## SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

## Content Area $\quad$ Grade 3 Math <br> Recommended Days of Instruction $\quad$ First Nine Weeks

Standard 3-2: The student will demonstrate through the mathematical processes an understanding of the representation of whole numbers and fractional parts; the addition and subtraction of whole numbers; accurate, efficient, and generalizable methods of multiplying whole numbers; and the relationships among multiplication, division, and related basic facts.

## Indicators:

3-2.1 Compare whole-number quantities through 999,999 by using the terms is less than, is greater than, and is equal to and the symbols $<,>$, and $=$. (B2)
3-2.2 Represent in word form whole numbers through nine hundred ninety-nine thousand. (A2)
3-2.12 Analyze the magnitude of digits through 999,999 on the basis of their place value. (B4)
3-2.3 Apply an algorithm to add and subtract whole numbers fluently. (C3)
3-2.4 Apply procedures to round any whole number to the nearest 10, 100, or 1,000. (C3)
3-2.9 Analyze the effect that adding, subtracting, or multiplying odd and/or even numbers has on the outcome. (This is also repeated under "Operations - Multiplication" below.) (B4)

Standard 3-3: The student will demonstrate through the mathematical processes an understanding of numeric patterns, symbols as representations of unknown quantity, and situations showing increase over time.

## Indicators:

3-3.1 Create numeric patterns that involve whole-number operations. (B6)
3-3.2 Apply procedures to find missing numbers in numeric patterns that involve whole-number operations. (C3)
3-3.3 Use symbols to represent an unknown quantity in a simple addition, subtraction, or multiplication equation. (B3)

* These indicators are covered in the following 3 Modules for this Nine Weeks Period.

Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.

Module 1-1 Number Structures and Relationships - Whole Numbers

| Indicator | Recommended Resources | Suggested Instructional Strategies | Assessment Guidelines |
| :---: | :---: | :---: | :---: |
| Module 1-1 Lesson A <br> 3-2.12 Analyze the magnitude of digits through 999,999 on the basis of their place value. (B4) | STANDARD SUPPORT DOCUMENT <br> http//:www.ed.sc.gov/apps/c so/standards <br> NCTM's Online <br> Illuminations http://illuminations.nctm.o rg | See Instructional Planning Guide Module 1-1 Introductory Lesson A | See Instructional Planning Guide Module 1-1 Lesson A Assessing the Lesson |
| Module 1-1 Lesson B <br> 3-2.2 Represent in word form whole numbers through nine hundred ninety-nine thousand. (A2) | NCTM's Navigations Series <br> Teaching StudentCentered Mathematics Grades K-3 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle | See Instructional Planning Guide Module 1-1 Introductory Lesson B <br> See Instructional Planning Guide Module 1-1, Lesson B Additional Instructional Strategies | See Instructional Planning Guide Module 1-1 Lesson B Assessing the Lesson |


| Module 1-1 Lesson C <br> 3-2.1 Compare wholenumber quantities through 999,999 by using the terms is less than, is greater than, and is equal to and the symbols $<,>$, and $=$. (B2) | NCTM's Principals and Standards for School Mathematics (PSSM) | See Instructional Planning Guide Module 1-1 Introductory Lesson C <br> See Instructional Planning Guide Module 1-1, Lesson C Additional Instructional Strategies | See Instructional Planning Guide Module 1-1 Lesson C Assessing the Lesson |
| :---: | :---: | :---: | :---: |
| Module 1-2 Operations - Addition and Subtraction |  |  |  |
| Indicator | Recommended Resources | Suggested Instructional Strategies | Assessment Guidelines |
| Module 1-2 Lesson A <br> 3-2.4 Apply procedures to round any whole number to the nearest 10,100 , or 1,000 . (C3) | STANDARD SUPPORT DOCUMENT <br> http//:www.ed.sc.gov/apps/c so/standards <br> NCTM's Online <br> Illuminations <br> http://illuminations.nctm.o rg | See Instructional Planning Guide Module 1-2 Introductory Lesson A | See Instructional Planning Guide Module 1-2 Lesson A Assessing the Lesson |


