MODULE

NINE

This module addresses quadratic functions.

SC Academic Elementary Algebra Indicators included in this module are:

EA-2.8 Carry out a procedure to factor binomials, trinomials, and polynomials by using various techniques (including the greatest common factor, the difference between two squares, and quadratic trinomials).

EA-6.1 Analyze the effects of changing the leading coefficient *a* on the graph of $y = ax^2$.
EA-6.2 Analyze the effects of changing the constant *c* on the graph of $y = x^2 + c$.
EA-6.3 Analyze the graph of a quadratic function to determine its equation.
EA-6.4 Carry out a procedure to solve quadratic equations by factoring.
EA-6.5 Carry out a graphic procedure to approximate the solutions of quadratic equations.
EA-6.6 Analyze given information to determine the domain of a quadratic function in a problem situation.

The resources provided in this module are not all inclusive. They are provided to begin to build the conceptual foundation students need. Additional resources will be required to develop the concepts.

Introductory Activity: The following activity is a continuation of Lighting up the Sky (Part I) from Module 4. Students will be introduced to quadratic functions by comparing the graph they create in this problem to the graph of the linear function from Part I.

LIGHTING UP THE SKY

Part II

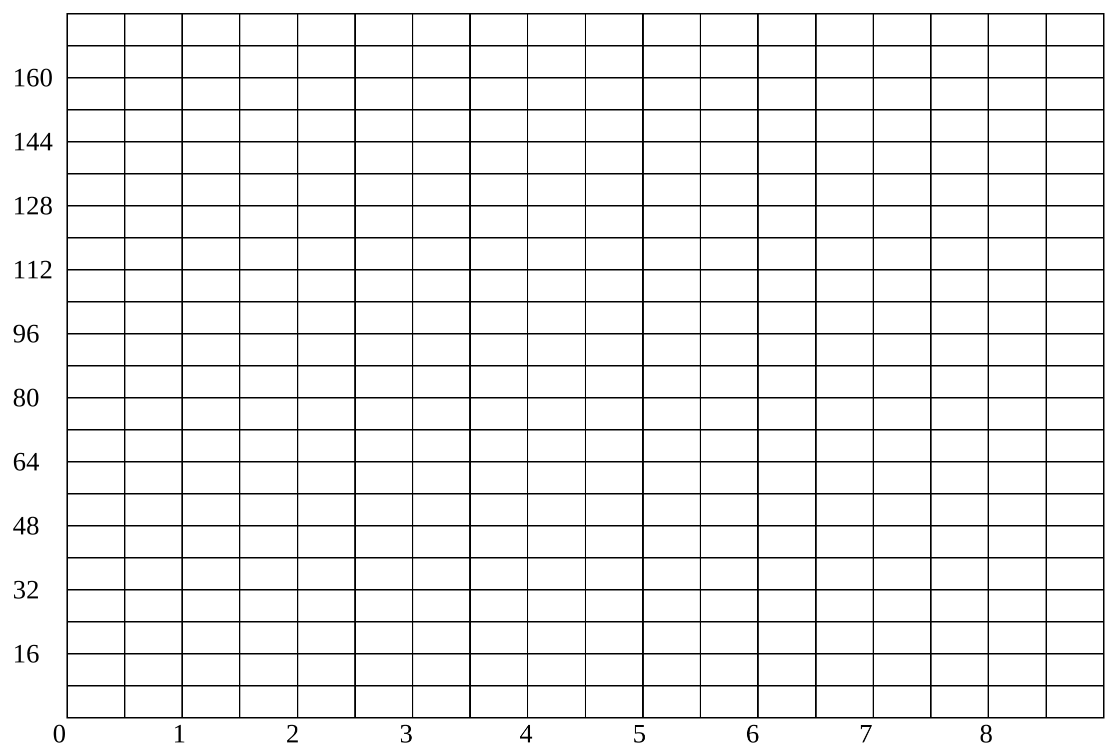
The school newspaper is doing an article on the Mr. Burton's science class project. The school photographer wanted to take a picture of the balloon launch but he needs to use a flare to create enough light to take a picture of the balloon as it rises.

The table below shows the height (feet) of the flare from the ground versus the time (seconds). The flare is set off 2 seconds after the balloon is launched.

Height of flare above the ground after 2 seconds	0 feet
Height of flare above the ground after 3 seconds	80 feet
Height of flare above the ground after 4 seconds	128 feet
Height of flare above the ground after 5 seconds	144 feet
Height of flare above the ground after 6 seconds	128 feet
Height of flare above the ground after 7 seconds	80 feet
Height of flare above the ground after 8 seconds	0 feet

1. Does the data in the table represent a **linear or quadratic** relationship? Explain your reasoning.

2. Label (name) the axes on the grid below. **Draw a graph** of the height (feet) of the flare from the ground versus time (seconds) from 2 seconds to 8 seconds.



3. What is the maximum height, in feet, that the flare reaches?

4. From the time the flare is set off, how many seconds will the flare take to reach its maximum height?

5. Redraw the graph for the height of the balloon from Part I question #2 onto the graph of the height of the flare (Part II # 2).

6. The photographer needs to take the picture when the flare is higher than the balloon. For approximately how many seconds will the height of the flare be greater than the height of the balloon? Explain how you determined your answer.

Lesson # 1
Topic: The effects of "a" in $y = ax^2$
Standard (s): EA – 6.1

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In eighth grade students translated between graphic and algebraic representations of linear functions (8-3.1) and identified the y-intercepts of linear equations from the graph (8-3.6)
  - In Elementary Algebra students investigate graphs of equations of the form $y = ax^2$ for different values of a in order to determine the result of a change in the leading coefficient on the graph of the equation.
  - In Intermediate Algebra, students graph transformations of parent functions (IA-2.8), Match the equation of a conic section with its graph (IA-5.7), and carry out a procedure to write an equation of a quadratic function when given its roots (IA-3.6).

