MODULE ONE

This module addresses the foundational concepts and skills that support <u>all</u> of the Elementary Algebra academic standards.

SC Academic Elementary Algebra Indicators included in this module are:

EA-2.1 Exemplify elements of the real number system (including integers, rational numbers and irrational numbers).
EA-2.5 Carry out a procedure using the properties of real numbers (including commutative, associative, and distributive) to simplify expressions.
EA-2.6 Carry out a procedure to evaluate an expression by substituting a value for the variable.
EA-2.7 Carry out a procedure (including addition, subtraction, multiplication, and division by a monomial) to simplify polynomial expressions.
EA-2.9 Carry out a procedure to perform operations with matrices (including addition, subtraction, and scalar multiplication).
EA-2.10 Represent applied problems by using matrices.

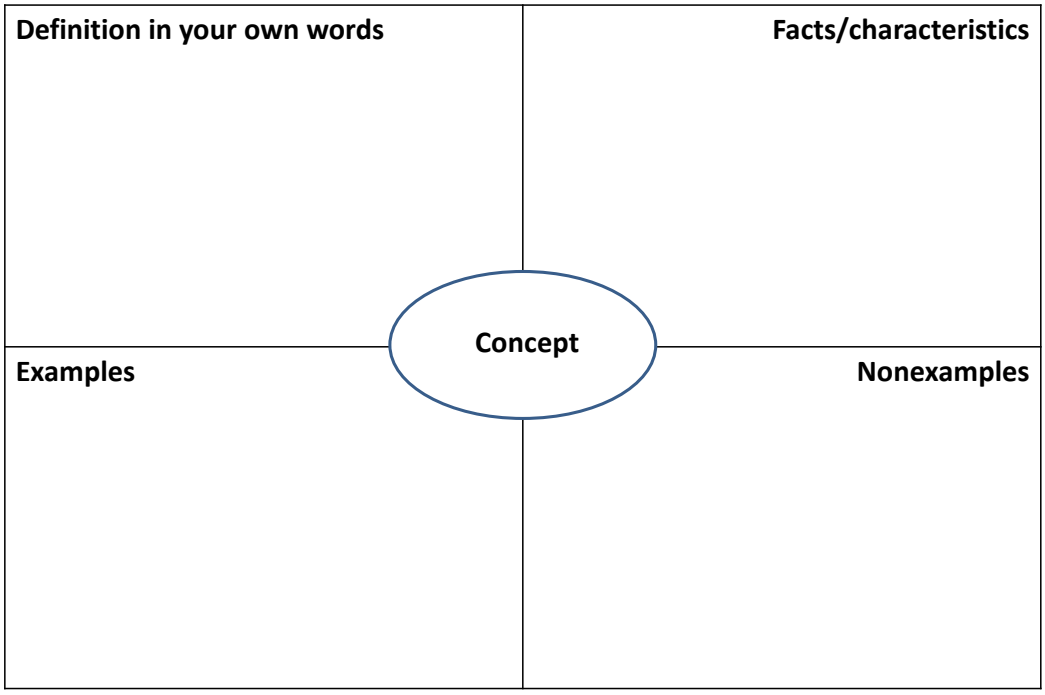
The resources provided in this module are not all inclusive. They are provided to begin to build the conceptual foundation students need. Additional resources will be required to develop the concepts.

The following Frayer model strategy can be used during Lesson #2 as a strategy to help students develop a conceptual understanding of the real number system and as a tool for keeping track of their learning. This strategy can be adapted to use with other Elementary Algebra Lessons.

What is a Frayer Model? A Frayer Model is a concept map that can help students develop an understanding of topics. This strategy provides students with an opportunity to develop both examples and nonexamples of a concept, helping them further refine their understanding of that concept.

