SOUTH CAROLINA SUPPORT SYSTEM INSTRUCTIONAL GUIDE

Content Area	**Sixth Grade Math**

Third Nine Weeks

Standards/Indicators Addressed:

Standard: 6-4 The student will demonstrate through the mathematical processes an understanding of shape, location, and movement within a coordinate system; similarity, complementary, and supplementary angles; and the relationship between line and rotational symmetry.

6-4.7* Compare the angles, side lengths, and perimeters of similar shapes. (B2)
6-4.8* Classify shapes as similar. (B2)
6-4.9* Classify pairs of angles as either complementary or supplementary. (A2)

Standard 6-5 The student will demonstrate through the mathematical processes an understanding of surface area; the perimeter and area of irregular shapes; the relationships among the circumference, diameter, and radius of a circle; the use of proportions to determine unit rates; and the use of scale to determine distance.

6-5.1* Explain the relationships among the circumference, diameter, and radius of a circle. (B2)
6-5.2 Apply strategies and formulas with an approximation of pi (3.14 , or. .) to find the circumference and area of a circle (C3)
6-5.3* Generate strategies to determine the surface area of a rectangular prism and a cylinder. (B6)
6-5.4* Apply strategies and procedures to estimate the perimeters and areas of irregular shapes. (C3)
6-5.5* Apply strategies and procedures of combining and subdividing to find the perimeters and areas of irregular shapes. (C3)

* These indicators are covered in the following 5 Modules for this Nine Weeks Period.
Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.

Module 3-1 Dimensional Geometry			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 3-1 Lesson A: **6-4.7** Compare the angles, side lengths, and perimeters of similar shapes. (B2)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – see Appendix A	See Instructional Planning Guide Module 3-1 Introductory Lesson A See Module 3-1, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 3-1 Lesson A Assessment
Module 3-1 Lesson B: **6-4.8** Classify shapes as similar. (B2)		See Instructional Planning Guide Module 3-1, Introductory Lesson B See Instructional Planning Guide Module 3-1, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 3-1 Lesson B Assessment
Module 3-1 Lesson C: **6-4.9** Classify pairs of angles as either complementary or supplementary. (A2)		See Instructional Planning Guide Module 3-1 Introductory Lesson C See Instructional Planning Guide Module 1, Lesson C Additional Instructional Strategies	See Instructional Planning Guide Module 3-1 Lesson C Assessment

Module 3-2 Circumference and Area			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 3-2 Lesson A: **6-5.1** Explain the relationships among the circumference, diameter, and radius of a circle. (B2)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 3-2 Introductory Lesson A See Instructional Planning Guide Module 3-2, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 3-2 Lesson A: Assessment
Module 3-2 Lesson B: **6-5.2** Apply strategies and formulas with an approximation of pi (3.14 , or. .) to find the circumference and area of a circle (C3)		See Instructional Planning Guide Module 3-2, Introductory Lesson B See Instructional Planning Guide Module 3-2, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 3-2 Lesson B Assessment
Module 3-2 Lesson C: **6-5.3** Generate strategies to determine the surface area of a rectangular prism and a cylinder. (B6)		See Instructional Planning Guide Module 3-2 Introductory Lesson C See Instructional Planning Guide Module 3-2, Lesson C Additional Instructional Strategies	See Instructional Planning Guide Module 3-2 Lesson C Assessment

Module 3-3 Perimeter and Area			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 3-3 Lesson A: **6-5.4** Apply strategies and procedures to estimate the perimeters and areas of irregular shapes. (C3)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations - See Appendix A	See Instructional Planning Guide Module 3-3, Introductory Lesson A See Instructional Planning Guide Module 3-3, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 3-3 Lesson A: Assessment
Module 3-3 Lesson B **6-5.5** Apply strategies and procedures of combining and subdividing to find the perimeters and areas of irregular shapes. (C3)		See Instructional Planning Guide Module 3-3, Introductory Lesson A See Instructional Planning Guide Module 3-3, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 3-3 Lesson A: Assessment

MODULE

3-1

This module addresses the following indicators:

6-4.7 Compare the angles, side lengths, and perimeters of similar shapes. (B2)

6-4.8 Classify shapes as similar. (B2)

6-4.9 Classify pairs of angles as either complementary or supplementary. (A2)

This module contains three lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

Planning the Module

- **Continuum of Knowledge**

  6-4.7 Compare the angles, side lengths, and perimeters of similar shapes. (B2)
  - In fifth grade the students compare angles, side lengths, and perimeters of congruent figures (5-4.2)
  - In seventh grade, students compare the areas of similar and congruent shapes (7-4.6) and apply their knowledge of proportional relationships to find missing attributes of similar shapes (7-4.8)

  6-4.8 Classify shapes as similar. (B2)
  - In fifth grade the students compare angles, side lengths, and perimeters of congruent figures (5-4.2)
  - In seventh grade, students compare the areas of similar and congruent shapes (7-4.6) and apply their knowledge of proportional relationships to find missing attributes of similar shapes (7-4.8)

  6-4.9 Classify pairs of angles as either complementary or supplementary. (A2)
  - In third grade students learn to identify angles as right, acute, or obtuse (3-4.4)

- **Key Concepts/Key Terms**
  *Similar shapes
  *Congruent angles
  Proportional relationships
  *Ratio
  Corresponding angles/sides
  *Complementary angles- complement
  *Supplementary angles-supplement
  *acute angle
  *congruent shapes
  degree symbol
  *obtuse angle
  *perimeter
  proportions
  *right angle
  *straight angle
  *sum

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

Teaching the Lessons

Teaching Lesson A "Similar shapes"

6-4.8

For this indicator, it is essential for students to:

- Understand the characteristics of similar shapes
- Know the difference between similar and congruent shapes.
- Identify a shape as similar using proportional reasoning

For this indicator, it is not essential for students to:

- Find the measure of missing attributes i.e. setting up a proportion to find a missing side, etc…

Indicators with Taxonomy

6-4.8 Classify shapes as similar. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

Introductory Lesson -
Materials: Draw 3 sets of similar rectangles on a piece of paper and make a copy for each student, scissors

Introductory Lesson: Look Alike Rectangles

Give each child a copy of the rectangles handout. Have them cut out each rectangle. Ask the students to group them in 3 groups of 3 "Look alikes" with one rectangle not in any group. When the students have decided on their groupings, stop and discuss the reasons for grouping them as they did. If no one demonstrates placing the rectangles on top of each other to see how they fit, show them. Tell them that they are similar shapes. Have them generate a definition of similar. Close the lesson by having a discussion about what words need to be included in the definition and post the definition on the wall for future reference.