- **Taxonomy**
  Cognitive Process Dimension: Analyze
  Knowledge Dimension: Conceptual Knowledge

- **Key Concepts**
  Leading coefficient
  Transformation
  The graph of a function
  The stretch of a graph
  The shrink of a graph

II. Teaching the Lesson

In this lesson, students examine how changes in the value of "a" affect the graph of $y = ax^2$. At this point, students may not have had experience with quadratic functions; therefore, an introductory activity that compares linear and quadratics functions may be needed. Exploring examples of quadratic functions in the real world such as the path of a baseball, the path of water from a fountain and the St. Louis Arch set the stages for this module.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

  - Interpret graphs including understanding how a graph is generated from a symbolic representation.
  - Recognize the leading coefficient in $y = ax^2$.
  - Determine the graphical effect of changing the sign of the leading coefficient in $y = ax^2$.
  - Determine the graphical effect of increasing the magnitude of the coefficient in $y = ax^2$.
  - Determine the graphical effect of decreasing the magnitude of the coefficient in $y = ax^2$.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

  - How do the graphs of $y = x^2$ and $y = -x^2$ differ?
  - How do the graphs of $y = 3x^2$ and $y = -3x^2$ differ?
  - How do the graphs of $y = ax^2$ and $y = -ax^2$ differ?
  - How do the graphs of $y = x^2$ and $y = 2x^2$ differ?
  - How do the graphs of $y = x^2$ and $y = \frac{1}{2}x^2$ differ?
  - How does changing the leading coefficient in $y = ax^2$ affect the graph?
  - What does halving the leading coefficient in an equation of the form $y = ax^2$ do to its graph?
  - Study the graphs of $y = ax^2$ for *a* = 1, *a* = -1, *a* = 2, *a* = -2, *a* = 0.5 and a = -0.5. Write a sentence describing how the sign of the leading coefficient affects the graph.
  - Study the graphs of $y = ax^2$ for *a* = 1, *a* = 2, *a* = 3, *a* = 0.5 and a = 1/3. Write a sentence describing how the relative magnitude of the leading coefficient affects the graph.

- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

  - Graph a large number of functions by hand.

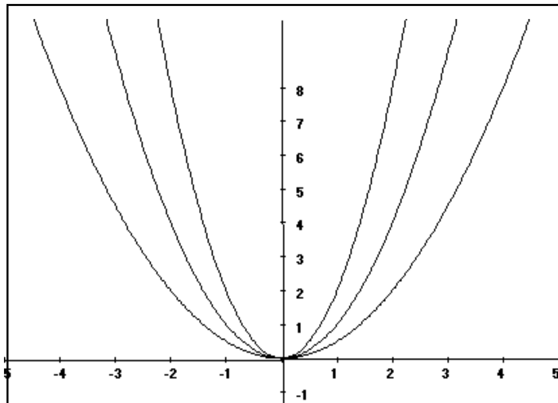
- Identify the components (e.g. vertex, directrix, focal point) of a quadratic graph.
- Analyze functions with irrational or imaginary coefficients

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

  - On graph paper, graph $y = ax^2$ for a = 1, a = 2, a = 3, a = 0.5 and a = 1/3. Be sure to label the vertex, the line of symmetry, the y-intercept and any zeros. Write a sentence describing how the relative magnitude of the leading coefficient affects the graph.
  - How do the graphs $y = \frac{5\Pi}{6}x^2$ and $y = ex^2$ differ?

- **Misconceptions/Common Errors**
  For $y = ax^2$, students assume the sign of the leading coefficient a is constant.

- **Technology**
  Students may use technology to graph multiple equations on the same set of axes for comparison. By examining differences among graphs on the same axes, students may more readily see the relationships between the equation and its graph.



The graphs of $y = ax^2$ **for** a = 0.5, a = 1 and a = 2

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to analyze the effects of changing the leading coefficient a on the graph of* $y = ax^2$***.*** *The primary focus of the assessment should be for students to determine how changes in leading coefficients of equations of this form result in differences among their graphs.*

- **Assessment Item Example**
  - How do the graphs of y = ax^2 and y = $-ax^2$ differ (a>0)?
    A. The graph of y = ax^2 opens up from the origin while the graph of y = $-ax^2$ opens down from the origin.
    B. The graph of y = ax^2 opens down from the origin while the graph of y = $-ax^2$ opens up from the origin.
    C. The graph of y = ax^2 opens right from the origin while the graph of y = $-ax^2$ opens left from the origin.
    D. The graph of y = ax^2 opens left from the origin while the graph of y = $-ax^2$ opens right from the origin.

Lesson # 2
Topic: The effects of "c" on $y = x^2 + c$
Standard (s): EA – 6.2

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In eighth grade students translated between graphic and algebraic representations of linear functions (8-3.1), and identified the y-intercepts of linear equations from the graph (8-3.6).
  - In Elementary Algebra students will investigate graphs of equations of the form $y = x^2 + c$ for different values of c in order to determine the result of a change in the constant in the equation.
  - In Intermediate Algebra, students graph transformations of parent functions IA-2.7, match the equations of a conic section with their graphs (IA-5.7), and carry out a procedure to write an equation of a quadratic function when given its roots (IA-3.6).

- **Taxonomy**
  Cognitive Process Dimension: Analyze
  Knowledge Dimension: Conceptual Knowledge

- **Key Concepts**
  Constant
  Graph of a function
  Vertical shift of a graph

II. Teaching the Lesson

In this lesson, students examine how changes in the value of "c" affect the graph of $y = x^2 + c$. The value of c affects the graph of quadratic functions in the same way c affect the graph of a linear function; therefore, it is important that students make this generalization. Technology may be used to illustrate the relationship among quadratic functions where only the value of c is changed (see Technology notes).

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

  - Interpret graphs including understanding how a graph is generated from a symbolic representation.
  - Recognize the constant in $y = x^2 + c$.
  - Determine the graphical effect of increasing the constant in $y = x^2 + c$.
  - Determine the graphical effect of decreasing the constant in $y = x^2 + c$.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

  - How do the graphs of $y = x^2 + 3$ and $y = x^2 + 5$ differ?
  - How does changing the constant c in $y = x^2 + c$ affect the graph?
  - What does decreasing the constant c by 2 units in an equation of the form $y = x^2 + c$ do to its graph?
  - Study the graphs of $y = x^2 + c$ for c = 1, c = 2, c = 3, c = -1 and c = -2. Write a sentence describing how changing the constant c in $y = x^2 + c$ affects the graph.

- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

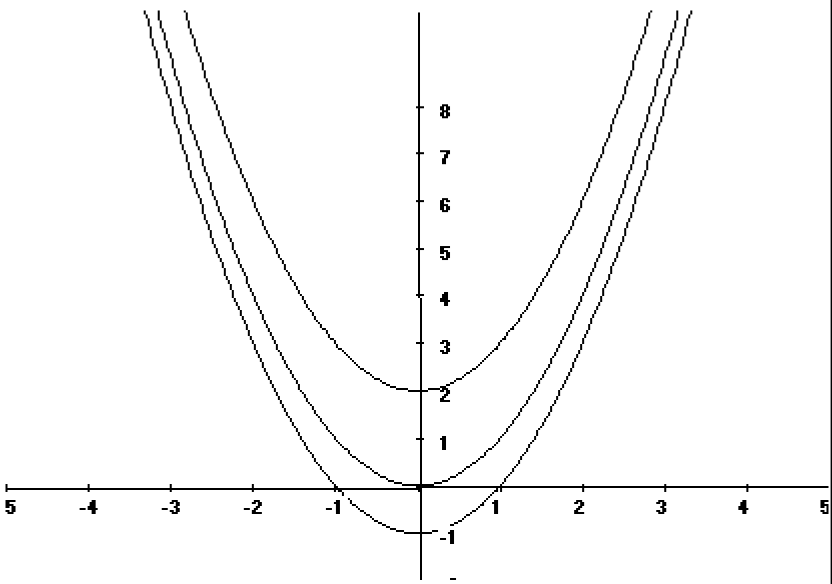
  - Graph a large number of functions by hand.
  - Identify the components (e.g., vertex, directrix, focal point) of a quadratic graph.
  - Analyze functions with irrational or imaginary coefficients.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

  - On graph paper, graph $y = x^2 + c$ for *c* = 1, *c* = 2, *a* = 3, *c* = 0.5 and *c* = 1/3. Be sure to label the vertex, the line of symmetry, the y-intercept and any zeros. Write a sentence describing how changing the constant c affects the graph.

- How do the graphs $y = x^2 + \frac{\Pi}{3}$ and $y = x^2 + \frac{22}{21}$ differ?

- **Misconceptions/Common Errors**
  For $y = x^2 + c$, students assume the sign of the constant c is positive.
- **Technology**
  Students may use technology to graph multiple equations on the same set of axes for comparison. By examining differences among graphs on the same axes, students may more readily see the relationships between the equation and its graph.



The graphs of $y = x^2 + c$ for *c* = 0, *c* = -1 and *c* = 2

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to* <u>*analyze*</u> *the effects of changing the constant term c on the graph of* $y = x^2 + c$. *The primary focus of the assessment should be for students to determine how changes in the constant term of equations of this form result on differences among their graphs.*

- **Assessment Item Examples**

  - What effect does the constant "c" have on the graph of the equation $y = ax^2 + c$?
    A. The constant c shifts the vertex of the graph left and right from the origin.
    B. The constant c shifts the vertex of the graph up and down from the origin.
    C. The constant c determines the magnitude of the graph.
    D. The constant c has no effect.

  - Choose the best description of the graph $y = x^2 + 5$.
    A. The graph opens up with the vertex at the point (0,-5).
    B. The graph opens down with the vertex at the point (0,5).
    C. The graph opens up with the vertex at the point (0,5).
    D. The graph opens down with the vertex at the point (0,-5).

  - Given the equation of the graph $y = x^2 + c$, how would increasing the value of c by 2 change the graph.
    A. The vertex would move right 2 units.
    B. The vertex would move up 2 units.
    C. The vertex would move down 2 units.
    D. The vertex would move left 2 units.

Lesson # 3
Topic: Determining a quadratic equation from the graph
Standard (s): EA – 6.3

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In eighth grade students translated between graphic and algebraic representations of linear functions (8-3.1), and identified the y-intercepts of linear equations from the graph (8-3.6).
  - In Elementary Algebra students will analyze the leading coefficient, constant and possibly the zeros of the graph of a quadratic function to determine its equation.
  - In Intermediate Algebra, students graph transformations of parent functions IA-2.7, match the equations of a conic section with their graphs (IA-5.7), and carry out a procedure to write an equation of a quadratic function when given its roots (IA-3.6).

- **Taxonomy**
  Cognitive Process Dimension: Analyze
  Knowledge Dimension: Conceptual Knowledge

- **Key Concepts**
  4.3 B
  Cognitive Process Dimension: Analyze
  Knowledge Dimension: Conceptual Knowledge

II. Teaching the Lesson

In this lesson, students analyze the graph of quadratic functions to determine its equation. Students are combining their understanding of the solutions of the equations and the effects of "a" and "c". Emphasizing that a quadratic function consists of two linear factors, $(x - x_1)(x - x_2)$ whose x-intercepts are x_1 and x_2, solidifies their conceptual understanding of how quadratic functions are structured.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

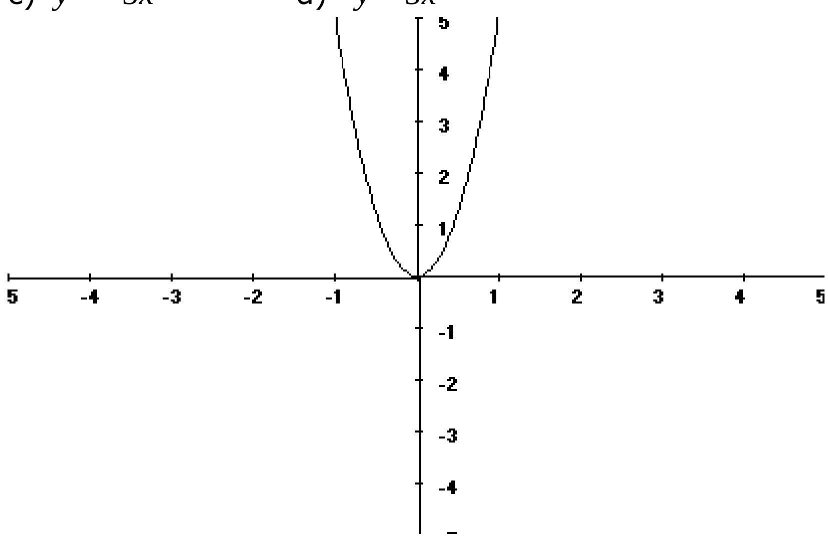
  - Associate the features of the graph with the leading coefficient or constant term of equations of the form $y = ax^2$ and $y = x^2 + c$.
  - Associate the x-intercepts of the graph with the factored form of the equation.
  - Interpret graphs including understanding how a graph is generated from a symbolic representation.
  - Determine the graphical effect of changing the sign of the leading coefficient in $y = ax^2$.
  - Determine the graphical effect of increasing the constant in $y = x^2 + c$.
  - Determine the graphical effect of decreasing the constant in $y = x^2 + c$.
  - Associate the x-intercepts of the graph of a quadratic function to its factored form.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