Example:
Below is a template that might be used in Elementary Algebra Lessons. Also, an example of how this template might be used has been inserted. In place of copying this template, students may also create their own Frayer Models. See: http://wvde.state.wv.us/strategybank/FrayerModel.html

Frayer Model



Frayer Model

Definition in your own words	**Facts/characteristics**
A number that can be expressed as the quotient of two integers x/y, y ≠0.	•Any number that can be written as a fraction •Any decimal that is either repeating or terminates
Rational Numbers	
Examples	**Nonexamples**
-3 -¼ -5 -1.25	1.333333…. √7 √-3

NOTE: *The responses in the above Frayer Model are not all inclusive. They are provided to give an example of possible student responses.*

Lesson # 1
Topic: Writing and Evaluating Algebra Expressions
Standard (s): EA – 2.6

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills related to this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 6th grade students apply order of operations to simplify whole-number expressions (6-3.2). Students write variable expressions to represent quantities (6-3.3).
  - In Elementary Algebra, students use substitution to find a numerical value for an expression (evaluate). Student understanding should exceed rote operational proficiency.
  - This essential skill is necessary in all subsequent study of mathematics.

- **Taxonomy**
  3 -C
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  Evaluation
  Simplification
  Substitution

II. Teaching the Lesson

Students have been writing algebraic expressions since 6th grade but may need review their conceptual understanding of variables and uses. They have also applied to order of operations to simplify expression and may need a review of this concept. In this lesson, students become fluent in evaluating algebraic expressions (EA-2.6) which an essential skill which is used to check solutions to linear equations, to determine points on a line and to write linear equations.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:
  - Understand how variables are used to represent numerical quantities.

- Substitute values for one or more variables.
- Evaluate algebraic expressions for specified real numerical values.
- Evaluate algebraic expressions that may involve square roots and/or exponents

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.
  - Evaluate x + y for x = 0.3, y = 2/5
  - Evaluate x^2 for x = $\sqrt{3}$
  - Evaluate $\frac{xy^2z^3}{y}$ for $x = 5$, $y = 3$, and $z = -2$

- **Non-Essential Learning and Understanding**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - Substitute and evaluate expressions that are not related to the indicators of Elementary Algebra.
  - Substitute and evaluate expressions involving imaginary numbers.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - Evaluate sin x for x = ∏/2
  - Evaluate log x for x = 100

- **Misconceptions/Common Errors**
  - Students may not recognize xy, (x)(y), and x•y, as x multiplied by y.
  - Students may misapply the Order of Operations.

- **Technology**
  - Students may use technology for complex computation.
  - Students may use the STO on TI graphing calculators to evaluate expressions.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to carry out a procedure to evaluate an expression by substituting values for*

variables. Therefore, the primary focus of the assessment should be for students to carry out such procedures.

- **Assessment Item Examples:**

  - If $a = 3$ and $b = {}^{-}2$, what is the value of the expression $\frac{2a+ab^3}{7(a-5b)}$?

    A. $\frac{-210}{31}$

    B. $\frac{-210}{91}$

    C. $\frac{-18}{31}$

    D. $\frac{-18}{91}$

  - The area of a circle is given by the formula $A = \pi r^2$. Find A when $r = 3$.

    A. 3π
    B. 6π
    C. 9π
    D. 36π

  - The formula for finding the perimeter of a square is P = 4s. Find P when s = 5.

    A. 9
    B. 4
    C. 20
    D. 16

Lesson # 2
Topic: The Real Number Line and Operations with Integers
Standard (s): EA – 2.1

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills related to this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 6^{th} grade, students applied order of operations (6-3.2). In 8^{th} grade students compare rational and irrational numbers by using the symbols ≤, ≥, <, >, and = (8-2.4). Also, students apply an algorithm to add, subtract, multiply, and divide integers (8-2.1).
  - In Elementary Algebra students find examples of the following subsets of the real numbers: integers, rational numbers, and irrational numbers.
  - In Intermediate Algebra, students build on their knowledge of the real number system by studying complex numbers. Intermediate Algebra students carry out a procedure to simplify expressions involving powers of *i* (IA-3.1). In addition, Intermediate Algebra students carry out a procedure to perform operations with complex numbers (including addition, subtraction, multiplication, and division) (IA-3.2).

