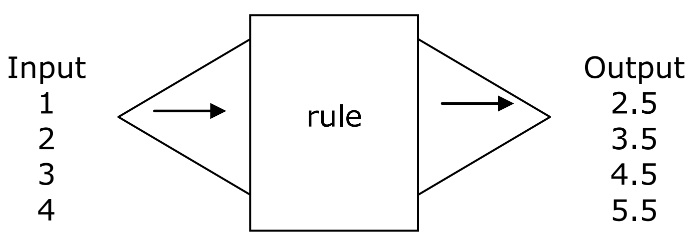
The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Write a rule for each of the following A and B. Explain how you determined your rule.

A.



B.



MODULE

2-2

Representations, Properties, and Proportional Reasoning

This module addresses the following indicators:

5-3.1 Represent numeric, algebraic, and geometric patterns in words, symbols, algebraic expressions, and algebraic equations. (B2)
5-3.3 Match tables, graphs, expressions, equations, and verbal descriptions of the same problem situation. (B2)
5-3.4 Identify applications of commutative, associative, and distributive properties with whole numbers. (A1)

This module contains 3 lessons. These lessons are **INTRODUCTORY ONLY.** Lessons in S^3 begin to build the conceptual foundation students need. **ADDITONAL LESSONS will be required** to fully develop the concepts.

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I. ***Planning the Module:*** Represent Numeric, Algebraic, and Geometric Patterns

- **Continuum of Knowledge**

5-3.1
In fourth grade, students analyzed numeric, nonnumeric, and repeating patterns involving all operations and decimal patterns through hundredths (4-3.1). They generalized a rule for numeric, nonnumeric and repeating patterns involving all operations (4-3.2).

In fifth grade, students represent numeric, algebraic, and geometric patterns using words, symbols, algebraic expressions and algebraic equations (5-3.1) and analyze patterns and functions with words, tables and graphs (5-3.2).

In sixth grade, students will analyze numeric and algebraic patterns and pattern relationships (6-3.1).

5-3.3
In fourth grade, students translated among, letters, symbols, and words to represent quantities in simple mathematical expressions or equations (4-3.4).
In fifth grade, students will match tables, graphs, expressions, equations, and verbal descriptions of the same problem situation (5-3.2). Students also analyze patterns and functions with words, tables, and graphs (5-3.2).
In sixth grade, students will represent algebraic relationships with variables in expressions, simple equations, and simple inequalities (6-3.3).

5-3.4
In first grade, students illustrated the commutative property based on basic facts but did not use the formal term.
In fifth grade, students will identify applications of commutative, associative, and distributive properties with whole numbers (5-3.4).
In sixth grade, students will use the commutative, associative, and distributive properties to show that two expressions are equivalent (6-3.4).

- **Key Concepts/ Key Terms**

These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use and conversation for students.

*numeric patterns
*algebraic patterns
*geometric patterns
*variable
*expression
*equation

Symbols
=
Verbal description
Multiple representation
*Verticle axis
*Horizontal axis

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Data
*commutative property
*associative property
*distributive property
*equivalent

II. Teaching the Lesson(s)

1. Teaching Lesson A: Identify Applications of Commutative, Associative, and Distributive properties

Fourth grade students used variables to represent an unknown quantity and to write a mathematical expression and equations in symbolic form. They had experience analyzing numeric, nonnumeric, and repeating patterns. Fifth grade students for the first time will represent numeric, algebraic, and geometric patterns using words, symbols, algebraic expressions and algebraic equations. The focus for new learning is on algebraic and geometric patterns. Geometric patterns could include triangular numbers and square numbers. Algebraic patterns include patterns with variables as well as numbers.

Fifth grade students should be able to make associations of tables, graphs, expression, equations, and verbal descriptions of the same problem situation. For example: Teachers could represent several problem situations using tables, graphs, expressions, and equations and cut them into "pieces". Students could match the tables, graphs, expressions, and equations that represented the same problem situation. Students should be able to verbally explain the matched pieces.

Students as far back as first grade were exposed to the concept of commutativity when working with basic facts for addition and subtraction. Although, they did not talk about fact families in terms of the commutative property. They simply experienced by identifying and using fact families. In fifth grade, students will identify applications of commutative, associative, and distributive properties with whole numbers. For example: Given 3+4 = 7 and 4+3 = 7 which property does this represent Identifying and extending patterns is important to algebraic thinking. The development of this process usually begins in kindergarten.

Tables and charts usually help students determine patterns and relationships. Once a table or chart is developed, students have two representations of the pattern: the one they created with manipulatives or drawings and the numeric one in the table or chart. It is important for students to focus on the relationships between these two representations of pattern.

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Properties are very useful in relating numbers and relationships. Focus on how these properties are related is key in helping develop more complex number sense.

For this indicator, it is **essential** for students to:

- Identify patterns
- Represent patterns using a concrete models, where appropriate
- Describe patterns in words, symbols, algebraic expressions, and algebraic equations
- Translate between different representations (words, symbols and algebraic)

For this indicator, it is **not essential** for students to:

- Solve algebraic equations

a. Indicators with Taxonomy

Indicator → 5-3.1 Represent numeric, algebraic, and geometric patterns in words, symbols, algebraic expressions, and algebraic equations. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

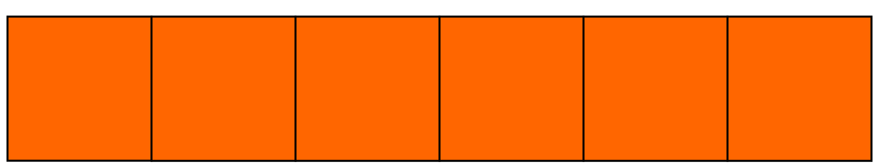
b. Introductory Lesson

Materials Needed

Orange square pattern blocks (commercial or teacher made)
Lesson

Part A:
Consider the following challenge:

How many one inch edges are there in the perimeter of this figure? (We are assuming the side of each square pattern block is an one inch edge) – *answer: 14*



How many one inch edges would there be in the perimeter of a figure with 20 squares? YOU CANNOT build it or draw it! Instead, complete the table starting with the number of edges on one square, then two squares, etc. Write a rule or algebraic expression for the rule and then use the rule to answer the question.

Number of squares (*n*)	Number of one-inch edges (perimeter)
1	4
2	6
3	8
4	10
5	12
6	14

Students should write the rule in words and an algebraic expression.

ANSWER: one possible rule is 2n + 2 since there are two inches of perimeter on the top and bottom of each square (*n*) plus 1 inch on both ends, left and right.

Part B:

Write a situation that corresponds to the each of the patterns below:

1) \$5, \$12, \$20, \$29
2) 56.5^0, 65.1^0, 73.7^0, 82.3^0, 90.9^0

c. *Misconceptions/Common Errors*

Students may treat letters as objects instead of values. For example: Shirts cost s dollars and pants cost p dollars. If I buy 3 shirts and 2 pairs of pants, what does 3s + 2p represent? (3 shirts and 2 pairs of pants)

Students may think that letters always have one specific value. Students need to understand the notion of variable as something that can vary or change. Students may think that letters always represent different numbers. For example, if y = 10 in one situation, y cannot = 10 in a different situation.

When asked to determine the nth figure in a pattern that is not consecutive, students may find the next consecutive figure instead of the nth figure. For example, John made the figures

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shown. The picture shows figures one through four. However, the question may ask what the 10th figures looks like.

d. Additional Instructional Strategies/Differentiation

Adapted from Anderson5:
Have students create a geometric pattern with colored pencils. Have them swap with another student and have that student draw and color the next three shapes in the sequence. Then explain the pattern they found.

Have students use attribute blocks to manipulate and create patterns. Give students examples of various patterns. Ask them to represent the pattern in a different way. For example: AA BBB CCC could be represented with numerals as 11 222 333. Have students share their representations. While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

There is no specific technology recommended for this lesson at this time.

f. Assessing the <u>Lesson</u>

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

1) Write a problem situation that corresponds to the time sequence below:

8:00 am, 12:05 pm, 4:10 pm, 8:15 pm, 12:20 am

Record your answer on an index card as an exit ticket for teacher review.

II. ***Teaching Lesson B*: Match Tables, Graphs, Expressions, Equations, and Verbal Descriptions**

For this indicator, it is **essential** for students to:

- Understand that all representations are a representation of the same relationship.
- Differentiate between the different types of representations.
- Analyze the multiple representation and determine relevant and irrelevant data

For this indicator, it is **not essential** for students to:
None noted

a. Indicators with Taxonomy

Indicator → 5-3.3 Match tables, graphs, expressions, equations, and verbal descriptions of the same problem situation. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

Materials Needed

Pattern blocks – yellow hexagons and orange squares (commercial or teacher made)

Lesson

(Adapted From Hands On Standards Grades 5-6, Learning Resources, pages 110-111.)

2 different people are planning some stone walkways through a flower garden.

Situation 1:
Mary is using a hexagon stone and a square stone represented by the yellow hexagon and the orange square. The six sides of the hexagon will have squares attached to them. Attach the next hexagon to the side of one of the squares in the direction you want the path to take. Continue adding squares and hexagons until the path is complete.