Misconceptions/Common Errors –

- Students often think all shapes that are the same are also always similar. Students will identify all rectangles as similar and forget to verify the corresponding sides.

Additional Instructional Strategies/Differentiation –

- Sixth grade is the first time students are formally introduced to the concept of similarity. Therefore, experiences should actively engage and enable students to discover that similar figures have the same shape, equal corresponding angle measures, and proportional corresponding side lengths. This can be accomplished through the use of similar geometric manipulatives and similar shapes formed on Geoboards or dot paper to compare angles, side lengths and perimeters.

- The exploration of similarity provides the opportunity to review and apply measurement skills as students measure side lengths and angles to determine if two shapes are similar. Make sure the students can identify the corresponding angles, they should also know that corresponding sides are connected by corresponding angles.

- Have the students create similar shapes on the geoboard then record their answers on dot paper.

- Give students a shape recorded on dot paper or grid paper and have them enlarge or shrink the shape.

- Have the students take a picture (comic strips work well) and enlarge it on grid paper.

Technology-

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and the virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- Harcourt online activity: http://www.harcourtschool.com/activity/similar_congruent/

- Power Point presentations: http://math.pppst.com/congruent.html

- Interactive tool with SC Mathematics standards; click on the indicator you are addressing. http://www.shodor.org/interactivate/standards/organization/19/

Assessing the Lesson

Ask the students to give real life examples of things that are similar.

Teaching Lesson B "Comparing similar shapes"

6-4.7 Compare the angles, side lengths, and perimeters of similar shapes. (B2)

For this indicator, it is essential for students to:

- Identify a shape as either similar or congruent.
- Identify the corresponding sides and angles of similar shapes.
- Know how increasing the side length affects the perimeter.
- Compare the corresponding side lengths using proportions.
- Compare the perimeters of similar shapes using proportions.
- Discover that the corresponding angle measures will be equal whereas the corresponding side lengths and perimeters are proportional.

For this indicator, it is not essential for students to:

- Identify missing attributes of similar shapes.

Indicators with Taxonomy

6-4.7 Compare the angles, side lengths, and perimeters of similar shapes. (B2)

Cognitive Process Dimension: Understand

Knowledge Dimension: Conceptual Knowledge

Introductory Lesson- "Comparing similar shapes"

Materials: shapes from lesson A, dot paper, triangle pattern blocks, scissors, tangrams, magazines

Using the similar shapes created or found in lesson A, have the students find and record the corresponding angle measurements, side lengths and perimeters. Ask them what they notice (similarities and differences). If no one mentions the equal proportions, point that out

in one of the similar shapes and ask if they think it will hold true for all.

Give students five identical triangles from pattern blocks and have them measure and prove that the triangles are congruent. Ask students to arrange four of the five triangles to form a larger triangle that is the same shape as one of the small triangles. When the large triangle is constructed, have students investigate and analyze the angles and sides of the two triangles, comparing the measurement with the one triangle that was not used from the original five. Students should also be asked to investigate and analyze the area and perimeter of the triangles. Have students discuss their conclusions.

Continue their investigation by asking students to draw sixteen equilateral triangles on dot paper and cut them out. Use the triangles to complete the same investigation again. This will show students another example of congruent and similar triangles of different sizes.

After the second investigation and a class discussion about their conclusions, have students investigate a set of tangrams to find similar and congruent figures. Close the lesson by asking students to generate mathematical statements about the relationships among similar shapes, perimeter, corresponding sides and angles.

Misconceptions/Common Errors –

- One of the most common mistakes students make is comparing sides or angles that are not corresponding. Therefore, it is important that students understand the concept of correspondence.

- While on the surface comparison of angles, side lengths, and perimeters of similar shapes may appear to be a simple concept, the indicator requires a more in-depth level of mathematical understanding. One of the most common mistakes students make is comparing sides or angles that are not corresponding. Therefore, it is important that students understand the concept of correspondence. Two polygons are in correspondence when consecutive vertices of one are matched with consecutive vertices of the other. It is critical that students discover that the corresponding angle measures will be equal whereas the corresponding side lengths and perimeters are proportional. With this knowledge students should determine and justify similarity of shapes.

Additional Instructional Strategies/Differentiation –

- Using grid paper, have the students construct 4 similar squares where the sides increase by one unit each time. Then have them find the perimeter of each. Have them graph the results. Discuss their findings. What patterns do they see? Do the same thing with other polygons. How are the patterns different with different polygons?

- Ask students to find pictures in magazines or on the Internet that include congruent and similar figures. (Example: Bridges, building, etc.).

Technology-

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and the virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- Interactive tool with SC Mathematics standards; click on the indicator you are addressing.
  http://www.shodor.org/interactivate/standards/organization/19/

Assessing the Lesson

Throughout the class activity look for the following in students…Are they able to identify the similar figures? Can they match the corresponding angles and determine the sides that are proportional?