- **Taxonomy**
  2 -B
  Cognitive Process Dimension: Understand
  Knowledge Dimension: Conceptual Knowledge

- **Key Concepts**
  Real Numbers
  Integers
  Rational Numbers
  Irrational Numbers

II. Teaching the Lesson

In this lesson, students not only focus on the four arithmetic operations but also transfer their understanding of the whole number system to integers, rational and irrational numbers. The real number line is an example tool for demonstrating the relationship among the types of numbers. Students performed operations with integers in 8^{th} grade but may require a short review of this concept.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:
  - Give examples of integers, rational numbers, and irrational numbers.
  - Understand how integers, rational numbers, and irrational numbers are interrelated.
  - Classify numbers as integers, rational numbers, or irrational numbers.
  - Perform the four arithmetic operations with integers

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that student should able to successfully complete.
  - Identifying 5 is an integer and a rational number.
  - Provide an example of a number that is an integer and a rational number, such as -7.
  - Identify $0.\overline{66}$ as a rational number.
  - Give an example of a rational number such as - ⅜.
  - Identify 0.25 as a rational number.
  - Provide an example of an irrational number, such as 0.676676667... (does not repeat in a pattern or terminate) or ∏.
  - Identify $\sqrt{2}$ as an irrational number.
  - Explain why $\sqrt{7}$ is irrational. (A formal proof is not required).
  - 5 + (-15)
  - $\frac{-20}{-10}$
  - $9 \bullet -5$

- **Non-Essential Learning and Understanding**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - Know additional subsets of the real number system (whole numbers and counting numbers).
  - Study imaginary numbers.
  - Study complex numbers.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - Identify 7 as a counting number.

- Know that infinitely many irrational numbers are between 0 and 1 on the number line.
- Give an example of a complex number, such as 5 + 3i.

- **Misconceptions/Common Errors**
  Students may not identify integers as a subset of the rational numbers. Therefore, students may fail to classify integers as rational numbers.

- **Technology**
  Students may use technology to identify the existence of a pattern when calculating a quotient.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to exemplify elements of the real number system. Therefore, the primary focus of the assessment should be for students to give examples of integers, rational numbers, and irrational numbers. Because this is conceptual knowledge, assessments should test the student's ability to apply this concept to any integer, rational number, or irrational number, not to be restricted to memorized examples. Students should understand the interrelationships among integers, rational numbers, and irrational numbers.*
In addition to exemplify, students should be able to:
Classify numbers as integers, rational numbers, or irrational numbers.

- **Assessment Item Examples**

  - (-15)(-3)
  - -15 ÷ 3
  - -15 + (-3)
  - -15 – (-3)

  - Which number is not rational?
    A. 4
    B. 0.3
    C. $\sqrt{3}$
    D. 6.17

  - Which number is rational?
    A. 2.151551555…
    B. $7.\overline{07}$
    C. $\sqrt{2}$
    D. π

  - Non-repeating, non-terminating decimals are always

A. Irrational
B. Rational
C. Natural
D. Integers

- The number -4 is which of the following?
  A. An irrational number
  B. A natural number
  C. An integer
  D. A positive number

IV. *Resources*

See Frayer model example on pages 1 and 2 of this module.

Lesson # 3
Topic: Combining like terms
Standards (s): EA – 2.7 (addition and subtraction)

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 8th grade students apply order of operations to simplify whole-number expressions (6-3.2). Apply an algorithm to add, subtract, multiply, and divide integers (8-2.1). Understand equivalent symbolic expressions as distinct symbolic forms that represent the same relationship (6-1.4, 7-1.4, and 8-1.4). Use commutative, associative, and distributive properties to examine the equivalence of a variety of algebraic expressions (8-3.3).
  - In Elementary Algebra students carry out a procedure (including addition, subtraction) to simplify polynomial expressions. NOTE: Polynomial multiplication will be addressed in Lesson #5 of Module 1 and division by a monomial will be addressed in Module Eight.
  - In Intermediate Algebra, students build on their knowledge of the real number system by studying complex numbers. Intermediate Algebra students carry out a procedure to perform operations (including multiplication, exponentiation, and division) with polynomial expressions (IA-4.1). Also, students carry out procedures to perform operations on polynomial functions (including $f(x) + g(x)$, $f(x) – g(x)$, $f(x) \bullet g(x)$, and $f(x)/g(x)$) (IA-2.5).

- **Taxonomy**
  3 -C
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  Expressions
  Terms
  Monomial
  Polynomial

II. Teaching the Lesson

In this lesson students become fluent in procedures to simplify polynomial expression using addition, subtraction. NOTE: Multiplication and division by a monomial will be addressed in Module Eight of Elementary Algebra. Students use their knowledge of operations with integers, real number properties and order of operations to simplify expressions.