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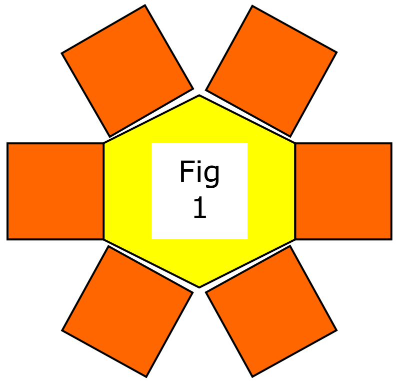
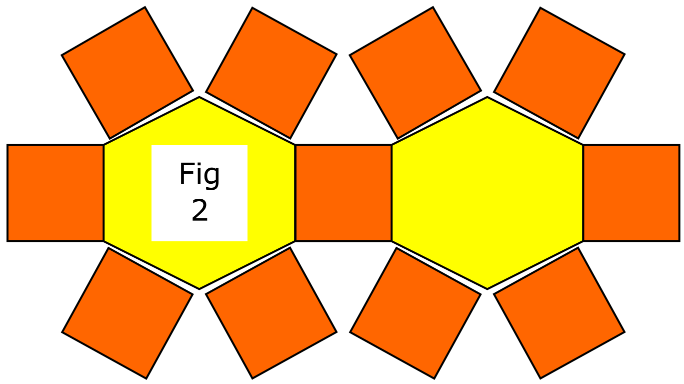
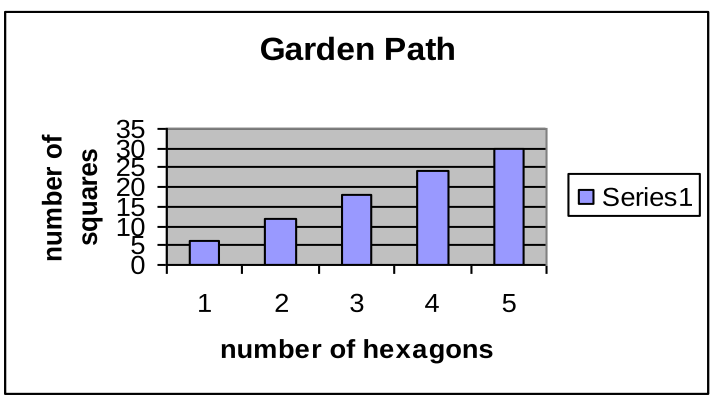




Match the table, the graph and the expression that goes with the situation of Mary's garden path. Choose from the 6 selections below.

a)



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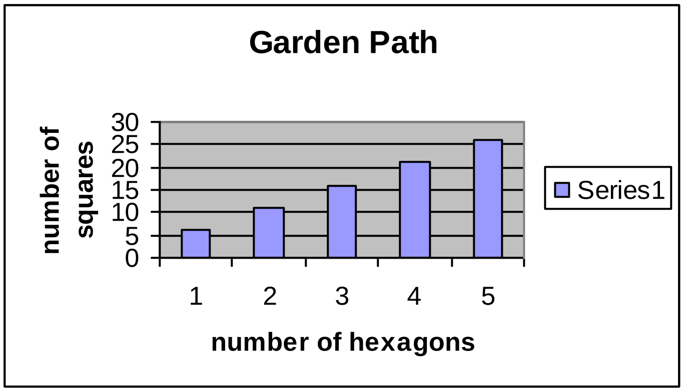
b)



c)

Hexagons	Squares
1	6
2	11
3	16
4	21
5	26

(d)

Hexagons	Squares
1	6
2	12
3	18
4	24
5	30

e) the rule is 6n where n = number of hexagons in the path

f) the rule is 5n + 1 where n = number of hexagons in the path

<u>Situation 2</u>:

Juan is using square stone and triangular stone. The squares are all connected for the length of the path. Each exposed side of the square has a triangle attached to it. Continue adding squares and triangles until the path is complete.

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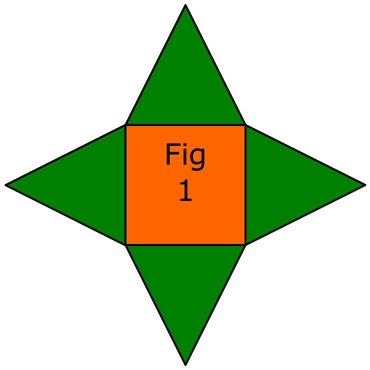
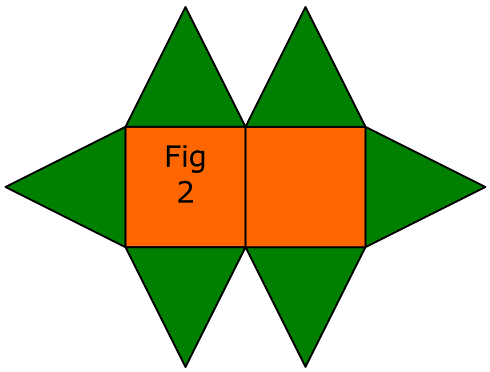
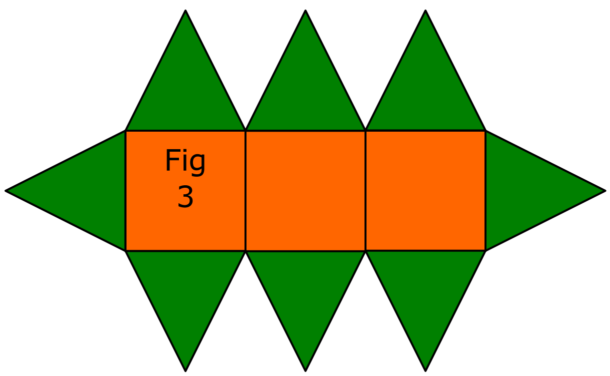
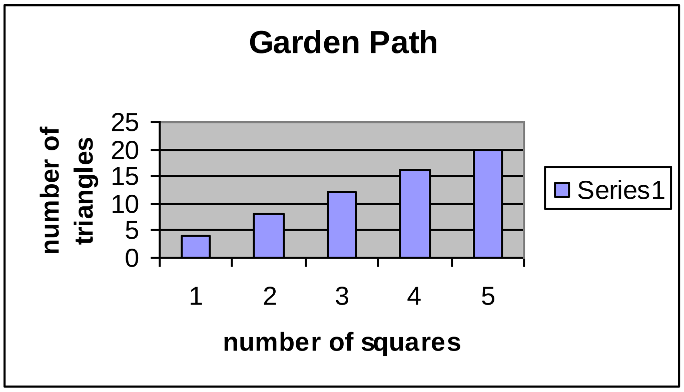




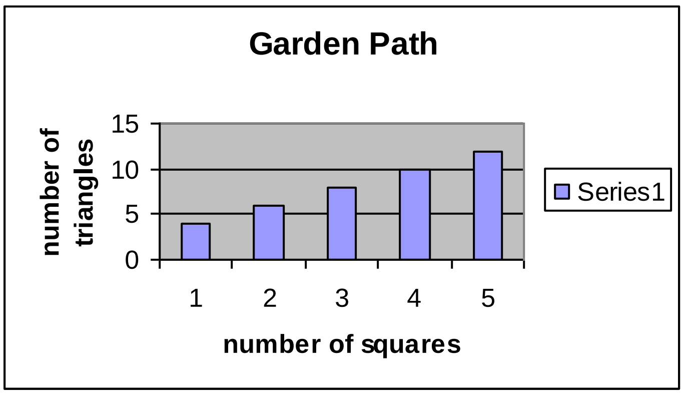


Match the table, the graph and the expression that goes with the situation of Juan's garden path. Choose from the 6 selections below.

a)



b)



c)

Squares	Triangles
1	4
2	8
3	12
4	16
5	20

(d)

Squares	Triangles
1	4
2	6
3	8
4	10
5	12

e) the rule is 2n + 2 where n = number of squares in the path

f) the rule is 4n where n = number of squares in the path

c. *Misconceptions/Common Errors*

Student may only use one piece of data when matching representations. They need to understand that they need to look at the entire scope of the data.

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d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

There is no specific technology recommended for this lesson at this time.

f. Assessing the <u>Lesson</u>

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

To monitor student learning for the lesson, have students choose one of the examples to refer to. Have them write what they have learned about the connections between tables, graphs and charts. They may do this in their mathematics notebook or as an exit slip.

3. ***Teaching Lesson C:*** Identify Applications of Commutative, Associative, and Distributive Properties with Whole Numbers

For this indicator, it is **<u>essential</u>** for students to:
- Understand commutative and associative properties of addition and multiplication
- Understand the distributive property of multiplication
- Create examples of each property
- Understand the these rules apply to all whole numbers
- Understand equivalent relationships (This is the same as …)

For this indicator, it is **<u>not essential</u>** for students to:
- Use these properties to solve equations

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- Write or understand these properties when they are given in algebraic form. For example, a x b = b x a or a + (b + c) = (a + b) + c.

a. Indicators with Taxonomy

Indicator → 5-3.4 Identify applications of commutative, associative, and distributive properties with whole numbers. (A1)
Cognitive Process Dimension: Remember
Knowledge Dimension: Factual Knowledge

b. Introductory Lesson

(Adapted from Hands on Standards Grades 5-6, Learning Resources, pages 94-99.)

Materials Needed

For Part A and B
Cuisenaire Rods (commercial or teacher made)
And/Or
Centimeter grid paper
& Colored Pencils
For Part C
Base 10 blocks (commercial or teacher made) –
50 units and 10 rods per pair

Lesson – *parts A, B and C will not be done back to back, there should be time in between each part to explore the concept of the property(ies) fully.*

Part A: Commutative and Associative for Addition

Pose the following situation: There is a box of 10 bouncy tennis balls, 6 bouncy racquetballs and 4 balls that don't bounce at all. Find two different ways to determine how many bouncy balls are in the box. Show your ways of solving with the Cuisenaire Rods or the centimeter grid paper and colored pencils. (*Students should come up with 10 + 6 and 6 + 10.)*
Afterwards, introduce the commutative property of addition.

Next, show the students the expressions:

(10 + 6) + 4 and 10 + (6 + 4)

Have them simplify each expression by first simplifying what is in parentheses (this is the order of operations).

The result should be:

16 + 4 and 10 + 10

Have them model these two with the Cuisenaire Rods OR centimeter grid paper and colored pencils.

The final result for each expression is 20.
Discuss why and introduce the associative property of addition.

Part B: Commutative and Associative for Multiplication

Pose the following situation: Chris and Josh have 3 cats. Each cat gets 10 treats a week. Chris and Josh use different ways to calculate how many treats the cats get all together in a week. How might they have solved this problem? Use the Cuisenaire Rods or the centimeter paper and colored pencils to represent your solution. *(Students should represent 3 x 10 and 10 x 3.)* Introduce the commutative property of multiplication.

Next ask: How many treats do the cats get all together in 2 weeks?
Then, show the following expressions and ask the following question:

3 x 10 x 2 and 3 x 10 x 2

How can one set of parentheses be placed in each expression so that the result is the same when simplified, but the expressions look different? *(Students should group (3 x 10) in one and group (10 x 2) in the other.)*
Have students prove their solutions.
Introduce the associative property of multiplication.

Part C: Distributive

Pose the following situation: Kimberly has 4 boxes of make up samples. Each box has 12 samples in it. How many samples does she have all together? Have students use models to solve the problem.

Students would have either done 4 x 12 or 12 x 4.
Discuss these two possibilities, but then have students model 4 x 12 with the base 10 blocks showing 4 groups of 12 units.

Write 4 x 12 for students to view, and ask, what is another way to represent 12? Students should say to break 12 down into 10 and 2. Ask them to model this with their blocks by making trades. They should exchange 10 units for 1 rod in each set.