Teaching Lesson C : "Complementary and supplementary angles"

6-4.9 Classify pairs of angles as either complementary or supplementary. (A2)

For this indicator, it is essential for students to:

- Understand the complementary angles add up to 90 degree
- Understand that supplementary angles add up to 180 degree
- Understand the complementary angles form a right angle
- Understand the supplementary angles form a straight angle
- Classify a pair of angles as either complementary or supplementary angles.
- Identify the missing angle measure when given one angle in a complementary or supplementary pair.

For this indicator, it is not essential for students to:

- Name angle relationships that involve parallel lines and transversals.

Indicators with Taxonomy

6-4.9 Classify pairs of angles as either complementary or supplementary. (A2)

Cognitive Process Dimension: Understand

Knowledge Dimension: Remember Knowledge

Introductory Lesson – "Complementary and supplementary angles"

Use strips of paper to make rays and brads to connect them to make angles. Have the students construct two angles that when put together make a right angle. Then have them find the measures of each. They should note that they add up to 90 degrees. Have them do this several times for right angles. Explain that these are complementary angles. Then do the same thing for supplementary angles. Close the lesson by allowing students to offer strategies for remembering the definitions of complementary and supplementary so they will not confuse the two terms.

Misconceptions/Common Errors

- Supplementary angles do not have to be linear angles. They can be two separate angles that add up to 180 degrees.

Additional Instructional Strategies/Differentiation –

- Have students identify complementary and supplementary angles in shapes they made in the previous lessons.
- A strong understanding of how to use a protractor is essential in drawing angles. This allows students to create mental models of the concept. The teacher can then move to the more abstract by giving students drawings with angle measurements and asking them to determine if the angles are complementary or supplementary.
- Students should also relate this to the inverse relations, i.e. if the students are given one angle measure they should know the difference

between either 180 degrees or 90 degrees will give the missing angle of a complementary or supplementary relationship. Questions like: If this angle measures 30°, what is the complement? If this angle measures 30°, what is the supplement?

Technology-

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and the virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- Interactive tool with SC Mathematics standards; click on the indicator you are addressing. http://www.shodor.org/interactivate/standards/organization/19/

Assessing the Lesson

Observe the angles students are making. Ask questions like: "If this angle measures 30 degrees, what will its complement measure?"; "So supplementary angles have to be linear?"

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

6-4.7 Compare the angles, side lengths, and perimeters of similar shapes. (B2)

The objective of this indicator is to compare which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To compare is to detect correspondences between ideas; therefore, student construct an understanding of similarity by exploring a variety of examples. The learning progression to **compare** requires students to recall the characteristics of congruent shapes. Students investigate and analyze a variety of shapes based on characteristics and generalize connections among these them (6-1.7). They use these generalizations to generate mathematical statements (6-1.5) about the relationships among similar shapes, perimeter, corresponding sides and angles. Students use these relationships to identify and generate examples of similar shapes. They *evaluate* their using by

posing questions to prove or disprove their conjecture (6-1.2). Students explain and justify their answers using correct and clearly written and spoken words and notation (6-1.6). As students compare, they write statements that summarize the relationship between specific angles, sides and perimeters.

6-4.8 Classify shapes as similar. (B2)

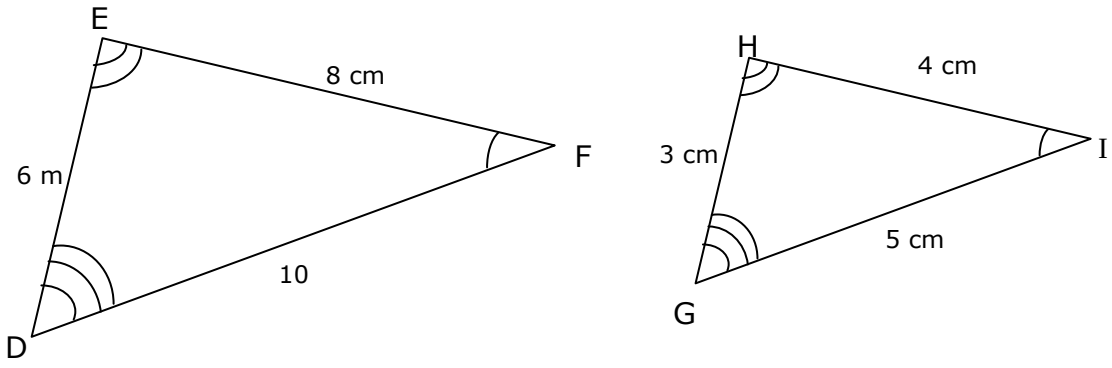
The objective of this indicator is classify which is in the "understand conceptual" knowledge cell of the Revise Taxonomy. To classify is to determine if something belongs to a category; therefore, students build a conceptual understanding of similarity by placing shapes in appropriate categories. The learning progression to **classify** requires students to recall the characteristics of congruent and similar shapes. Students use inductive and deductive reasoning to analyze problems (6-1.3). They recognize these characteristics when given examples and explain and justify their classifications using correct and clearly written or spoken words and notations (6-1.6).