A possible equation for the graph below is:

a) $y = x^2 - 5$ b) $y = 5x$

c) $y = -5x^2$ d) $y = 5x^2$



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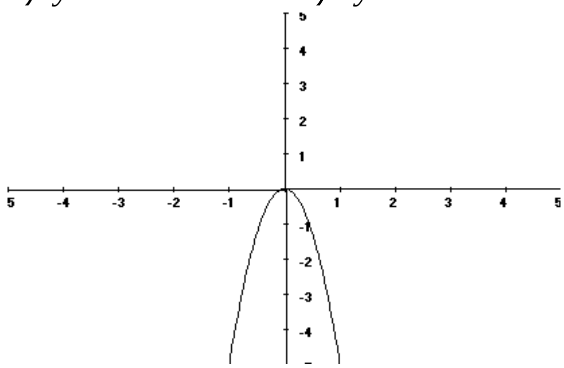
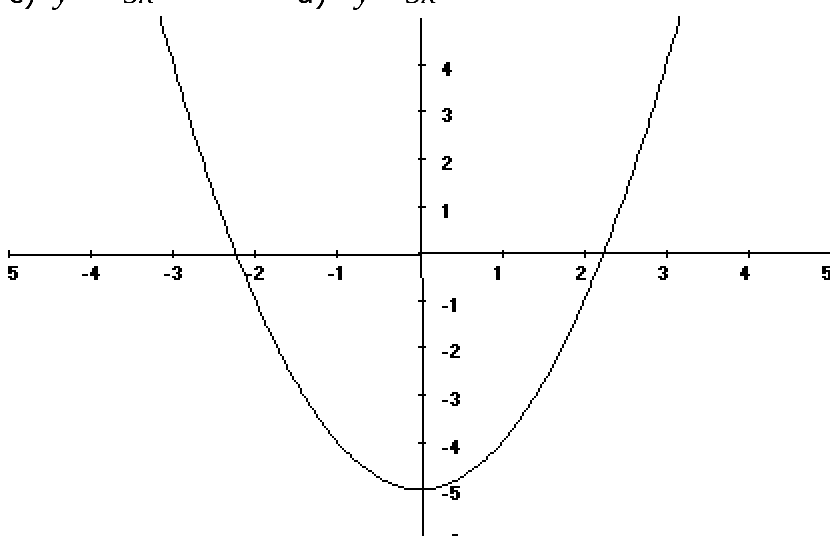


A possible equation for the graph below is:

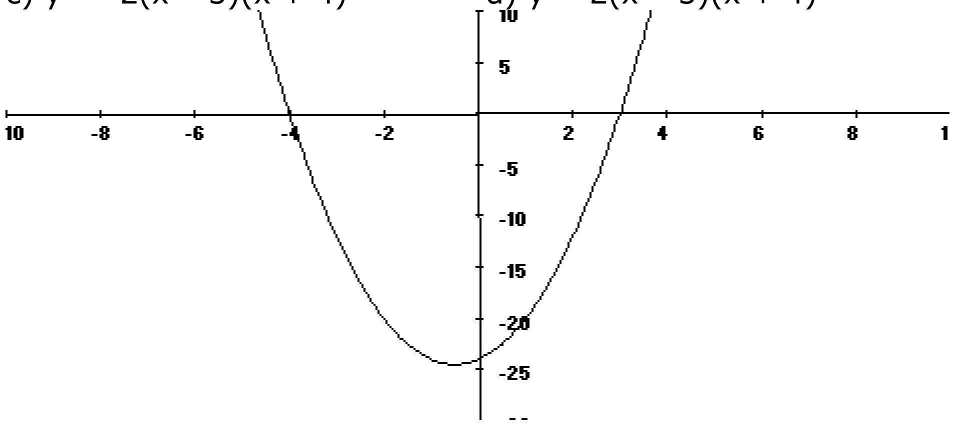
a) $y = x^2 - 5$ b) $y = x^2 + 5$

c) $y = -5x^2$ d) $y = 5x^2$



A possible equation for the graph below is:

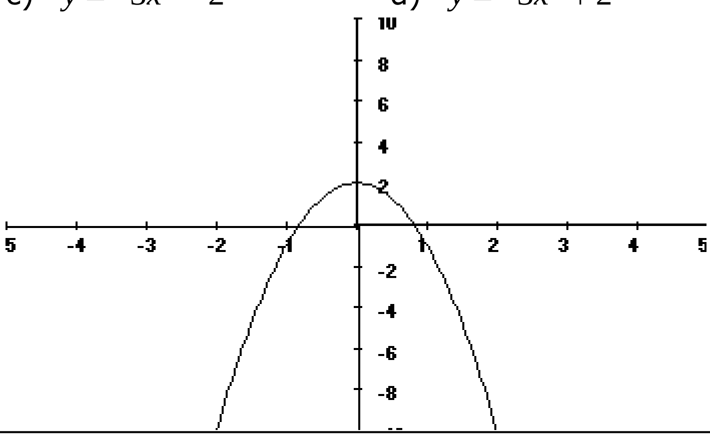
a) y = -2(x + 3)(x – 4) b) y = 2(x +3)(x – 4)
c) y = -2(x – 3)(x + 4) d) y = 2(x – 3)(x + 4)



Note: The intent of problems involving matching the graph to the factored form of the equation is that they are done by observation. The factorization should be simple enough that the roots can be determined without algebraic manipulation of the given equation.

A possible equation for the graph below is:

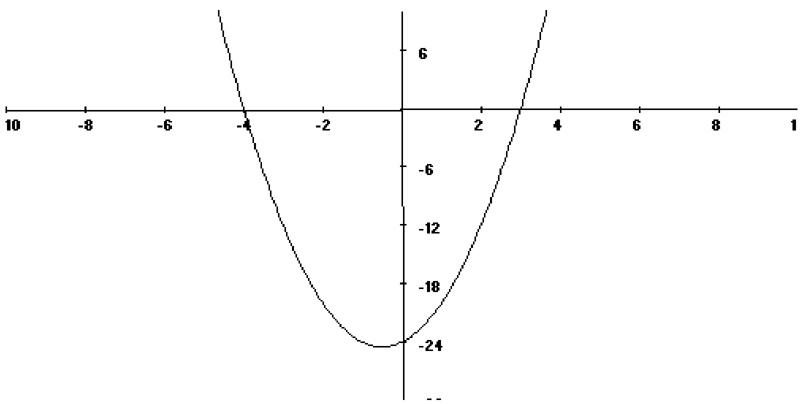
a) $y = 3x^2 - 2$ b) $y = 3x^2 + 2$
c) $y = -3x^2 - 2$ d) $y = -3x^2 + 2$



- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - Determine the graphical effect of increasing the magnitude of the leading coefficient in $y = ax^2$.