| Module 1-2 Lesson B <br> 3-2.3 Apply an algorithm to add and subtract whole numbers fluently. (C3) | NCTM's Navigations Series <br> Teaching StudentCentered Mathematics Grades K-3 and Teaching Elementary and Middle School Mathematics Developmentally 6th | See Instructional Planning Guide Module 1-2 Introductory Lesson B | See Instructional Planning Guide Module 1-2 Lesson B Assessing the Lesson |
| :---: | :---: | :---: | :---: |
| Module 1-2 Lesson C <br> 3-2.9 Analyze the effect that adding, subtracting, or multiplying odd and/or even numbers has on the outcome. (This is also repeated under "Operations - <br> Multiplication" below.) (B4) | Walle <br> NCTM's Principals and <br> Standards for School <br> Mathematics (PSSM) | See Instructional Planning Guide Module 1-2 Introductory Lesson C | See Instructional Planning Guide Module 1-2 Lesson C Assessing the Lesson |
| Module 1-3 Patterns, Relationships and Functions/Representations, Properties and Proportional Reasoning |  |  |  |
| Indicator | Recommended Resources | Suggested Instructional Strategies | Assessment Guidelines |


| Module 1-3 Lesson A <br> 3-3.1 Create numeric patterns that involve whole-number operations. (B6) <br> 3-3.2 Apply procedures to find missing numbers in numeric patterns that involve whole-number operations. (C3) | STANDARD SUPPORT DOCUMENT <br> http//:www.ed.sc.gov/apps/c so/standards <br> NCTM's Online <br> Illuminations http://illuminations.nctm.o rg <br> NCTM's Navigations Series <br> Teaching Student- | See Instructional Planning Guide Module 1-3 Introductory Lesson A | See Instructional Planning Guide Module 1-3 Lesson A Assessing the Lesson |
| :---: | :---: | :---: | :---: |


| Module 1-3 Lesson B <br> 3-3.3 Use symbols to represent an unknown quantity in a simple addition, subtraction, or multiplication equation. (B3) | Centered Mathematics Grades K-3 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle <br> NCTM's Principals and Standards for School Mathematics (PSSM) | See Instructional Planning Guide Module 1-3 Introductory Lesson B <br> See Instructional Planning Guide Module 1-3, Lesson B Additional Instructional Strategies | See Instructional Planning Guide Module 1-3 Lesson B Assessing the Lesson |
| :---: | :---: | :---: | :---: |

## MODULE

## 1-1

## Number Structure and Relationships- Whole Numbers

## This module addresses the following indicators:

3-2.1 Compare whole-number quantities through 999,999 by using the terms is less than, is greater than, and is equal to and the symbols $<,>$, and =. (B2)
3-2.2 Represent in word form whole numbers through nine hundred ninetynine thousand. (A2)
3-2.12 Analyze the magnitude of digits through 999,999 on the basis of their place value. (B4)

This module contains 3 lessons. These lessons are INTRODUCTORY ONLY. Lessons in S3 begin to build the conceptual foundation students need. ADDITIONAL LESSONS will be required to fully develop the concepts.

## I. Planning the Module

- Continuum of Knowledge

3-2.1 and 3-2.12
In second grade, students compared whole number quantities through 999 by using the terms is less than, is greater than and is equal to and the symbols $<,>$ and $=(2-2.4)$. Students also analyzed the magnitude of digits through 9,999 on the basis of their place value (2-2.10)

In third grade, students analyze the magnitude of digits through 999,999 on the basis of their place value (3-2.12) and compare whole number quantities through 999,999 by using the terms is less than, is greater than and is equal to using the symbols $<,>$ and $=(3-2.1)$.

In fourth grade, students analyze the magnitude of digits through hundredths on the basis of their place value (4-2.6)

3-2.2
In kindergarten, students translate between numeral and quantity through 31 ( $\mathrm{K}-2.2$ ). In first grade, students represent quantities in word form through ten (1-2.3). In second grade, students represent quantities in word form through twenty (2-2.2).

In third grade, students will represent in word form whole numbers through nine hundred ninety-nine thousand. (3-2.2)

In fourth grade, students recognize the period in the place-value structure of whole numbers: units, thousands, millions and billions (4-2.1) and analyze the magnitude of digits through hundredths on the basis of their place value (4-2.6). They also compare decimals through hundredths by using the terms is less than, is greater than, and is equal to and the symbols $<,>$ and $=(4-2.7)$

## - Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.
- *Digit
- *Hundreds
- *Ten thousands
- *Hundred thousands
- *Expanded form
- *Standard form
- *Value
- *Compare
- *Is greater than
- *Is less than
- *Is equal to
- *Place value
- *Magnitude


## Math Notation/Symbols

- <
- >
- =


## II. Teaching the Lesson(s)

## 1. Teaching Lesson A-Place Value

This is difficult to do because physical models are not commonly available. One idea that should be extended is the idea that each place value position is ten times greater than the position to the right.