6-4.9 Classify pairs of angles as either complementary or supplementary. (A2)

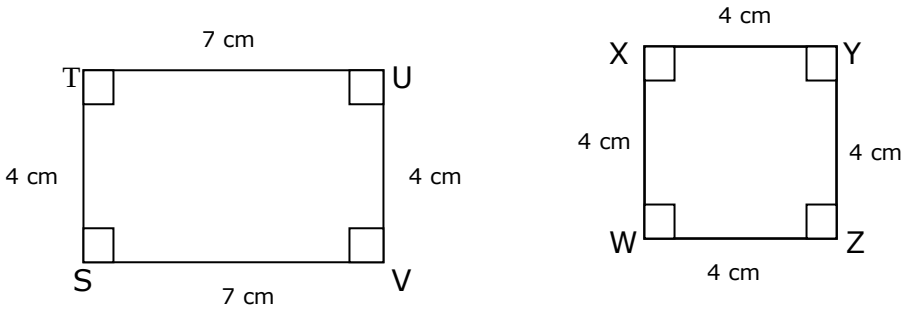
The objective of this indicator is classify which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To classify is to determine if something belongs to a category; therefore, students build a conceptual understanding of supplementary and complementary angles by placing pairs of angles in appropriate categories. The learning progression to **classify** requires students to recall the definition of complementary and supplementary angles. Students construct numerical (numbers only), concrete and pictorial representations of pairs of angles that are complementary and supplementary. Students analyze these constructions to generalize connections (6-1.7) between complementary angles and a right angle and the connection between supplementary angles and a straight angle. They use their understanding of these relationships to classify angles as supplementary or complementary when given the numerical representation (numbers only) and the pictorial representation. They explain and justify their answers using correct and clearly written or spoken word and notations (6-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

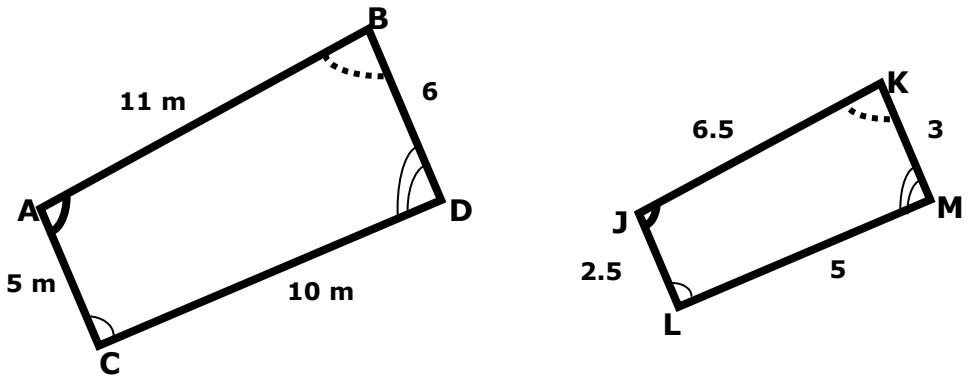
1. Determine if ΔDEF is similar to ΔGHI. Explain your answer.



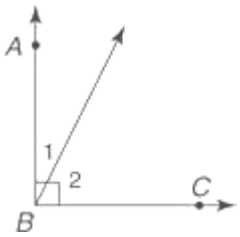
2. Is Quadrilateral *STUV* similar to quadrilateral *WXYZ?* Explain your answer.



3. Is Quadrilateral ABCD similar to quadrilateral JKLM? Explain your answer.

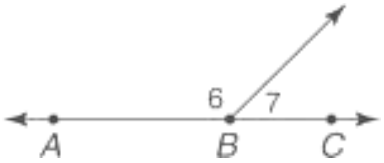


4 a. If the m∠1 is 22 degrees, what is the m∠2? _____



b. Are angles 1 and 2 classified as complementary, supplementary, or neither? Explain.

5 a. If the m∠6 is 115°, what is the m∠7? _____



b. Are angles 6 and 7 classified as complementary, supplementary, or neither? Explain.

MODULE

3-2

This module addresses the following indicators:

6-5.1 Explain the relationships among the circumference, diameter, and radius of a circle. (B2)

6-5.2 Apply Strategies and formulas with an approximation of pi (3.14 or 22/7) to find the circumference and area of a circle. (C3)

6-5.3 Generate strategies to determine the surface area of a rectangular prism and a cylinder. (B6)

This module contains three lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Lesson

- **Continuum of Knowledge**

6-5.1 Explain the relationships among the circumference, diameter, and radius of a circle. (B2)

- In third grade students learned the attributes of a circle, including: center, radius, circumference, and diameter (3-4.1).

6-5.2 Apply Strategies and formulas with an approximation of pi (3.14 or 22/7) to find the circumference and area of a circle. (C3)

- In third grade, students identified the attributes of circles: center, radius, circumference and diameter (3-4.1).

- In eighth grade, students apply formulas to determine the exact (pi) circumference and area of a circle (8-5.4)

6-5.3 Generate strategies to determine the surface area of a rectangular prism and a cylinder. (B6)

- In fourth grade, students analyzed the relationship between three-dimensional geometric shapes in the form of cubes, rectangular prisms, and cylinders, and their two-dimensional nets (4-4.2)

- In seventh grade, students apply formulas to determine the surface area and volume of prisms, pyramids and cylinders (7-5.2).

- **Key Concepts/Key Terms**

*Circumference
*Diameter
*Radius
*Pi (∏)
*Cylinder
*Rectangular prism
*Area
*Surface Area

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lessons

1. Teaching Lesson A

6-5.1 Explain the relationships among the circumference, diameter, and radius of a circle. (B2)

For this indicator, it is **essential** for students to be able to:

- Recall the meaning of circumference, diameter and radius
- Understand the relationship between the diameter and the radius
- Recall the formula for circumference
- Understand that pi is the relationship of $\frac{Circumference}{d}$.
- Understand that the diameter of a circle will fit around the circumference of the circle about three and a little more (3.14)

For this indicator, it is **not essential** for students to:

- Calculations involving circles (circumference and area)

a. Indicators with Taxonomy

6-5.1 Explain the relationships among the circumference, diameter, and radius of a circle. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson A–

Materials:

- ***Assorted circular objects (jar lids, tubes, wastebaskets, etc.)***
- ***Measuring tape and/or string with rulers***
- ***Trundle wheels***

Literature Connections:

- *Sir Cumference and Dragon of Pi* by Cindy Neuschwander
- *Sir Cumference and the First Round Table* by Cindy Neuschwander

Introductory Lesson A: "Discovering Pi"

Have groups of students carefully measure the circumference and diameter of many different circles. Each group measures different circles.