- Determine the graphical effect of decreasing the magnitude of the coefficient in $y = ax^2$.
- Carry out a procedure to translate between the factored form and standard form of a quadratic equation.
- Determine the equation of the quadratic function in standard form.
- Factor the quadratic equation.s

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

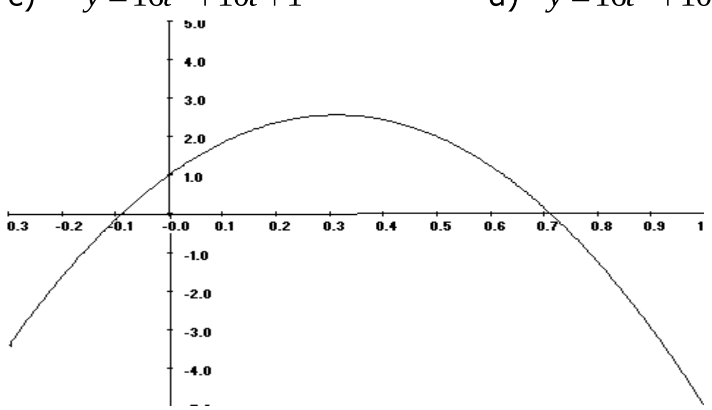


Write the equation for the graph above.

A possible equation for the graph below is:

a) $y = -16t^2 + 10t - 1$ b) $y = -16t^2 + 10t + 1$

c) $y = 16t^2 + 10t + 1$ d) $y = 16t^2 + 10t + 1$



This example is not essential because it requires matching an equation that contains a middle term. This is beyond the scope of the essential expectations of Elementary Algebra.

- **Misconceptions/Common Errors**
  None noted

- **Technology**

  - Graphing technology can be used for the converse skills of this indicator:
  - Graphing $y = ax^2$ or $y = x^2 + c$ and seeing the effect of changing the leading coefficient or constant of the equation is addressed in indicators 6.1 and 6.2, and may be helpful for conveying the concepts involved in this standard.
  - Graphing equations that are in factored form and having student finde the x-intercepts graphically will help establish the understanding needed to address this indicator.

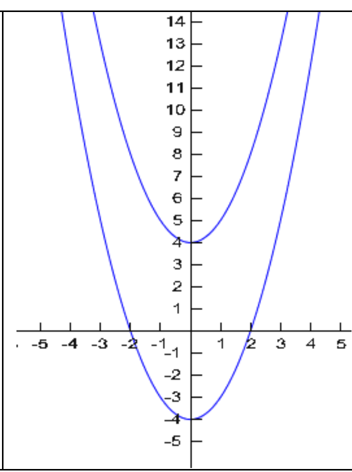
III. Assessing the Lesson

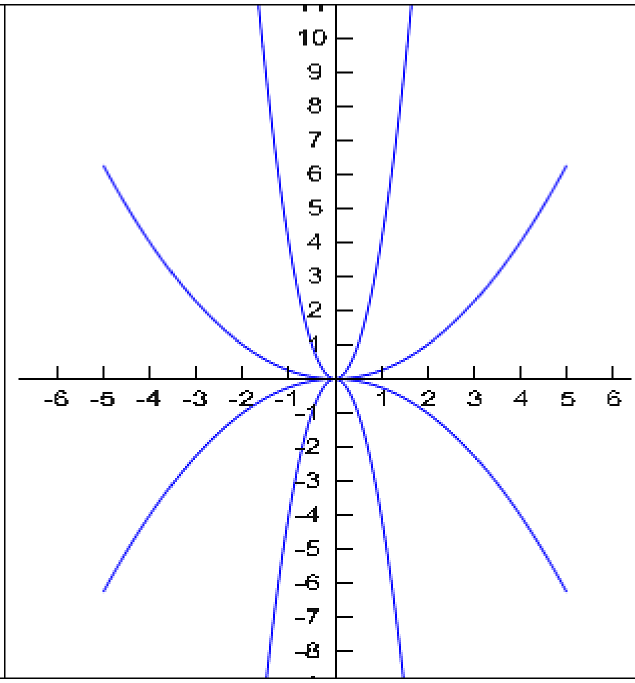
Assessment Guidelines: *The objective of this indicator is to analyze the graph of a quadratic equation to determine its equation.*
The primary focus of assessment should be for students to determine how specific qualities of the graph are related to values of leading coefficients, factors and constants in a quadratic equation.

Assessments may require students to use knowledge of how a and c affect the graph and may also focus on distinguishing a possible equation from the graph by using the relationship between the factors of the quadratic equation and the x-intercepts of its graph.

Assessments should not require students to go from a quadratic graph to the standard form of its equation because in elementary algebra students are not required to use algebraic techniques to determine a specified value of a.

- **Assessment Item Examples**

Match the equations with the possible graphs of those equations. A. $y = x^2 + 4$ B. $y = x^2 - 4$	

Match each equation with the possible graph of that equation. A. $y = 4x^2$ B $y = -4x^2$ C. $y = \frac{1}{4}x^2$ D. $y = -\frac{1}{4}x^2$	

Lesson # 4
Topic: Factoring methods
Standard (s): EA – 2.8

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 5^{th} grade students generate strategies to find the greatest common factor and the least common multiple of two whole numbers (5-2.7). Also, in 6^{th} grade students represent the prime factorization of numbers by using exponents (6-2.8).
  - In Elementary Algebra students carry out a procedure to factor binomials, trinomials, and polynomials by using various techniques (including the greatest common factor, the difference between two squares, and quadratic trinomials).
  - In Intermediate Algebra, students carry out a procedure to solve quadratic equations algebraically (including factoring, completing the square, and applying the quadratic formula) (IA-3.3). In addition, students carry out a procedure to solve polynomial equations (including factoring by grouping, factoring the difference between two squares, factoring the sum of two cubes, and factoring the difference between two cubes) (IA-4.3).

- **Taxonomy**
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  Factor
  Greatest Common Factor
  Binomial
  Trinomial
  Polynomial

II. Teaching the Lesson

In this lesson, students factor expressions involving greatest common factor, difference of squares and quadratic trinomial. Students become fluent in the procedures of factoring in order to apply this procedure in the next lesson on solving quadratic functions by factoring.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

  - Remove the greatest common factor (GCF).
  - Apply the difference of two squares formula: $x^2-y^2 = (x+y)(x-y)$.
  - Factor quadratic trinomials by using a trial-and-error method or another suitable method.
  - Recognize a polynomial as being prime.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should able to successfully complete.

  Factor:

  - $14x^3+4$
  - $-3x^2-3$
  - $6x-6y$
  - X^2-25
  - $5x^2-75$
  - $81c^2-49d^2$
  - $X^2+8x+15$
  - y^2-2y+1
  - $3x^2-9x+6$
  - $x^2-6x-27$
  - $n^2+5n-36$
  - x^2+2x+5 (Students should recognize as prime.)
  - $7x^2+7x+7$ (GCF only and will not factor any more)
  - $3x^2y-21xy+3xy^2$ (GCF only and will not factor any more)
  - $5x^3-10x^2+5x-50$ (GCF only and will not factor any more)
  - $2x^2y+4xy-30y$ [$2y(x^2+2x-15) = 2y(x+5)(x-3)$]

- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

  - Factor difference of two squares with more that one variable in a term
  - Factor by grouping

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

  Factor
  - $81x^2c^2-49d^2$
  - x^4+2x^3-x-2

  Solution by factor by grouping: $x^3(x+2)-1(x+2)$
  $(x+2)(x^3-1)$

- **Misconceptions/Common Errors**
  - Students fail to first remove greatest common factors, provided one exists.
  - When factoring quadratic trinomials, students sometime write factors that are not correct for the middle term. Therefore, students may wish to use multiplication to check the factored form, giving special attention to the middle term.
  - Students may not recognize that x^2+y^2 is prime.

