SOUTH CAROLINA SUPPORT SYSTEM INSTRUCTIONAL GUIDE

Content Area	**Seventh Grade Math**

First Nine Weeks

Standards/Indicators Addressed:

Standard: 7-2: The student will demonstrate through the mathematical processes an understanding of the representation of rational numbers, percentages, and square roots of perfect squares; the application of ratios, rates, and proportions to solve problems; accurate, efficient, and generalizable methods for operations with integers; the multiplication and division of fractions and decimals; and the inverse relationship between squaring and finding the square roots of perfect squares.

7-2.1 Understand fractional percentages and percentages greater than one hundred

7-2.2 Represent the location of rational numbers and square roots of perfect squares on a number line.

7-2.3 Compare rational numbers, percentages, and square roots of perfect squares by using the symbols ≤, ≥, <, >, and =.

7-2.4 Understand the meaning of absolute value.

7-2.6 Translate between standard form and exponential form.

7-2.7 Translate between standard form and scientific notation

7-2.9 Apply an algorithm to multiply and divide fractions and decimals.

7-2.10 Understand the inverse relationship between squaring and finding the square roots of perfect squares.

*** These indicators are covered in the following 3 Modules for this Nine Weeks Period.**

Module 1-1 Rational Numbers

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines

Module 1-1 Lesson A: **7-2.1** Understand fractional percentages and percentages greater than one hundred.	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 1-1 Introductory Lesson A See Module 1-1, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 1-1 Lesson A Assessment
Module 1-1 Lesson B: **7-2.10** Understand the inverse relationship between squaring and finding the square roots of perfect squares.		See Instructional Planning Guide Module 1-1, Introductory Lesson B See Instructional Planning Guide Module 1-1, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 1-1 Lesson B Assessment
Module 1-1 Lesson C **7-2.2** Represent the location of rational numbers and square roots of perfect squares on a number line.		See Instructional Planning Guide Module 1-1 Introductory Lesson C See Instructional Planning Guide Module 1-1, Lesson C Additional Instructional Strategies	See Instructional Planning Guide Module 1-1 Lesson C Assessment
Module 1-1 Lesson D		See Instructional Planning Guide	See Instructional

7-2.3 Compare rational numbers, percentages, and square roots of perfect squares by using the symbols ≤, ≥, <, >, **and =.**		Module 1-1, Introductory Lesson D See Instructional Planning Guide Module 1-1, Lesson D Additional Instructional Strategies	Planning Guide Module 1-1 Lesson D Assessment

Module 1-1 Continued

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines
Module 1-1 Lesson E: **7-2.4** Understand the meaning of absolute value.	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See	See Instructional Planning Guide Module 1-1 Introductory Lesson E See Instructional Planning Guide Module 1-1, Lesson E Additional Instructional Strategies	See Instructional Planning Guide Module 1-1 Lesson E Assessment

	Appendix A		

Module 1-2 Number Structure

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines

Module 1-2 Lesson A: **7-2.6** Translate between standard form and exponential form.	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 1-2 Introductory Lesson A See Instructional Planning Guide Module 1-2, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 1-2 Lesson A Assessment
Module 1-2 Lesson B: **7-2.7** Translate between standard form and scientific notation		See Instructional Planning Guide Module 1-2, Introductory Lesson B See Instructional Planning Guide Module 1-2, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 1-2 Lesson B Assessment
Module 1-3 Operations on Fractions/Decimals			
Module 1-3 Lesson A: **7-2.9** Apply an algorithm to multiply and divide fractions and decimals.	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics	See Instructional Planning Guide Module 1-3 Introductory Lesson A See Module 1-3, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 1-3 Lesson A Assessment

Module 1-3 Lesson B: **7-2.9** Apply an algorithm to multiply and divide fractions and decimals	Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 1-3, Introductory Lesson B See Instructional Planning Guide Module 1-3, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 1-3 Lesson B Assessment

Module 1-3 Continued

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines
Module 1-3 Lesson C: **7-2.9** Apply an algorithm to multiply and divide fractions and decimals	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See	See Instructional Planning Guide Module 1-3 Introductory Lesson C See Instructional Planning Guide Module 1-3, Lesson C Additional Instructional Strategies	See Instructional Planning Guide Module 1-3 Lesson C Assessment
Module 1-3 Lesson D: **7-2.9** Apply an algorithm		See Instructional Planning Guide Module 1-3,	See Instructional Planning Guide

to multiply and divide fractions and decimals	Appendix A	Introductory Lesson D See Instructional Planning Guide Module 1-3, Lesson D Additional Instructional Strategies	Module 1-3 Lesson D Assessment

MODULE

1-1

Rational Numbers

This module addresses the following indicators:

7-2.1 Understand fractional percentages and percentages greater than one hundred. (B2)

7-2.10 Understand the inverse relationship between squaring and finding the square roots of perfect squares. (B2)

7-2.2 Represent the location of rational numbers and square roots of perfect squares on a number line. (B2)

7-2.3 Compare rational numbers, percentages, and square roots of perfect squares by using the symbols ≤, ≥, <, >, and =. (B2)

7-2.4 Understand the meaning of absolute value. (B2)

This module contains 5 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- ***Continuum of Knowledge***

7-2.1 Understand fractional percentages and percentages greater than one hundred. (B2)

In the third grade, students worked with fractions that were less than or greater than one (3-2.5 and 3-2.6).

In sixth grade, students are first introduced to the concept of percents (6-2.1).

In seventh grade, students' knowledge is extended to percentages greater than one hundred (7-2.1).

7-2.10 Understand the inverse relationship between squaring and finding the square roots of perfect squares. (B2)

In the third grade students had their first experiences with perfect squares as they learned basic multiplication facts such as 4 x 4 and 7 x 7 (3-2.7). Students developed understanding of the inverse relationship between addition and subtraction in first grade (1-2.7) and multiplication and division in third grade (3-2.8).

In seventh grade, students develop an understanding of the inverse relationship between squaring and finding square roots of perfect squares (7-2.10).

In the eighth grade, students will begin applying strategies and procedures to approximate the square roots or cube roots of numbers less than 1,000 between two whole numbers (8-2.6). Cube roots will be a new topic in eighth grade but requires an understanding of square roots.

7-2.2 Represent the location of rational numbers and square roots of perfect squares on a number line. (B2)

In the third grade students were first exposed to perfect square numbers while learning their basic multiplication facts such as 3 x 3, 6 x 6, etc. (3-2.7). In fifth grade, students compared whole numbers, decimals, and fractions (5-2.4). In sixth grade, students compared rational numbers and whole number percentages through (6-2.3).

In seventh grade, students represent the location of rational numbers and square roots of perfect squares on a number line (7-2.2).

In the eighth grade, students compare rational and irrational numbers (8-2.4).

7-2.3 Compare rational numbers, percentages, and square roots of perfect squares by using the symbols ≤, ≥, <, >, and =. (B2)

Since kindergarten, students have used the terms *more than, less than,* and *the same as* to describe sets of objects (K-2.3). In first grade, students are introduced to the terms *is greater than, is less than,* and *is equal to* compare whole number quantities through 100 (1-2.5). In second grade, order symbols <, >, and = are used along with appropriate terms to compare whole numbers quantities through 999 (2-2.4), in third grade through 999,999 (3-2.1). In fourth grade, students compare fractions to benchmark fractions (4-2.9) and compare decimals through hundredths using terms *less than, greater than,* and *equal to* along with order symbols (4-2.7). In fifth grade students compare whole numbers, decimals, and fractions using order symbols (5-2.4). In the sixth grade students are first introduced to the concept of percents and learned to compare whole number percents through 100. They also learn the new symbols ≤ and ≥ (6-2.3).

In seventh grade, students compare rational numbers, percentages, and square roots of perfect squares by using the symbols <, >, ≤, ≥, and = (7-2.3). They also represent the location of rational numbers and square roots of perfect squares on a number line (7-2.2).

7-2.4 Understand the meaning of absolute value. (B2)

In seventh grade, students understand the meaning of absolute value. The concept of absolute value is introduced for the first time in eighth grade.

In eighth grade, students apply the concept of absolute value (8-2.5).

- **Key Concepts/Terms**

*These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and in use in conversation with students.

∗ Percent meaning *per hundred*
∗ % as a symbol
∗ base
∗ exponent
∗ power
∗ √ (square root)

* x^2 (a number squared)
* perfect square
* inverse relationship
* <, >, $\leq$, $\geq$, =
* rational number
* absolute value
* absolute value symbol | |

II. Teaching the Lessons

1. Teaching Lesson A – Fractional Percentages and Percents Greater than 100%

In fifth grade students compared whole numbers, decimals, and fractions. In sixth grade the comparisons were continued and whole number percents were included.

Seventh grade students should extend this knowledge to percentages less than one and percentages greater than one hundred. Using concrete models with enable the students to connect the new learning to prior knowledge. Since students worked with fractions that are less than or greater than one in third grade, they should now be given opportunities to line that to fractional percents and percents greater than 100%. Students should be given opportunities to develop models using materials such as base-ten blocks, 10 x 10 grid paper, or graph paper to help students visualize the connection of fractions to percents less than one percent and mixed number to percents greater than 100%.

a. Indicators with Taxonomy

7-2.1 Understand fractional percentages and percentages greater than one hundred. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

For this indicator, it is **essential** for students to:

- Recall that a fraction is part of a whole
- Recall and understand how to represent fractions and percentages less than 100
- Understand that with percent the whole is represented by 100%, no matter how many parts it includes.

- Represent percentages using concrete and/or pictorial models
- Dividing one percent (1%) is the same as a fractional percent (a part of a part).
- Mixed numbers and improper fractions will always be a percent greater than 100.

For this indicator, it is **not essential** for students to:

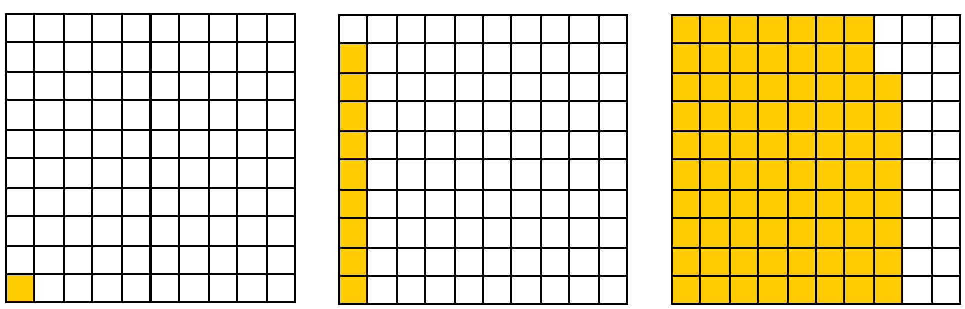
None noted

b. Introductory Lesson A – Fractional Percentages and Percents Greater than 100%

Materials:

cm grid paper (if it's laminated, students can use fine tip dry erase markers and old socks to work)

- Ask students to give several examples that show how percents are used in everyday life. Ask students for a definition of "percent". Remind students that the word percent means "per 100" or "out of 100". Model how to mark off a 10 x 10 grid on their cm grid paper. In 6th grade students had experiences in shading 10 × 10 grids to represent given percents.
- Model percents using 10 x 10 grids to help students understand the concept of "percent" meaning "for each 100." Assess the students understanding of whole number percents by asking them to shade the percents shown below on the grid worksheet.



1% 9% 78%

- Renaming percents as fractions

  Ask students: **"How might you write a percent as a fraction?"** If guidance is needed, follow up with: **"If 'percent' means 'out of 100 parts' how might you write 10% as a fraction that represents 10 out of 100 parts?"** This is a good opportunity to review the functions of the numerator and denominator. Be sure to ask students to simplify their fractions.

- Renaming fractions as percents

  Now ask students how they might rename a fraction as a percent. Remind them that before they simplified the fractions in Step 3, all the denominators were 100. Ask: **How might you use what you did to rename percents as fractions help you rename a fraction as percent?"** (Because 7-2.9 has not been introduced all denominators must be factors of 100, unless they are benchmark fractions.)

- Renaming percents as decimals

  Ask: **What does 78% mean?** [It means 78 out of 100 parts.] **So, how can it be renamed as a fraction?** [78/100] **What operation does the fraction bar signify?** [division] **So 78/100 is the same as 78 ÷ 100?** [yes]

  Have students use calculators to do the division for several examples. They should keep track of their work in their notebooks. You want them to see that every time they divide by 100, the decimal in the quotient is 2 places to the left of where it was in the dividend. They probably need to be reminded that all whole numbers can be written with a decimal at the end. If they do the math and see the pattern themselves, they are much more likely to remember that the shortcut for renaming a percent as a decimal is as simple as moving the decimal point 2 places to the left.

- Renaming decimals as percents

  Without using the "move the decimal 2 places to the right" rule, students have to go from decimal to fraction to percent. But if they do examples in this fashion (decimal → fraction of 100 → percent), they'll discover the rule rather than you telling it to them.

  Start with decimals that are hundredths. Ask: **"How can you rename .45 as a fraction?"** [45/100] **"Now, how can you rename that fraction as a percent?"** [45%] **"How do you know it's 45%?"** [The % symbol means out of 100, and my denominator is 100.] Have the students work on several examples and keep track of the work in their notebooks. When they are finished, ask: **"What do you notice about the placement of the decimal point in the decimal name as compared to the placement of the decimal point in the percent name?"** [The decimal point moves 2 places to the right.]

Do some examples using tenths and challenge the students to find out if the rule holds. They'll need to think about **.**2 being the same as **.**20.

Another way to think about this: **"If you divide by 100 to rename a percent as a decimal, what operation would make sense for renaming a decimal as a percent?"** [Multiply by 100, because it's the inverse of dividing by 100.]

- Up to this point, students should be familiar with the percents themselves because of the work they did in 6th grade with whole number percents. They now need to begin thinking about fractional percents and percents greater than 100%. Before thinking about different forms of either, they need to have a concept of the meaning of fractional percents and percents greater than 100%.