Some groups measure both the circumference and diameter of circular items such as jar lids, tubes, cans, and wastebaskets. To measure circumference, wrap string once around the object and then measure that length of string.

Other groups measure large circles marked on gym floors and playgrounds. Use a trundle wheel or rope to measure the circumference.

Collect measures of circumference and diameter from all groups and enter them in a table. Ratios of the circumference to the diameter should also be computed for each circle. A scatter plot of the data should be made with the horizontal axis representing diameters and the vertical axis circumferences.

Most of the ratios should be in the neighborhood of 3.1 or 3.2. The scatter plot should approximate a straight line through the origin. The slope of the line should be close to 3.1. The graphs of equivalent ratios are always straight lines through the origin. The exact ratio is an irrational number, about 3.14159, represented by the Greek letter pi, ∏.

What is most important here is that students develop a clear understand of ∏ as the ratio of circumference to diameter in any circle. The quantity ∏ is not some strange number that appears in math formulas; it is a naturally occurring and universal ratio.

c. Misconceptions/Common Errors –

Students often confuse radius and diameter.

Students may not realize that when they use non-standard ways to measure (string, jar lids, etc.) circumference, that their measurements will be inaccurate.

d. Additional Instructional Strategies –

To build conceptual understanding of these relationships, the following activity may be used. Have groups of students carefully measure the diameter of many different circles (jar lids, tubes, cans, and wastebaskets) using string. Then have the students figure out how many times the diameter will fit around the circumference of the circle. It should fit three times with a little bit left over. So what's the little bit?

To get more a more accurate answer, have student measure the length of the diameter using a ruler. To measure circumference, wrap string once around the object and then measure that length of string. Collect measures of circumference and diameter from all groups and enter them in a table. Use the relationship that the diameter fits into (or divides into) the circumference about three and a little bit more based on their estimates. Now they will see what the answer is if they divide the measurements. It

should be also be three and a little bit more but in decimal form. Students discover that ∏ = C/D, the circumference divided by the diameter. From this, the students should be able to come up with the circumference = ∏ D. Half the diameter is the radius (r), so the same equation can be written C = 2∏r.

So what does C = 2∏r mean? Let student explore the relationship between the radius and circumference using string to estimate. The radius fits around the circumference six and a little bit more because 2∏ ≈ 6.28.

Also measure large circles marked on gym floors and playgrounds. Use a trundle wheel or rope to measure the circumference. Ratios of the circumference to the diameter should also be computed for each circle.

e. Technology

The Joy of Pi: History and Facts Related to Pi
http://www.joyofpi.com/index.html

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Observe the students' measuring techniques and correct use of the tools. Ask them to describe the patterns they are discovering with the table and the scatter plot.

- ***Teaching Lesson B***

6-5.2 Apply Strategies and formulas with an approximation of pi (3.14 or 22/7) to find the circumference and area of a circle. (C3)

For this indicator, it is **essential** for students to be able to:

- Distinguish between radius and diameter
- Understand the concept of pi and different forms of approximations
- Given real-world situation, determine when to use which formula
- Understand that area is square units and circumference is linear units
- Substitute values into the formulas without simplifying
- Set up the formulas when given a picture, a story problem or the values (numbers with units)

- Develop fluency in setting up formulas

For this indicator, it is **not-essential** for students to be able to:

- Compute the value of the circumference and area without a calculator because students are not fluent in multiplication of decimals and fractions.
- Find the diameter or radius given the circumference or area
- Calculate using irrational numbers

a. Indicators with Taxonomy

6-5.2 Apply Strategies and formulas with an approximation of pi (3.14 or 22/7) to find the circumference and area of a circle. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson – "Area of a Circle"

In lesson A, students discovered that ∏ = C/D, the circumference divided by the diameter. From this, the students should be able to come up with the circumference = ∏ D. Half the diameter is the radius (r), so the same equation can be written C = 2∏r.

Students should be challenged to figure out the area formula on their own. To help them generate a strategy for finding the area of a circle, give each student a paper circle. Have them divide it into at least 8 sectors. Then cut out each sector and arrange them side by side in a "near parallelogram". The more sectors that the circle is divided into, the closer the shape is to a parallelogram. Since the 2 long sides of the parallelogram are made up with the circumference of the circle, one side would be half of the circumference or ∏r. The short side is the length of the radius. This task should be used as a hint toward development of an area formula for the circle. But the complete argument for the formula should come from your students.

c. Misconceptions/Common Errors –

- Students often confuse the concepts of diameter and radius.
- Students often confuse when to use square units and linear units.

d. Additional Instructional Strategies –

- Since sixth graders are only generating strategies to multiply and divide fractions and decimals, they do not have to compute the value of the circumference or area without a calculator. The emphasis is on students understanding how to set up the formulas.

- Students will need to review the concepts of pi, diameter, radius, and circumference learned in 6th grade.
- Students should explore investigations to review conceptual knowledge of the formulas for circumference and area of circles.
- Students should set up formulas when given real world examples.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

No suggestions for use of technology or websites are included at this time.

f. Assessing the Lesson

Assess student understanding in this lesson by listening to student dialogue around creation of the formula for the area of a circle.

- ***Teaching Lesson C***

6-5.3 Generate strategies to determine the surface area of a rectangular prism and a cylinder. (B6)

For this indicator, it is **essential** for students to be able to:

- Recall area of rectangles, squares and circles
- Recall and understand the concept of nets
- Fluent computation with whole numbers (only)

For this indicator, it is **not essential** for students to be able to:

- Gain computational fluency with calculate the surface area
- Use side lengths other than whole numbers

a. Indicators with Taxonomy

6-5.3 Generate strategies to determine the surface area of a rectangular prism and a cylinder. (B6)

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson – "Wrapping Gifts"

Materials:
Assorted boxes and containers in shapes of rectangular prisms and cylinders
Measuring tools: rulers, tape measures, string, etc.