3-2.12 Analyze the magnitude of digits through 999,999 on the basis of their place value. (B4)

For this indicator (3-2.12), it is essential for students to:

- Understand place value
- Understand that each place value ten times greater than the position to the right.
- Model place value relationships i.e. what does ten times one place look like, what does ten times the hundreds place look like, etc...
- Expand number in order to analyze place value

For this indicator, it is not essential for students to:
None noted

## a. Indicators with Taxonomy

3-2.12 Analyze the magnitude of digits through 999,999 on the basis of their place value. (B4)

Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

## b. Introductory Lesson

Lesson adapted from Elementary and Middle School Mathematics Teaching Developmentally by John A. Van de Walle (6 ${ }^{\text {th }}$ Edition) What Comes Next? page 211

## Materials:

Centimeter grid paper cut into unit pieces ( 1 cm square), ten strips ( 10 cm x 1 cm long), and hundreds pieces ( $10 \mathrm{~cm} \times 10 \mathrm{~cm}$ square).
Extra centimeter grid paper
Class Place Value chart divided into 6 columns
6 column Place Value chart for each student
For children to have a good grasp of numbers beyond 1000, the conceptual ideas that were developed in second grade must be extended. This is difficult to do because physical models are not commonly available. One idea that should be extended is the idea that each place value position is ten times greater than the position to the right.

Lesson A: "What Comes Next?"
Have a "What Comes Next?" discussion to review place value concepts learned in grades one and two. The teacher should place a unit cm square in the first column of a class place value chart. Ask: What is this square is called? Label the column "Ones." Place a ten strip in the next column. What is this strip called? How does it compare to the unit square? Label this column "Tens." Ask "What comes next?" Place a $10 \mathrm{~cm} \times 10 \mathrm{~cm}$ square in the column. How does it relate to the tens strip? To the units square? Label this column "Hundreds." Ask "What Comes Next?" Student responses should reflect an understanding that it would be a strip that is made of 10 hundred squares or 10 hundreds. Have students make a thousand strip by taping 10 hundred squares together. Make sure the squares are taped end to end to form a strip that is $10 \mathrm{~cm} \times 100 \mathrm{~cm}$ instead of a $100 \mathrm{~cm} \times 100 \mathrm{~cm}$ square. Label this column "Thousands." Ask "What Comes Next?" Let small groups work on the dimensions of a ten-thousand piece. Ten one-thousand strips would make a square measuring 1 meter on a side. Display the ten-thousand square so that students can have a visual of the magnitude of ten-thousand.

For further practice on understanding the magnitude of the digits in a number, write numbers on the board and have students place the digits in the correct column on their place value charts. Numbers should have 4 to 6 digits.
Suggested numbers:
3,427 4,273 46,732 425,367
Ask students to give the value of the 4 in each number. Refer to the class place value chart and have students use the squares, strips, and cubes to explain the value of the number. For example: 3,427: The 4 would be
four hundred squares. In 46,732: The 4 would be four of the metersized squares.
Have students write each of the numbers in expanded form. As students are working, monitor student understanding of number value by asking them to explain the difference between similar digits in different numbers. (Explain the value of the 2 in 3,427 and 4,273)

## c. Misconceptions/Common Errors

- As students write large numbers in expanded form to represent the magnitude of the digits, they should translate between number and expanded form. Some students can write the expanded form of a number but have difficulty writing the standard form of a number when given the expanded form. A common error is for students to forget to use zero to represent the value of a particular place. For example: students may write 34,56 for $30,000+4,000+50+6$. This can be avoided through the use of a Place Value Chart with marked columns.
- Another common error occurs when students are given representations such as 2 ten-thousands 4 thousands 13 hundreds 7 tens and 6 ones. Students will write the standard form as 241376 because they are only looking at the digits instead of considering the value of the digits. Students should have plenty of experiences using base ten blocks as they decompose 4-digit numbers so that they can extend the concept of decomposed numbers to 5 - and 6 -digit numbers.