  So let's go back to the 10 x 10 grid. Present the situation to the students and pose the questions. Let them work in pairs so that they can think together and communicate with each other. Keep a record of the dialogue on chart paper so that the thinking of the class can be preserved and displayed over time.

  If the grid represents a seating capacity of 200 people in an auditorium…

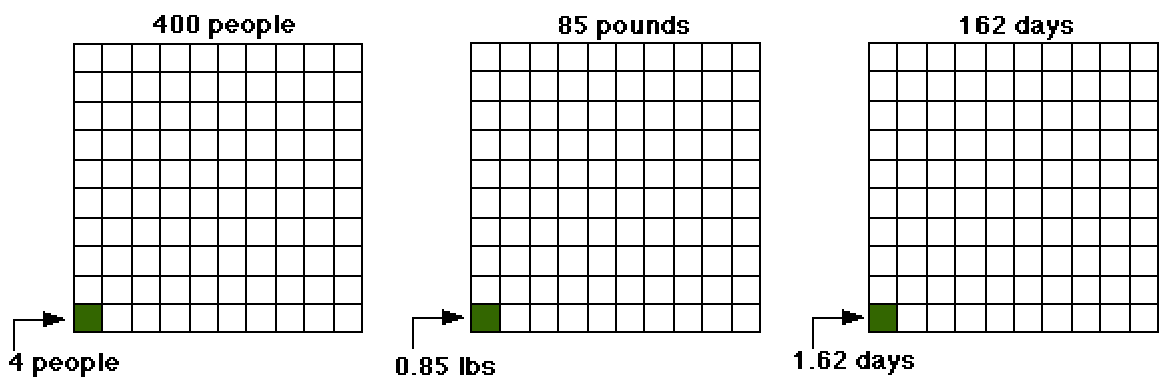
  - What does one small square in the grid represent? How do you know?
  - What do 10 small squares represent? How do you know?
  - What about 5½ squares?
  - What if you wanted to represent 400 people using the same model? How would you do that? How do you know?
  - What percent are 400 people? How do you know?
  - What about 300 people? How do you know?
  - What percent are 300 people? How do you know?
  - Have the students pose some questions of their own. Share with the class and work through them.

If the grid represents my cat's weight of 25 pounds…

- What does one small square in the grid represent? How do you know?
- What do 10 small squares represent? How do you know?
- What about 5½ squares?
- If he gained 10% of his body weight, how much would he weigh?
- If the vet says my cat needs to lose 4% of his body weight to be healthy, how much weight does he need to lose?

Now, they work and you circulate. Pay attention to the conversations students are having. Ask questions to probe for understanding. You'll need to split the material up into manageable chunks for your students. *This is the formative assessment for this lesson.*

Let the unit square (10 × 10 grid) represent given amounts, and then determine the value of one of the small squares (1 percent). To help students determine the value of 1 percent of the unit square, they can think of sharing the given amount equally among the 100 parts of the unit square. Dividing by 100 can be done by mental computation. As examples, if the unit square represents 400 people (that is, if 400 people represents 100% in some situation), then each small square represents 4 people. Similarly, if the unit square represents 85 pounds, then each small square has a value of 0.85 pounds; and, if the unit square represents 162 days, then the value of each small square is 1.62 days. Each of these can easily be related to *mental* division by 100. *(Relate division of money - $1 divided into 10 parts would be 10 cents or 0.10 which moves the decimal 1 place. $1 divided into 100 parts would be 1 cent or 0.01 which moves the decimal 2 places. Several examples of this type should help.)*



Successfully determining the value of one square (1%) is key to solving percent problems. To prepare students for this task, it may be helpful to pose questions that involve convenient fractional parts of the grid. For example, if the unit square represents 400 people:

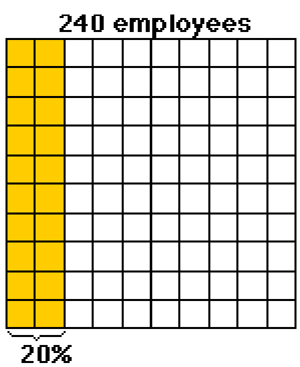
- How many people would be represented by half of the unit square?
  … by one-fourth of the unit square?
  … by twenty small squares?
  … by ten small squares?
- What part of the unit square would represent 200 people?
  …100 people?
  …40 people?
  …4 people?

The following percent problems can be solved using the unit square grids. The first three problems use a two-part approach: first, students are asked to use a percent grid to represent the given information; then, students use the sketch to answer a question related to that information. This approach emphasizes the importance of thinking through the given information before attempting to obtain an answer.

Problems 1-3 are typical percent problems. Problem 4 involves percents greater than 100. Problems 5-6 deal with percent increase, a situation that often causes anxiety in students. Finally, problem 7 uses the grid method to solve a problem involving a percent discount.

1. Twenty percent of a company's 240 employees are classified as minorities.
   a. Use a percent grid to show that 20 percent of a business's 240 employees are classified as minorities.
   b. How many employees are classified as minorities?

[The given information can be represented by letting the unit square represent 240 employees and shading 20 percent of the square.]



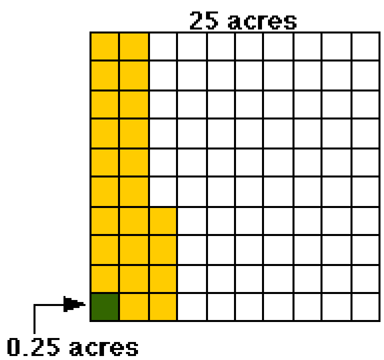
Students may notice immediately from the shaded grid that one shaded column represents 1/10 of 240, or 24, so two shaded columns—which is 2/10, or 20%—represent 48.

[*Only if 7.2.9 has been introduced*…Alternatively, if the whole square represents 240 employees, then one small square represents 2.4 employees, so 20 squares represent 20 × 2.4 = 48. Therefore, 48 employees are classified as minorities.]

2. Twenty-five acres of land are donated to a community, but the donor stipulates that six acres of this land should be developed as a playground.
   a. Use a percent grid to represent this situation.

b. What percent of the land is to be used for playground?

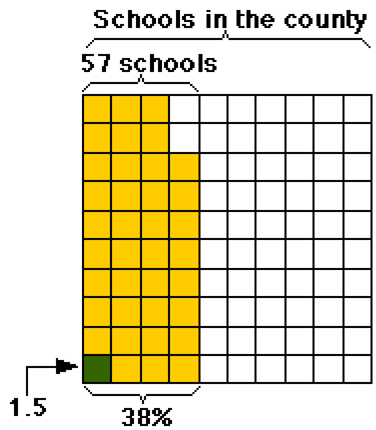
[If the unit square represents 25 acres, we need to determine how many small squares represent 6 acres. Each small square represents 0.25 = ¼ acres, so 4 small squares represent 1 acre, and 24 small squares represent 6 acres. Students have not been exposed to division of decimals except with models so they will not understand since each small square has a value of 0.25 acres, then 6 ÷ 0.25 = 24 squares are needed to represent 6 acres.



The playground is represented by 24 shaded squares, so 24 percent of the land is to be used for the playground.]

3. In Hazzard County, 57 of the schools have a teacher-to-student ratio that meets or exceeds the requirements for accreditation. These 57 schools represent 38 percent of the schools in the county.
   a. Use a unit square sketch to represent this situation.
   b. How many schools are in Hazzard County?

[The unit square represents the number of schools in the county, and 38 small squares (38 percent) represent 57 schools.



Because 38 small shaded squares represent 57 schools, each small shaded square represents 57 ÷ 38 = 1.5 schools. So the unit square represents 100 × 1.5 = 150, which is the number of schools in the county.]

4. The school population in Provincetown is 135 percent of the school's population from the preceding year. The school population in Provincetown is 135 percent of the school's population from the preceding year. The new student population is 270. How many students did the school have the previous year?

[The current population can be represented by 135 small squares, so each small square represents 270 ÷ 135 = 2 students.

The school population for the preceding year is the value of one unit square, so the number of students the previous year was 100 × 2 = 200.]

c. *Student Misconceptions/Errors*

- It is difficult for students to understand how a percent can be greater than 100. Real world examples such in 2006, 4000 people answered yes to the survey question. In 2009, 10,000 people answered yes. Explain that 100% increase would be 8000 people but it increased over 100% because there's more than 8000 people.

- Students who memorize rules for moving decimals may incorrectly move the decimal the wrong direction, or simply leave the decimal as it is and add a percent sign, or not understand decimal placement at all.

d. *Additional Instructional Strategies –*

While additional learning opportunities are needed, no suggestions are included at this time.

e. *Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

http://nlvm.usu.edu

f. *Assessing the Lesson*

NOTE: This is here because students need to be able to rename the different forms in order to complete this module successfully.

Complete any three rows from the table above and explain how you know your renaming of the different forms is correct.

	Percent	**Decimal**	**Fraction**
1)			$\frac{2}{5}$
2)		0.75	
3)	50%		
4)		0.6	
5)			$\frac{1}{3}$
6)		0.05	

	Percent	**Decimal**	**Fraction**
7)			$2\frac{9}{10}$
8)		0.03	
9)	19%		
10)		0.275	
11)	588%		
12)		0.003	

2 . Teaching Lesson B – Squares and Square Roots

In third grade, students had their first experiences with perfect squares as they learned basic multiplication facts such as 4 x 4, 7 x 7, etc. In fifth grade, students were exposed to the concept of squares when they determined the area of geometric squares. Students have been exposed to inverse relationships for addition and subtraction in first grade and multiplication and division in third grade. These relationships can be used to help explain these inverse relationships.

When the terms *squares* and *square roots* are introduced, it is essential that the connection is made between the squared number and the corresponding geometric square. In other words, students should understand that "find the length of a side of a square with area equal to 25 units" and "find the square root of 25" are basically the same question. Students have been exposed to inverse relationships for addition and subtraction in first grade and multiplication and division in third grade. In seventh grade, the concept of inverse relationships is expanded to include squaring and finding square roots of perfect squares. Learning opportunities should include both models and numbers.

For this indicator, it is **essential** for students to:

- Understand the concept of squaring
- Understand the concept of square root
- Use concrete/pictorial models to connect the concept of a squared number and a corresponding geometric square. For example, finding the length of a side of a square with area equal to 25 units and finding the square root of 25 are basically the same question.

For this indicator, it is **not essential** for students to:

- Know whether or not the square root can be positive or negative. For example, the square of 25 can be -5 or +5.

a. Indicators with Taxonomy

7-2.10 Understand the inverse relationship between squaring and finding the square roots of perfect squares.

Cognitive Process Dimension: Understand

Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson –Squares and Square Roots
****This will likely take two class periods.***

Materials Needed:

- cm grid paper (2 sheets for each student)
- color tiles (1 bag of 100 tiles for each pair of students)
- If you are short color tiles, put the students in quads and have 1 bag of 100 tiles for each group of 4.
- inch grid paper (for specific groups to use for the class chart)
- chart paper or something like it to use for class charts
- tape or glue sticks (1 roll or stick for each pair of students)
- scissors (1 pair for each pair of students)

Students should be paired/grouped so that they can talk about the math they are doing.

Give each pair/quad of students a bag of color tiles. Ask them to use their color tiles to build squares with areas equal to the numbers 1 – 100. When they find those squares, they should record them on grid paper by drawing and labeling them. NOTE: If your students use math notebooks, they can cut out and paste/tape those squares into their notebooks, leaving space to write. Otherwise, they should cut out and paste/tape the squares onto a clean sheet of paper to keep with their math work, using whatever method you have in place in your classroom.

If students are struggling with finding the squares, give them a hint: There are 10 squares between 1 and 100.

As groups finish, give them a piece of inch grid paper to use to draw and cut out one square. You need only choose one group per square.

When students are finished finding and recording their squares, create a class chart with the students' help, displaying each of the squares pictorially. When each of the squares is on the chart, begin with the square that has an area of 4 $units^2$. Have students find that square in their own work. Then ask: **"What number sentence can you write to describe this square?"** [2 x 2 = 4] Have students write the number sentence that describes each of their squares. Come back together as a class and label the rest of the squares on the class chart.

Ask: **"What things do you notice about the figures and the number sentences that describe them?"** [The figures are squares; all the sides are equal;

the number sentences have the same factor 2 times.] Be sure to record both the question and the answers students give on a class chart.

[Good stopping place if you have to split this lesson into two parts.]

Put the following "fact families" on chart paper.

A) 6 + 5 = 11 5 + □ = 11 □ − 6 = 5 11 − □ = 6	B) 3 x 7 = 21 7 x □ = 21 □ ÷ 7 = 3 21 ÷ □ = 7

Ask students to work with their partners to fill in the missing numbers. When they're finished, bring the class back together to discuss the following questions:

How did you know what number was missing in each number sentence for group A?

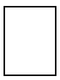
How did you know what number was missing in each number sentence for group B?

BIG IDEAS: Through this discussion, you want students to remember what they already know about using inverse operations. They also need to review some vocabulary that will be helpful in discussing square numbers and square roots: factor, product, dividend, quotient, and divisor.

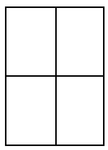
Now, revisit the class chart with the squares and number sentences as well as the second chart with the student responses to the question: **"What things do you notice about the figures and the number sentences that describe them?"** [All of the figures are squares and the number sentences have the same factor two times. The products are the areas of the squares.]

Begin with the square that's 4 units^2. Ask: **"What's the inverse of multiplication?"** [division] **"So how do you write a number sentence that's the inverse of the one we already have?"** [4 ÷ 2 = 2] Have the students help you do the same thing with each of the other squares. They should be recording the same information in their notebooks/notes.

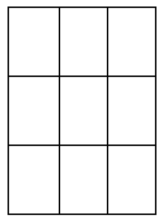
Examples of what should be included in the class chart follow.