Introductory Lesson: "Wrapping Gifts"
Provide student groups with 3-4 assorted boxes and containers and tell them that you need to wrap each of these gifts and need to know how much wrapping paper to buy. Their task is to determine how much wrapping paper each gift will require to cover all of the sides. They should keep detailed notes of their work to defend their method of determining the amount of wrapping paper when they share their solutions with the entire class.

After each group has determined how much wrapping paper is needed for each box, create a class list of strategies sorted by type of container (cylinder and rectangular prism) to identify a generalized formula for the surface area of each.

c. Misconceptions/Common Errors –

- Students usually confuse area and perimeter, as well as, the units for these calculations.
- When given picture models of rectangular prisms, student find it difficult to match up dimensions for the areas of each side.

d. Additional Instructional Strategies –

Exploring the concepts of surface area is easier when students have concrete models but it is important for them to transfer that understanding to pictorial models. When given a concrete model, have student label corresponding sides as A's, B's and C's. They know that they are always two A sides, two B sides and two C sides. When given the pictorial model have them label sides in the same manner. The big difference is that they can't see the other sides but know they exist.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Assess student understanding of the formulas created for surface area through an exit slip, a notebooking prompt, or paired dialogue around the question: "How might you explain to someone who was absent today how to determine the surface area of a rectangular prism and a cylinder?"

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

6-5.1 Explain the relationships among the circumference, diameter, and radius of a circle. (B2)

The objective of this indicator is to explain which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To explain is to construct a cause and effect model; therefore, students demonstrate their understand of these relationships by using statements such as "the circumference is ∏ times D because the diameter will fit around etc.." Because conceptual knowledge is not bound by specific examples, students should build understanding by exploring a variety of examples. The learning progression to **explain** requires students to recall parts of a circle (radius, diameter and circumference). Students investigate relationships using standard and nonstandard representational forms (6-1.8) that allow them to construct an understanding of the number *pi* by using inductive reasoning (details to generalization). They formulate an argument regarding the relationship among pi, circumference and diameter (6-1.3) and pose follow questions to prove or disprove their argument (6-1.2). Students explain the relationship among pi, circumference, diameter, and area using correct and clear written or spoken words, variable, and notations (6-1.6).

6-5.2 Apply Strategies and formulas with an approximation of pi (3.14 or 22/7) to find the circumference and area of a circle. (C3)

The objective of this indicator is apply, which is in the "apply procedural" cell of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus is to gain fluency with setting up circumference and area formulas, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **apply** requires students to recall and understand the concepts of pi, diameter, radius, and circumference. Students explore a variety of situations that involve both computational and application problems. Students analyze these situations to determine which formula is appropriate based on the given information. They explain and justify their answers using

correct and clearly written or spoken words (6-1.6) and check the reasonableness of their solutions.

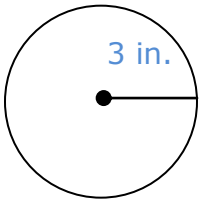
6-5.3 Generate strategies to determine the surface area of a rectangular prism and a cylinder. (B6)

The objective of this indicator is to generate, which is in the "create conceptual" knowledge cell of the Revised Taxonomy. To create is to reorganize elements (areas of square, circles and rectangles) into a new pattern or structure (surface area). The learning progression to **generate** requires the students to recall the formulas for area of squares, rectangles and circles. They understand that the surface area is the sum of the areas of all faces. As students explore the concept of surface area, they should generate conjectures (6-1.2) and exchange mathematical ideas with classmates. They evaluate those conjectures and pose questions for further understanding (6-1.2). Students use correct and clearly written or spoken words to explain their reasoning for their answers (6-1.6). By using deductive reasoning (specific to general), students generalize mathematical statements (6-1.5) about surface area and how to find it.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. The circumference of a circle is 8π cm. What is its diameter? What is its radius? Explain.

2. **Use the circle to answer the next two questions.**



a. Find the circumference of the circle.

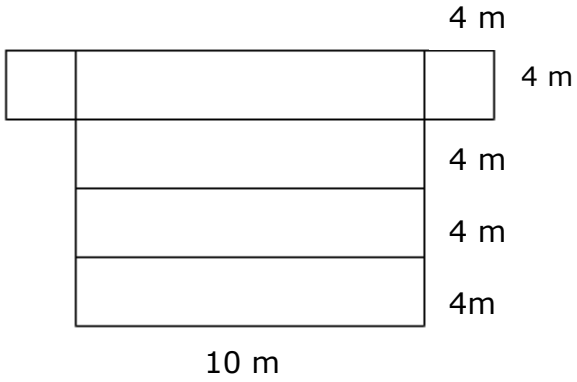
A. 3π in.
B. 6π in.
C. 9π in.

b. Find the area of the circle.

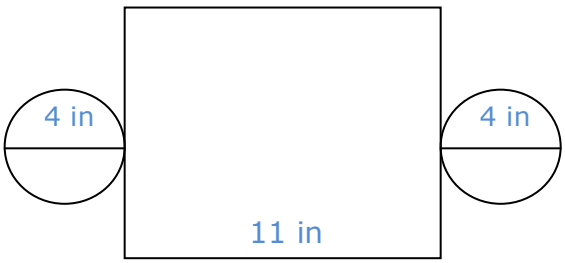
A. 3π in^2
B. 6π in^2
C. 9π in^2

3. Rodney's bicycle tire has a diameter of two feet. Rodney knows that the circumference of a circle is about three times its diameter. About how far will Rodney's bike travel when the tire has made seven revolutions? *Adapted from http://razorbackslv.com*