- **Technology**
  Students may use computer algebra system technology, which is capable of performing symbolic manipulations, to verify solutions.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to carry out a procedure to factor binomials, trinomials, and polynomials by using various techniques (including the greatest common factor, the difference between two squares, and quadratic trinomials). Therefore, the primary focus of the assessment should be for students to carry out such procedures.*

- **Assessment Item Examples**
  - Factor completely: $x^2 - 25$
    A. $(x - 5)(x + 5)$
    B. $(x - 5)^2$
    C. $(x + 5)^2$
    D. $X^2(25)$
  - Factor completely: $7x^2 - 7$
    A. $7(x^2 - 1)$
    B. $7(x + 1)(x - 1)$
    C. $(7x + 1)(x - 1)$
    D. $(7x - 1)(x - 1)$

- Which of the following is a factor of $n^2 + 3n + 2$?
    - A. (n – 2)
    - B. (n + 3)
    - C. (n – 3)
    - D. (n + 2)

Lesson # 5
Topic: Solving Quadratic Equations by Factoring
Standard (s): EA – 6.4

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In the eighth grade, students apply procedures to solve multistep linear equations (8.34).
  - In Elementary Algebra, students will solve quadratic equations by the method of factoring.
  - In Intermediate Algebra, students will carry out a procedure to solve polynomial equations (IA 4.3).

- **Taxonomy**
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  Factoring
  Roots of a Quadratic Equation

II. Teaching the Lesson

In this lesson, students solve quadratic functions by factoring. In the previous lesson, student factored expression involving the greatest common factor, difference of squares and quadratic trinomials. Emphasizing that a quadratic function consists of two linear factors, $(x - x_1)(x - x_2)$ whose x-intercepts are x_1 and x_2, solidifies their conceptual understanding of how quadratic functions are structured. Student use the solution to each linear factor to determine the solution to the quadratic equation.
Although the indicator focuses on procedural knowledge, contextual problems that require students to solve quadratic functions will give purpose to the process.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

  - Place a quadratic equation in $0 = ax^2 + bx + c$ form.
  - Multiply two binomials.
  - Factor a monomial or constant out of an expression.
  - Factor a quadratic expression into two binomial factors.

- Recognize and factor the difference of two squares.
- Recognize and factor the result of squaring a binomial.
- Solve a quadratic equation by factoring.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

  - Solve by factoring: $y = x^2 - 2x + 1$ when y = 0
  - Solve by factoring: $0 = x^2 - 3x + 2$
  - Solve by factoring: $y = 5x^2 - 15x + 10$ when y = 0
  - Solve by factoring: $-1 = 6x^2 + x - 3$
  - Solve by factoring: $0 = 3x^2 - 12$
  - Solve by factoring: $y = 4x^2 - 9$ when y = 0
  - Solve by factoring: $-9 = 4x^2$
  - Solve by factoring: $y = x^2 - 3x$ when y = -2
  - Solve by factoring $-2 = x^2 - 3x$

- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

  - Know and/or apply the quadratic formula.
  - Factor a quadratic expression that has a large composite leading coefficient and a composite constant.
  - Factor a variable to reduce a higher order power to a quadratic.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

  - Solve by using the quadratic formula: $0 = x^2 - 2x - 2$
  - Solve $0 = 20x^2 + 7x - 6$ by factoring.
  - Solve by factoring $y = 3x^3 - 6x$ when y = 0

- **Misconceptions/Common Errors**

  - Students may use graphing utilities to check for the existence of real number solutions.

  - Students may use computer algebra system technology (calculators or software with symbolic manipulation capabilities) to verify solutions.

- **Technology**

  - Students may use graphing utilities to check for the existence of real number solutions.
  - Students may use computer algebra system technology (calculators or software with symbolic manipulation capabilities) to verify solutions.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to carry out a specific procedure, factoring, to solve a quadratic equation. Therefore, the primary focus of the assessment should be for students to carry out such procedures in the context of solving an equation.*

- **Assessment Item Examples**

  - Solve by factoring: $x^2 - 16 = 0$
    A. x = 4, -4
    B. x = 16, -16
    C. x = 4
    D. x = 0, 4

  - Solve by factoring: $x^2 - 5x = 0$
    A. x = 0, 5, -5
    B. x = 5
    C. x = 0
    D. x = 0, 5

  - What are the solutions to the quadratic equation $9x^2 - 25 = 0$?
    A. x = 25, 9
    B. x = 5, 3
    C. x = -5, -3
    D. x = 5/3, -5/3

Lesson # 6
Topic: Estimating solutions to a quadratic from the graph
Standard (s): EA – 6.5

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In eighth grade, students identify the coordinates of the *x*-intercepts of a linear equation from a graph (8-3.6).
  - In Elementary Algebra students will estimate the solutions of a quadratic function graphically.
  - In Pre-Calculus, students will carry out a procedure to solve polynomial equations graphically (PC-3.7).

- **Taxonomy**
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  Solutions of a quadratic equation
  Quadratic functions

II. Teaching the Lesson

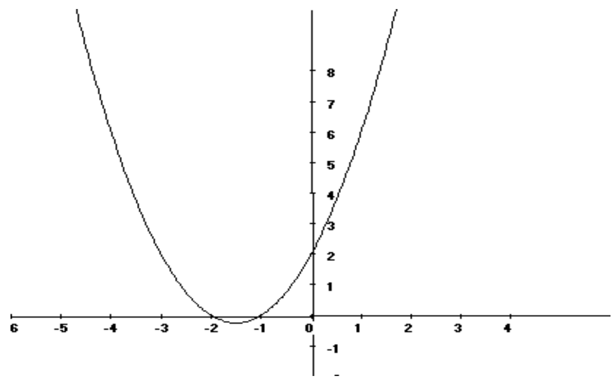
In this lesson, students approximate quadratic solutions from a graph. This lesson can be integrated into the previous lesson on solving quadratic equation by factoring. From previous lessons, student explored how a quadratic function consists of two linear factors, $(x - x_1)(x - x_2)$ whose x-intercepts are x_1 and x_2. Proficiency in this procedure will allow students to verify solutions that are found using algebraic methods.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