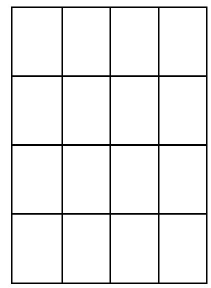
1 unit^2
1 x 1 = 1
1 ÷ 1 = 1



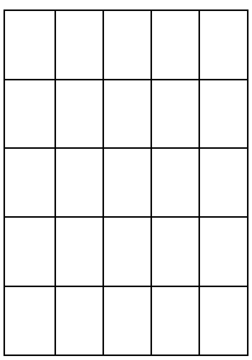
4 units^2
2 x 2 = 4
4 ÷ 2 = 2



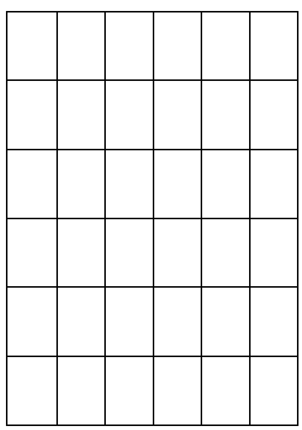
9 units^2
3 x 3 = 9
9 ÷ 3 = 3



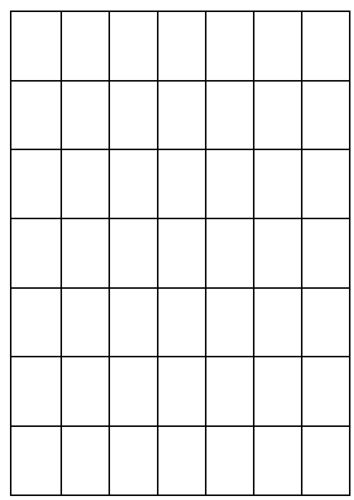
16 units^2
4 x 4 = 16
16 ÷ 4 = 4



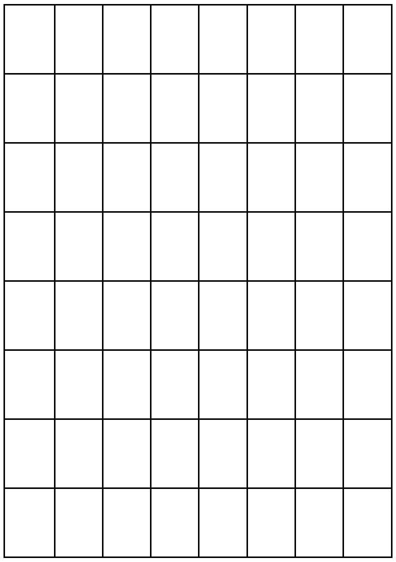
25 units^2
5 x 5 = 25
25 ÷ 5 = 5



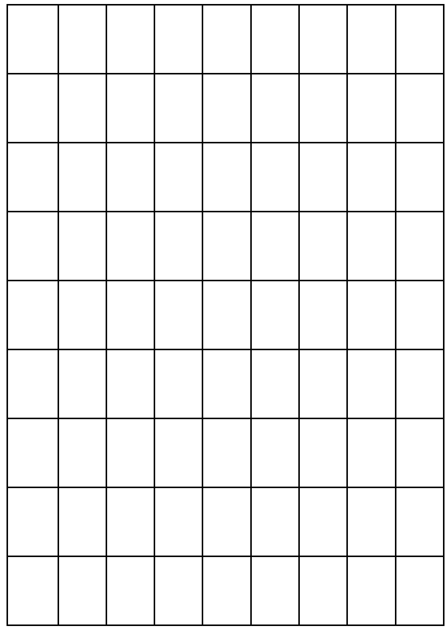
36 units^2
6 x 6 = 36
36 ÷ 6 = 6



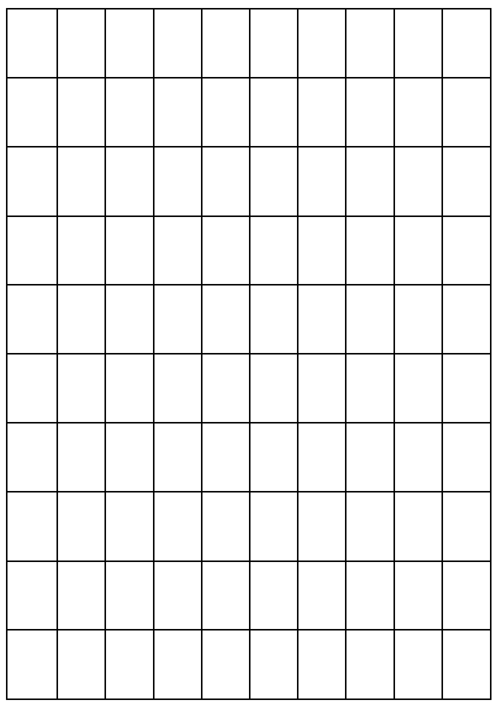
49 units^2
7 x 7 = 49
49 ÷ 7 = 7



64 $units^2$
8 x 8 = 64
64 ÷ 8 = 8



81 $units^2$
9 x 9 = 81
81 ÷ 9 = 9

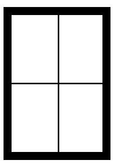


100 $units^2$
10 x 10 = 100
100 ÷ 10 = 10

Hi-lite or circle the products. Tell students that these are special numbers called "perfect squares." Ask: **"Why do you think these are called perfect squares?"** [Because the areas of the squares match the numbers.] **"What do you notice about the factors?"** [In each sentence, it's a number times itself that makes the product/area of the square.] Tell the students they're going to make a new kind of fact family, and that they need some new symbols to do it. Students used exponents in 6th grade. Thus the new concept here is that squaring a number has an inverse operation.

Review the exponent 2 (square) and introduce the square root symbol - $\sqrt{}$.

Start with the square that's 4 $units^2$.

2

2

Two units on each edge of the square →

2 squared is 4 <AND> the square root of 4 is 2

This is a really BIG mathematical idea and connecting the concrete and pictorial models to the mathematical symbols will help students distinguish between square root and dividing by 2.

$2^2 = 4$

So, you use the exponent "2" to show that the factor is used two times.
You read this as "2 squared" or "2 to the second power."

2 squared is equal to 4

$\sqrt{4} = 2$

So, you use the root symbol –" $\sqrt{}$ " to show that you want to think about which factor is used two times to give you the product 4.
You read this as "the square root of 4."

The square root of 4 is equal to 2

The New Fact Family!!

$2^2 = 4$

$\sqrt{4} = 2$

Have the students write the fact family for each of the other squares. Be sure to emphasize the following:

- the inverse relationship between x^2 and $\sqrt{x}$
- the connection between the squared number and the geometric square
- that perfect squares have whole number square roots
- "square root" DOES NOT mean "divide by 2" (have students examine their original multiplication sentences)

c. *Misconceptions/Common Errors –*

- Students may have the misconception that square root means to divide by 2.

d. *Additional Instructional Strategies –*

While additional learning opportunities are needed, no suggestions are included at this time.

e. *Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

http://www.math.com/school/subject1/lessons/S1U1L9GL.html

http://www.mathsisfun.com/square-root.html

f. *Assessing the Lesson*

Formative assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

1. **Exit Slip:** Explain why this expression is true.

$$4^2 = \sqrt{16}$$

2. Sketch a model that represents 11^2

3. The length of a square can be represented by $\sqrt{36}$. What is another way to represent the length of this square?

4. Explain the relationship between squaring a number and finding the area of a square. Draw a model if you need to.

3. Teaching Lesson C – Rational numbers and square roots on the number line

In seventh grade the expectation is to locate rational numbers and square roots of perfect squares on a number line. Students should explore the location of fractions, decimals, percents, and square roots of perfect squares on a number line. It is equally important that students justify the placement of these representations on a number line, as well as understand the relationship to the numbers between which a given value lies. Being able to justify the placement on a number line will enable students to compare and order rational numbers, percentages, and square roots of perfect squares using the symbols ≤, ≥, <, >, and =.

For this indicator, it is **essential** for students to:

- Understand the meaning of rational numbers
- Find the square root
- Understand the difference between ≤ and ≥
- Translate numbers to same form, where appropriate before comparing numbers
- Translate between the fraction and percents
- Recall the benchmark fractions and common fraction – decimal equivalents.

For this indicator, it is **not essential** for students to:

- None noted

Suggested Literature

A Grain of Rice, Helean Clare Pitman, Bantam Skylark, 1986

The King's Chessboard, David Burch, Dial Books, 1988

a. Indicators with Taxonomy

7-2.2 Represent the location of rational numbers and square roots of perfect squares on a number line.

Cognitive Process Dimension: Understand

Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson – Rational numbers and square roots on the number line

Materials:

- large number line at front of room **(keep this for Lesson D – 7-2.3 Comparing rational numbers)**
- rational numbers handout (print the handout on mailing labels and put the labels on index cards)
- transparency of rational numbers handout
- chart paper or something like it to make a class chart

There are 32 rational numbers on the handout. Divide your class into small groups (between 2 and 4 students in each group). Give each group an equal number of cards.

Tell the class that there are 32 cards distributed throughout the room. Display the transparency of the numbers. Tell them that their job is to put the numbers in the correct order on the class number line. Ask them to brainstorm ways that this might be done. Record their responses on a chart. One response that you

definitely want to hear is: *"Rename all the numbers so that we're working with a common form."* If they don't volunteer this, try to get it from them by asking questions like: **"What's the easiest way to compare a fraction and a percent? Or a decimal and a fraction? What should you do with the numbers that are squared or have a square root symbol?"** Once you've helped them establish that they may need to do some renaming, tell them they should write the other forms of their numbers on the blank space on their cards. Have the small groups order their own cards and, when they're comfortable with their order, go to the class number line and tape their cards up. Let them know they have the freedom to check back and make changes as more cards are placed. Move around the room, listening to their thinking, asking questions, and helping clear up problems.
When all groups have placed their cards, take one section of the number line at a time and have the students help you decide whether the cards in that section are in the right order. Students should put the finished number line in their notebooks.

As part of a debrief for this lesson, pose the following question: **"Can all of these numbers be written as a fraction?"** Have students justify their answers. It may be necessary to go through the transparency and prove to them that the answer is yes. This provides the opportunity to introduce the definition of rational number, a number that can be written as a ratio of two integers – more commonly known as a fraction.

c. Misconceptions/Common Errors –

- Confusing the positive and negative "sides" of zero.
- Students may not understand that rational numbers include integers
- Some students may not realize repeating decimals are rational numbers.
- Students can also misread the order symbols and not include "or equal to" for $\leq$ and $\geq$.
- Misunderstanding of comparisons of negative rational numbers.
- Not converting improper fractions or finding square roots before comparing.

Stress to students that often it is easier to change each of the given numbers to a common form to order then place on the number line in its original form.

d. Additional Instructional Strategies –

- Play a game of "Who Am I?" Students have a rational number written on a post-it note placed on their back upon entering the classroom. Students walk around asking yes/no questions to determine what their number is. Questions can include, “Am I greater than or equal to__?” Students will order themselves from left to right, least to greatest in the front of the room, according to the responses they get to their questions. This can also be the starting point for using the symbols **≤, ≥, <, >,** and **=** to compare numbers**. (Lesson D)**

- Provide true/false examples and give students the opportunity to justify their responses with evidence.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

http://www.lessonplanspage.com/MathCILAIdentifyingAndOrderingRationalAndIrrationalNumbers8.htm

f. Assessing the Lesson

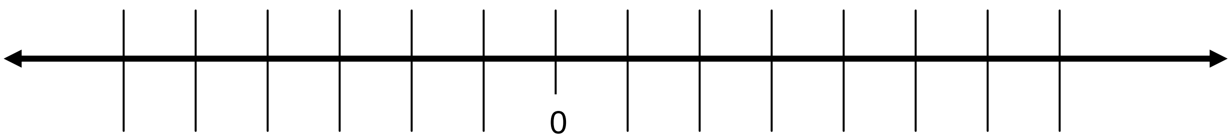
Formative assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

- Ask students to choose any section of the number line containing at least 5 numbers and explain how they know the numbers are in the right order. Have them come up with 2 more numbers on their own and place them in the correct places relative to the section of the number line with which they are working. Students turn in their work for the teacher to read and provide feedback.

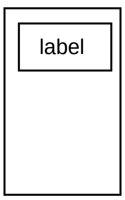
- Give each student a 8 ½ by 11 sheet of paper or a large index card. Have them put the rational number of their choice on their card. Then have the class stand in a circle. Pass around a card that has a > on one side and an = on the back. (Students flip the > symbol card when needed to become a < symbol.) When a student gets the symbol card, they must manipulate it to make it the symbol that will make the statement true, comparing the 2 rational numbers of the people on either side of them. Everyone should have their cards facing in to the circle for the class to see. The class will give thumbs-up or a thumbs-down for agree or disagree. Pass the symbol card around for everyone to get a turn. (Can also be used with 7-2.3)

- Place each number in the correct position on the number line.

a) $2\frac{1}{4}$ b) $\sqrt{36}$ c) - 3.5 d) 100% e) - 250%



Rational Numbers Handout (for use with the number line in the lesson)



Print this handout on mailing labels. Turn the index cards so that they're "tall" and put the label on the top end of the card. This will leave room for students rename their numbers if they need to.

Also make a transparency for use with the whole class. It could also be given to the students as a separate handout for them to put in their notebooks, along with the class number line. Adjust the numbers to fit the needs of your class or to create a second set for later use.