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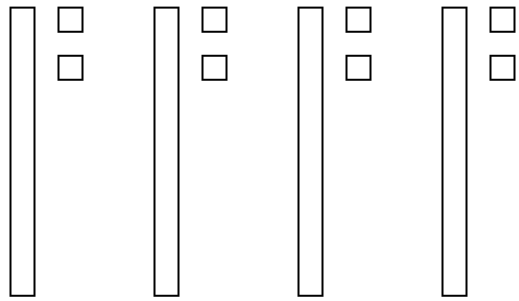

Write 4 x (10 + 2) on the board.
Show the two expressions:
4 x 12 and 4 x (10 x 2)
Students should continue to discuss the similarities and/or equivalences. Then, introduce the distributive property.

c. *Misconceptions/Common Errors*

Students may believe that these properties are true for all operations.

d. *Additional Instructional Strategies/Differentiation*

While additional learning opportunities are needed, no suggestions are included at this time.

e. *Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.
There are no specific recommendations for technology for this lesson at this time.

f. *Assessing the <u>Lesson</u>*

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Students should respond to these in their mathematics notebooks or as a journal entry. Have students pair up and

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discuss their solutions. A few solutions should be shared by volunteers or by pulling popsicle sticks to call on students.

1. Write two expressions demonstrating the associative property. Write a problem situation to correspond to your expression and explain why it could be "set up" in the two ways.

2. Using expressions, demonstrate the distributive property. What problem situation could correspond to your expression?

3. Why does the distributive property work? For example:
   3 x (5 + 4)

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

5-3.1
The objective of this indicator is to represent which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand means to construct meaning; therefore, the students' focus is on building conceptual knowledge of the relationships between the forms. The learning progression to **represent** requires students to analyze patterns (5-1.1) to determine known and unknown values and the operations involved. They generate word descriptions (5-1.4) of the observed pattern and generalize the connection (5-1.6) between the words and structure of expression and equations. Students explain and justify their ideas (5-1.3) with their classmates and teachers using correct, complete and clearly written and oral language to communicate their ideas (5-1.5). Students then compare the pattern form, word form and algebraic form (equation or expression) to verify that each form conveys the same meaning (4-1.5).

5-3.3
The objective of this indicator is to match which is in the "understand conceptual" knowledge cell of the Revised Bloom's Taxonomy. Conceptual knowledge is the interrelationships among the basic elements within a larger structure that enable them to function together; therefore, the student's conceptual knowledge of these multiple representations (words, table and graph) should be explored using a variety of examples. The learning progression to **match** requires students to analyze each representation and determine relevant and irrelevant data. They use this data to generalize connections (5-1.6) among the multiple representations and communicate their understanding using correct, complete and clearly written and oral mathematical language (5 – 1.5). Students match representations based on this analysis and explain and justify answers

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(5 – 1.3) placing an emphasis on the similar meaning that is conveyed by each representation.

5-3.4

The objective of this indicator is to identify which is in the "remember factual" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is to recall which is to retrieve from long term memory learning experiences should integrate both memorization and concept building strategies to support retention. The learning progression to **identify** requires student to explore a variety of examples of these number properties. They analyze these examples and generate descriptions (5-1.4) of what they observe using correct, complete and clearly written and oral language (5-1.5) to communicate their understanding. Students translate these descriptions into mathematical statements and connect these statements to the terms commutative, associative and distributive. Students then develop meaningful and personal strategies that enable them to recall these relationships.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Complete the chart below for the number of edges of each figure in the pattern.

Figure 1:

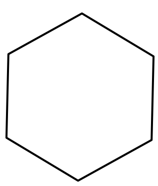

Figure 2:

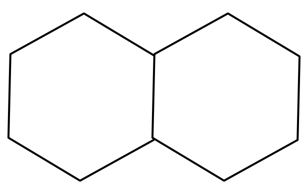

Figure 3:

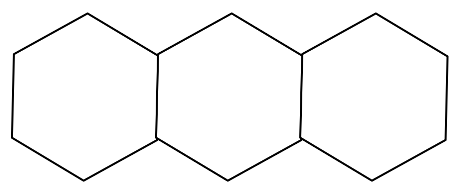

Chart:

Figure	1	2	3	4	5
Number of edges (perimeter)	6	10	14		

a. What is the rule for finding the perimeter?
b. What would the perimeter of the 8^{th} figure be?

2. Which property is represented by the following?

a. 6 x 4 = 4 x 6
b. (4 + 5) + 8 = 4 + (5 + 8)

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c. 6(4 + 5) = (6 x 4) + (6 x 5)

3. Name two arrays using the commutative property of multiplication, that could represent a rectangle with an area of 12 square inches.

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MODULE

2-3

Change in Various Contexts

This module addresses the following indicators:

5-3.5 Analyze situations that show change over time. (B4)

This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY.** Lessons in S^3 begin to build the conceptual foundation students need. **ADDITONAL LESSONS will be required** to fully develop the concepts.

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I. *Planning the Module*

- **Continuum of Knowledge**

First grade students classified change over time as qualitative or quantitative (1-3.6). Second grade students identified (2-3.4) and analyzed (2-3.5) qualitative and quantitative change over time. In third grade, students illustrate situation that show change over time as increasing (3-3.4). Students also interpret data in tables, bar graph, pictographs and dot plots (3-6.3). In fourth grade, students illustrate situation that show change over time as either increasing, decreasing or varying (4-3.6)
In fifth grade, students analyze situations that show change over time (5-3.5).

- **Key Concepts/ Key Terms**

These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use and conversation for students.

Change	*Horizontal axis
*Increasing	*Vertical axis
Varying	*Line Graph
Trends	Data
*Decreasing	*Coordinates

II. Teaching the Lesson(s)

1. Teaching Lesson A: Change Over Time

Fourth grade students illustrated situations that show change over time as increasing, decreasing, or varying. Students in fifth grade are expected to analyze situations that show change over time. In other words, they should examine situations that show increasing, decreasing, or varying change over time and describe the relationship between time and the change. Students should be provided with multiple opportunities to analyze real world situations and describe the change that occurs. ***Situations that show no change over time should also be analyzed*** so that students have a solid understanding of the concept of change over time versus no change.

Be sure to point out to students when reading a line graph that it is read from left to right. If not, students will misinterpret the increasing value for decreasing if read backwards.

Also, when looking at relationships, usually you look at y-values as x-values increase. As x-values increase, y-values will increase, decrease or vary (both increase and decrease on the same graph).

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For this indicator, it is **essential** for students to:
- Understand change over time
- Determine if change has occurred
- Understand the concepts of increasing, decreasing and varying
- Use their understanding of change over time to find examples increasing, decreasing and varying change
- Describe observed change in words
- Recognize counter examples (no change)

For this indicator, it is **not essential** for students to:
None noted

a. Indicators with Taxonomy

Indicator → 5-3.5 Analyze situations that show change over time. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

TEACHER NOTE: A motion detector can sense distance at given intervals of time.

Give handouts or handout printed front and back to students. Have them discuss the drawings and the graphs to create matches of the situation.

Answers are as follows:
- *Figure A matches graph 1*
- *Figure B matches graph 4*
- *Figure C matches graph 2*
- *Figure D matches graph 3*

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HANDOUT (2 pages)

If the following pictures illustrate Eric walking either away from or towards a motion detector at a steady rate, or just standing in front of it without moving – which graphs depict the relationship?

Figure A

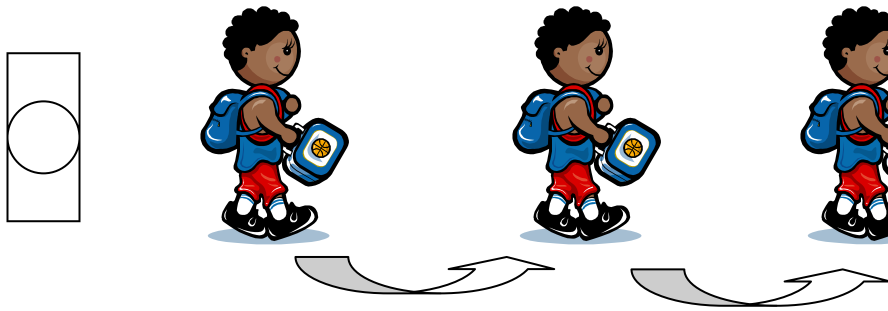


Figure B

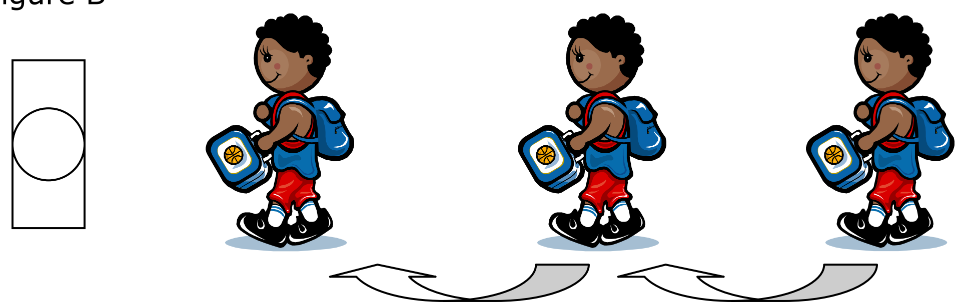
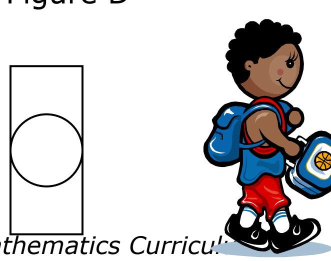


Figure C

Figure D



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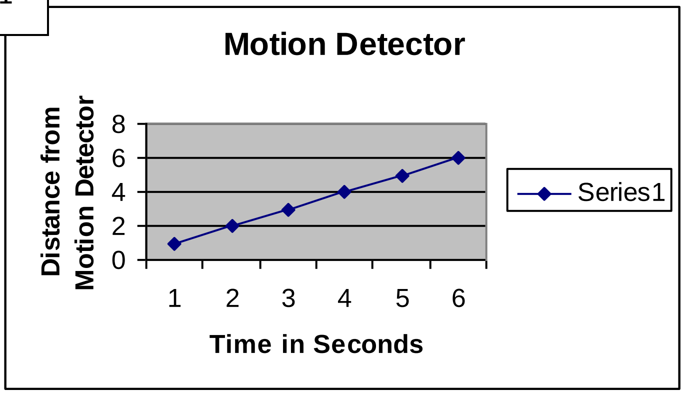
1



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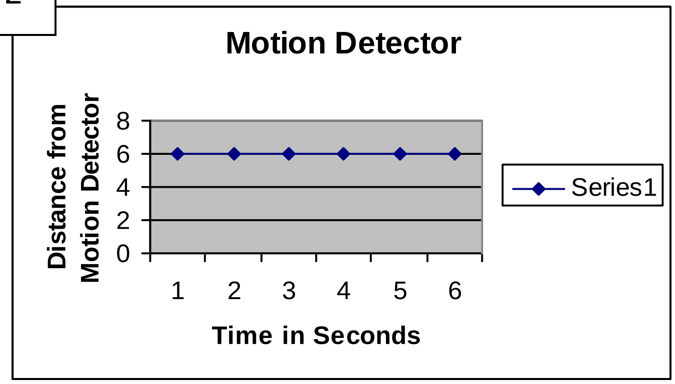