4. Find the surface area of the rectangular prism.



5. Find the expression to compute surface area of the cylinder.



A. $(\pi*4) + (\pi*4) + 2\pi*4*11$
B. $(\pi*2) + (\pi*2) + 2\pi*2*11$
C. $\pi*(2)^2 + \pi*(2)^2 + 2\pi*2*11$
D. $(\pi*16) + (\pi*16) + 2\pi*2*11$

MODULE

3–3

This module addresses the following indicators:

6-5.4 Apply strategies and procedures to estimate the perimeters and areas of irregular shapes. (C3)

6-5.5 Apply strategies and procedures of combining and subdividing to find the perimeters and areas of irregular shapes. (C3)

This module contains two lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Lesson

- **Continuum of Knowledge**

6-5.4 Apply strategies and procedures to estimate the perimeters and areas of irregular shapes. (C3)

- In fifth grade, students applied formulas to determine the perimeters and areas of triangles, rectangles, and parallelograms (5-5.4).
- Seventh grade students will generate strategies to determine the perimeters and areas of trapezoids (7-5.3).

6-5.5 Apply strategies and procedures of combining and subdividing to find the perimeters and areas of irregular shapes. (C3)

- In fifth grade, students applied formulas to determine the perimeters and areas of triangles, rectangles, and parallelograms (5-5.4).
- Seventh grade students will generate strategies to determine the perimeters and areas of trapezoids (7-5.3).

- **Key Vocabulary**
  *Perimeter
  *Area
  Irregular shapes

  ** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lesson

1. Teaching Lesson A

6-5.4 Apply strategies and procedures to estimate the perimeters and areas of irregular shapes. (C3)

For this indicator, it is **essential** for students to:

- Students need to recall formulas for area of squares, rectangles, triangles, and parallelograms.
- Know to subdivide using familiar shapes
- Know how to calculate perimeter
- Record final answer using correct units
- Estimate perimeter and area using terms like at least, a little more, about etc..

For this indicator, it is **not essential** for students to:

- Irregular shapes should not include trapezoid and circles/semi-circles as a sub-divided piece.
- Calculate area with side lengths measured in rational numbers (fractions, decimals)

a. Indicators with Taxonomy

6-5.4 Apply strategies and procedures to estimate the perimeters and areas of irregular shapes. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson –

Materials:

Copies of model of pond on graph paper
Measuring tools: tape measures, string, rulers, etc.

Pose this problem: "Linda bought a house with a pond in the back yard. She wants to put a small fence up around it to keep small children from accidentally falling into the pond. She also wants to know how much of her yard is taken up with the pond."

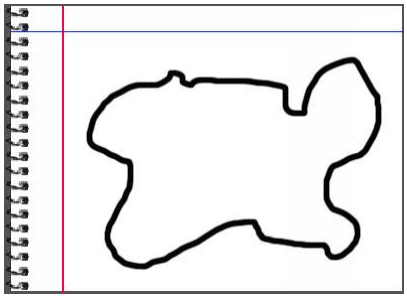
Look at the model of the pond, (this should be a pond with curve sides, not straight lines, drawn on grid paper) and estimate the perimeter and area of the pond. Students should generate strategies to estimate. After they have worked on this a few minutes, ask them what strategies they have tried so far. (For area, they might count all the complete squares on the inside and then estimate portions of the other squares. For perimeter, they might compare the sides to the lines in the squares or they may trace the outside of the pond with string and measure the string with the lines on the grid paper). Have them continue generating strategies to solve, share their answers with other students and defend their answers.

c. Misconceptions/Common Errors –

Students often confuse the correct form of the units on perimeter and area problems. For example, students place square units (cm^2) on perimeter answers and vice versa.

d. Additional Instructional Strategies –

The focus of the indicator is for students to use their knowledge of areas and perimeter of known shapes such as squares, rectangles, triangles, etc… to estimate the area of irregular shapes. These include combinations of polygons puddles, shoeprints, etc… For example, given the following irregular shapes, students would develop strategies and procedures for estimating the areas. Some students may overlay centimeter grid paper on top of the shape and then count squares. Others may draw squares, rectangles, and triangles within their shape and calculate the area of each polygon. Still others may compare their shape with other objects for which they know the exact dimensions and area. The key is for them to come up with the strategies.



Students can use the same idea with a shapes made up of polygons. The focus is on the estimation. Students will use formulas to compute the exact area and perimeter in Indicator 6-5.5.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the <u>Lesson</u>

Assessment is embedded in this lesson through the questioning of the teacher during the student dialogue and student understanding of strategies for determining area and perimeter.

2. Teaching Lesson B "Changing Areas"

6-5.5 Apply strategies and procedures of combining and subdividing to find the perimeters and areas of irregular shapes. (C3)

For this indicator, it is **essential** for students to:

- Recall formulas for area of squares, rectangles, triangles, and parallelograms.
- Know how to subdivide a shape using familiar shapes
- Know how to calculate perimeter
- Record final answer using correct units
- Calculate perimeter using whole and rational numbers.

For this indicator, it is **not essential** for students to:

- Irregular shapes should not include trapezoid and circles/semi-circles as a sub-divided piece.
- Calculate area with side lengths measured in rational numbers (fractions, decimals)

a. Indicators with Taxonomy

6-5.5 Apply strategies and procedures of combining and subdividing to find the perimeters and areas of irregular shapes. (C3)

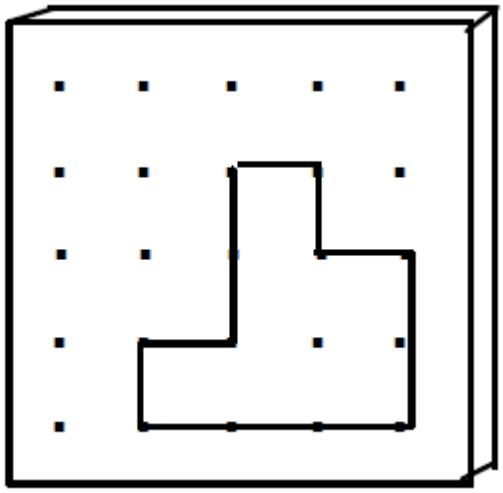
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson

Materials:
Geoboards
Rubber Bands
Geoboard Dot Paper

Lesson

Have students copy the shape below on their geoboard and on geoboard dot paper.