- **Essential Learning and Understanding**
  It is essential for students to understand the following for the attainment of this indicator:
  - Real number properties
  - Adding, subtracting polynomial expressions

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should able to successfully complete.
  - $(x + 5) + (2x + 3)$
  - $(2x^3 + 5x^2 - 3) + (-4x^3 + 9x^2 - 1)$
  - $(3x^4 - 8x^3 + 2x^2 - 3) + (6x^3 - 2x^2 + 5)$
  - $(x + 5) - (2x + 3)$
  - $(2x^3 + 5x^2 - 3) - (-4x^3 + 9x^2 - 1)$
  - $(3x^4 - 8x^3 + 2x^2 - 3) - (6x^3 - 2x^2 + 5)$
  - NOTE: Polynomial multiplication will be addressed in Lesson #5 of Module 1 and division by a monomial will be addressed in Module Eight.

- **Non-Essential Learning and Understanding**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - Multiplication and division by a monomial do not need to be addressed in this module. Polynomial multiplication will be addressed in Lesson #5 of this Module and division by a monomial will be addressed in Module Eight.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - $x^2(2x^2 - 3x - 9)$
  - $3(x + 8) - 2(x^2 - 4x - 12)$

- $2x(x^3 - 2x - 3) + x^2(-3x^2 + 5x - 10)$
- $\frac{2x^2 - 14x + 6}{2}$
- $\frac{-3x^3 + 21x^2 - 15x}{x}$

- **Misconceptions/Common Errors**
  - When subtracting two polynomials students may forget to distribute the negative thus subtracting only the first term of the subtrahend rather than the entire polynomial provided that the polynomial has more than one term. (minuend – subtrahend = difference)

- **Technology Note**
  - Students may use computer algebra system technology, which is capable of performing symbolic manipulations, to verify solutions.

III. Assessing the Lesson

Assessment Guidelines *The objective of this indicator is to carry out a procedure (including addition and subtraction) to simplify polynomial expressions. Therefore, the primary focus of the assessment should be for students to carry out such procedures.*

- **Assessment Item Examples**

  - What is $(x^2 + 3x + 4) - (3x^2 + x - 1)$?
    A. $4x^2 + 4x + 3$
    B. $-2x^2 + 2x + 5$
    C. $-2x^2 + 4x + 3$
    D. $4x^2 + 2x + 3$

  - What is $(x^4 - 3x^2 + 7) + (2x^3 - 1)$?
    A. $x^4 + 2x^3 - 3x^2 + 6$
    B. $3x^7 - 3x^2 + 6$
    C. $x^4 + 2x^3 - 3x^2 + 8$
    D. $3x^7 - 3x^2 + 8$

IV. Resources

Activity: Students may have difficulty determining which terms are alike. Using different color markers to identify like terms or writing the terms on different colors squares may help. For example, all the x^2 terms can be on red squares, the constants on blue squares and the x terms on white squares. Student sort them by color and then discuss common characteristics (other than color).

Lesson # 4
Topic: Real Number Properties (Including Identity, Commutative and Associative)
Standards (s): EA – 2.5

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills related to this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 8th grade students use commutative, associative, and distributive properties to examine the equivalence of a variety of algebraic expressions (8-3.3).
  - In Elementary Algebra students carry out a procedure using the properties of real numbers (including identity, commutative and associative) to simplify expressions (EA-2.5). The distributive property will be addressed in Lesson #5 of this module.
  - These essential skills are necessary in all subsequent study of mathematics.

- **Taxonomy**
  3 -C
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  Properties of real numbers
  Commutative Property
  Associative Property
  Expressions
  Terms

II. Teaching the Lesson

In this lesson, students gain an understanding of commutative and associative properties by applying them to simplify algebraic expressions. Students also gain a deeper conceptual understanding of the concept of equivalency. Students used these properties, in the previous lesson, to address the addition and subtraction portions of indicator EA – 2.7 (carry out addition and subtraction to simplify polynomial expressions). In Lesson #5, students will apply the distributive property to multiply polynomial

expressions. In Module Eight, students will carry out a procedure to divide a polynomial by a monomial (the division portion of EA-2.7).

