MODULE

EIGHT

This module addresses exponents.

SC Academic Elementary Algebra Indicators included in this module are:

EA-2.2 Apply the laws of exponents and roots to solve problems.

EA-2.3 Carry out a procedure to perform operations (including multiplication and division) with numbers written in scientific notation.

EA-2.7 Carry out a procedure (including addition, subtraction, multiplication, and division by a monomial) to simplify polynomial expressions.

The resources provided in this module are not all inclusive. They are provided to begin to build the conceptual foundation students need. Additional resources will be required to develop the concepts.

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Lesson # 1
Topic: Law of exponents
Indicator (s): EA – 2.2 and EA - 2.7

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 6th grade students represent whole numbers in exponential form (6-2.9). In 7th grade students translate between standard for and exponential form (7-2.6), represent the location of rational numbers and square roots of perfect squares on a number line (7-2.2), compare rational numbers, percentages, and square roots of perfect squares by using the symbols ≤, ≥, <, >, and =, and Understand the inverse relationship between squaring and finding the square roots of perfect squares. In 8th grade students apply strategies and procedures to approximate between two whole numbers the square roots or cube roots of numbers less than 1,000.
  - In Elementary Algebra students apply the laws of powers (exponents) and roots (radicals) to solve problems. Students also simplify expressions that involve division by a monomial. Note: Carry out a procedure (including addition, subtraction and multiplication) to simplify polynomial expressions was addressed in Module One.
  - Intermediate Algebra students carry out a procedure to perform operations (including multiplication, exponentiation, and division) with polynomial expressions (IA-4.1). Carry out a procedure to simplify algebraic expressions involving rational exponents (IA-4.5). Carry out a procedure to simplify algebraic expressions involving logarithms (IA-4.6). Carry out a procedure to perform operations with expressions involving rational exponents (including addition, subtraction, multiplication, division, and exponentiation) (IA-4.7). Carry out a procedure to solve radical equations algebraically (IA-4.9). Carry out a procedure to graph exponential functions (IA-4.14).

- **Taxonomy**
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

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- **Key Concepts**
  Exponents
  Powers
  Base
  Reciprocal
  Radical
  Radicand
  Index
  Root
  Square root

II. Teaching the Lesson

In this lesson, students learn how to apply to laws of exponents to simplify expressions. In the next lesson, students combine their understanding of operations with integers and the laws of exponents to multiply and divide expressions written in scientific notation.
In addition to working with the laws of exponents, students simplify expressions involving roots, specifically, square roots.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

  - Evaluate expressions and simplify expressions using the following properties. The values of *a* and *b* are integral.
    - Product of powers with like bases ($x^a x^b = x^{a+b}$)
    - Power of a power ($\left(x^a\right)^b = x^{ab}$)
    - Power of a product ($(xy)^a = x^a y^a$)
    - Zero power of a nonzero number is 1 ($x^0 = 1, if\ \ x \neq 0$)
    - Reciprocal ($x^{-a} = \frac{1}{x^a}, if\ \ x \neq 0$)
    - Quotient of powers with like bases ($\frac{x^a}{x^b} = x^{a-b}, if\ \ x \neq 0$)
    - Powers of quotient ($\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}, if\ \ y \neq 0$)
    - The principle square root is positive.
    - $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ for a, b > 0
    - $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ for a, b > 0.

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- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should able to successfully complete.

  - Simplify $x^3 \cdot x^5$
  - Simplify $y^6 \cdot y^{-10}$
  - Simplify (- 5 rm)^2 ($\frac{1}{2}$ r^{3}m^4)^2
  - Simplify (7a)$^{-3}$
  - Evaluate (4^{-1})$^{-2}$
  - Evaluate 9^0
  - Evaluate $\left(\frac{2}{5}\right)^{-2}$
  - Simplify $\frac{-2x^{-1}}{8x^2y^{-3}}$
  - Simplify $\left(\frac{zy^3}{-5z^2y^{-3}}\right)^3$

  The next three problems require students to either 1) factor out a common factor and simplify or 2) separate the numerator using the common denominator and simplifying using the law of exponents.