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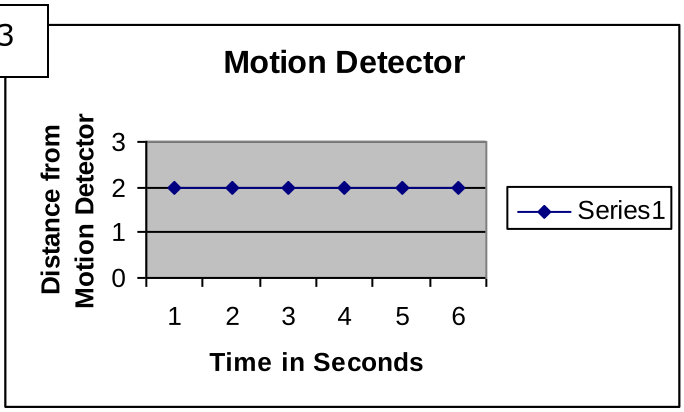
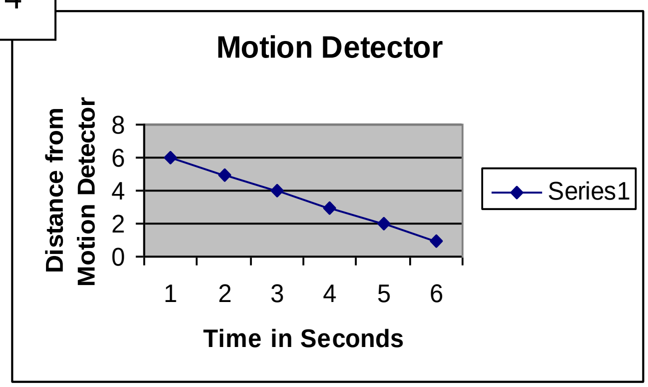


4



What would happen if Eric walked away from the motion detector, then stopped for a second and started walking backwards towards it again? What would the graph look like? Sketch the graph. Would this be described as increasing, decreasing, varying, or no change?

What would it look like if Eric started walking slowly, then increased his speed? Sketch the graph. Would this be described as increasing, decreasing, varying, or no change?

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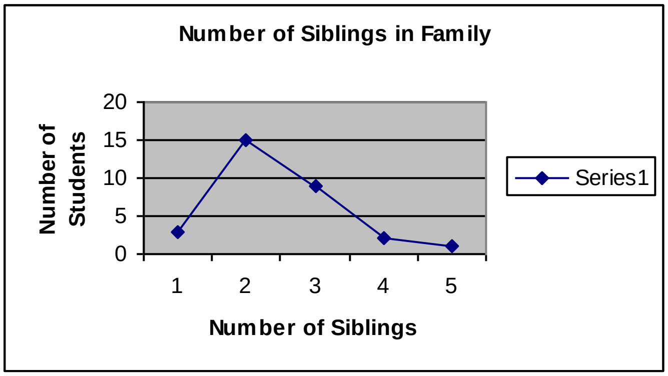
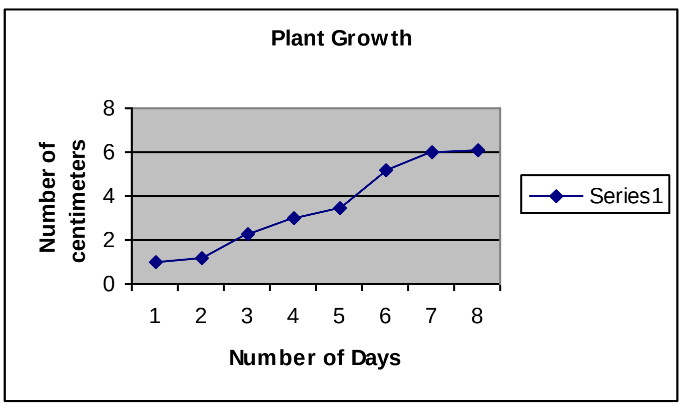
c. Misconceptions/Common Errors

Students sometimes tend to graph discrete data using a line graph which is for continuous data. Remember in a line graph, EVERY point on the line should have a value. Don't use a line graph when a bar graph may be a better choice. Consider the graph below (it is an **INCORRECT** use of a line graph. For example, is there a value at 1½ siblings? NO, but it looks like there is on the graph. (Van de Walle, page 334)



The following is a correct use of a line graph. Notice how it grows every day and in between whole number of days, as plants do – they continually grow.



d. Additional Instructional Strategies/Differentiation

Adapted from Anderson V:

Discuss how things are changing constantly. Brainstorm areas where people need to use charts and graphs to help them interpret information (e.g., attendance at sporting events, extra sales in the lunchroom). Look at examples of graphs. Analyze the data to determine how it changed over time. Use images

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from Google to find examples of graphs. Write a story about what is changing in their graph.

While additional learning opportunities are needed, no suggestions are included at this time.

e. *Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:
MOTION DETECTORS – CBLs and software to display graphs

f. *Assessing the <u>Lesson</u>*

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Answer the following on an exit slip.

1. What would happen if you jumped up and down in front of the motion detector? (Teacher note: this is to see that the students understand that the motion detector is measuring distance from the detector and has nothing to do with height.)

III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

5-3.5

The objective of this indicator is to <u>analyze</u> which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To analyze conceptual knowledge is to determine relevant and irrelevant data about a concept; therefore, students should explore a variety of examples to build understanding of the concept of increasing, decreasing, varying and no change. The learning progression to **analyze** requires students to <u>understand</u> change over time and characteristics of increasing, decreasing and varying change. Students <u>explore</u> teacher generated examples and <u>analyze</u> information (5-1.1) from those examples to determine if change has occurred. They

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generate descriptions (5-1.4) of the observed change then explain and justify their answer on the basis of mathematical relationships (5-1.3). Students use this understanding to find other examples of increasing, decreasing and varying change and analyze counter-examples (no change) to support conceptual understanding.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Given the situation, what type of change is represented: increasing, decreasing or varying. Sketch a graphical representation.

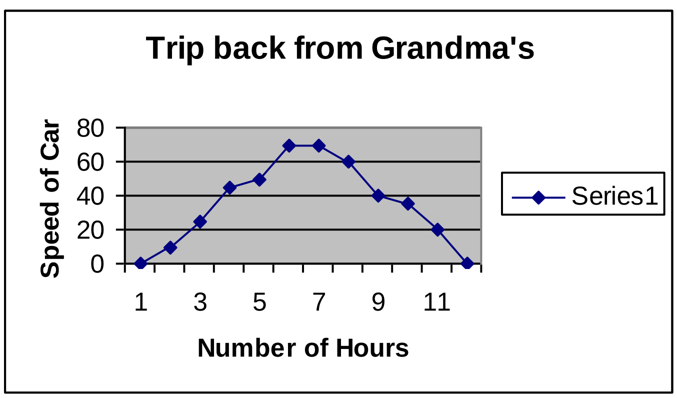
a. As the time of day passes, the temperature increases from 8am to 3pm, then decreases from 3pm to 8pm.

b. As a baby grows into a teenager, his or her weight increases.

c. As trees grow larger, the number of tree rings they contain grows as well so you can determine age of trees.

d. As every day for 10 days yields no rain, the level of the lake is dropping due to evaporation.

e. On a long trip back from Grandma's house, the following graph shows the speed of our car over time.



MODULE

2-4

Angles

This module addresses the following indicators:

5.5.2 Use a protractor to measure angles from 0 to 180 degrees. (C3)

This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY.** Lessons in S^3 begin to build the conceptual foundation students need. **ADDITONAL LESSONS will be required** to fully develop the concepts.

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I. *Planning the Module*

- **Continuum of Knowledge:**

In third grade, students classified angles as either right, acute or obtuse (3-4.4). They also classified triangles by the length of their sides and by the size of their angles (3-4.5).In fourth grade, students compare angle measures with referent angles of 45 degrees, 90 degrees and 180 degrees to estimate angle measures (4-5.2).
In fifth grade, students compare the angles, side lengths and perimeters of congruent shapes (5-4.2) and use a protractor to measure angles from 0 to 180 degrees (5-5.2). This is the first time students are introduced to the protractor.

- **Key Concepts/ Key Terms**

These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use and conversation for students.

*Angle
*Degree
Measure
*Right
*Vertix
Ray
*Protractor
*Acute
*Obtuse
*Straight Line

II. Teaching the Lesson

1. Teaching Lesson A: **Measure Angles from 0 to 180 Degrees with Protractor**

In third grade geometry students classified angles as either right, acute, or obtuse. Students did not measure angles to classify. They simply compared acute and obtuse angles to right angles to make the classification. In other words, fourth grade students compared angle measures with referent angles of 45 degrees, 90 degrees, and 180 degrees to estimate measures. Fifth grade is the first time students are introduced to the measurement tool protractor. A connection should be made between the fourth grade referent angles knowledge and the actual measuring which will enable students to avoid the common mistake of reading a protractor from the wrong direction when measuring angles.

Think about angles as a composition of two rays infinite in length with a common vertex. The size is how far apart the two rays are spread.

(Adapted From Teaching Student Centered Mathematics Grades 3-5, Van de Walle, John A. and Lovin, LouAnn, Pearson Learning, page 272)

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Also you can think of an angle being formed by one ray rotating away from the other. Demonstrate this with two sticks or two rulers.

Students should understand that angles can open from the left or open from the right. The scales are what are difficult for them… which scale do you use – the top or the bottom?

For this indicator, it is **essential** for students to:

- Recall the characteristic of an angle
- Recall the meaning of acute, right and obtuse angles
- Understand how to use referent angles (4^{th} grade) to verify measurements
- Understand how to measure angles given in context (i.e. not just a picture of an angle).
- Find the measure of angles using concrete and pictorial models

For this indicator, it is **not essential** for students to:

- Measure angles larger than 180 degrees

a. Indicators with Taxonomy

Indicator → 5-5.2 Use a protractor to measure angles from 0 to 180 degrees. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson

Materials Needed

Index card per student
Ruler per student
Scissors per student

Suggested Literature Connection

Sir Cumference and the Great Knight of Angleland by Cindy Neuschwander

Radius, the son of Sir Cumference and Lady Di of Ameter, ventures on a heroic quest to earn his knighthood. He first proves his ability to make a "knightly right angle," as Sir D'Grees has trained him, and then doubles the right angle to make a straight angle. So he is sent off with the family medallion, in the shape of a circle (cardboard medallion

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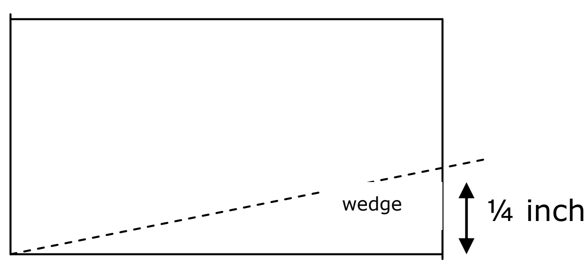
included), to rescue the missing King Lell. Falling bridges, a cryptic riddle, a crocodile-infested moat, and a winding labyrinth all must be mastered before finding the king and his twin dragons, known as "Pair of Lells."