Students work with partners to determine the area and perimeter of the shape. Pairs share with another pair to check their work.

Students change the figure to create another figure that has the same area and a larger perimeters, recording it on dot paper with its area and perimeter.

Students change the figure to make another figure that has the same area and a larger perimeter, recording it on dot paper with its area and perimeter.

Students change the figure to make another figure that has the same area and a smaller perimeter, recording it on dot paper with its area and perimeter.

Students make three more figures that have different perimeters but the same area, recording them on dot paper. Discuss the results as a class.

c. Misconceptions/Common Errors –

- Students often have difficulty applying area measurement formulas and that formulas for area and perimeter are often confused. Part of the problem may be the premature jump to formulas to determine these measurements and the fact that the formulas for both area and perimeter refer to linear measurements. This hides what is being measured in the area situation. Another cause of this difficulty may be students' belief that the formula is the definition of the measurement. Students need experience with problems that require an understanding of concepts and how formulas work.
- Students often confuse the correct form of the units on perimeter and area problems. For example, students place square units (cm^2) on perimeter answers and vice versa.
-

- Manipulatives may be useful as students try to visualize how the shapes are subdivided and combined.

d. Additional Instructional Strategies –

The focus of this indicator is to find the exact area (not estimated) of irregular shapes. Students should explore real world examples as well such as pools, gardens, fences, etc…. These examples do not include puddle, shoeprints, etc..

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

No suggestions for use of technology or websites are included at this time.

f. Assessing the Lesson

Assess student understanding of this lesson by observing the area and perimeter determined for the shapes created on the geoboards.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

6-5.4 Apply strategies and procedures to estimate the perimeters and areas of irregular shapes. (C3)

The objective of this indicator is apply which is in the "apply procedural" cell of the Revised Taxonomy. Although the focus of the indicator is to apply, the learning progression should include opportunities for students to generate strategies for estimating the area and then apply it other shapes. The learning progression to **apply** requires students to recall and understand formulas for the areas and perimeter of squares, rectangles, triangles, circles, etc.. Given an irregular shape, students generate ideas related to how they could estimate the area using their prior knowledge. They explore these strategies using a variety of examples. They explain and justify their strategy using correct and clearly written or spoken words (6-1.6). Students

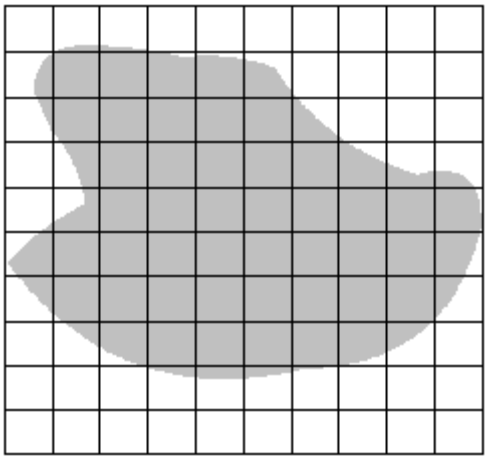
should generalize mathematical statements (6-1.5) summarizing strategies used to estimate the area and perimeter of irregular shapes.

6-5.5 Apply strategies and procedures of combining and subdividing to find the perimeters and areas of irregular shapes. (C3)

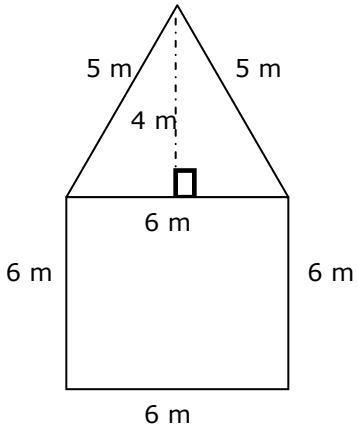
The objective of this indicator is apply which is in the "apply procedural" cell of the Revised Taxonomy. The focus of the indicator is to apply; therefore, students should gain computational fluency with finding perimeter and area of irregular shapes. The learning progression to **apply** requires students to recall and understand formulas for the areas and perimeter of squares, rectangles, triangles, circles, etc.. Given an irregular shape, students explore how the shapes can be divided or combined using manipulatives, where appropriate. They develop strategies for computing the area and perimeter. They explain and justify their strategy and their answers using correct and clearly written or spoken words (6-1.6). Students engage in repeated practice to support retention and understanding of their strategy.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Estimate the perimeter and area of the figure. Explain your estimates.



2. Find the perimeter and area of each shape.



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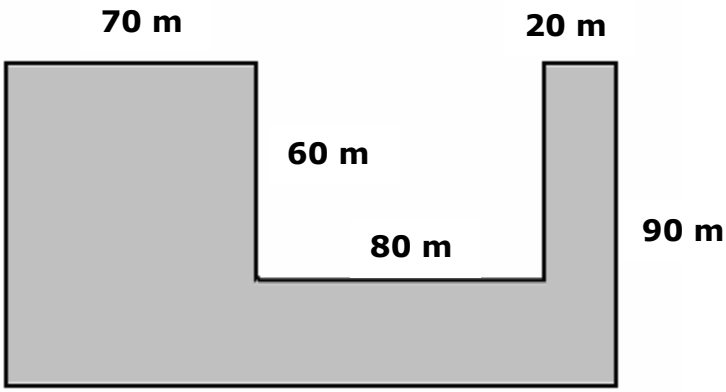

P = _____

A = _____