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:
  - Use the following properties of the real number system to simplify expressions: Commutative and Associate Properties.
    Identity Property of Addition
    $a + 0 = a$ and $0 + a = a$
    Identity Property of Multiplication
    $a \bullet 1 = a$ and $1 \bullet a = a$
    Commutative Property of Addition
    $a + b = b + a$
    Commutative Property of Multiplication
    $ab = ba$
    Associative Property of Addition
    $(a + b) + c = a + (b + c)$
    Associative Property of Multiplication
    $(ab)c = a(bc)$
  - Simplify algebraic expressions, including numerical expressions.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.
  Write an equivalent expression:
  - $x + (y + 2) =$
  - $a(bc) =$
  - $y + z =$
  - $ts =$
  - $a \bullet 1 =$
  - $0 + a =$

- **Non-Essential Learning and Understanding**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - Identify which property is used to justify equivalent expressions.
  - Demonstrate knowledge of additional properties of the real number system beyond the Associative, Commutative, and Identity Property. The Distributive Property will be addressed in Lesson #5 of Module One.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - a(x + y) = ax + ay is true by what property?
  - (m + n) + p = m + (n + p) according to what property?
  - 7 = 7 according to what property?
  - Create an example of the Commutative Property of real numbers using symbols or numbers

- **Misconceptions/Common Errors**
  - None noted.

- **Technology**
  - Students may use technology to verify equivalent expressions by substituting values and/or using computer algebra systems (CASs).
  - Students may use computer algebra system technology, which is capable of performing symbolic manipulations to verify solutions.

III. Assessing the Lesson

Assessment Guidelines: The objective EA-2.5 is to carry out a procedure using the properties of real numbers (including commutative and associative) to simplify expressions. Therefore, the primary focus of the assessment should be for students to carry out such procedures.

- **Assessment Item Examples**

  - Write an equivalent expression for each of the following:
    - (xy)z =
    - (x + y) + z =
    - 1•x =
    - 0 + z =
    - d + (c + b) =

IV. *Resources*

- **Activity:** Have students create models of the properties using objects or pictures instead of numbers. They can cut out pictures from magazines or draw them. For example, ♥ + ♦ = ♦ + ♥.

Lesson # 5
Topic: Distributive Property
Standard (s): EA-2.5 and EA-2.7

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills related to this standard. It is recommended that students are pre-assessed on this prior knowledge.

- Continuum of Knowledge
  - In 8th grade students apply order of operations to simplify whole-number expressions (6-3.2); apply an algorithm to add, subtract, multiply, and divide integers (8-2.1); understand equivalent symbolic expressions as distinct symbolic forms that represent the same relationship (6-1.4, 7-1.4, and 8-1.4); use commutative, associative, and distributive properties to examine the equivalence of a variety of algebraic expressions (8-3.3).
  - In Elementary Algebra students carry out a procedure (the distributive property) to simplify polynomial expressions. In Module 8 of Elementary Algebra, students carry out a procedure to divide polynomials by a monomial.
  - In Intermediate Algebra, students build on their knowledge of the real number system by studying complex numbers. Intermediate Algebra students carry out a procedure to perform operations (including multiplication, exponentiation, and division) with polynomial expressions (IA-4.1). Also, students carry out procedures to perform operations on polynomial functions (including $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \bullet g(x)$, and $f(x)/g(x)$) (IA-2.5).

  - **Taxonomy**

    Indicator EA-2.5:
    3 -C
    Cognitive Process Dimension: Apply
    Knowledge Dimension: Procedural Knowledge

    Indicator EA-2.7 (multiplication):
    3 -C
    Cognitive Process Dimension: Apply
    Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  Distributive Property
  Expressions
  Terms
  Monomial
  Binomial
  Trinomial
  Polynomial

II. Teaching the Lesson

In this lesson, students use the distributive property to multiply polynomial expressions.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:
  - Add, subtract and/or multiply polynomial expressions.
  - Simplify algebraic expressions, including numerical expressions.

- **Examples of Essential Tasks**
  - These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.
    - $-2x^2 (x^2 - 10)$
    - $3x(x^2 - 2x + 9) - 2(x^3 + 8x - 12)$
    - $(x - 3)(2x + 4)$
    - $(4x + 3)(-x - 7)$
    - $(x + 5)(x - 5)$
    - $(x - 1)(-4x^2 + x + 12)$

- **Non-Essential Learning and Understanding**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - Multiply polynomial expressions that exceed the number of terms in a binomial times a trinomial.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for

the attainment of other indicators within Elementary Algebra.