$\frac{1}{3}$	$-\frac{8}{9}$	0.4	20%	$\sqrt{4}$	$\frac{9}{2}$	$2\frac{3}{8}$
-1.6	125%	$\sqrt{16}$	$\frac{15}{5}$	$-4\frac{2}{5}$	3.85	450%
$\sqrt{9}$	- 30%	$\sqrt{25}$	0.01	-2.5	$\frac{0}{2}$	2^2
0.5%	3.9	3^2	2^3	100%	- 5	$-\frac{1}{3}$

4. Teaching Lesson D – Using Symbols to Compare Numbers

Notes from previous lesson:

In seventh grade the expectation is to locate rational numbers and square roots of perfect squares on a number line. Students should explore the location of fractions, decimals, percents, and square roots of perfect squares on a number line. It is equally important that students justify the placement of these representations on a number line, as well as understand the relationship to the numbers between which a given value lies. **Being able to justify the placement on a number line will enable students to compare and order rational numbers, percentages, and square roots of perfect squares using the symbols ≤, ≥, <, >, and =.**

For this indicator, it is **essential** for students to:

- Understand the meaning of rational numbers
- Find the square root
- Understand the difference between ≤ and ≥
- Translate numbers to same form, where appropriate before comparing numbers
- Translate between the fraction and percents
- Recall the benchmark fractions and common fraction – decimal equivalents.

For this indicator, it is **not essential** for students to:

None noted

a. Indicators with Taxonomy

7-2.3 Compare rational numbers, percentages, and square roots of perfect squares by using the symbols ≤, ≥, <, >, and=.

Cognitive Process Dimension: Understand

Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson – Comparing using symbols

Materials:

- Post-it notes with rational numbers, percentages, and square roots of perfect squares written on them.
- White boards, markers, and erasers

A common activity to compare rational numbers, percentages, and perfect squares is to play "Who Am I?" As the students enter the room, each student is given a label to be placed on their back. A post-it note is often used for this activity. The label will have a rational number, a percentage or a perfect square written on it. The students will circulate among each other asking questions of their classmates to find out "who" they are. They will assemble themselves in order from least to greatest. Ordering the students according to the numbers that were on their backs simulates locating rational and perfect squares on a number line.

Note: If this is more complicated than you like, give each student a card that they can examine with a number on it rather than taping them to their backs. Tell students they must place themselves in order from least to greatest.

Continue the lesson with a review of the symbols $\leq$, $\geq$, $<$, $>$, and $=$ and their meanings. Make sure students are reading the inequalities from left to right. Have the students write each symbol at the top of their note page, along with how each is read. Ask each student to write at least one true statement using each symbol then share with their neighbor or group.

Ask for 2 student volunteers to come to the front of the room. Question the class which symbol should be used to compare the 2 rational numbers. (Be sure to question students which sign would be used if the students switched sides.)

Ask 2 other student volunteers to come to the front and show the class their numbers. Have the rest of the class use the whiteboards to write the symbol they would use to compare the two numbers. Discuss the correct answer with the class. Continue by alternating 1 and 2 until comfortable with your students understanding.

Have students number their paper from 1 to 10. Create problems for the students to complete. Write these on the overhead or promethean. (Students may also be asked to draw a number line during this lesson to again illustrate their understanding of 7-2.2). Ask students to pair up to check their work. If there are disagreements, they should discuss their thinking with each other. Listen carefully to their conversations.

c. *Misconceptions/Common Errors –*

- Students may not understand that rational numbers include integers
- Some students may not realize repeating decimals are rational numbers.
- Students can also misread the order symbols and not include "or equal to" for $\underline{<}$ and $\underline{>}$.
- Misunderstanding of comparisons of negative rational numbers.
- Not converting improper fractions or finding square roots before comparing.

d. *Additional Instructional Strategies*

Give each student a 8 ½ by 11 sheet of paper or a large index card. Have them put any rational number on their paper. Then have the class stand in a circle. Pass around a card that has a >, <, ≥, ≤ and an = symbol. (Students may flip the > symbol card when needed to become a < symbol.) When a student gets the symbol card, they must manipulate it to make a true statement by comparing the 2 rational numbers of the people on both sides. Everyone should have their numbers facing in to the circle for the class to see. The class will give thumbs-up or a thumbs-down for agree or disagree. Pass the symbol cards around for everyone to get a turn discussing any misconceptions as you progress.

"Rational Number War" – Have cards with decimals, fractions, percents, and square roots. Students will play "war" in pairs or quads, with the largest number winning the round.

e. Technology

No suggestions for use of technology or websites are included at this time.

f. Assessing the <u>Lesson</u>

The assessment is embedded in the conversations as well as in the last part of the lesson where students first complete then discuss their working of 10 problems.

5. Teaching Lesson E – Absolute Value

Seventh grade is the first time students are introduced the concept of absolute value. Instruction should focus on the fact that the absolute value of a number is the distance of the number from zero. The absolute value of any number except zero is a positive value. An understanding that distance is always a positive value is essential to develop a solid understanding. Students will need to be reminded that positive and negative indicate DIRECTIONS from zero.

For this indicator, it is **essential** for students to:

- Understand that absolute value is a distance from zero, not a direction.
- Understand that distance is always a positive value; therefore, the absolute value is always positive

For this indicator, it is **not essential** for students to:

- Be able to solve problems involving absolute value.

a. Indicators with Taxonomy

7-2.4 Understand the meaning of absolute value.

Cognitive Process Dimension: Understand

Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson E – Absolute Value

Ask students for several real-life examples of opposites.

- Driving to work and then driving home: If you leave home to come to school and then go back home, did you travel zero miles? What if you drove to work and then drove the car back home in reverse? Even if you backed all the way home, the odometer in the car still moves forward because distance is always positive.
- What if you walk two miles to the park? Does walking the two miles back mean you walked negative two miles making the distance you traveled equal zero? You cannot walk a negative two miles. You just walk two miles in the opposite direction.

- Solicit and record other examples from students.

Use a number line and a piece of string to demonstrate the meaning of absolute value and stress that it is the distance from zero. Measure the string from 0 to positive 10. Place a knot in the string at each number interval. Measure the string from 0 to negative 10. Use an overhead sheet and the knotted string to demonstrate that 10 and -10 are the same distance from zero and a distance (or the piece of string) cannot be negative.

Use the following questions to engage students in discussion:

- If four red chips represent a negative four, what would its opposite look like?
- What changed? (The direction to move from zero)
- Where would that be on the number line?
- How far are both numbers from zero?
- Do you see a pattern with regard to opposite integers and their distance from zero?

Write the following on the board, "The absolute value of an integer is its relative distance from zero". Show and explain the notation for absolute value. (Examples might include $| 4 | = 4$ and $| {}^{-}4 | = 4$.) Ask what they notice about the absolute value of each? (They are both the same distance from zero.) Ask students to explain in writing what this means to them. Collect explanations to assess for understanding.

c. *Misconceptions/Common Errors –*

- Students often have the misconception that distance can be a negative number when determining absolute value of a positive number, confusing the concept with opposite/additive inverse. The absolute value bars do not simply change the sign of the number inside the bars.
- Students may mistakenly use parentheses or brackets for the absolute value thinking that it doesn't matter but these symbols do not mean the same thing.

d. *Additional Instructional Strategies –*

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

No suggestions for use of technology or websites are included at this time.

f. Assessing the Lesson

Formative assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Exit Slip:

Explain the meaning of absolute value as you understand it. Use examples to demonstrate.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-2.1 Understand fractional percentages and percentages greater than one hundred. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

The objective of this indicator is to understand, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge of percents greater than 100 and less than one should include a variety of examples. The learning progression to **understand** requires students to recall the concept of a percent as a part of a whole and to understand the types of fractions and their meaning. Students use standard and nonstandard representations (7-1.8) to convey to support mathematical relationships between fractions and fractional percents, mixed number fractions and percents greater than 100 by exploring concrete and pictorial models. They interpret their use in the real world and *generate* meaningful concepts for fractional percents, for percents greater than one-hundred. They will also evaluate their real world situations and pose questions to prove or disprove their conjectures about percentages (7-1.2). Students should use correct and clearly written or spoken words to communicate their understanding of these relationships (7-1.6).

7-2.10 Understand the inverse relationship between squaring and finding the square roots of perfect squares.

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

The objective of this indicator is to understand, which is in the "understand conceptual" knowledge cell of the Revised Bloom's Taxonomy. To understand is to construct meaning. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge should include a variety of examples. The learning progression to **understand** requires students to recall and understand the meaning of inverse relationships, squaring and square roots. Students also recall what is unique about the side lengths of squares. Students explore the relationship between area and square roots using inductive and deductive reasoning (7-1.3). They generalize this connection using correct and clearly written or spoken words (7-1.6)to demonstrate their understanding of the inverse relationship.

7-2.2 Represent the location of rational numbers and square roots of perfect squares on a number line.

Cognitive Process Dimension: Understand

Knowledge Dimension: Conceptual Knowledge

The objective of this indicator is to represent which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning. To represent means to change from one representation to another; therefore, students construct conceptual understanding of rational numbers by changing them from numerical (number) to graphical form (number line). The learning progression to **represent** requires students to recall the concept of percents, fractions, and decimals as part of a whole and to make the connection to prior knowledge that the value of each part lies between two whole quantities. Students should explore how to represent these types of numbers in the correct location on a number line and justify their placement of a rational number on a number line using general mathematical statements (7-1.5) based on inductive and deductive reasoning (7-1.3).

7-2.3 Compare rational numbers, percentages, and square roots of perfect squares by using the symbols ≤, ≥, <, >, and =.

Cognitive Process Dimension: Understand

Knowledge Dimension: Conceptual Knowledge

The objective of this indicator is to compare, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge should include numerous examples. The learning progression to **compare** requires students to recall common fraction/decimal equivalents and identify square roots. They understand equivalent symbolic expressions (7-1.4) and translate numbers to common form, if necessary. Students use their conceptual understanding to compare without dependent on a traditional algorithm and use concrete models to support understanding where appropriate. Students recognize mathematical symbols <, >, ≥, ≤ and = and their meanings. As students analyze the relationships to compare percentages and rational numbers, they evaluate conjectures and explain and justify their answer to classmates and their teacher. Students should use correct and clearly written or spoken words, variables and notation to communicate their reasoning (7-1.6).

7-2.4 Understand the meaning of absolute value.

Cognitive Process Dimension: Understand

Knowledge Dimension: Conceptual Knowledge

The objective of this indicator is to understand, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning; therefore, the student's conceptual knowledge of absolute value should include numerous real world examples and non-examples. The learning progression to **understand** requires students to recall the concept of integers and their position in relation to zero. When directed to determine the absolute value of a number, students should understand that absolute value is a distance away from zero and does not include which direction (positive or negative) away from zero. They illustrate this understanding by representing absolute value relationship on the number line. Students use their understanding to generate and solve complex problems to deepen conceptual knowledge. They use correct and clearly spoken words, variables and notation to communicate their understanding (7-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

Teacher Resource for Mathematics Blackline Masters and Templates

http://lrt.ednet.ns.ca/PD/BLM/table_of_contents.htm

Assessing the Module

1. The change in value of a share of stock was -$0.65 on Monday, -$1.43 on Tuesday, $0.27 on Wednesday, and $1.05 on Thursday. On which day was the absolute value of the change the lowest?

   a) Monday b) Tuesday c) Wednesday d) Thursday

2. Arrange the numbers below in order, from least to greatest.

   3^2, 4.5, $\sqrt{25}$, 10^1

3. Which of the following is not a perfect square?

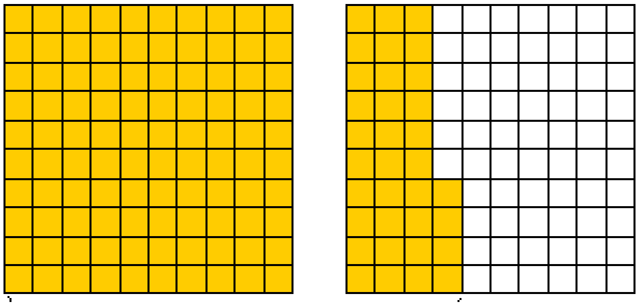
   a) 25 b) 100 c) 31 d) 144

4. Draw a model to represent 225%.

5. What is 0.9% expressed as a decimal? Explain your answer.

6. What is 0.237 written as a percent? Explain your answer.

7. Use these figures to answer the question below.



What percent of the figures is shaded?

a. 0.134%

b. 1.34%

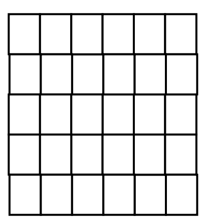
c. 13.4%

d. 134%

8. How are squaring a number and finding the square root of a number related? Use an example.

9. Explain why the model represents both expressions:

$6^2 = 36$ and $\sqrt{36} = 6$

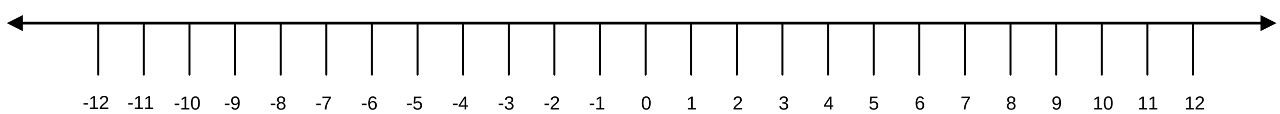


10. What is a perfect square?

11. Explain the relationship between squaring a number and finding the area of a square.

12. Place each number on the number line:

a) $\sqrt{144}$ b) 2½ c) | - 9 | d) 100%



13. Which statement is true?

a) $\sqrt{36} < 25$
b) $8 < \sqrt{64}$
c) $\sqrt{64} < \sqrt{36}$
d) $\sqrt{100} > \sqrt{144}$

14. Circle the true statements.

a) $|4| = 4$
b) $|0| = 0$
c) $|-10| = -10$
d) $|4| = -4$
e) $|-10| = 10$

MODULE

1-2

Number Structure

This module addresses the following indicators:

7-2.6 Translate between standard form and exponential form. (B2)

7-2.7 Translate between standard form and scientific notation. (B2)

This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. *Planning the Module*

- **Continuum of Knowledge**

7-2.6 Translate between standard form and exponential form. (B2)

In sixth grade, students applied strategies and procedures to determine values of powers of 10 up to 10^6 (6-2.7) and represented whole numbers in exponential form (6-2.9).