Lesson

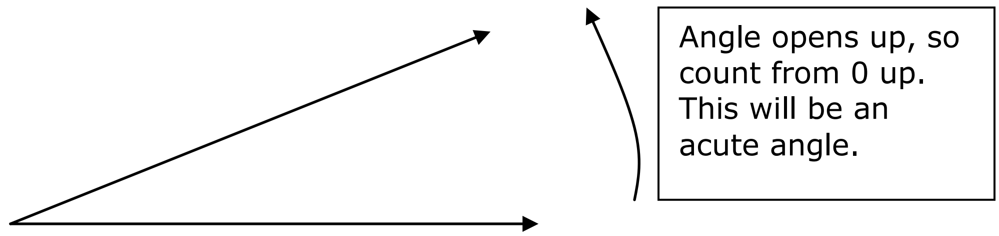
This lesson will allow students to visualize the measurement of angles as a measurement of area in a way. Students need to understand what an angle is before they can understand how to measure one with a protractor. Following the lesson are some tips on measuring with a standard protractor.

Angle Wedges → Give students an index card and a ruler. Have them measure ¼ inch from the bottom of one of the short sides up and make a mark. Then, have them connect the bottom left corner to the mark made ¼ inch up on the opposite side. Cut along that line and they have an angle wedge. (see diagram)



Pass out a sheet with assorted angles on it. There should be some that open left and some that open right. Some with short rays, some with long… just vary several factors. Students should measure how many wedges make up each angle.

Moving on to the standard protractor: The biggest difficulty is knowing which scale to use – the top or bottom on a standard protractor. Hints can be given to students to first identify what angle type they are measuring: acute, obtuse, right or straight. They need to place the protractor on the angle and note whether it opens right or left. An arrow drawn in the direction the angle opens may prove helpful to students when looking at the scales on the protractor. As they follow the arrow up, they should use the scale starting with 0 degrees and count up. Then they should compare the result they get with the type of angle classification they found. See Diagram.



c. Misconceptions/Common Errors

Students may be confused by the set up of the protractor because angle measures runs both clockwise and counterclockwise.
Students may be confused about which set of numbers to use when measuring.
Angles can be compared by cutting one out and laying it on top of another one. Be sure to have students compare angles with rays of different lengths. A wide angle with short sides may seem smaller than a narrow angle with long sides. This is a common misconception for students. Once students can distinguish between a large angle and a small angle, they can move to measuring angles. (Adapted from Teaching Student Centered Mathematics Grades 3-5, Van de Walle, John A. and Lovin, LouAnn, Pearson Learning, 2006, page 272.)

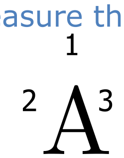
d. Additional Instructional Strategies/Differentiation

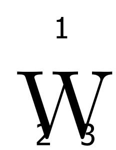
Adapted from Anderson 5:

Have students find and identify angles in the classroom.

In your math note book, have students gather pictures that have examples of angles. On the back of each picture outline the angles seen in black marker. Teams challenge each other to find and measure angles.

Have students take dye cut capital letters to **spell out their names** and measure the angles of the letters.





e. Technology

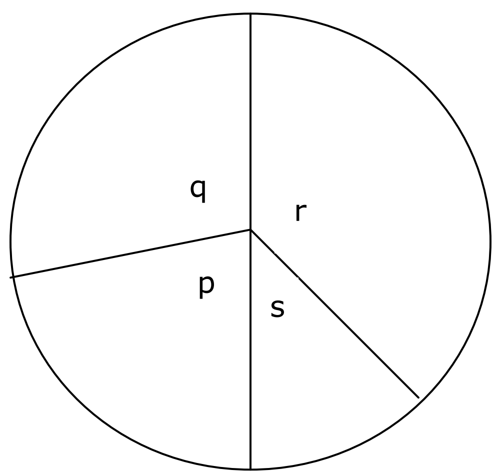
Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

There are no specific recommendations for technology for this lesson at this time.

f. Assessing the <u>Lesson</u>

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Pose the following for a different way to look at angle comparisons: What do you notice about the following angles p, q, r and s?



(Adapted from Mathematics Assessment Sampler Grades 3-5, National Council of Teachers of Mathematics, 2005, page 123)

III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

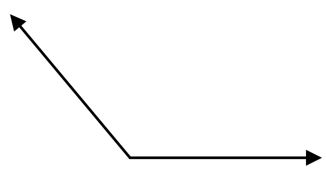
5-5.2

The objective of this indicator is to <u>use</u> which is in the "apply procedural" knowledge cell of the Revised Taxonomy. To apply is to have knowledge of steps and the criteria for when to use those steps. The learning progression to **use** requires students to recall and understand the meaning of degrees. Students also understand the referent angles of 45, 90 and 180 degrees and use that understanding to make estimations of angle measure. They understand the structure of a protractor and how to use it. They explore angle measures in a variety of real world situations. Students explain and justify their answers using correct, clear and complete oral and written language (5-1.5).

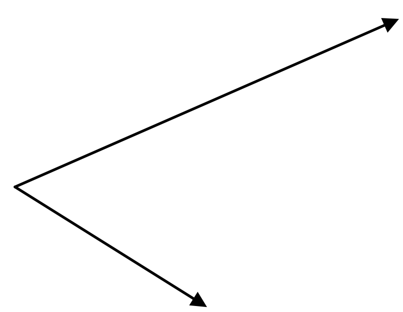
The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

Measure the following angles:

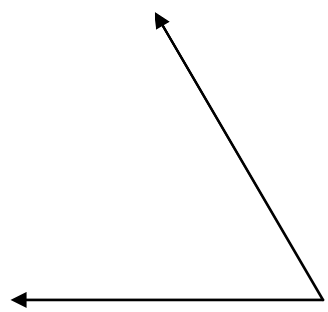
1.



2.



3.



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MODULE

2-5

Dimensionality

This module addresses the following indicators:

5-4.1 Apply the relationships of quadrilaterals to make logical arguments about their properties. (B3)
5-4.2 Compare the angles, side lengths, and perimeters of congruent shapes. (B2)
5-4.3 Classify shapes as congruent. (B2)

5-4.4 Translate between two-dimensional representations and three-dimensional objects. (B2)

This module contains 3 lessons. These lessons are **INTRODUCTORY ONLY.** Lessons in S^3 begin to build the conceptual foundation students need. **ADDITONAL LESSONS will be required** to fully develop the concepts.

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I. *Planning the Module*

- **Continuum of Knowledge:**

5-4.1

In fourth grade, students analyzed the quadrilaterals squares, rectangles, trapezoids, rhombuses, and parallelograms according to their properties (4-4.1).

In fifth grade, students apply the relationships of quadrilaterals to generate descriptions and mathematical statements about relationships between quadrilaterals (5-1.4).

In sixth grade, students compare the angles, side lengths, and perimeters of similar shapes (6-4.7).

5-4.2

In fourth grade, students used transformation(s) to prove congruency (4.4.5).

In fifth grade, students will compare the angles, side lengths and perimeters of congruent shapes (5-4.2) and classify shapes as congruent (5-4.3).

In sixth grade, students will compare the angles, side lengths, and perimeters of similar shapes (6-4.7).

5-4.3

In fourth grade, students used transformations to prove congruency (4-4.5).

In fifth grade, students classify shapes as congruent (5-4.3) and compare the angles, side lengths and perimeters of congruent shapes (5-4.2).

In sixth grade, students will compare the angles, side lengths, and perimeters of similar shapes (6-4.7).

5-4.4

In second grade, students identified and analyzed three-dimensional shapes spheres, cubes, cylinders, prisms, pyramids, and cones according to the number and shape of the faces, edges, corners, and bases of each (2-4.1). In fourth grade, students analyzed the relationship between three-dimensional geometric shapes in the form of cubes, rectangular prisms, and cylinders and their two-dimensional nets (4-4.2).

In fifth grade, students translate between two-dimensional representations and three-dimensional objects.

In seventh grade, students translate between two- and three-dimensional representations of compound figures (7-4.4).

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- **Key Vocabulary/Concepts**

**Properties*
**Angles*
*Right angle
*Acute angle
*Obtuse angle
*Congruent
*Perimeter
*Attribute
Sides
*Vertices
Net (open 3D Shape)
Notation or Symbols
* ≈ Congruent
***Quadrilaterals**
Examples include:
Rhombuses (rhombi)
Squares
Rectangles
Parallelograms
Trapezoids
***Two Dimensional Shapes**
Examples include:
Square
Rectangle
Trapezoid
Triangle
Hexagon
Octogon
***Three Dimensional Objects**

Examples include:
Rectangular Prism
Cylinder
Cube
Triangular Prism
Square Pyramid
Rectangular Pyramid

II. Teaching the Lessons

1. ***Teaching Lesson A:*** Relationships of Quadrilaterals to Make Logical Arguments about their Properties

In 5th grade, students apply the relationships of quadrilaterals to make logical arguments about their properties. This will include making and testing conjectures and explaining conclusions about quadrilateral properties and relationships. For example, are all squares rectangles? Are all rectangles squares? Why or why not?

Students in 5th grade must also **classify** shapes as congruent. The definition of congruent is factual understanding. For students to classify shapes as congruent, they will need to have a conceptual understanding.

In 4th grade students used transformations to prove congruency. In 5th grade, students will compare the angles, side lengths and perimeters of congruent shapes. Given congruent shapes, students will discover that the corresponding angles are the same, the corresponding side lengths are the same, and the perimeters are the same. Therefore, students will conclude that congruent shapes have the same shape and same size. In 6th grade, students will compare the angles, side lengths, and perimeters of similar shapes. It is essential that students understand congruent shapes in order to progress to the concept of similarity in shapes.

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It is in 5th grade that students begin exploring methods for translating between two-dimensional representations and three-dimensional objects. In 5th grade, students should sketch the front, top, and side views of a three-dimensional object built with cubes. Students should be able to draw a net for a given three-dimensional shape and construct and/or state the three-dimensional shape when given its two-dimensional representation (net). Teachers should incorporate isometric dot paper in guiding the students to draw three-dimensional objects.