## d. Additional Instructional Strategies/Differentiation

- Read Earth Day Hooray, by Stuart Murphy, Literature Link. Use as an interactive read aloud.
Use sticky notes to cover up the bubbles on the following pages:
- 359 on page 11
- 56 on page 16
- 635 on page 20
- 1,483 on page 22
- 2,852 on page 28
* Before uncovering the bubbles, have students write the number on paper. Then uncover the bubbles as you read.
- Review a place value chart through hundreds.
- Review the importance of "0" as a "place holder" in some numbers.
- Review the proper use of hyphens in number words (ex. Forty-two).


## Journal / Writing Connection:

Retell the story, making up different amounts of cans collected. For example, tell the child that the characters have collected 5 bags of 100, 6 bags of 10, and 3 single cans.

## e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- Base 10 Blocks Virtually (Use to represent numbers virtually after using concrete base 10 blocks. Students are limited to the number of columns and blocks that can be added.) http://nlvm.usu.edu/en/nav/frames asid 152 g 2 t 1.html?from= category g 2 t 1.html
- Place Value Game to Hundred Thousands (Students either type in the digit for the place value or type out the number. There are 3 levels.)
http://www.toonuniversity.com/flash.asp?err=503\&engine=15
- Place Value Tutorial, Practice, and Games http://www.aaamath.com/plc31e-placevalue-w2n.html
- Place Value Puzzler (Click on Medium Level. This game is place value recognition.) http://www.funbrain.com/tens/index.html
- Place Value Playoff! (Students match standard form to expanded form.) http://www.quia.com/mc/279741.html
- Place Value Playoff Concentration (Students play a concentration game matching standard form to expanded form.) http://www.quia.com/cc/279741.html
- Space Chase Place Value (Students use strategies to capture creepy space creatures while learning about place value.) http://teacher.scholastic.com/lessonrepro/reproducibles/profbooks/ m970818c.htm
- SMART Notebook Lessons/Activities (This site offers choices of grade levels, subject, and teachers can choose state standard correlations. Browse Educator Resources, Lesson Activities for more than Notebook Lessons. This site also offers SMART Response question sets, teacher-created lessons and activities, SMART sync collaboration activities, and SMART Ideas Software activities. Browse by curriculum standards and the website will find correlated activities for the standard you choose.) http://education.smarttech.com/ste/en-


## US/Ed+Resource/Lesson+activities/Notebook+Activities/Correlated +Search+us.htm

- SMART Board Interactive Whiteboard Lessons and Resources http://www.scholastic.com/interactivewhiteboards/


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

In the number 327, what number is in the tens place?
A. 0
B. 2
C. 7
D. 3

Which place value contains the underlined digit?
$1, \underline{2} 24$
A. ones
B. tens
C. hundreds
D. thousands

## 2. Teaching Lesson B-Writing Numbers

3-2.2 Represent in word form whole numbers through nine hundred ninety-nine thousand. (A2)

## Teacher Note:

Place value is the value given to a digit by virtue of the place it occupies in the number relative to the units place. When speaking of the base ten system each place to the left of the units place represents ten times the value of the place to the immediate right. The position of the digits in a number will determine the value of the digit.

As students continue to work with word form in future lessons, a common error is for students to forget to use a zero to represent the value of a place that is not named in word names. For example, a student would write 23,46 for twenty-three thousand forty-six, omitting the zero for the hundreds place. This too can be avoided by
having students write numbers within a labeled grid. Another common mistake is to say "and" where commas occur in a number. A special effort should be made to avoid that mistake. For example, one thousand five hundred sounds very close to one thousand and 5 hundredths. Correcting that error immediately will avoid later confusion for the student.
Model for students how they might self-assess whether they have written the number correctly by reading their number and comparing it to the word form. "Is this two thousand three hundred thirty-six?" Another way to self-assess is for students to examine the value of each digit. Is the 2 in the thousands place? Is the 3 in the hundreds place?

For assessment purposes, students should be able to translate words to numbers and numbers to words.
Although not essential, using concrete representations at this stage will help students continue to build connecting between the word, number and quantity; thereby, deepen their number sense about the magnitude of numbers.