In seventh grade, students translate between standard form and exponential form.

7-2.7 Translate between standard form and scientific notation. (B2)

In sixth grade, students applied strategies and procedures to determine values of powers of 10 up to 10^6 (6-2.7) and they represented whole numbers in exponential form (6-2.9).

In seventh grade, students translate between standard form to exponential form and to scientific notation (7-2.7). They also translate between standard form and scientific notation (7-2.6). This is the first time students transfer numbers between standard form and exponential form and scientific notation.

- **Key Vocabulary**

∗ factor
∗ prime
∗ composite
∗ standard form
∗ exponential form
∗ scientific notation
∗ base number
∗ base
∗ exponent
∗ squared
∗ cubed
∗ prime factorization

II. Teaching the Lesson

1. Teaching Lesson A – Standard and Exponential Form

In sixth grade, students applied strategies and procedures to determine values of powers of 10 up to 10^6. In sixth grade, students represented whole numbers in exponential form by finding the prime factorization of whole numbers.

In seventh grade, students build on strategies to determine values of powers of 10 by translating between standard form to exponential form and to scientific notation. Seventh grade is the first time students transfer numbers between standard form and exponential form and scientific notation. Students need to understand that in scientific notation the first number should be greater than or equal to one and less than ten. Students need to work with a variety of numbers, both very large and very small, as well as decimal and whole numbers.

For this indicator, it is **essential** for students to:

- Understand the structure of exponential form
- Recall how to apply the procedure of prime factorization
- Understand that any number to the power of one is the number
- Understand that any number to the zero power is one except 0^0

For this indicator, it is **not essential** for students to:

- Perform operations with numbers in exponential form

a. Indicators with Taxonomy

7-2.6 Translate between standard form and exponential form.

Cognitive Process Dimension: Understand

Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson – Translating between exponential and standard form

Materials Needed:

- paper
- pencil
- powers of 10 handout (1/2 sheet will do; or students can copy the table)
- scientific calculator for Part 2 (needs an exponent key)

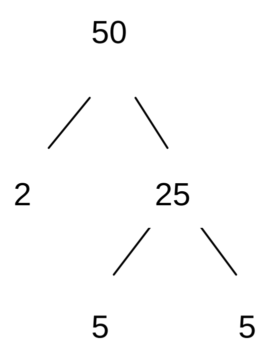
Part 1 – Prime factorization

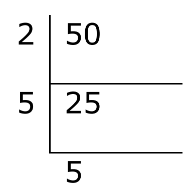
In sixth grade, students started with the standard form and found the prime factorization, writing it in exponential form. Now, students need to be able to "go both ways:"

- Start with standard, factor it, and write the exponential.
- Start with exponential and multiply the factors to find the standard.

Ask students to find the prime factorization of 50. This is a prior skill, but they may need some prompting.

Two possible procedures: factor tree and factor ladder.





$2 \bullet 5^2 = 25$

Reminders:

- prime factorization ($2 \bullet 5^2$)
- exponential form (2^1, 5^2)
- standard form (25)

Pair students and assign the following problems before moving on to Part 2.

- 15
- 40
- 60
- 100
- 35

Review the answers to the five problems and answer any questions.

Part 2 – Powers of 10

Students worked with powers of 10 in sixth grade. Have pairs of students work together using a scientific calculator to find the standard form for each power of 10.

Exponential form	Standard form
10^6	
10^5	
10^4	
10^3	
10^2	
10^1	
Before you use the calculator, what do you think the standard form for 10^0 is? Why?	
10^0	*Were you right?*

Debrief the students' findings. Guiding questions:

- What patterns do you see?
- When were you able to stop using the calculator? Why?
- What's the standard form for 10^8? 10^{10}?
- Can you write a rule for finding any power of 10?

c. Misconceptions/Common Errors –

- Students multiply the base number by the exponent. For example, $2^3 = 2 \times 2 \times 2$ *not* 2 x 3.
- Misinterpreting the terms *squared* or *cubed*.
- Students may multiply the base by the exponent. Ex. $2^3 = 2 \times 3 = 6$.
- Students may not understand the terminology, such as "squared" or "cubed".
- Student may incorrectly think that a base with an exponent of zero equals zero. (such as $4^0 = 0$). The student may incorrectly think *every* number raised to the zero power is zero forgetting that there is the exception of 0^0

d. Additional Instructional Strategies –

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

No suggestions for use of technology or websites are included at this time.

f. Assessing the <u>Lesson</u>

The assessment is embedded in the practice problems in Part 1 and the debrief of the table in Part 2.

2. Teaching Lesson B – Scientific Notation

For this indicator, it is **<u>essential</u>** for students to:
- Understand and expand powers of 10
- Understand the purpose of scientific notation
- Understand the structure of scientific notation
- Determine if a number is written in scientific notation
- Recognize small and large numbers
- Connect the sign of the exponent to the direction in which the decimal is moved
- Convert from standard to scientific and vice-versa

For this indicator, it is **<u>not essential</u>** for students to:

- Perform operations with numbers in scientific notation

a. Indicators with Taxonomy

7-2.7 Translate between standard form and scientific notation.

Cognitive Process Dimension: Understand

Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson –Scientific Notation

Materials:

- 25 m of string per group or 25 m of cash register paper per group
- Masking tape
- Index cards – 10 per group (can use 5 per group and have students cut in half)
- Solar System Chart below – one per group

To introduce the concept of translating very large numbers into standard form only the solar system chart is necessary. However, creating a scale model will help students begin to understand the relative magnitude of those very large numbers. In this lesson students will create a scale model of distances from the sun of various planets with the solar system. Give each group of students the materials listed above. Instruct the students to wrap a piece of masking tape around the string at one end and attach an index card labeled *Sun*. At the other end of attach an index card labeled *Pluto*. Use the solar system chart to discuss the relationship of the distance between the Sun and Pluto. Instruct student groups to use the scale distance information (they are familiar with comparing decimals) on the solar system chart to estimate the location for the other planets and then attach index cards with the planets' names.

Example:

Allow student groups to compare their estimations. Call on a couple of groups to share the strategies they used to estimate the location of planets. Next, use Pluto's information (5.9 09; 5.9×10^9; 5,900,000,000) to introduce the students to proper calculator notation and scientific notation. DO NOT go into a lot of detail, simply point the information out on the chart. Instruct student groups to discuss how they might use the given information to complete the chart. DO NOT complete the chart, simply discuss how the calculator and scientific notation can be used to determine actual distance.

Allow student groups to share their strategies. Instruct groups to complete the chart.

SOLAR SYSTEM CHART

Planet	Scale distance from the sun (meters) *	Distance to the planet in calculator notation	Distance to the planet in Scientific Notation	Actual distance to the planet -km (Standard form)
Mercury	.25	5.8 07	5.8×10^7	58,000,000
Venus	.45	1.08 08	1.08×10^8	
Earth	.63	1.5 08		
Mars	.96	2.28 08	2.28×10^8	
Jupiter	3.3		7.78×10^8	
Saturn	6	1.43 09	1.43×10^9	1,430,000,000
Uranus	12.1	2.87 09		
Neptune	19		4.5×10^9	
Pluto	25	5.9 09	5.9×10^9	5,900,000,000

***Note to teacher: This scale distance was calculated by dividing the useable distance (25 meters) by the distance each planet is from the sun in astronomical units to get a scaling factor. Once the scaling factor of .63 was determined, it was multiplied by the actual distance a planet is from the sun in kilometers to get this scale distance. Ask, "What is the relationship between the calculator notation and the scientific notation? … the scientific notation and the standard form? …the standard form and the calculator notation?"**

Planet	Scale distance from the sun (meters) *	Distance to the planet in calculator notation	Distance to the planet in Scientific Notation	Actual distance to the planet -km (Standard form)
Mercury	.25	5.8 07	5.8×10^7	58,000,000
Venus	.45	1.08 08	1.08×10^8	108,000,000
Earth	.63	1.5 08	1.5×10^8	150,000,000
Mars	.96	2.28 08	2.28×10^8	228,000,000
Jupiter	3.3	7.78 08	7.78×10^8	778,000,000
Saturn	6	1.43 09	1.43×10^9	1,430,000,000
Uranus	12.1	2.87 09	2.87×10^9	2,870,000,000
Neptune	19	4.5 09	4.5×10^9	4,500,000,000
Pluto	25	5.9 09	5.9×10^9	5,900,000,000

Other real-world examples of very large numbers which might be written in scientific notation.

- The chances of winning a Pick 6 Lottery are 1 in 7,059,052
- An estimate of the size of the universe is 40 billion light-years. One light-year is the distance light travels in one year. The speed of light is
  186, 281.7 miles per second. This means light travels 16,094,738,880 miles in a single day.
- There are about 100 billion cells in the human body.
- The population of the world is not more than 6 billion.

c. *Misconceptions/Common Errors –*

- Students may be confused by which direction they should move the decimal point when translating from one form to another/

- The student may not understand prime and composite numbers. The student may not understand the correct placement of a decimal in a whole number. The student may incorrectly think 7.69×10^5 simply means add 5 zeroes and not move the decimal. (as in 7.6900000) The student may add the zeroes and move the decimal to the end without regard to the original placement of the decimal. (as in $7.69 \times 10^5 = 76,900,000$)

d. Additional Instructional Strategies –

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

No suggestions for use of technology or websites are included at this time.

f. Assessing the <u>Lesson</u>

These might be for practice or homework. Use the answers to assess where students are on target or having difficulty.

1. Write 258,000 in scientific notation. Then explain how you did it.

2. Write $5.71 \bullet 10^8$ in standard form. Then explain how you did it.

3. If writing a one digit number in scientific notation, what would be the exponent of the 10? Write an example if you need to.

4. Does the value of a number change when it is written in scientific notation? Explain your answer.

5. What are some real-world examples where people might use scientific notation?

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-2.6 Translate between standard form and exponential form. (B2)

The objective of this indicator is to translate, which is in the "understand procedural" knowledge cell of the Revised Bloom's Taxonomy. To understand a procedural means that students not only know how to translate but they also develop the conceptual relationship between standard form and exponential form using a variety of examples. The learning progression to **translate** requires students to recognize the standard and exponential form of a number. Students explore patterns to generalize mathematical statements about any number to the power of one. They explain and justify their answers using correct and clearly written or spoken words and notation (7-1.6).

7-2.7 Translate between standard form and scientific notation. (B2)

The objective of this indicator is to translate, which is in the "understand procedural" knowledge cell of the Revised Taxonomy. To understand a procedural means that students not only know how translate but they also develop the conceptual relationship between standard form and scientific notation using a variety of examples. The learning progression to **translate** requires students to recognize the standard form and scientific notation form of a number. Students need to explore and connect numbers that are very large or very small to meaningful points of reference to understand their magnitude in the real world. They analyze numbers to determine the number part and the power of 10. They explain and justify their answers to their classmates and teacher using correct and clearly written or spoken words, variables and notation to communicate their understanding (7-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Which expression below shows 7,698,000 written in scientific notation?

   a) 76.98×10^4 b) 7.698×10^5 c) 7.698×10^6 d) 7.698×10^7

2. What is half of 2^6?

3. What is another way to write $3^4 \times 4^2$?

a) 3 x 3 x 4 x 4 x 4 x 4

b) 4 x 3 x 2 x 4

c) $(3 \times 4)^6$

d) 3 x 3 x 3 x 3 x 4 x 4

4. The distance between Johnsonville and Macadenville is 5^3 miles.

Written in standard form, what is the distance between the two towns?

a.) 15 miles

b.) 25 miles

c.) 75 miles

d.) 125 miles

5. Express 3500 in scientific notation. Explain how you did it.

6. Express 7.3×10^7 in standard form. Explain how you did it.

7. Translate the exponential form to standard form.

a) 10^7 =
b) 1^{10} =
c) 4^0 =

MODULE

1-3

Operations on Fractions/Decimals

This module addresses the following indicators:

7-2.9 Apply an algorithm to multiply and divide fractions and decimals. **(C3)**

7-5.1 Use ratio and proportion to solve problems involving scale factors and rates. **(C3)**

7-2.5 Apply ratios, rates, and proportions to discounts, taxes, tips, interest, unit costs and similar shapes. **(C3)**

This module contains 6 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. *Planning the Module*

- **Learning Continuum**

7-2.9 Apply an algorithm to multiply and divide fractions and decimals. **(C3)**

In sixth grade, students generated strategies to multiply and divide fractions and decimals. Sixth grade was the first time the concept of multiplying and dividing fractions and decimals was introduced (6-2.5).

In seventh grade, students apply an algorithm to multiply and divide fractions and decimals (with numbers only).

In eighth grade, students understand the effect of multiplying and dividing rational numbers by another rational number (8-2.2).

7-5.1 Use ratio and proportion to solve problems involving scale factors and rates. (C3)

In sixth grade students used proportions to determine unit rates (6-5.6). Sixth grade students also used a scale to determine distance.

In seventh grade, students extend their understanding of and use of proportional reasoning (unit rates and scale) to solve problems using ratio and proportion involving scale factors and rates (7-5.1).

In eighth grade, students use proportional reasoning and the properties of similar shapes to determine the length of a missing side (8-5.1).

7-2.5 Apply ratios, rates, and proportions to discounts, taxes, tips, interest, unit costs and similar shapes. **(C3)**

In sixth grade, students understand the relationship between ratio/rate and multiplication/division (6-2.6).

In seventh grade, students apply ratios, rates, and proportions to discounts, taxes, tips, interest, unit costs and similar shapes (7-2.5).

In eighth grade, students apply ratios, rates and proportions.

- **Key Concepts/Key Terms**

  *These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and in use in conversation with students.