- $(5x^2 - x + 3)(-7x^2 + 8x + 12)$

- **Misconceptions/Common Errors**
  - Students may not properly apply the distributive property stating that $3(x + 2) = 3x + 2$ instead of the correct answer, which is $3(x + 2) = 3x + 6$.
  - Students may not properly distribute a term if the term is found at the end of the expression. Example: $(x - 3)x$

- **Technology**
  - Students may use technology to verify equivalent expressions by substituting values and/or using computer algebra systems (CASs).
  - Students may use computer algebra system technology, which is capable of performing symbolic manipulations to verify solutions.

III. Assessing the Lesson

Assessment Guidelines: The objective of EA-2.7 is to carry out a procedure to use multiplication to simplify polynomial expressions. Therefore, the primary focus of the assessment should be for students to carry out such procedures.

- **Assessment Item Examples**

  - Simplify: $x^2(2x^2 - 3x - 9)$
    A. $2x^4 - 3x^2 - 9$
    B. $2x^2 - 3x - 9x^2$
    C. $2x^4 - 3x^3 - 9x^2$
    D. $2x^2 - 3x^3 - 9$

  - Simplify: $3(x + 8) - 2(x^2 - 4x - 12)$
    A. $-2x^2 - 5x$
    B. $-2x^2 + 11x + 48$
    C. $-2x^2 - x - 4$
    D. $-2x^2 + 7x + 20$

  - Simplify: $(x - 9)(x + 8)$
    A. $X^2 - x - 72$
    B. $X^2 + 17x + 72$
    C. $X^2 - x + 72$
    D. $X^2 + 17x - 72$

- Simplify: $(2x + 3)(3x – 8)$
  A. $5x^2 + 12x + 5$
  B. $5x^2 – 5$
  C. $6x^2 + 25 + 24$
  D. $6x^2 – 7x – 24$

- Simplify: $(x + 5)(x^2 – 5x – 1)$
  A. $X^2 – 26x – 5$
  B. $X^3 + 10x^2 + 26x + 5$
  C. $X^3 – 26x – 5$
  D. $X^3 + 10x^2 – 26x - 5$

Lesson # 6
Topic: Operations with matrices
Standard (s): EA–2.9 and EA–2.10

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 8th grade students organize data in matrices or scatterplots as appropriate (8-6.2).
  - In Elementary Algebra students carry out a procedure to perform operations with matrices (including addition, subtraction, and scalar multiplication) and represent applied problems using matrices.
  - In Geometry, students apply transformations (including translation and dilation) to figures in the coordinate plane by using matrices (G-6.4).

- **Taxonomy Level**
  Indicator EA-2.9:
  3-C
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

  Indicator EA-2.10:
  2-C
  Cognitive Process Dimension: Understand
  Knowledge Dimension: Conceptual Knowledge

- **Key Concepts**
  Matrix
  Row
  Column
  Dimension (size of a matrix)
  Matrix addition
  Matrix subtraction
  Scalar
  Scalar multiplication

II. Teaching the Lesson

In this lesson, students apply their knowledge of operations with integers to perform operations with matrices. Although students have organized data into matrices in the eighth grade, a review of this concept will be useful in setting the context for operations with matrices.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

  Indicator EA-2.9:
  - Add two matrices of size no larger than a 3x3 matrix.
  - Subtract two matrices of size no larger than a 3x3 matrix.
  - Multiply a matrix by a scalar for matrices of size no larger than a 3x3.

  Indicator EA-2.10:
  - Represent real-world data using matrices, no larger than a 3x3 matrix
  - Distinguish relevant from irrelevant data.
  - Represent data using matrices, understanding the meaning of columns and rows in the applied situation.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

  Indicator EA-2.9:
  Perform the operation or explain why it is not possible.
  - $\begin{bmatrix} 7 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -5 \end{bmatrix}$
  - $\begin{bmatrix} 6 & 0 \end{bmatrix} - \begin{bmatrix} 3 & -4 \end{bmatrix}$
  - $\begin{bmatrix} 2 & -1 \\ -8 & 7 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix}$
  - $\begin{bmatrix} 2 & -1 \\ -8 & 7 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix}$
  - $\begin{bmatrix} 2 & -1 \\ -8 & 7 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 8 & -7 \end{bmatrix}$