For this indicator, it is essential for students to:

- Connect the word form to the numeral
- Translate between and among words, quantities and numeral
- Understand place value
- Write numbers in expanded form
- Correctly read word representations of numbers

For this indicator, it is not essential for students to:
None noted

## a. Indicators with Taxonomy

3-2.2 Represent in word form whole numbers through nine hundred ninety-nine thousand. (A2)

Cognitive Process Dimension: Knowledge
Knowledge Dimension: Conceptual Knowledge

## b. Introductory Lesson

## Materials Needed

Chart paper
Glue stick
Newspapers
Magazines

## Lesson B: Writing Real-World Numbers

Have students work collaboratively in groups of 2-4 students to locate numbers or number words using informational text such as newspapers or magazines. Glue the words or numbers on the chart paper. Students should write the number for each number word found or write the number word if a number was found. If there are no newspapers available, have students look through classroom books and copy the numbers and number words onto the chart. As students work, the teacher should preassess their understanding of numbers by observing whether they are able to write the correct number for each number word or write the number word correctly.
After an appropriate amount of time, gather the class together to share their charts on numbers and number words. The teacher should list 3 or 4 examples of each size of number given by students on a class chart. (This is one suggestion for the type of chart the teacher may create.)

|  | Number | Word Form |
| :--- | :--- | :--- |
| 2-digit Numbers |  |  |
| 3-digit Numbers |  |  |
| 4-digit Numbers |  |  |
| 5-digit Numbers |  |  |
| 6-digit Numbers |  |  |

Begin with 2-digit numbers and their word form. Direct students' attention to the hyphen in the word form of 2-digit numbers. .Proceed to 3 -digit, 4 -digit, 5 -digit, and 6 -digit number words.
Have students practice reading the numbers on the chart. Listen to see if they insert "and" into numbers such as 123,456 and repeat the correct way to read the number without "and"
Look for opportunities for students to practice reading large numbers in other content areas such as Social Studies and Science.

## c. Misconceptions/Common Errors

- When writing large numbers, it is important to note that a common student misconception is that two thousand five is written as 20005. This can be avoided by writing numbers within a labeled grid. The student will immediately observe that there is not the physical space to record the number when beginning with the thousands place.
- Students sometime to forget to use a zero to represent the value of a place that is not named in word names. For example, a student
would write 23,46 for twenty-three thousand forty-six, omitting the zero for the hundreds place.
- Another common mistake is to say "and" where commas occur in a number.


## d. Additional Instructional Strategies/Differentiation

For additional practice: "Memory game"
Materials: Set of 10 pairs of numbers and the corresponding word form.
Objective: Match number word cards to the correct standard form. Have students shuffle the cards. Place them face down in a $4 \times 5$ array. Students take turns turning over 2 cards in order to find a matching pair. As students turn over a card, they should read the number or number word on the card. As students play, the teacher should monitor students and assess whether they are reading the numbers correctly--avoiding the use of "and" and using the correct place value terms for larger numbers.
Variation: Have students match word form to expanded form.

1. Write a 3 digit number on the board in standard and word form. Focus on the different ways to write a number. Place a Number Word Bank in the room for all students to see and use. Demonstrate proper use of hyphens (ex. forty-six).
2. Play "Go Go Go." Directions: The teacher will divide the class into two groups (A and B). Students in each group will receive an index card with one digit written on it (0-9). The teacher will start with "Group A." The teacher will call out a three digit number. The students who possess a card that contains one of the digits in the called number will move quickly to the front of the room. These students will discuss how to use their cards to form the called number, position themselves in the correct order in a large place value chart (on floor using tape, on board for them to stand in front of, etc.), and present the number to the class for evaluation. Group B will construct the next number. Continue alternating.
3. Play "Pass It." Each student will receive an index card displaying the numeric form of a number on one side and the word form of that same number on the other side. Students bring their cards to a class circle. One by one students call out the number on their card. Students present both versions of the number to the class as they read aloud. After all students have read the numbers on their cards, students pass their card to the person to their right. The process begins again, but should get faster. After each round, students pass to the right.
4. Play "Match Game." Half of the class will receive an index card with a numeric form of a number written on it. The other half of the class will receive cards with the same numbers, only in word form. When the teachers says, "GO!", the students find their matching partners.
5. Play "Number Concentration." In small groups of four or less students, 16 Cards will be shuffled and placed face down between the players. Players will take turns flipping two cards at a time trying to match the numeric form of a number to its word form. The player collecting the most matches wins!