  - $\frac{-10x+5}{5}$
  - $\frac{9x^3+3x-6}{3x}$
  - $\frac{6x^4-9x^3+3x^2}{3x^2}$
  - Simplify $\sqrt{2} \bullet \sqrt{3}$
  - Simplify $\sqrt{2} \bullet \sqrt{4}$
  - Simplify $\sqrt{90}$
  - Simplify $\sqrt{\left(\frac{81}{16}\right)}$
  - Simplify $\frac{\sqrt{27}}{\sqrt{3}}$

- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

- Problems involving rational exponents
- $\sqrt{a^2} = |a|$. This is non-essential knowledge for this indicator because the expectation is for students to simplify problems where the radicand will be numerical rather than a variable expression.
- Rationalizing a denominator

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - Evaluate $8^{\frac{1}{2}}$
  - Simplify $\frac{1}{\sqrt{2}}$
  - $\sqrt{7}, \sqrt{2}, etc$
  - $\frac{x^2 - x - 12}{x + 3}$
  - $\frac{-2x^2 + 33x - 54}{2x + 3}$

- **Misconceptions/Common Errors**
  - Sometimes students have difficulty determining the value of the expression x^0 ($x \neq 0$). By using the division rule $\frac{x^4}{x^4} = x^{4-4} = x^0$. If $x \neq 0$, then $\frac{x^4}{x^4} = 1$. Therefore, $x^0 = 1$, if $x \neq 0$.
  - If students do not conceptually understand how to simplify polynomial expressions they may mistakenly divide only a portion of the numerator by the denominator. That is,

$$\frac{2x^2 + 4x - 7}{2} = x^2 + 2x - 7$$

Or $$\frac{2x^2 - 6x + 9}{4} = \frac{x^2 - 3x + 9}{2}$$

  - If we go with the above radical laws, then note the following: Radicals with the same index can be multiplied and divided. However, radicals can not be directly added. That is, $\sqrt{9} + \sqrt{16} \neq \sqrt{25}$. In general, $\sqrt[n]{a} + \sqrt[n]{b} \neq \sqrt[n]{a+b}$.

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- **Technology**
  Students may use technology to verify solutions.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to carry out or use the laws of powers (exponents) and roots (radicals) to evaluate and to simplify expressions. Therefore, the primary focus of the assessment should be for students to carry out or use such procedures in given situations.*

- **Assessment Item Examples**

Simplify: x^5x^4

A. x^{20}
B. x
C. x^9
D. x^5
E.

Simplify: $(-9xy^5)^2(\frac{1}{3}x^2y^4)^2$

A. $2x^8y^{80}$
B. $2x^6y^{18}$
C. $9x^6y^{18}$
D. $9x^8y^{15}$

Simplify: 12^0

A. 12
B. 1
C. 0
D. -1

Simplify: $(6w)^{-1}$

A. $\frac{1}{6w}$
B. 6w
C. $\frac{1}{6}$ w
D. $\frac{6}{w}$

Simplify: $\frac{\sqrt{27}}{\sqrt{3}}$

A. 3 B. $\sqrt{3}$ C. 9 D. 1

Simplify: What is $\frac{14x^4 - 6x^2 + 12x}{2x}$

A. $7x^3 - 3x^2 + 6$
B. $7x^4 - 3x^2 + 6x$
C. $7x^3 - 3x + 6$
D. $14x^3 - 6x + 6$

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Lesson # 2
Topic: Scientific Notation
Indicator (s): EA – 2.3

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 7^{th} grade students translate between standard form and scientific notation (7-2.7).
  - In Elementary Algebra, students carry out a procedure to perform the operations of multiplication and division with numbers written in scientific notation. Student understanding should exceed rote operational proficiency.
  - This skill is necessary in the subsequent study of mathematics and other disciplines such as science and technology.