Translating between two-dimensional representations and three-dimensional objects means being able to identify and draw 2-d images of a 3-d figure AND being able to build a 3-d figure from a 2-d drawing.

For this indicator, it is **essential** for students to:

- Identify the properties of quadrilaterals.
- Identify examples and non-examples of different types of quadrilaterals.
- Compare examples and non-examples of quadrilaterals.
- Make and test conjectures and explain conclusions about quadrilateral properties and relationships.
- Create a quadrilateral given its properties

For this indicator, it is **not essential** for students to:

- Find the measure of missing angles in quadrilaterals.

a. Indicators with Taxonomy

Indicator → 5-4.1 Apply the relationships of quadrilaterals to make logical arguments about their properties. (B3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

Materials Needed

- *shape sheets (in lesson)*
- *index cards (to check for right angles, mark side lengths, draw straight lines, etc.*
- *tracing paper (to check for angle congruence and rotational symmetry)*
- *mirrors (to check for line symmetry)*

Lesson

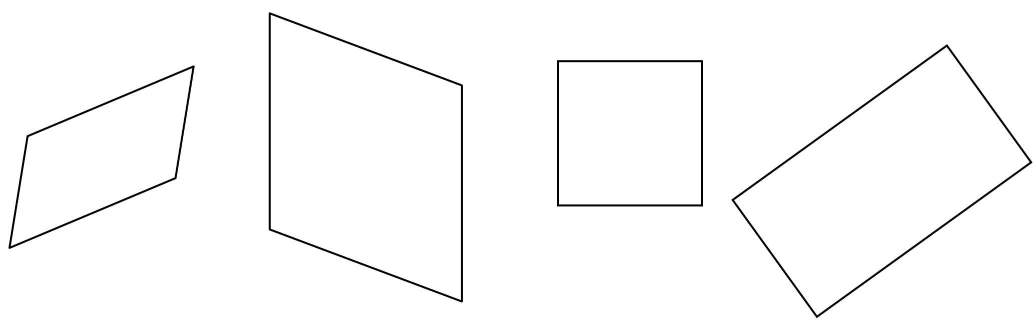
<u>Property Listings</u> → Give students worksheets of shapes and all materials. Students should organize their notes as a table to record their findings. See "notes table" example in this lesson. Assign students working in trios or quads to one type of quadrilateral. Their task is to list as many properties as they can in their notes table. Each property listed must be applicable to all shapes on their sheet. Encourage them to use all the materials available. Encourage them to be specific in their property listings.

This activity may take longer than one day to complete.

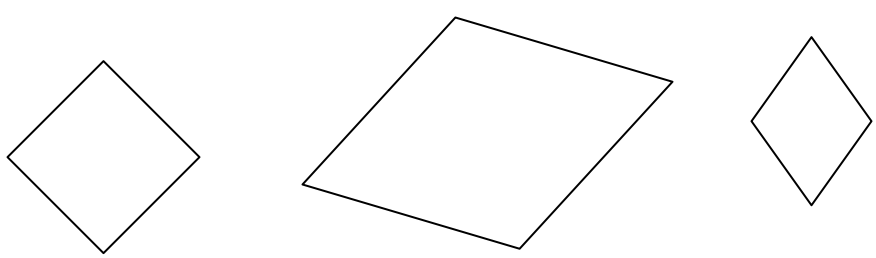
For share out, have students share properties from the parallelograms first, then the rhombi, rectangles, squares and last, the trapezoid. Have one parallelogram group present first, then the other group of parallelograms (if any) add to or dispute properties given.

As the teacher, clarify any terms needed to be formally introduced such as right/acute/obtuse angle, parallel, perpendicular, midpoint, etc.

Sheet A: Parallelogram



Sheet B: Rhombuses (Rhombi)



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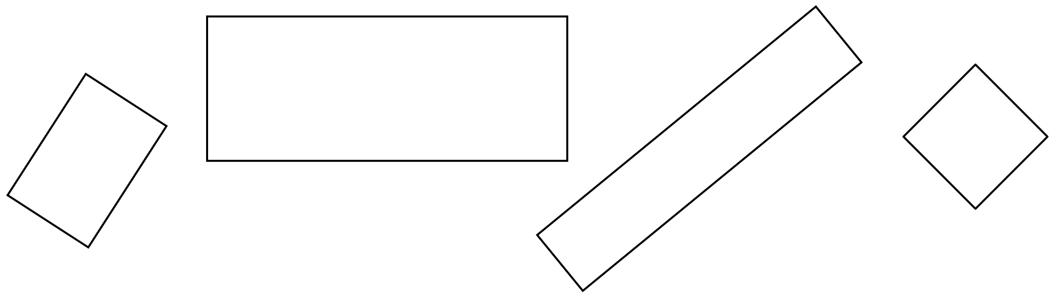
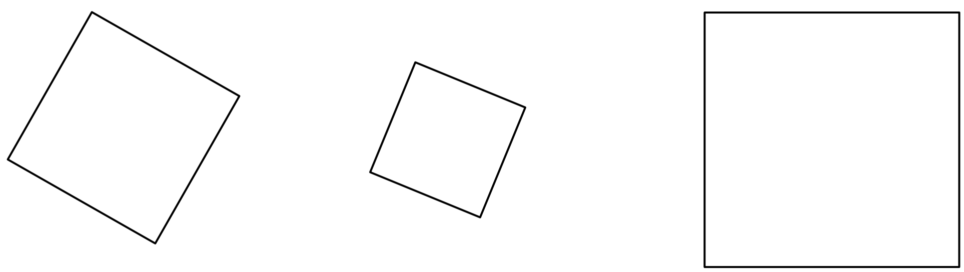
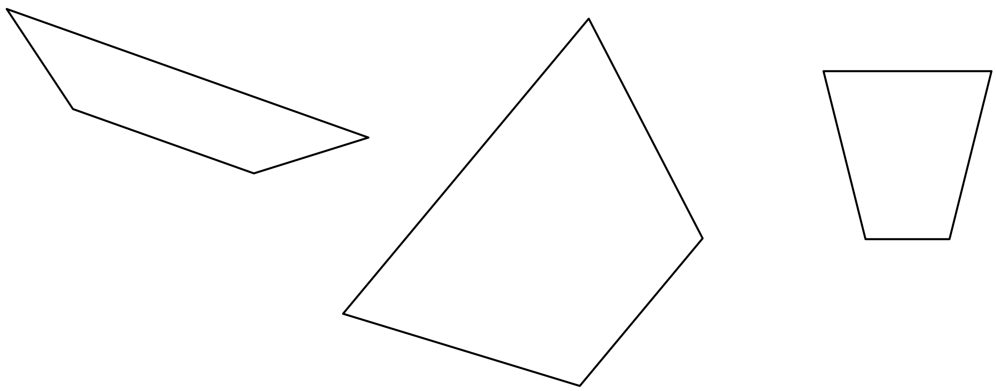
Sheet C: Rectangles



Sheet D: Squares



Sheet E: Trapezoid



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Notes Pages Set Up

Shape	*Sides*	*Angles*	*Diagonals*	*Symmetry*
Parallelogram				
Rhombuses (Rhombi)				
Rectangles				
Squares				
Trapezoids				

Students should use the chart above to make arguments about the properties of quadrilaterals. For example, ask them the following (do not accept yes or no as answers, but rather have them prove their response!):

- *How are squares and rectangles related?*
- *Can a square be a rectangle?*
- *Can a rectangle be a square?*
- *What is the relationship between a parallelogram and a rhombus?*
- *Is a rhombus a parallelogram?*
- *Are all parallelograms rhombuses?*
- *Can a trapezoid have 2 right angles?*
- *Can a parallelogram have 4 acute angles?*
- *Does a square have rotational symmetry?*
- *Can a trapezoid have symmetry?*
- *Can a trapezoid have rotational symmetry?*

Continue the questions so that students are applying the list of properties in order to answer the questions.

c. Misconceptions/Common Errors

Understanding the relationships among and between quadrilaterals is very challenging for students. Having students create Venn diagram or other organizational charts may be helpful.

d. Additional Instructional Strategies/Differentiation

1. To review geometric properties of quadrilaterals, have students work in pairs or small groups and sort the quadrilateral attribute blocks by their properties. Each group should discuss the strategy used for sorting their attribute blocks. Repeat the exercise, sorting the quadrilateral blocks by other properties such as the number of equal sides, the number of sets of parallel lines, or types of angles.

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Adapted from Anderson District 5

2. Play "Name That Quadrilateral." One person calls out a 4 sided geometric shape and someone else must name an object in the room and explain why it is that particular shape.

3. Give students a description of a shape and have them identify the shape.
Example: The shape has two sets of parallel lines with two obtuse angles and two acute angles. All four sides are equal in length. What is my shape? (rhombus)
My shape has four equal sides and all four angles are 90 degrees. What is my shape? (square)

4. Play "Make That Quadrilateral." Divide class into groups of four. Someone calls out a 4 sided geometric shape and the groups must make the shape with their bodies.

5. Relate to objects in real life (Ex: A trapezoid looks like a Pizza Hut sign. I wouldn't mind getting trapped (trapezoid) in Pizza Hut. Others: dry erase board = rectangle).

6. Tour the school building and identify examples of quadrilaterals. Discuss characteristics and properties of each.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

There is no specific technology recommended for this lesson at this time.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Students should respond to the following in their journals or notebooks.. or turn them in to the teacher for review.

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1. What property or properties of a square make it a rectangle? Why can't a rectangle be a square?

2. What is the difference in symmetry and rotational symmetry? Give an example of a shape that has both. Explain why.

3. Give an example of a shape that has symmetry, but not rotational symmetry. Explain why.

2. *Teaching Lesson B*

5-5.2
For this indicator, it is **essential** for students to:
- Know how to measure angles, side lengths, and perimeters of congruent shapes.
- Create a one-to-one mapping between congruent parts
- Understand that congruent shapes have the same shape and size

For this indicator, it is **not essential** for students to:
None noted
5-4.3:
For this indicator, it is **essential** for students to:
- Understand that congruent shapes are exactly the same shape and size.
- Create a one-to-one mapping between corresponding parts
- Identify non-examples of congruent shapes

For this indicator, it is **not essential** for students to:
- Identify and understand the characteristic of similar shapes

a. Indicators with Taxonomy

Indicator → 5-4.2 Compare the angles, side lengths, and perimeters of congruent shapes. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

Indicator → 5-4.3 Classify shapes as congruent. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson B: Compare Angles, Side Lengths, and Perimeters of Congruent Shapes and Classify as Congruent

Materials Needed

Handouts of shapes described below… (these will need to be teacher created)

Give students handouts of squares, rectangles, parallelograms, rhombuses (rhombi) and trapezoids… MAKE SURE that there are at least 2 congruent shapes of each type of quadrilateral.