* divisor
* dividend
* denominator
* numerator
* quotient
* product
* ratio
* rate
* proportion
* scale
* unit rate

II. *Teaching the Lessons*

For this indicator, it is **essential** for students to:

- Understand that the product of fractions and decimals does not always result in a larger number
- Understand the meaning and concept of fractions and decimals
- Connect their generated strategies and their concrete/pictorial models with their algorithm
- Understand that multiplication does not always result in a larger answer and division does not result in a smaller answer
- Gain computational fluency
- Estimate products and quotients of fractions and decimals to determine reasonableness of answers once a problem is solved.

For this indicator, it is **not essential** for students to:

None noted

1. Teaching Lesson A – Multiplying Fractions

In sixth grade, students generated strategies to build conceptual understanding of multiplying and dividing fractions and decimals.

Seventh grade is the first time students are required to multiply and divide fractions and decimals symbolically (numerals only). As a result, students should be given opportunities to relate their prior concrete and pictorial experiences to the new symbolic operations. In addition to building on those previous experiences, students should estimate the products and quotients of problems involving fractions and decimals and use those estimations as the basis for explaining the reasonableness of results after actually solving. Furthermore, students should be given opportunities to apply multiplication and division of fractions and decimals in context – not merely perform the operations for the sake of multiplying or dividing.

a. Indicators with Taxonomy

7-2.8 Apply an algorithm to multiply and divide fractions and decimals.

Cognitive Process Dimension: Apply

Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson – Multiplying Fractions

Materials needed:

- manipulatives students can use to model fractions (fraction strips, pattern blocks, fraction squares and circles, counters, etc.)
- paper
- colored pencils

Use only proper fractions to get students started. Once they have established an algorithm for proper fractions, try a mixed number times a fraction. Ask students what they can do to the mixed number to make the number sentence "look like" the ones they already know how to do. [Rename the mixed number as an improper fraction.]

Students should be paired so they can talk about the math. Present the problems to students and challenge them to solve using models. They need to record their strategies. As pairs finish, have them form quads to compare their work. Move around the room, checking student progress and interacting with them. When students have completed the problems, bring the class back together and discuss the strategies used. Then look for patterns in the work and challenge students to find a rule (algorithm) for multiplying fractions.

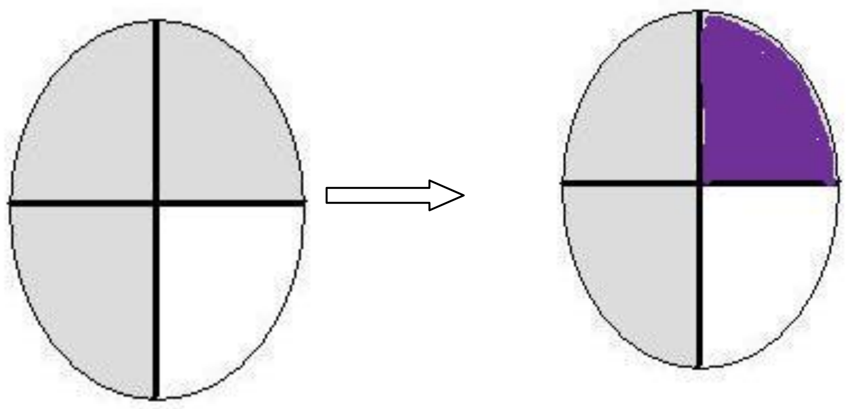
Follow-up questions for students as you observe them working:

- Why did you draw this model?
- How did you decide what to section and what to shade?
- Can you write a multiplication sentence to describe your model?

Problem #1

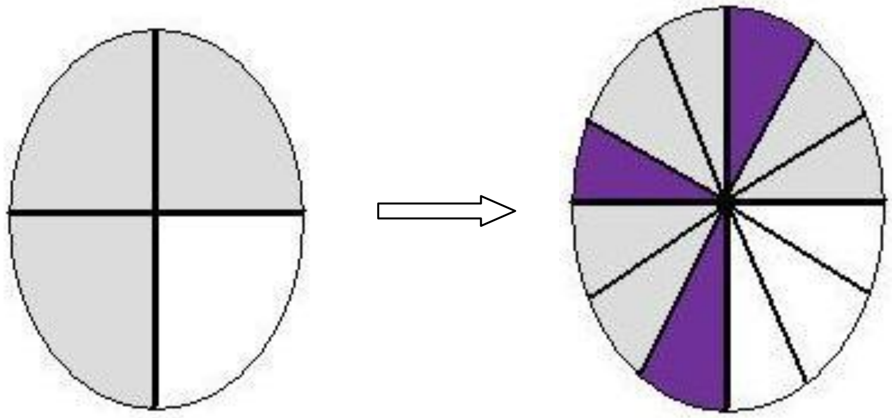
There was ¾ of a pizza left in the fridge. If you eat 1/3 of what's there, how much of the whole pizza did you eat?

Possible models:



¾ of a pizza left in the fridge

1 of the 3 leftover pieces is 1/3 of the leftover pizza. That's ¼ of the whole thing.



1/3 of the ¾ leftover pizza is 3/12 of the whole pizza. 3/12 is the same as ¼.

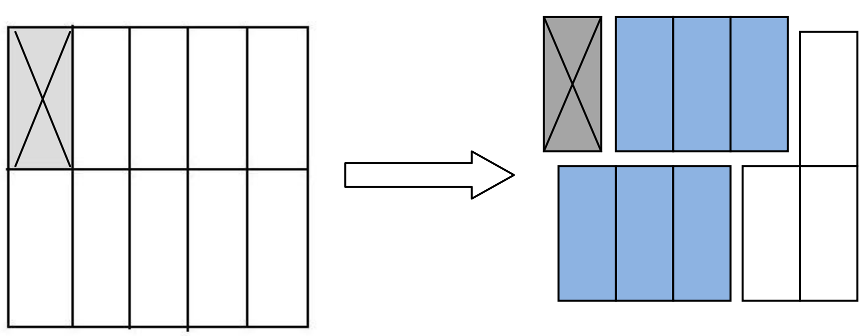
¾ of a pizza left in the fridge

Section each slice (even the eaten one) into 3 parts. Take 1/3 of each remaining slice – that's ¼ of the whole pizza.

Problem #2:

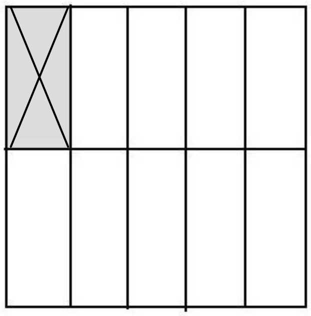
Someone ate 1/10 of a cake. If you eat 2/3 of the cake that's left you'll be really full! How much of the whole cake will you have eaten?

Possible models:

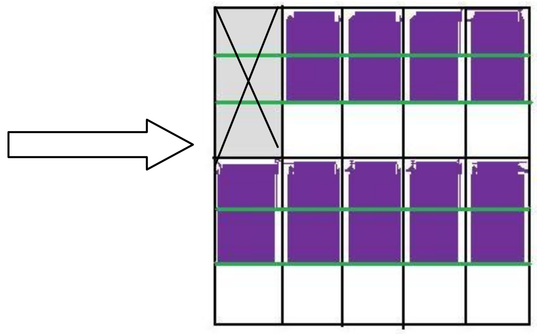


1 slice out of 10 is eaten (1/10 of the cake)

9 slices are left (9/10 of the cake). There are 3 groups of 3 (3/3) in those 9 pieces. 2 of those 3 groups (2/3) is 6 pieces.



1 piece out of 10 is eaten (1/10 of the cake)

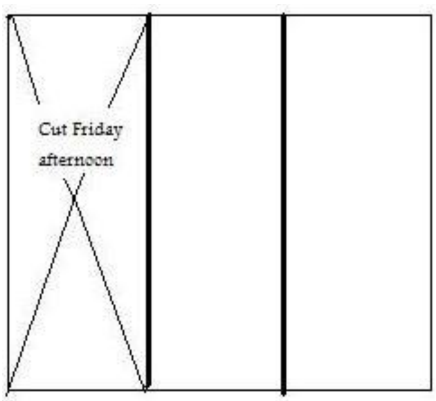


Section the whole cake into thirds. Then take 2/3 of each remaining piece. That's 18/3 or 6 pieces of cake.

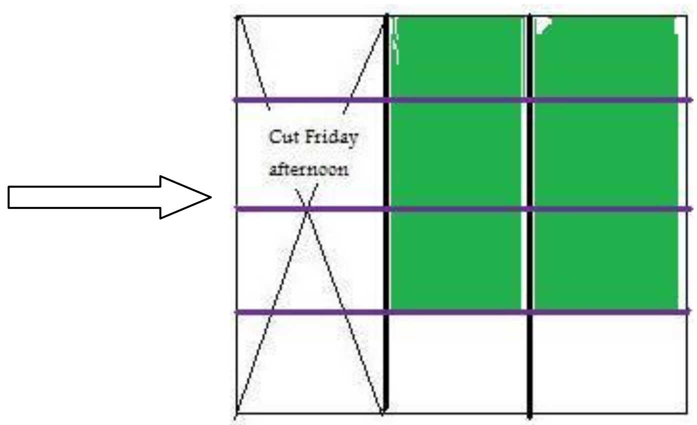
Problem #3:

Gloria cut 1/3 of her yard Friday afternoon. Saturday morning, she had 2/3 of the yard left to cut. After lunch, she cut ¾ of the grass she had left. How much of the whole yard did Gloria cut after lunch?

Possible model:



2/3 of the yard to cut on Saturday morning



Section the whole yard into fourths. Take 3 of the 4 smaller sections in the 2/3 of the yard she had left in the morning. So…she cut 2/8 or ¼ of the yard *before* lunch and 6/8 or ¾ of the yard *after* lunch.

c. Misconceptions/Common Errors –

In a problem such as 4 ½ ● 3 1/4, the student multiplies the whole numbers then multiplies the fractions. Ex., A student mistakenly thinks 4 ½ * 3 ¼ means 4*3 + ½ * ¼ to find a product of 12 1/8. Teachers should be alert to the misconception that all multiplication results in "a bigger number".

d. Additional Instructional Strategies

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

http://www.learner.org/courses/learningmath/number/session9/part_a/index.html

> Part A of this interactive lesson is good for refreshing students' memories of the area model of multiplication.

http://www.visualfractions.com/multiply.htm

f. Assessing the Lesson

Formative assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Provide other opportunities for students to model fraction multiplication to prove the algorithm works.

2. Teaching Lesson B – Dividing Fractions

a. Indicators with Taxonomy

7-2.9 Apply an algorithm to multiply and divide fractions and decimals.

Cognitive Process Dimension: Apply

Knowledge Dimension: Procedural Knowledge

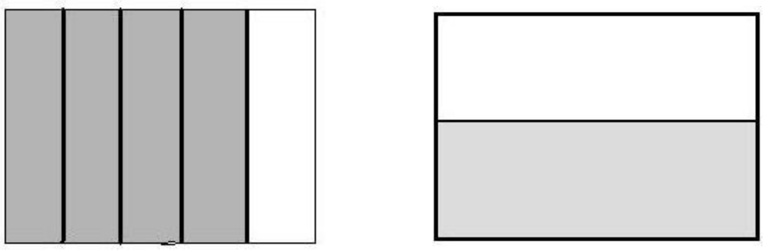
b. Introductory Lesson –

NOTE: It's very difficult to model fraction division. Two possibilities are offered here, adapted from Teaching Student-Centered Mathematics Grades 5-8, John A. Van de Walle and LouAnn H. Lovin, Pearson, 2006. It would be very beneficial to take time to study this text before teaching fraction division if you want to help your students understand it conceptually, rather than just being able to crunch the numbers.

Students will need multiple opportunities to model and solve problems in order for either algorithm to make sense to them.

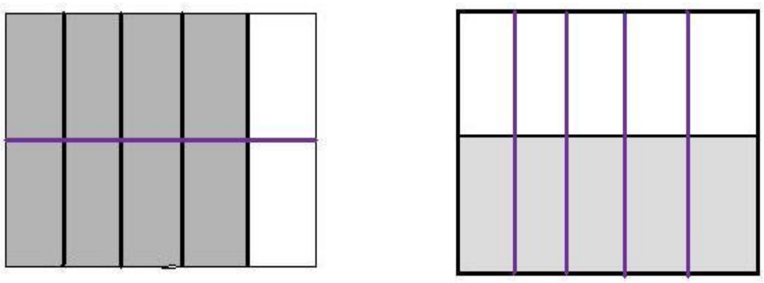
The Common Denominator Algorithm

4/5 ÷ ½ means, "How many sets of ½ are in 4/5?"

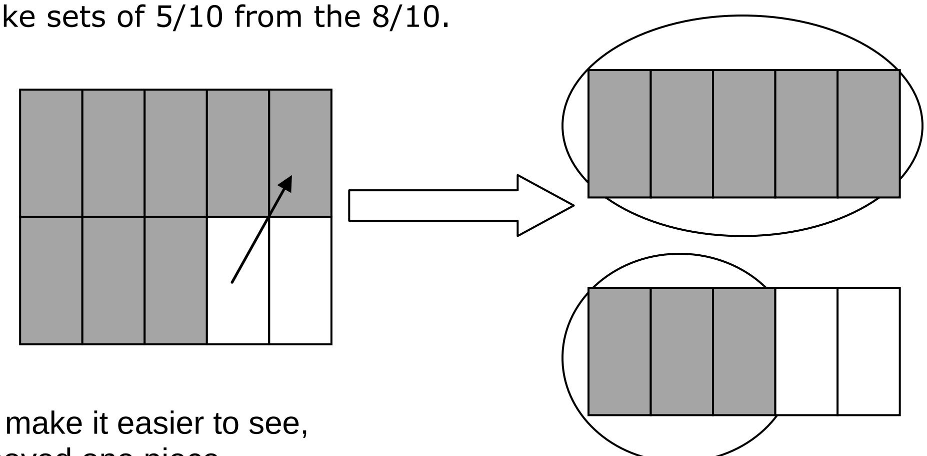


Restate the problem with common denominators.