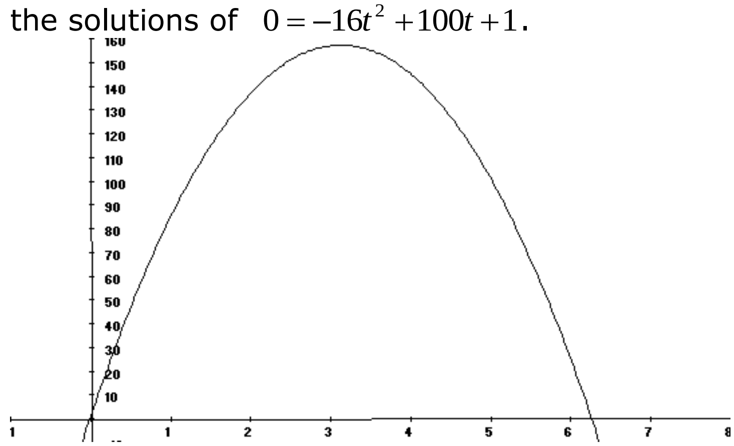
  - Graph a quadratic function.
  - Estimate the zeros of a function from a graph.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

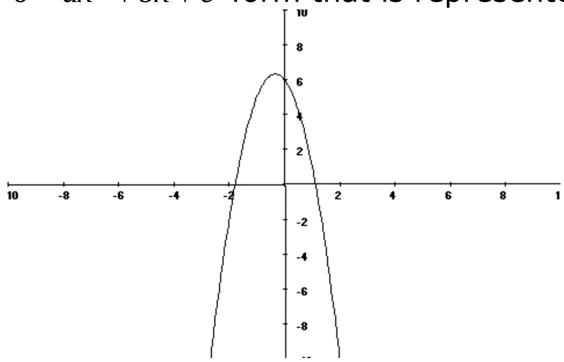
The graph of $y = x^2 + 3x + 2$ is given below. Estimate the solutions of $0 = x^2 + 3x + 2$.



The graph of $y = -16t^2 + 100t + 1$ is given below. Estimate the solutions of $0 = -16t^2 + 100t + 1$.



Estimate the solutions of the quadratic equation in $0 = ax^2 + bx + c$ form that is represented by the graph:



Estimate the solutions to $0 = -16t^2 + 10t + 1$ graphically.

- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

  - Place the equation in $0 = ax^2 + bx + c$ form.
  - Have irrational answers in radical form. An approximation fulfills the indicator.
  - Convert repeating decimal answers to fractions. An approximation fulfills the indicator

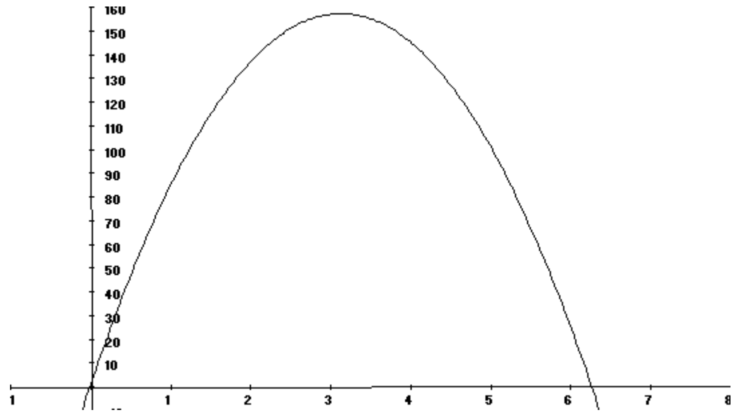
- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

Estimate the solution to $6 = -16t^2 + 10t + 7$ graphically.

Estimate the solution of $0 = x^2 - 2$ graphically. State your answer in radical form.

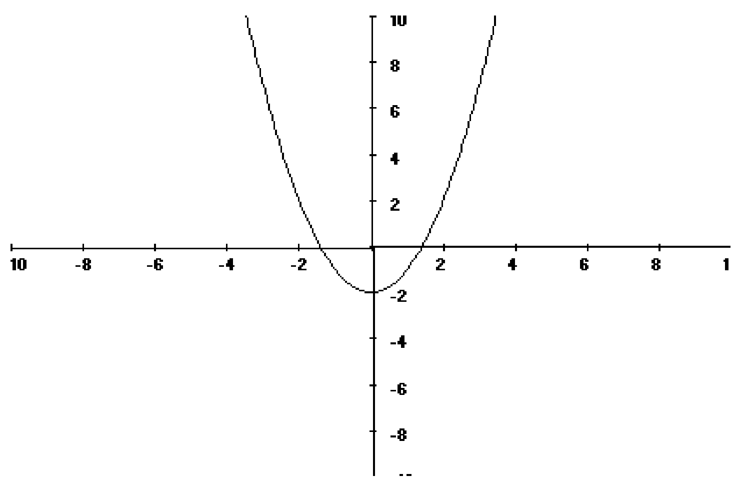
Estimate the solution of $0 = 2x^2 + 5x + 3$ graphically. State any rational non-integer answers in fractional form

The graph of $y = -16t^2 + 100t + 1$ is given below. Estimate the solutions of $6 = -16t^2 + 100t + 7$.



Choose the solution of a quadratic equation in $x^2 = 2$ represented by the graph of the related function $f(x) = x^2 - 2$ shown below:

a) x = -2   b) x = ± 1 c) x = $\pm\sqrt{2}$ d) x = ± 1.414



- **Misconceptions/Common Errors**
  None noted.

- **Technology**
  Students may use graphing calculators or software to graph and solve the equation. When using technology, the approximate answer given fulfills the indicator. Students are not expected to, and technically should not, assume radical or fractional values from an apparently non-terminating the approximation.

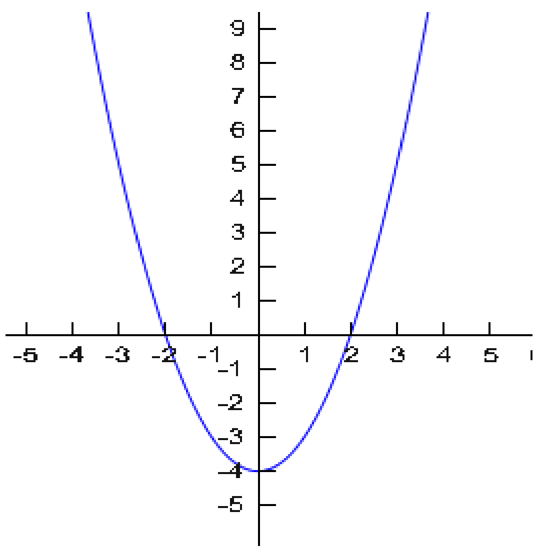
III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to carry out a procedure to estimate the solutions of a quadratic equation from the graph of its related function. Therefore, the primary focus of the assessment should be for students to estimate the solutions from a graph. Finding the related function from a given quadratic equation and graphing the function may be necessary steps in this procedure.*

- **Assessment Item Examples**

Estimate the solution to the graph at the right.