- $\begin{bmatrix} 2 & -1 \\ -8 & 7 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -8 & 7 \end{bmatrix}$
- $\begin{bmatrix} 2 & -1 \\ -8 & 7 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- $\begin{bmatrix} 2 & -13 \\ 0 & 17 \\ -1 & 20 \end{bmatrix} - \begin{bmatrix} 1 & -4 \\ 6 & -1 \\ 5 & 0 \end{bmatrix}$
- $\begin{bmatrix} 2 & 6 & 0 \\ 4 & 1 & -8 \end{bmatrix} + \begin{bmatrix} 9 & -6 & 1 \\ 7 & 0 & -1 \end{bmatrix}$
- $\begin{bmatrix} 2 & 6 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 9 & -6 & 1 \\ 7 & 0 & -1 \end{bmatrix}$ (Explain why it is not possible to perform the operation.)
- $\begin{bmatrix} 1 & 7 & -6 \\ 9 & 1 & 10 \\ -11 & 12 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 7 & 19 \\ 3 & 0 & 7 \\ -5 & 12 & 1 \end{bmatrix}$
- $2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $-7\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- $4\begin{bmatrix} 3 & 11 & -5 \\ -6 & 0 & 10 \\ 8 & 2 & 1 \end{bmatrix}$
- $-4\begin{bmatrix} 3 & 11 & -5 \\ -6 & 0 & 10 \\ 8 & 2 & 1 \end{bmatrix}$

Indicator E-2.10
Example 1: The Table A gives the number of people (in thousands) who visited Australia and South Africa in 1998. Figures are rounded to the nearest 10,000. Source: The New York Times, January 14, 2000.

Table A	*To*	
From	**Australia**	**South Africa**
North America	440	190
Europe	950	950
Asia	1,790	200

Represent the tourism data in Table A using a 3x2 matrix.

Example 2: The following is sales data from a department store sales person who works part time.

Sales Data:
Monday: 10 shirts, 8 pants, 10 shorts, 2 hats
Tuesday: 9 shirts, 13 pants, 7 shorts, 0 hats
Wednesday: 15 shirts, 14 pants, 6 shorts, 7 hats

Represent the sales data using a matrix.

Example 3: In a discount department store, similar items sale for the same price. Shirts are $18, pants are $25, shorts are $13, and hats are $10. Represent the price of the items using a matrix.

Example 4: The number of grams of protein, carbohydrates, and fats are given for three samples of food. Each ounce of Food I contains 6 grams of protein, 12 grams of carbohydrates, and 37 grams of fat. Each ounce of Food II contains 10 grams of protein, 5 grams of carbohydrates, and 32 grams of fat. Each ounce of Food III contains 12 grams of protein, 18 grams of carbohydrates, and 77 grams of fat. Represent the number of graphs of protein, carbohydrates, and fats for each ounce of the three food samples using a matrix.

- **Non-Essential Learning and Understanding**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

Perform matrix multiplication.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - $\begin{bmatrix} 3 & -1 \\ 8 & 7 \end{bmatrix} x \begin{bmatrix} -1 & 5 \\ 1 & 3 \end{bmatrix}$

- **Misconceptions/Common Errors**
  - Students may forget to distribute the negative to each term in the subtrahend when subtracting matrices. [minuend – subtrahend = difference]

- **Technology Note**
  - Students may use technology to verify solutions.

III. Assessing the Lesson

Assessment Guidelines: The objective of this indicator is to carry out a procedure to perform operations with matrices (including addition, subtraction, and scalar multiplication). Therefore, the primary focus of the assessment should be for students to carry out such procedures.

- **Assessment Item Examples**
  - Explain why each example is or is not possible. If possible, perform the operation:

$$\begin{pmatrix} 7 & 9 \\ 2 & 7 \\ 5 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 8 \\ 9 & 0 & 3 \end{pmatrix} =$$

$$4 \begin{bmatrix} 1 & 3 & 5 \\ -1 & -8 & 10 \\ -7 & -5 & 13 \end{bmatrix}$$

$$-3 \begin{bmatrix} 1 & 3 & 5 \\ -1 & -8 & 10 \\ -7 & -5 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$$