The orientation of the shapes should vary…. Rotating two congruent shapes makes it more difficult to classify and that' s OKAY!

For example, you <u>may</u> have

- *3 squares, 1 pair that is congruent.*
- *2 rectangles congruent to each other*
- *4 parallelograms, 2 pairs that are congruent*
- *3 rhombuses, 1 pair that is congruent*
- *3 trapezoids, 1 pair that is congruent*

Suggested Literature Connections

What's Your Angle, Pythagoras? by Julie Ellis

Meet Pythagoras, a young Greek boy born around 563 BCE. This engaging story of discovery is told through a fictionalized account of curiosity and persistence. The father of Pythagoras wants his son to be a merchant, but the son has other dreams. He likes to ponder the geometrical puzzles he sees before him. Why do columns lean, why is the ladder too short, and what is that specially knotted rope used by Neferheperhersekeper the builder? Left alone in a garden with a statue base shaped like a triangle, Pythagoras experiments by placing different colored tiles around the sides. Through visual discovery, the young Greek begins to understand the laws governing the right triangle and how that knowledge may help him. From this inquiry, the Pythagorean Theorem is born. Although a sophisticated concept to understand, the endearing storyline and delightful cartoon pictures help the youngest reader to grasp the concept behind "a squared + b squared = c squared."

Lesson

Students should take the characteristics chart from the first lesson in this module and use it, a protractor, a ruler, and any other tool needed to compare the shapes on the handout.

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Students should be encouraged to write the angle measures and side measures on the handout itself. Generalizations should be made and students should discover shapes that are congruent and be able to classify them as such.

c. Misconceptions/Common Errors

Students may confuse similar shapes (same shape, but different size) with congruent shapes (same shape and size). Students have difficulty working with shapes that are not in the upright position.

Student experiences are sometimes limited to figures with the exact same orientation, or with a 90 degree turn. Try orienting the figures at a 45 degree angle or different angle rotations. Students need to be matching up sides or angles and using properties to make determinations of congruency.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Activity on creating congruent triangles from the National Library of Virtual Manipulatives.

http://nlvm.usu.edu/en/nav/frames_asid_165_g_2_t_3.html?open=instructions&from=category_g_2_t_3.html

f. Assessing the <u>Lesson</u>

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Have students draw three of the same shapes (using protractors, rulers and grid paper if needed). Two should be congruent to each other and the other not. They should be

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labeled A, B and C or 1, 2 and 3. They should trade their three shapes with a partner and see if their partner can find the two congruent shapes.

3. ***Teaching Lesson C:*** Translate between Two-dimensional Representations and Three-dimensional Objects

For this indicator, it is **essential** for students to:

- Identify and recognize three-dimensional shapes
- Visualize mentally objects and spatial relationships
- Identify relationships between two- and three-dimensional objects.
- Draw and recognize objects from different perspectives (front, back, right and left)
- Draw a net for a given three-dimensional shape
- Understand that different views of a three dimensional objects built with cubes will be represented with squares
- Translates from two dimensional (nets) to three-dimensional (objects) and vice-versa

For this indicator, it is **not essential** for students to:
None noted

a. Indicators with Taxonomy

Indicator → 5-4.4 Translate between two-dimensional representations and three-dimensional objects. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

Materials Needed

Handouts will need to be created as needed from the examples below.
1 inch cubes (not interlocking)

Lesson

(Adapted From Teaching Student Centered Mathematics Grades 3-5, Van de Walle, John A. and Lovin, LouAnn, pages 245-246)

What's Your Viewpoint? →

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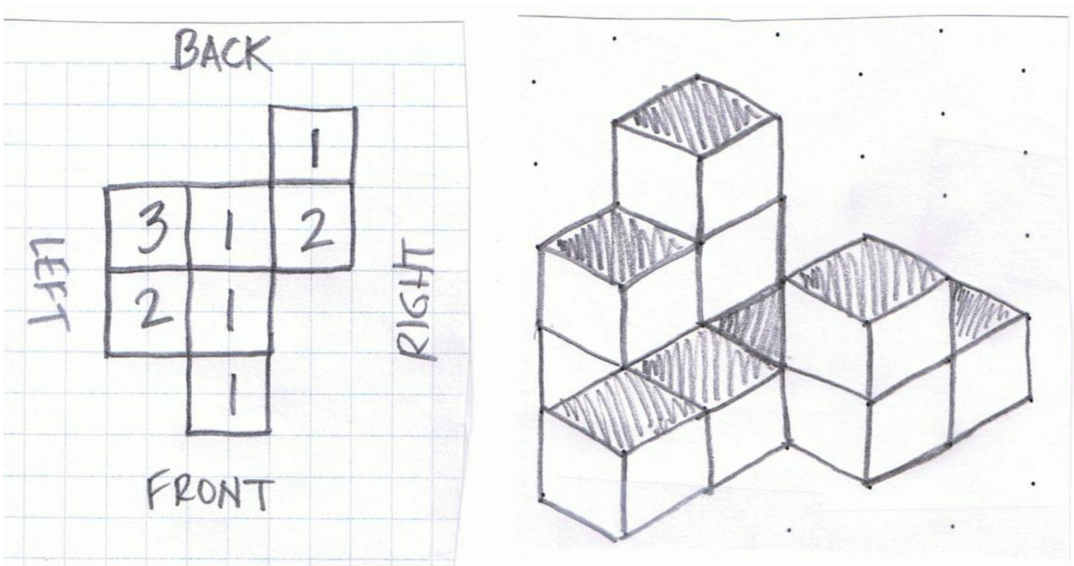
Part A:
Start with a drawing of a cube structure and a building plan. The building plan shows a top view and the numbers indicate how many blocks are in the given column.

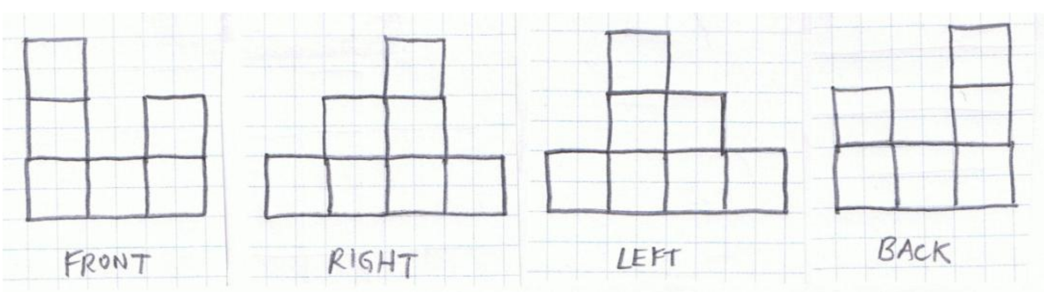
For example:



Students should first build the structure with cubes.

Then, the task is to draw or sketch the views from the front, the back, the right and the left sides.

For the example above, the views are as follows:



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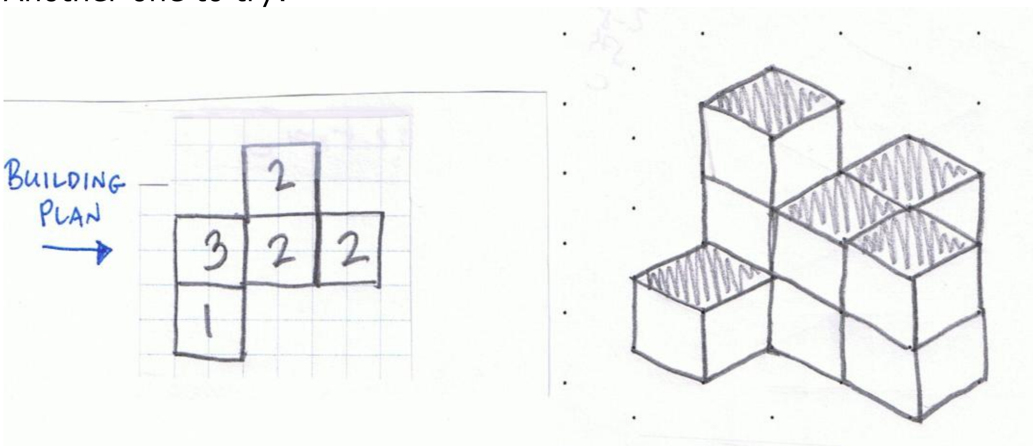
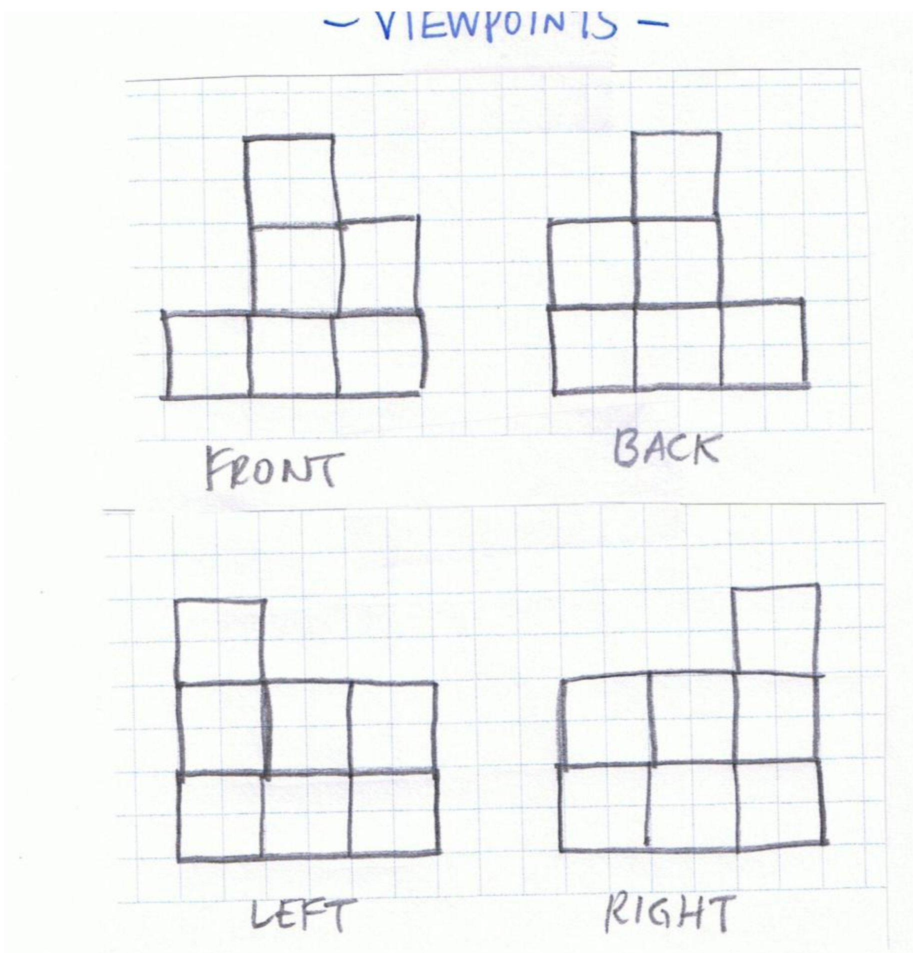
Another one to try:



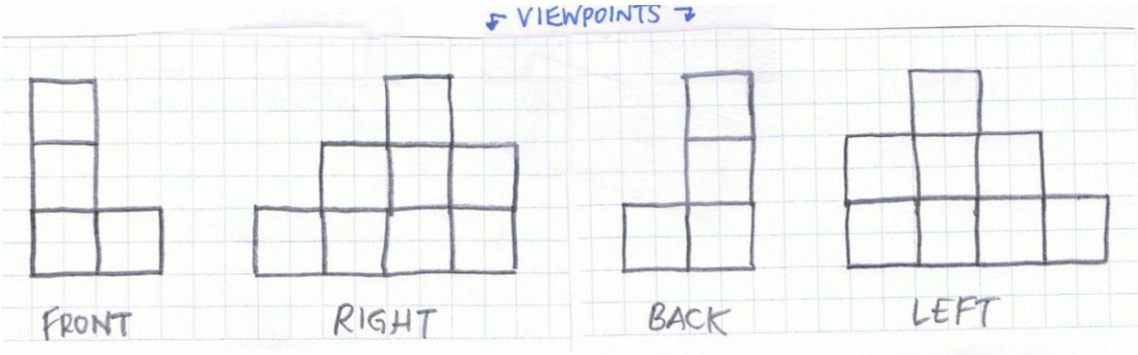
Students should first build the structure with cubes.

Then, the task is to draw or sketch the views from the front, the back, the right and the left sides.

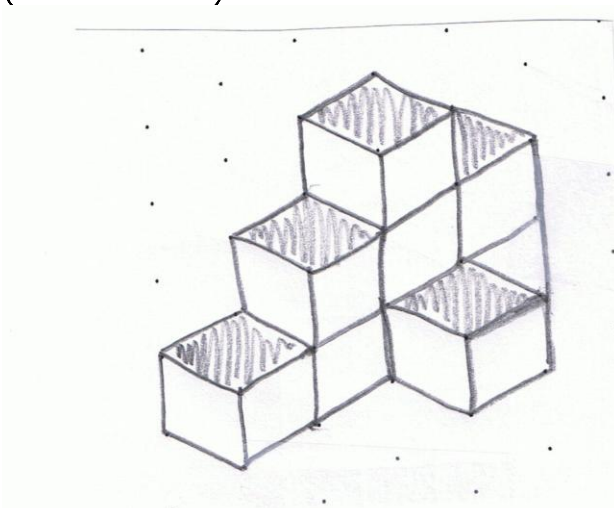
For the example above, the views are as follows:



Part B:
Now, let's start with the views and build the structure.

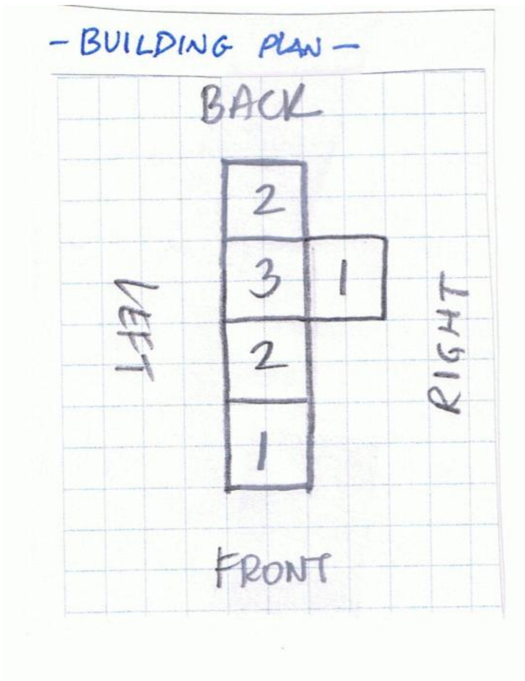


(Teacher Note)



From the structure, make a building plan from the top view.

(Teacher Note)



c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Isometric Dot Paper Virtually…

http://nlvm.usu.edu/en/nav/frames_asid_129_g_2_t_3.html?open=activities&from=category_g_2_t_3.html

http://illuminations.nctm.org/ActivityDetail.aspx?ID=125

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*f. Assessing the **Lesson***

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Have students discuss the following questions with a partner.

- What do you notice about the number of cubes in the structure?
- How does the building plan help you build the structure?
- How do the different viewpoints combine to give you one structure?

*III. Assessing the **Module***

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

5-4.1

The objective of this indicator is to apply which is in the "apply conceptual" knowledge cell of the Revised Taxonomy. To apply conceptual knowledge means to use an understanding of the interrelationships among and between mathematical ideas to solve problems. The learning progression to **apply** requires students to recall and understand the properties of quadrilaterals. Students generate descriptions of the relationships between and among quadrilaterals (5-1.4) and apply these relationships to construct arguments about their properties. Based on these relationships, students generate mathematical statements (5-1.4) and explain and justify their statements (5-1.3) to their classmates and teacher using correct, complete, and clearly written and oral mathematical language (5-1.5).

5-4.2

The objective of this indicator is to compare which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To compare means to detect similarities and difference between and among ideas; therefore, students build conceptual understanding of congruent shapes by exploring a variety of examples. The learning progression to **compare** requires students to recall and understand the meaning of angles, sides and perimeter. Students then use their understanding to explore a variety of examples of congruent shapes. Based on simple observations, students construct arguments about the possible relationship between congruent shapes as it relates to their angles, sides and perimeter. Students explain and justify their arguments (5-1.3) to their classmates and teacher. They use appropriate problem solving strategies such as measuring the angles,

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sides and perimeter to explore the validity of their argument. Students analyze information (5-1.1) from these explorations and generate mathematical statements (5-1.4) about the relationships between angles, sides and perimeter of congruent shapes.

5-4.3

The objective of this indicator is to classify which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning and conceptual knowledge is not bound by specific examples; therefore, students should gain a conceptual understanding of congruency in order to classify any shape. The learning progression to **classify** requires students to understand the characteristics of congruent shapes. Students analyze a variety of shapes and compare the corresponding parts of each shape. As students analyze these shapes, they categorize them as congruent or not congruent (non-examples). Students explain and justify their answers (5-1.3) using correct, clear and complete oral and written mathematical language (5-1.5).

5-4.4

The objective of this indicator is to translate which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Translate means to change from one form to another; therefore, the learning progression to **translate** requires students to use flexibility in mathematical representations (5-1.7). Students analyze (5-1.1) the given representation (2-D net or 3-D object) and generate a description of the relationship between the two dimensional net and the three dimensional object. After using this description to create the three dimensional object or two-dimensional net, the student uses their understanding view and perspective to translate it to the other form. They explain and justify their answers (5-1.3) to their classmate and teacher using correct, clear and compete oral and written mathematical language (5-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

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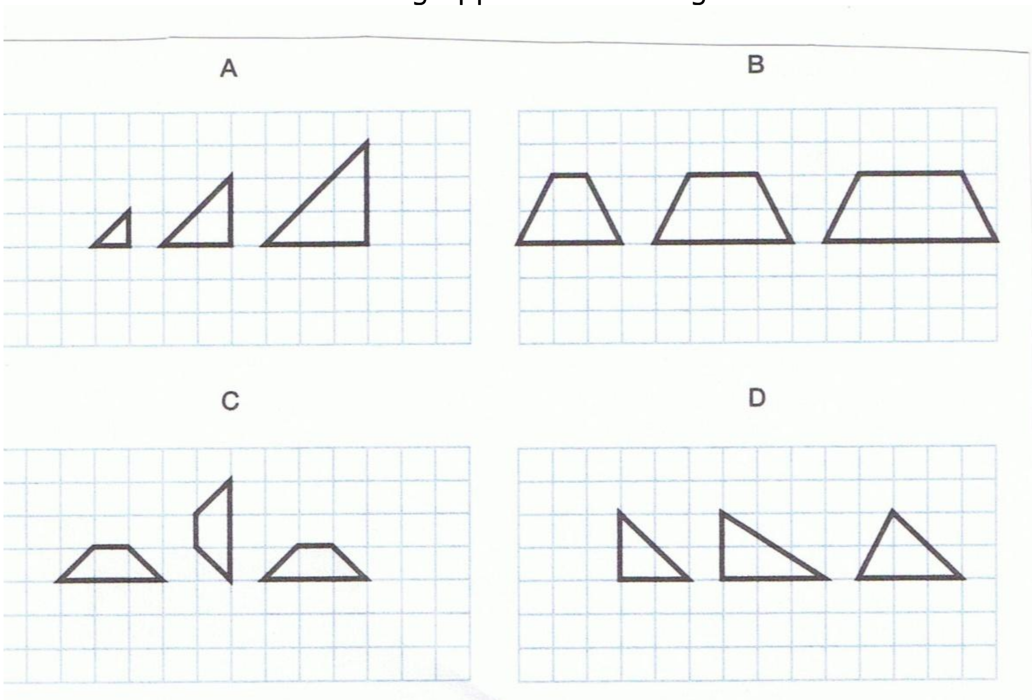
1. a. Which of the following appear to be congruent?



   b. What can you say about the side lengths of the congruent figures?
   c. What can you say about the angle measures of the congruent figures?

2. Students should be able to explain and justify answers based on mathematical properties, structures, and relationships. For example,

   - Are all squares rectangles?
   - Are all rectangles squares? Why or why not?
   - Can a trapezoid have 2 right angles?
   - Can a parallelogram have 4 acute angles?

3. True or False (explain your answers):

   a. If it is a square, then it is a rhombus.
   b. All squares are rectangles.
   c. Some parallelograms are rectangles.

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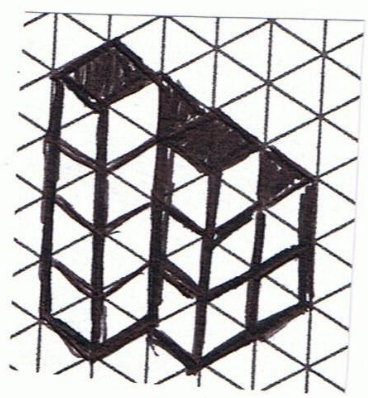
4. The figure below is made with 12 blocks. Build the figure, then draw the building plan and all 4 views.



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