A. x = 2, -4
B. x = 2, -2
C. x = 0, -4
D. x = -4, 0



Lesson # 7
Topic: Finding the domain of quadratic functions in problem situations
Standard (s): EA – 6.6

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In seventh grade, students generate and solve complex abstract problems that involve modeling physical, social, or mathematical phenomena (7.11). In eighth grade, students identify the coordinates of the *x*- and *y*-intercepts of a linear equation from a graph, equation, and/or table (8-3.6).
  - In Elementary Algebra students will find the domain of linear and quadratic functions in a problem situation.
  - In Intermediate Algebra, students will carry out a procedure to determine the domain and range of discontinuous functions (including piecewise and step functions) (IA-2.10).

- **Taxonomy**
  Cognitive Process Dimension: Analyze
  Knowledge Dimension: Conceptual Knowledge

- **Key Concepts**
  Domain of a function
  Modeling

II. Teaching the Lesson

In this lesson, students determine the domain of quadratic functions in problem situations. At this point, students have examined the graphs of quadratic functions whose domain was all real numbers. This lesson builds students conceptual knowledge of domain by examining restrictions and the reasonableness of domain values based on the context of the problem.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

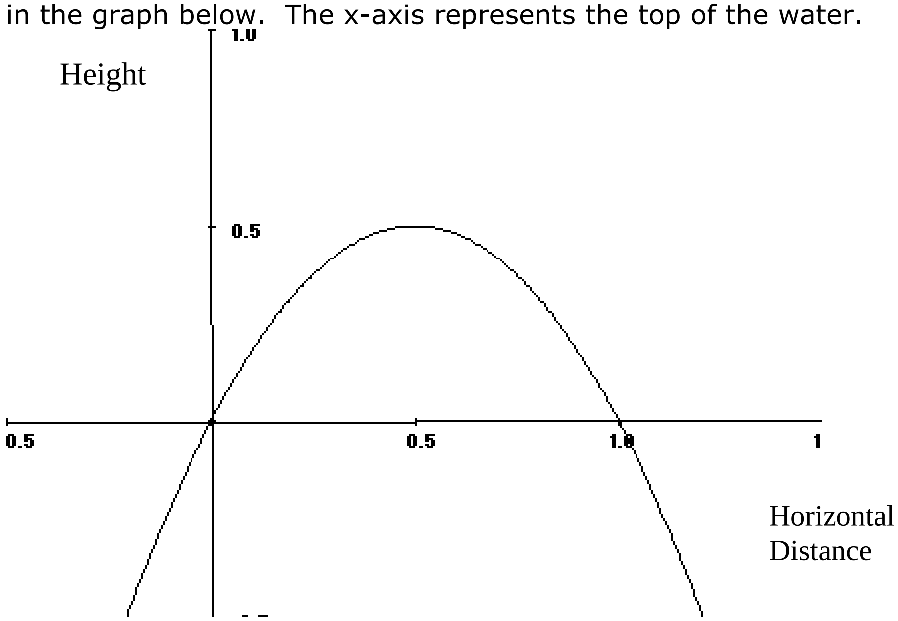
  - Interpret quadratic functions that model a problem situation.
  - Find the set of x-coordinates that determines the domain of a quadratic function.

- Determine the meaningful domain of a quadratic function for a given problem situation.
- Some problems may require finding the zeros of the function to determine the reasonable domain.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

  - The area of a rectangle with a perimeter of 500 cm is modeled by the function $A = l^2 - 500l$ where *l* is the length of the rectangle. What is the reasonable domain for this problem?
  - A steel wrecking ball is dropped from the roof of a 64 foot tall building. The ball's height is modeled by $h = -16t^2 + 64$ where t is time and h is height. Find the domain of the function in this context.

A fish jumps out of the water. His path is given by $h = -2d^2 + 2d$ as shown in the graph below. The x-axis represents the top of the water.



What is the domain of the function $h = -2d^2 + 2d$ when the fish is above the water?

- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

  - Generate the modeling function from a verbal description of the problem.
  - Find the maximum value of the function.
  - Find the input value that maximizes the function.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

  - A projectile is shot into the air. Its initial velocity is 10 ft/sec. The projectile is released at a height of 7 feet. The height of a projectile is modeled by the general equation $h = -16t^2 + v\ t + h_0$.
    What is the domain of the function that models this particular situation?
  - The area of a rectangle with a perimeter of 500 cm is modeled by the function $A = l(500 - l)$ where *l* is the length of the rectangle and
    w = 500 - *l* is the width. What length maximizes the area?
  - The revenue for a product is given by
    R = (1.25 + .01x)(1000 – 0.20x) where x represents a one-cent increase in price resulting in the loss of 0.2 customers. What price maximizes profit?
  - A projectile is shot into the air. Its height is modeled by $h = -16t^2 + 10t + 7$ where t is time and h is height. When does the projectile reach its maximum height? What is the maximum height reached?

- **Misconceptions/Common Errors**
  Students often misinterpret the meaning of the variables. Specifically, in vertical motion problems where height is a function of time, students believe the graph of the function is the path of the projectile.

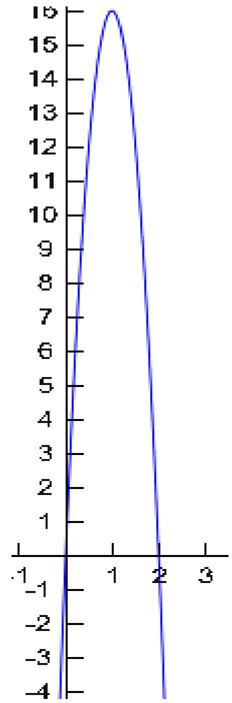
- **Technology**
  Students may use technology (graph or table) to find or estimate zeros.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to analyze given information to determine the domain of a quadratic function in a problem situation. Assessment should focus on determining the domain from the constraints of the problem situation.*

- **Assessment Item Examples**

Throwing a ball straight up in the air is modeled by a quadratic equation where y is the height of the ball after t seconds. Use the graph of the function at the right to determine the domain of the function in this problem situation.



A. $t > 0$
B. $t < 2$
C. $0 \leq t \leq 2$
D. $0.3 \leq t \leq 3$