8/10 ÷ 5/10 means, "How many sets of 5/10 are in 8/10?"



Make sets of 5/10 from the 8/10.



There's one set and 3/5 of another set.

There are 1 3/5 sets of 5/10 in 8/10.

To make it easier to see,
I moved one piece.

If you look again at the restated sentence

$$8/10 \div 5/10 \Rightarrow \frac{8 \div 5}{10 \div 10} = \frac{1\ 3/5}{1} \Rightarrow 1\ 3/5$$

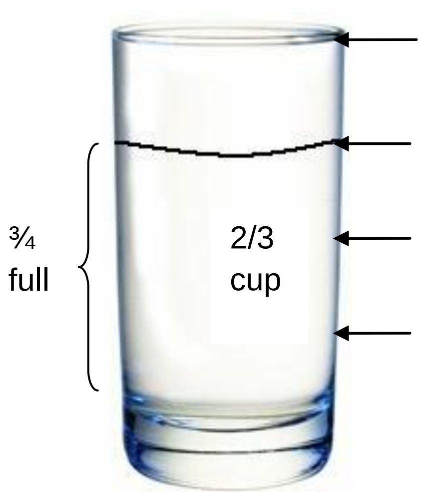
Once a common denominator has been found, and the fractions have been renamed, just divide the numerators to find the quotient.

Students should solve problems in context and the numbers should be “friendly” as they begin working with division using an algorithm.

<u>The Invert-and-Multiply Algorithm</u>

Consider this problem:

A glass can be filled to ¾ full with 2/3 of a cup of juice. How much will the glass hold if filled completely?



2/3 ÷ ¾ will tell us how much the whole glass will hold. The whole glass is 4/4. Since the juice in the glass is 3 of the 4 parts it takes to fill the glass, we can divide the juice by 3 (which tells us how much 1 part is) and multiply by 4 (to get the whole).

The denominator in a fraction divides the whole into parts – it is a divisor. The numerator tells how many of those parts are counted – it is a multiplier. In this problem, we divided the 2/3 by 3 and then multiplied by 4, so we multiplied by 4/3.

c. Misconceptions/Common Errors –

No typical student misconceptions are noted at this time.

d. Additional Instructional Strategies –

While additional learning opportunities are needed no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

No suggestions for use of technology or websites are included at this time.

f. Assessing the Lesson

Provide story problems for students to model and solve. Assess student understanding through questions and observations.

3. Teaching Lesson C – Multiplying Decimals

a. Indicators with Taxonomy

7-2.9 Apply an algorithm to multiply and divide fractions and decimals.

Cognitive Process Dimension: Apply

Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson –Multiplying Decimals

Materials Needed:

- multiplication decimal patterns handout
- calculators (1 for each pair of students)
- colored pencils (optional)

Pair students so they can talk about the math they are doing. Give each student a handout and each pair a calculator. Have them use the calculator to find the products.

Then have students answer the thinking questions on the handout. As pairs finish, have them form quads to discuss their work. Move around the room checking student progress and asking questions to guide their thinking. If students are having difficulty finding the pattern, suggest they circle decimal digits in the factors and products to make them more visible.

When students have had time to work in both pairs and quads, bring the class back together to discuss what they found about decimal patterns. The handout and any notes you want them to keep should be placed in their math notebooks or in whatever format you use for keeping work in your classroom.

Students should find that they can multiply decimal numbers just as they would whole numbers, then place the decimal in the product to match the number of decimal digits in the factors. Students need to be reminded that all whole numbers end with a decimal.

Multiplication Decimal Patterns

- Use the calculator to find the products of the multiplication sentences.
- Be careful to correctly enter decimals in the factors.
- Be careful to correctly record decimals in the products.

FACTORS	PRODUCT
32 • 48	
0.32 • 4.8	
32 • 0.48	
3.2 • 4.8	
0.32 • 0.48	

Thinking Questions

1. Place the decimals in the factors and product in the first row.

2. What do you notice about the products in the table? Why do you think this is so?

3. What do you notice about the placement of the decimals in the factors and products in each row?

4. Write a rule you think will work for placing decimals in the products of multiplication problems.

5. On the back of this paper or on another sheet of paper, make a table similar to the one you've completed to see if your thinking works for other numbers.

c. Misconceptions/Common Errors –

No typical student misconceptions are noted at this time.

d. Additional Instructional Strategies –

Ask students to bring in sales papers to generate problem solving opportunities involving decimal operations.

Decimal problems should be contextual.

Students should also practice estimation skills to determine whether or not their answers are reasonable.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

No suggestions for use of technology or websites are included at this time.

f. Assessing the Lesson

Formative assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

4. Teaching Lesson D – Dividing Decimals

a. Indicators with Taxonomy

7-2.9 Apply an algorithm to multiply and divide fractions and decimals.

Cognitive Process Dimension: Apply

Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson – Dividing Decimals

This lesson is very much like Lesson C – Multiplying Decimals

Materials Needed:

- division decimal patterns handout
- calculators (1 for each pair of students)
- colored pencils (optional)

Pair students so they can talk about the math they are doing. Give each student a handout and each pair a calculator. Have them use the calculator to find the products.

Then have students answer the thinking questions on the handout. As pairs finish, have them form quads to discuss their work. Move around the room checking student progress and asking questions to guide their thinking. If students are having difficulty finding the pattern, suggest they circle decimal digits in the problems and quotients to make them more visible.

When students have had time to work in both pairs and quads, bring the class back together to discuss what they found about decimal patterns. The handout and any notes you want them to keep should be placed in their math notebooks or in whatever format you use for keeping work in your classroom.

Students should find that they can divide decimal numbers very much like they do whole numbers, placing the decimal in the quotient according to the decimal in the dividend. Again, students need to remember that all whole numbers end with a decimal.

NOTE: The one "catch" is when the divisor is a decimal number. Such problems should only be given in context. For example, If Tyreese has $20, how many Gatorades can he buy if they cost $1.25 each? Encourage students to solve this problem in more than one way. THEN show them the "move the decimal" rule so they have a concrete experience on which to base it.

Division Decimal Patterns

- Use the calculator to find the quotients of the multiplication sentences.
- Be careful to correctly enter decimals in the dividends.
- Be careful to correctly record decimals in the quotients.

PROBLEM	QUOTIENT
3456 ÷ 72	
345.6 ÷ 72	
34.56 ÷ 72	
3.456 ÷ 72	
0.3456 ÷ 72	

Thinking Questions

1. Place the decimals in the dividend, divisor, and quotient in the first row.

2. What do you notice about the quotients in the table? Why do you think this is so?

3. What do you notice about the placement of the decimals in the dividend and quotient in each row?

4. Write a rule you think will work for placing decimals in the quotients of division problems.

5. On the back of this paper or on another sheet of paper, make a table similar to the one you've completed to see if your thinking works for other numbers.

c. Misconceptions/Common Errors –

The student divides decimal by a decimal and does not move the decimal point in the divisor or the dividend. Stress to students to check to make sure the decimal point in the quotient is directly above the decimal point in the dividend.

d. Additional Instructional Strategies –

Ask students to bring in sales papers to generate problem solving opportunities involving decimal operations.

Decimal problems should be contextual.

Students should also practice estimation skills to determine whether or not their answers are reasonable.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

No suggestions for use of technology or websites are included at this time.

f. Assessing the Lesson

Formative assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

5. Teaching Lesson E, Part 1 – Using Proportional Reasoning to Explore Ratio

7-5.1 Use ratio and proportion to solve problems involving scale factors and rates.

For this indicator, it is **essential** for students to:

- Understand the meaning of ratio
- Understanding the meaning of proportion
- Set up a ratio
- Set up a proportion
- Interpret their answers. For example, $\frac{250 miles}{5 hour}$ means he drove 250 miles in 5 hours.
- Work with answers that are in whole number, decimal or fractional form
- Use an appropriate strategy to solve the proportion

For this indicator, it is **not essential** for students to:

- Solve problems that involve missing pieces of similar figures (8-5.1).

a. Indicators with Taxonomy

7-5.1 Use ratio and proportion to solve problems involving scale factors and rates.

Cognitive Dimension: Apply
Knowledge Dimension: Procedural

***b*. Introductory Lesson E, Part 1:** Using Proportional Reasoning to explore ratio
Lesson adapted from: ETA Hands-on Standards, Grades 7-8, Lesson 5 and 9
Materials needed, part 1: Groups of three, 75 color tiles per group

Warm up: Questions below Adapted from: Van De Walle, 5th edition, Elementary and Middle School Mathematics

Solve these 2 problems using an approach for each that seems most reasonable to you.

1- Selena bought 3 widgets for $2.40. At the same price, how much would 10 widgets cost? (student determines unit rate or unit price, i.e., cost of one item)

2-Selena bought 4 widgets for $3.75. How much would a dozen widgets cost?

Introduce problem: Several student volunteers from Mrs. Thompson's 7th grade math class have agreed to help improve the outdoor play area for the kindergarten classes at the elementary school. They have been asked to paint 3 rectangular areas on the concrete. Each rectangle is identical and has an area of 75 square feet. Marie and Joe learned that 3 pints of paint would cover an area of 45 square feet. What was the ratio of pints to square feet of coverage? How many pints will be needed to paint one rectangle in the outdoor activity area?

Students Solve with materials, record on paper

1- How many pints of paint are needed to cover 45 square feet? Write as a ratio.

3 pints

45 Sq feet

(Students make a rectangular area with 45 tiles.)

2- Based on this ratio, how many square feet will 1 pint of paint cover? Write as a ratio.

1 pint

15 Sq feet

(Students divide their tiles into three equal groups to model answer.)

3- What is the area, in square feet, of the painted rectangle?

(Students create rectangle with 75 tiles.)

4- If 1 pint of paint covers 15 square feet, how many pints will cover 75 square feet?
(Students write a proportion reflecting this information and solve the problem.)

Recorded answers on paper above sketch of manipulatives.

Answer 1) 3 pints
45 Sq feet

Answer 2) 1 pint
15 Sq feet
[add Answer 3) beside 2] =
X pints
75 square feet

Answer 4) 5 pints
75 square feet

Class discussion should include:

What is a ratio? (summarize clearly with a verbal and written example)

What is a proportion? (summarize clearly with a verbal and written example)

Describe how you determined the ratio of pints of paint to square feet of coverage?

What are the two ratios in the proportion? What is the value of x, in ¼ = $^{x}/_{2}$*?*

A) $^{4}/_{1}$, $^{8}/_{x}$:=2   B) $^{1}/_{x}$, $^{4}/_{8}$:=4   C) $^{x}/_{4}$, $^{8}/_{4}$:x=8   D) ¼, $^{x}/_{8}$: x=2

Lesson E, Part 2 - Using proportional reasoning to explore scale factor

Suggested literature connection: *One Inch Tall* by Shel Silverstein (poem) (Here students can use ratios and proportionality to make decisions about the statements made in the poem based on mathematics.)

Materials needed: students pairs,

- AngLegs™ (3 green, 3 yellow, 6 orange) or
- 1 prepared set of cut straws (or something similar- see prep below).
- *Material prep- Straws should be precut into 3 different lengths and packaged as:*
  - *6 pieces, each = approx 1 ½ inches;*
  - *3 pieces, each = approx 2 inches;*
  - *3 pieces, each = approx 2 ½ inches.*

The Boy Scout troop is designing and building a dirt bike course. They want to include 3 jump ramps that are similar in shape, but are different sizes. The smallest ramp will be 5.75 feet. What will be the lengths of the second and third ramps, if the scouts use factors of 2 and 3 to build them?

Construct a right triangle with the medium length as the base, and the long length as the height. (Students will see and use the longest length for hypotenuse.)

This is the first and ‘smallest jump ramp’.

The second jump ramp is larger and will need to be designed like the first but with a scale factor of…? (Students should remember the scale factor increase of 2).

What will you need to do to your first ramp so that it increases by a scale factor of 2? (Students will discuss and determine that each side of the triangle should be two times the original, i.e., contain 2 pieces of the same length). Remind the students that the jump ramps are like these 2 similar triangles.

What will need to happen so that the 3rd jump ramp is increased from the original ramp by a scale factor of 3? (Students will discuss and determine that each side of the triangle should be two times the original, i.e., contain 3 pieces of the same length).

You have constructed models of 3 jump ramps that are similar in shape, but are different sizes.

Recorded answers on paper above sketch of manipulatives.

What is the length of the first Jump Ramp? $5.75\ ft^2$

What is the length of the second Jump Ramp? $2 \times 5.75\ ft^2 = 11.5\ ft^2$

What is the length of the third Jump Ramp? $3 \times 5.75\ ft^2 = 17.25\ ft^2$

Have students explain the relationship between the first and second triangles.

What is the scale factor of the third Jump Ramp compared to the second Jump Ramp?

Perez is building a 1/8 scale model of hang glider. If the actual hang glider has a wingspan of 28 feet, what will the wingspan of the model be?

A) 3.75 feet B) 3.5 feet C) 2.25 feet D) 8 feet

***c*. Additional instructional strategies/resources, etc**

LESSON: Understanding proportional reasoning

Engage questions Adapted from: Van De Walle, 5th edition, Elementary and Middle School Mathematics

Lesson adapted from: Figure This! Math Challenges for Families

http://www.figurethis.org/challenges/c11/challenge.htm

Lesson activity:

What's round, hard, and sold for 3 million dollars?

Which is worth more today, Babe Ruth's 1927 home-run record-breaking ball or Mark McGwire's 70th home-run record-breaking ball that sold in 1999 for three million dollars?

1- Let's assume that Babe Ruth's ball was valued at $3000 in 1927. What was its value seven years later? Make a t-chart or table.

Quick Answer:
You decide. If the value of Babe Ruth's ball **started at $3,000 and doubled** every seven years since 1927, its value in 1997 would be approximately **$3,072,000**.