- **Taxonomy**
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  Standard notation
  Scientific notation
  Coefficient
  Base
  Power

II. Teaching the Lesson

In this lesson, student use their knowledge of operation with integers and the laws of exponents to multiply and divide expression written in scientific notation.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

  - Carry out a procedure to multiply numbers written in scientific notation.
    $(N \times 10^{x})(M \times 10^{y}) = (N)(M) \times 10^{x+y}$

- Carry out a procedure to divide numbers written in scientific notation. $\frac{N\,x10^{x}}{M\,x10^{y}} = \frac{N}{M}x10^{x-y}$
- Be able to compute with integers.
- Express the final product or quotient in scientific notation.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should able to successfully complete.

  - $(5.0 \times 10^3)(2.0 \times 10^6)$

    First, $5.0 \times 2.0 = 10.0$
    Second, $10^3 \times 10^6 = 10^{3+6} = 10^9$
    Finally, 10.0×10^9

  - $(2.3 \times 10^4)(6.1 \times 10^{-8})$

    First, $2.3 \times 6.1 = 14.03$
    Second, $10^4 \times 10^{-8} = 10^{4+(-8)} = 10^{-4}$
    Then, 14.03×10^{-4} (answer is not in scientific notation)
    Finally, 1.403×10^{-3} (answer is in scientific notation)

  - $\frac{(6.3 \times 10^7)}{(2.1 \times 10^4)}$

    First, $\frac{6.3}{2.1} = 3$

    Second, $\frac{10^7}{10^4} = 10^{7-4} = 10^3$

    Finally, 3.0×10^3

  - $\frac{(8.0 \times 10^4)}{(2.0 \times 10^{-2})}$

    First, $\frac{8.0}{2.0} = 4$

Second, $\frac{10^4}{10^{-2}} = 10^{4-(-2)} = 10^{4+2} = 10^6$

Finally, 4.0 x 10^6

- $\frac{(8.58 \times 10^{11})}{(3.3 \times 10^{18})}$

  First, $\frac{8.58}{3.3} = 2.6$

  Second, $\frac{10^{11}}{10^{18}} = 10^{11-18} = 10^{-7}$

  Finally, 2.6 x 10^{-7}

- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

  - To carry out procedures to add, subtract, or exponentiate numbers expressed in scientific notation.
  - To know how to express exponents using other notations, such as 345,000,000 written as 3.45E +8 or as 3.45 X 10^8.
  - To translate between standard form and scientific notation. The numbers will be given in scientific notation.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

  - 4.5 x 10^{11} + 3.29 x 10^9
  - Express 345,000,000 as 3.45E +8 or as 3.45 X 10^8.
  - (3,540,000,000) (7.22 x 10^6)

- **Misconceptions/Common Errors**
  Students may misapply the laws of exponents and/or misapply the rules for performing operations with integers.

- **Technology**
  Students may use technology to verify solutions. When using technology to verify solutions, students need to be familiar with the scientific notation used for the calculator. Students should be aware of the limitations of some calculators to perform multiplication or division of numbers written in scientific notation. Sometimes the answer will "overflow" the calculator. For example 3.1×10^{75} times 2.6×10^{35} will produce an error message on some scientific calculators and graphing calculators. In such a case, students could not use the technology to verify solutions. For example, when using the TI-84 plus Silver edition graphing calculator when the answer has an exponent that is greater than 99, then the answer will "overflow" the calculator producing and error message.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to carry out a procedure to perform the operations of multiplication and division with numbers written in scientific notation. Therefore, the primary focus of the assessment should be for students to carry out such procedures.*

- **Assessment Item Examples**

  - What is 547,000,000 in scientific notation?
    A 5.47×10^8
    B 5.47×10^9
    C 25.3×10^7
    D 253×10^6

  - What is $(2.0 \times 10^4)(3.0 \times 10^3)$?
    A. 5×10^7
    B. 6×10^7
    C. 5×10^{12}
    D. 6×10^{12}

  - What is $\frac{(2.4 \times 10^7)}{(1.2 \times 10^5)}$?

    A. 2.0×10^2
    B. 2.0×10^{12}
    C. 0.2×10^2
    D. 0.2×10^{12}

- What is $\frac{(3.6 \times 10^2)}{(1.8 \times 10^6)}$?

  A. 2.0×10^8
  B. 2.0×10^{-4}
  C. 2.0×10^{12}
  D. 2.0×10^{-12}

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