## e. Technology

- Cookie Dough (Writing numbers on checks. This game only goes to 10, 000.) http://www.funbrain.com/cgibin/shtml.cgi?A1 = ../numwords/index.html
- Place Value Games (Edit these games to fit $3^{\text {rd }}$ grade indicator for digits through 999,999.) http://www.mathwire.com/games/pvgames.html
- SMART Notebook Lessons/Activities (This site offers choices of grade levels, subject, and teachers can choose state standard correlations. Browse Educator Resources, Lesson Activities for more than Notebook Lessons. This site also offers SMART Response question sets, teacher-created lessons and activities, SMART sync collaboration activities, and SMART Ideas Software activities. Browse by curriculum standards and the website will find correlated activities for the standard you choose.) http://education.smarttech.com/ste/enUS/Ed+Resource/Lesson+activities/Notebook+Activities/Correlat ed+Search+us.htm
- SMART Board Interactive Whiteboard Lessons and Resources http://www.scholastic.com/interactivewhiteboards/
- What's Your Name? (Students practice numbers in standard, word, and expanded form.) http://www.beaconlearningcenter.com/WebLessons/WhatsYourN ame/default.htm


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.


What number does the picture represent?
A. three hundred thirty-five
B. one hundred fifty-five
C. one hundred thirty-five
D. one hundred five

What is the word form for the numeral 723,508 ?
A. Seven thousand, twenty-three, five hundred eight
B. Seventy-two thousand, three five hundred eight.
C. Seven hundred twenty-three thousand, five hundred eight.
D. Seven hundred thousand, twenty-three and five hundred eight.

Which shows 604,109 in word form?
A. sixty-four thousand, one hundred nine
B. sixty-four, one hundred nine
C. six hundred forty thousand, one hundred nine
D. six hundred four thousand, one hundred nine

## 3. Teaching Lesson C

3-2.1 Compare whole-number quantities through 999,999 by using the terms is less than, is greater than, and is equal to and the symbols <, >, and $=$. ( B 2 )

## Teacher Note:

Do not limit students' experiences to comparing numbers with the same number of digits. Experiences should include comparing numbers that do not have the same number of digits. Students often focus on the first digit when comparing numbers. When they are comparing a five-digit number and a four-digit number, they do not consider the value of the digit. For example, a student may state that 7,509 > 34,658 because they know 7 > 3 .

For this indicator, it is essential for students to:

- Recognize the place value of digits through 999,999
- Understand the magnitude of digits
- Compare the place value of digits through 999,999
- Recognize mathematical symbols $<,>$, and $=$ and their meanings
- Read whole numbers using appropriate terminology
- Compare numbers that do not have the same number of digits

For this indicator, it is not essential for students to:

- Represent these quantities with concrete materials


## a. Indicators with Taxonomy

3-2.1 Compare whole-number quantities through 999,999 by using the terms is less than, is greater than, and is equal to and the symbols $<,>$, and $=$. (B2)

Cognitive Process Dimension: Understand Knowledge Dimension: Conceptual Knowledge

## b. Introductory Lesson

## Materials

- Base ten blocks: thousands cubes, hundreds flats, tens rods, and ones units
- Place value chart divided into thousands, hundreds, tens, and ones for each student
- Wipe-off boards and dry erase markers or paper
- Each pair of students should have at least 3 thousands cubes, 15 flats, 15 rods, and 15 units.
- ${ }^{* *}$ If there is a limited number of base ten blocks, students could work in groups of 4 or you could adjust the digits in each number to match the base ten blocks available.


## Lesson 3: Comparing Numbers

Write two 4-digit numbers on the board and have students model each number using base ten blocks on separate place value charts. Direct students to begin with the thousands place in each model and have them compare the base ten blocks displayed in each place until there is a difference. Example: 2,345 and 2,351 " 5 tens is greater than 4 tens" Have students write a number sentence using > or < to compare the numbers.
Continue to model different 4-digit numbers and have students identify the place that determines which number is greater. Students should also write the inequality on their wipe-off boards. Teachers can vary this by telling the students which inequality sign they should use in their number sentence. Students would then have to consider which number to write first, instead of always writing the largest number first.
Once students have had sufficient experience comparing two 4-digit numbers, the teacher should have students compare a 4-digit number
and a 3-digit number to provide students with practice to prevent the common error described below. Write the following numbers on the board: 2,361 and 423. Which number is greater? If students correctly identify 2,361, ask "How can a number that begins with a 2 be larger than a number that begins with a 4?" Have students model both numbers with their base ten blocks and place value charts. Ask students to identify the largest place where the two numbers differ. Return to the question that was asked earlier and ask how they might use their models to answer the question.
Allow time for pairs of students to model numbers on their place value charts using base ten blocks or picture drawings. Each student will model their own number and write the number on their wipe-off board. They will then compare their model and number to their partner's model and number. Students should write their partner's number after the number that is on their own wipe-off board and use the correct inequality sign to compare.