Complete Solution:
Suppose Babe Ruth's ball had a value of $3,000 in 1927. If the price doubled in seven years, the ball would be worth $6,000 in 1934. In seven more years, its value would double again.

Year	Value
1927	3000
1934	$2 \times 3000 =$ **6000**
1941	$2 \times 2 \times 3000 = 2^2 \times 3000 =$ **12,000**
1948	$2 \times 2 \times 2 \times 3000 = 2^3 \times 3000 =$ **24,000**
1955	$2 \times 2 \times 2 \times 2 \times 3000 = 2^4 \times 3000 =$ **48,000**
...	...
1997	$2^{10} \times 3000 =$ **3,072,000**

The year 1997 was 70 years after 1927, so there would be 10 sets of 7 years during that time. By 1997, Ruth's ball would have a value of $3,072,000. Since it would have a greater value than McGwire's in 1997, it would have a greater value in 1999.
Extension: What would be the approximate value for each baseball in 2007? Current year?

*Fun Fact: The $3,000,000 price of Mark McGwire's baseball was **23 times** that of any baseball previously sold and five to six times the **highest price paid** for any other sports artifact.*

- This indicator should not only focus on setting up proportions and using the algorithm to solve for the missing piece, but also, focus on building students' ability to use proportional reasoning. Proportional reasoning is hard to teach and should be developed over time using a variety of activities.
- Explore examples where scale and rates are used in the real-world (i.e. blueprints, speed, models)
- A hands-on activity is a scale drawing project. This can be done outside of school or as an in-class project. Give students a variety of coloring books (or the student can bring in a picture). Once they have selected their pictures, you will need to superimpose a grid on their picture. They will need your help will this next part because it requires measuring and partitioning off their squares. Given the dimensions of the paper (posterboard, 11 x 17 paper, etc..) on which they will be drawing, students determine what size squares they will need so their enlarged picture will fit on the paper. They determine their scale factor. Next, they have to numbers the squares on their picture and number the squares on their drawing paper. They have to redraw each square so that it is proportional to what is in the square on their picture. The key is that they only drawn one square at a time.
- The introduction and use of the procedural concept of the cross-product algorithm should only be introduced after students have had considerable amount of time with developing and experimenting with more intuitive and conceptual methods for solving proportions.
-

d. Misconceptions/Common Errors

Van De Walle, 5th edition, p 309: "Traditional textbooks show students how to set up an equation of two ratios involving an unknown- *"cross-multiply" and solve for the unknown.* This can be a very mechanical approach and will almost certainly lead to confusion and error". Students must have opportunities to explore proportions and find informal ways to solve proportions using their own ideas. Providing these foundational experiences will increase the student's capacity for new learning while *greatly reducing* misconceptions and errors.

e. Technology

Virtual Manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictoral representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Figure and Ratio of Area A page shows two side-by-side grids, each with a blue rectangle inside. Students can change the height and width of these blue rectangles and then see how their ratios compare — not only of height and width but also, most important, of area. The exercise becomes most impressive visually when a tulip is placed inside the rectangles. As the rectangles' dimensions are changed, the tulips grow tall and widen or shrink and flatten. An excellent visual!
What is the relation between ratio of lengths of width and height and the one of rectangle areas?
Applet http://www.ies.co.jp/math/java/geo/ratioAB/ratioAB.html

Cylinders and Scale Activity
http://nsdl.exploratorium.edu/nsdl/showRecord.do?id=10911
Using a film canister as a pattern, students create a paper cylinder. They measure its height, circumference, and surface area, then scale up by doubling and even tripling the linear dimensions. They can track the effect on these measurements, on the surface area, and finally on the amount of sand that fits into each module (volume). The lesson is carefully described and includes handouts.

Scaling Away
http://illuminations.nctm.org/LessonDetail.aspx?id=L584
For this one-period lesson, students bring to class either a cylinder or a rectangular prism, and their knowledge of how to find surface area and volume. They apply a scale factor to these dimensions and investigate how the scaled-up model has changed from the original. Activity sheets and overheads are included, as well as a complete step-by-step procedure and questions for class discussion. (From Illuminations, National Council of Teachers of Mathematics Vision for School Mathematics)

f. Assessing the Lesson

Formative assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

6. Teaching Lesson F – Rates and Ratios

For this indicator, it is **essential** for students to:

- Understand the meaning of ratio, rate and proportions
- Develop strategies for determining discounts
- Connect the similar ideas computing discounts, taxes, tips and interest in order to develop strategies
- Understand that with discounts, they pay less and with taxes, tips and interest they pay more

- Understand the concept of percents
- Multiply by a decimal
- Understand the relationship between unit cost and ratio and rate
- Understand the proportionality of similar shapes
- Solve proportions using an appropriate strategy
- Explore real world examples of discounts, taxes, tips and interest
- Determine the final cost after a discount, taxes and interest are applied

For this indicator, it is **not essential** for students to:

- Apply ratios, rates and proportions for problems beyond those situations outlined in the indicator

a. Indicators with Taxonomy

7-2.5 Apply ratios, rates, and proportions to discounts, taxes, tips, interest, unit costs and similar shapes

Cognitive Process Dimension: Apply

Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson –

Although students are not required to compute the final cost, it may helpful for them to understand that discounts (subtracting from original amount) and taxes, tips and interest (add to the original amount). Representing these concepts as pictorial or concrete models may help students deepen their understanding of these procedures. For example, using a hundreds charts
to represent $100 dollars. Shade appropriate blocks to represent discounts like 20% off mean subtracting 20 blocks or interest of 30% means adding 30 more blocks.

Showing students how to set up an equation of two ratios involving an unknown, "cross multiply" and solve for the unknown can be a very mechanical approach and will almost certainly lead to confusion and error. Although you may wish to eventually introduce the cross-product algorithm, it is well worth the time for students to find ways to solve proportions using their own ideas first. The cross multiplying algorithm for solving proportions is a valid strategy that should be taught with understanding. Using proportional reasoning is another powerful strategy that focuses more on a conceptual understanding of proportional relationship as opposed to a traditional algorithm. Proportional reasoning always allows students make estimates.

Pose problems to students similar to the following. Allow them to solve the problems in any manner they wish as long as they can explain why their answers make sense.

At the Office Store, you can buy #2 pencils, four for 59 cents. The store also sells the same pencils in a large box of 5 dozen pencils for $7.79. Which is the better deal? How do you know? How much do you save?

Brian can run 5 km in 18.4 minutes. If he keeps on running at the same speed, how far can he run in 23 minutes?

You have $30 to spend at a sporting goods store. You find a pair of running shoes that you want to purchase. The original cost of the shoes is $42, but they are on sale for 30% off. It is tax-free week, so you will not be charged sales tax. Can you purchase the shoes with your gift card? Explain your answer.

You have been saving your money to buy a skateboard. The skateboard costs $89, but you have a coupon for 25% off. Sales tax for your area is 8%. If you have saved $71 so far, how much more money must you save before you can buy the skateboard? Explain your answer.

Tommy mows lawns in the summer to earn spending money. He charges his customers $15 per lawn. Tommy wants to buy an MP3 player from his favorite electronics store. The player that he wants usually costs $75, but it is on sale for 20% off its regular price. Tommy must also consider the local sales tax of 6% he must pay. How many lawns will Tommy have to mow before he can purchase his MP3 player? Justify your answer.

c. Misconceptions/Common Errors –

Students may still struggle with converting decimals.

d. Additional Instructional Strategies –

- Use maps and scale drawings to solve problems.
- Using a menu and a set amount of money, ask students to purchase multiple food items, add sales tax and a tip, and find the cost.

Unit Costs

Materials Needed:
None

Pose the following to students groups/pairs:

I've decided to change my cell phone carrier so I started comparing prices. I was mainly concerned with the cost if I go over the limit on my minutes. One phone company charges $.70 for every 15 minutes over and another charges $1.00 for 20 minutes over. Which company has the cheaper rate?
(.70 for 15 minutes equals $2.80 per hour; $1 for 20 minutes equals $3 per hour)

Allow groups/pairs to solve the problem any way they wish as long as they can explain their reasoning. Have groups/pairs share their solution strategies. Listen for opportunities to probe student thinking with regard to proportional reasoning. Listen for opportunities to probe student thinking with regard to unit rate.

Similar Shapes

Materials Needed:

- Metric Tape Measures-two for each group
- Meter Sticks-two for each group
- Spool of string- one for each group
- Model car for each group
- Calculator-one for each group
- Group-recording sheet- one for each group
- Class-recording sheet

Prior to this lesson, select a car from the teacher parking lot for which a scale model can be found in the local Wal-Mart, Toys R Us, etc. Models come in various scales. In this activity, students will work in groups of five to measure and record measurements from the model car to the nearest tenth of a centimeter.

Using the scale from the model that has been purchased (There are many different sizes on the market i.e. 1:10 1:24 1:3) set up a proportion for determining the diameter of the actual tire. Discuss the ways to solve this proportion. It is important to discuss the units used in both the model and in the actual car. The scale factor may or may not be a unit rate depending on the units used in both the model and the car. Have the groups work together to predict the actual measures on car.

$$\frac{1 \text{ (scale)}}{24 \text{ (actual)}} = \frac{\text{scale's tire measure}}{\textbf{X} \text{ car's actual tire measure}}$$

When groups have completed measuring the model and using their results to make a proportion, take the class out to measure the actual measure of the car in the parking lot. Rather than having each group individually measure the car, the teacher should lead the entire class with the aid of individuals in finding the actual measures. These results should be reasonably close. Discuss the reasons for the difference in the measures.

Model Measurements

Group	**Diameter of the tire**	**Length of the car**	**Length of the back bumper**	**Diameter of the headlight**	**Height of the front door**
1					
2					
3					
4					
5					

	Model	**Proportion**	**Actual Car**
Diameter of the tire			
Length of the car			
Length of the back bumper			
Diameter of the headlight			
Height of the front door			

e. Technology

No technology resources noted at this time to address the intent of this indicator.

f. Assessing the Lesson

Formative assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-2.9 Apply an algorithm to multiply and divide fractions and decimals. **(C3)**

The objective of this indicator is apply, which is in the "apply procedural" of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus is to gain computational fluency with addition and subtraction of fractions, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **apply** requires students to understand decimals and fractional forms such as mixed numbers, proper fractions, and improper fractions. Students should apply their conceptual knowledge of fractions and decimals to transfer their understanding of concrete and/or pictorial representations to symbolic representations (numbers only) by generalizing connections among a variety of representational forms and real world situations (6-1.7). Students use these procedures in context as opposed to only rote computational exercises and use correct and clearly written or spoken words to communicate about these significant mathematical tasks (6-1.6). Students engage in repeated practice using pictorial models, if needed, to support learning. Lastly, students should evaluate the reasonableness of their answers using appropriate estimation strategies.

7-5.1 Use ratio and proportion to solve problems involving scale factors and rates. **(C3)**

The objective of this indicator is use, which is in the "apply procedural" cell of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus is to gain computational fluency with problems involving the use of proportions to solve problems with scale and rates, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **use** requires students recall the definition of ratio and proportion and understand how to use proportions (equivalent ratios) to solve simple problems involving unit rates.

Students explore a variety of situations that involve scale factors and rates and generalize connections among real-world situations (7-1.7). They develop strategies for solving problems and explain and justify their strategies using their understanding of proportional reasoning. They use these strategies (conceptual and procedural) to generate and solve other problems. They explain and justify their answers to their classmates and teacher using correct and clearly written or spoken (7-1.6).

7-2.5 Apply ratios, rates, and proportions to discounts, taxes, tips, interest, unit costs and similar shapes. **(C3)**

The objective of this indicator is to apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is procedural, students will need a conceptual understand of the concepts of discount, taxes, tips and interest. The learning progression to **apply** requires students to recall and understand the meaning of the concepts of discounts, taxes, tips, interest, unit cost and similar shapes. Students explore a variety of problems in context to generalize connections (7-1.7) between these concepts, appropriate computational procedures and these real world situations (7-1.7). They analyze pictorial and/or concrete models to gain a conceptual understanding of these procedures. They generate mathematical statements (7- 1.5) related to how to use ratios, rates and proportions to solve problems and use correct and clearly written and spoken words, variable and notation to communicate their understanding (7-1.6). Students should engage in repeated practice to gain fluency in these procedures.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized test.

1. Sally buys a pair of shoes that are $49.99. The sales tax is 8%. What is the amount of sales tax?

2. Cathy bought gas for $3.41 a gallon. She drove 80 miles in 2.5 hours. How many gallons did she use per hour? What is the rate of miles per hour?

3. Mary drinks 6½ glasses of water every day. How many glasses does she drink in a week?

4. Vanosia wants to rent a SUV for $160.00 for 5 days. What is the daily car rental?

5. It takes 23 minutes per pound to cook a turkey. Mrs. Hinson bought a 9.8 pound turkey. How many minutes will it take to cook?

6. At the Bay City Restaurant Jonnie and her 3 friends each orders Shrimp Caesar salad. If each salad costs $9.95, what is the total bill for the meals, excluding tax?

7. If each person in North America produces $3\frac{2}{3}$ pounds of garbage each day, how many pounds of garbage does each person produce in a week? In 30 days?

8. A recipe for a large batch of cookies calls for 3¼ cups of flour. How much flour would be needed to make half of a batch of cookies?

9. Alexis and Cathy went to the mall. Alexis spent ½ of her money, and Cathy spent ¼ of her money. Is it possible for Cathy to have spent more money than Alexis? Explain your reasoning.

10. If you have 1¼ hours to finish 4 chores, how long will it take you to do each task if you divide the time evenly?

11. You have 6 pints of ice cream. How many servings are there if one serving is ¾ of a pint?

12. Sam has 2½ gallons of water at the campsite. It takes ¾ of a gallon to make a batch of Gatorade. How many batches Gatorade can he make?