## c. Misconceptions/Common Errors

- It is difficult for students to conceptual such large numbers. Using real world illustrations of these quantities give them meaning and a lasting point of reference.
- To deepen student's conceptual understanding of comparison, let students create inequalities or equivalent relationships. For example, each student rolls their die four times and records the digit they rolled. Student A creates a four digit number and records it then the teacher tell Student B to create either a number that is less than or greater than that number. Student should record their four digit numbers and the appropriate relationship. Switch roles and continue to play for at least five rounds. So if student A creates 3213 and student B creates 5361 then they should write on their paper 5361 > 3213.
- Another misconception is for students to begin in the ones place to compare numbers. A student may state that 113,975 < 103,459 because $9>5$.


## d. Additional Instructional Strategies/Differentiation

## Materials Needed

- 2 sets of digit cards (0-9) - per group
- 6 column Place Value chart labeled through the Hundred Thousands - one per person
- Divide the class into pairs of students. Shuffle the digit cards and place face down. The first player draws a card and places the card in any column on his Place Value chart. The second player draws a card and places it on his own chart. Players continue to take turns drawing cards to make the largest 6-digit number possible. Once a card is placed in a column it may not be moved. After students have identified the largest number that was created, they write an inequality sentence to compare the numbers using < or >.
- As the students work with their partner to create numbers, monitor the groups to see if they are correctly identifying the greatest number. Ask students the value of a particular digit in a number that was created. Students should also be able to explain why one number is greater than the other and use correct math vocabulary. Example: 34,567 > 23,546 because 34,567 has 3 ten thousands and 23,546 only has 2 ten thousands.
- Variations: Have three students work together so that students can compare more than 2 numbers. Students can arrange the three numbers created in order from least to greatest or greatest to least.


## Another Idea:

Play "Around the World" to compare numbers. Teacher will distribute cards with numbers in the thousands through the hundred thousands. Students should sit in rows or in a line. The first two students will stand up and compare the numbers on their cards. The student with the higher number will remain standing. The next student in line will stand up and compare the number on his or her card to the number on the card of the student who is still standing. Once again, the student with the higher card remains standing. The game continues until each student has stood up and compared the number on his or her card. The teacher should distribute the highest number cards toward the end of the line or rows of students so that students toward the front of the line won't remain standing throughout most of the game. The student with the highest card will be the last student standing.

## e. Technology

- Genius Boxing (Students choose a famous "genius" to box. Each round increases levels of difficulty. Not only will students compare numbers, but addition and subtraction problems are included.) http://www.mrnussbaum.com/geniusboxing.htm
- SMART Notebook Lessons/Activities (This site offers choices of grade levels, subject, and teachers can choose state standard correlations. Browse Educator Resources, Lesson Activities for more than Notebook Lessons. This site also offers SMART

Response question sets, teacher-created lessons and activities, SMART sync collaboration activities, and SMART Ideas Software activities. Browse by curriculum standards and the website will find correlated activities for the standard you choose.) http://education.smarttech.com/ste/enUS/Ed+Resource/Lesson+activities/Notebook+Activities/Correlat ed+Search+us.htm

- SMART Board Interactive Whiteboard Lessons and Resources http://www.scholastic.com/interactivewhiteboards/
- Base 10 Blocks Virtually (Use to represent numbers virtually after using concrete base 10 blocks. Students are limited to the number of columns and blocks that can be added. Have students create numbers and compare them.) http://nlvm.usu.edu/en/nav/frames asid 152 g 2 t 1.html?fro m=category g 2 t 1.html
- Compare two numbers (Compare numbers from 4 to 6 digits.) http://www.quia.com/pop/7512.html
- Comparing Number Values (Three levels. Students compare three digit numbers.) http://www.toonuniversity.com/flash.asp?err=509\&engine=9
- Compare Numbers: Three Levels (Students play a game to compare numbers up to 100, 1000 or 10000.) http://www.crickweb.co.uk/assets/resources/flash.php?\&file=nc menu
- Number Sense (Create a number that is greater than the given.) http://pbskids.org/cyberchase/games/numbersense/index.html
- Greatest to Least (Compare numbers by ordering them from greatest to least or least to greatest. Students also have to include $<$ or $>$.) http://www.starrmatica.com/standalone/starrMaticaComparingN umbersCometoOrder.swf
- For Place Value overall: Promethean Planet (Register for free!) http://www.prometheanplanet.com/server.php?ResourceSearch \%5Bsearch text\%5D=place+value\&ResourceSearch\%5Bsubject \%5D=00200n009002002002\&ResourceSearch\%5Bgrade\%5D=0 0200n009002003002\&display=006007001\&ResourceSearch\%5B action\%5D=advanced\&change=ResourceSearchResults\&catMatc hType=includeChildren\&searchType=basic\&x=31\&y=11


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

Which statement is true?
A. $8,156<973$
B. $73,914>69,002$
C. $19,510<5,603$
D. $5,600=560$

Which number is between 349,614 and 372,005 ?
A. 380,141
B. 360,000
C. 321,073
D. 400,000

Compare these numbers:

A. >
B. +
C. <
D. =

Which number is 4,000 less than 9,214?
A. 4,000
B. 5,214
C. 5,000
D. 5,412

## III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

## Assessment Guidelines

3-2.1
The objective of this indicator is to compare which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning therefore, students should are not just learning procedural strategies for comparing numbers but they are also building number sense. The learning progression to compare requires students to compare the place value of digits through 999,999, compare the place value of digits through 999,999, recognize mathematical symbols $<,>$, and $=$ and their meanings. Students analyze place value patterns (3-1.4), construct arguments (3-1.2) and exchange mathematical ideas with classmates about which symbol is appropriate. Students use correct, complete and clearly written and oral language communicate these ideas.

## 3-2.2

The objective of this indicator is to represent which is in the "understand factual" knowledge cell of the Revised Taxonomy. Although this indicator requires student to represent a finite set of number, this is not a simple memorization task because to understand means to construct meaning. The learning progression to represent requires students to understand the place value system for whole numbers. Students write numbers in expanded form and use multiple informal representations (3-1.7) such concrete models and pictorial models to bridge the gap between whole number and word. Students examine the word, whole number and representation simultaneously and analyze the relationship among the word, the numeral and the representation to construct a deeper understanding. Students use this understanding to represent quantities as words and vice-versa. Students explain and justify their answers on the basis of mathematical properties, structures and relationships (3-1.3).

3-2.12
The objective of this indicator is to analyze which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To analyze means to determine relevant features and relationships. The learning progression to analyze requires students to understand place value and be able to locate the correct place value. Students represent the place value using concrete and/or pictorial models and generalize the connections (3-1.6) between place value and the multiple of ten. They compare the magnitude of digit and use these connections to generate statements (3-1.4) about the magnitude of numbers. Students explain and justify their answers (3-1.3) and use correct, complete and clearly written and oral language to communicate their ideas (3-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Use $<,>$ or $=$ in the $\bigcirc$ to compare each pair of numbers.
a. $41,005 \bigcirc 41,050$
b. 175,000 175,001
2. Use >, < or = to compare each number. $400,000+30,000+600+50+3$ forty-three thousand six hundred fifteen
3. Write the following in standard form: "Fifty Two Thousand, Three Hundred Seventy Four"
4. Write the following in expanded form: 140, 722
5. What is the value of the " 2 " in the number: 42,387 ?
6. Explain why the following is equal to 76,325 .

7 ten thousands 6 thousands 2 hundreds 11 tens 15 ones

## MODULE

## 1-2

## Operations - Addition and Subtraction

## This module addresses the following indicators:

3-2.3 Apply an algorithm to add and subtract whole numbers fluently. (C3)
3-2.4 Apply procedures to round any whole number to the nearest 10, 100 , or 1,000 . (C3)
3-2.9 Analyze the effect that adding, subtracting, or multiplying odd and/or even numbers has on the outcome. (This is also repeated under "Operations - Multiplication" below.) (B4)

This module contains 3 lessons. These lessons are INTRODUCTORY ONLY. Lessons in S3 begin to build the conceptual foundation students need.
ADDITIONAL LESSONS will be required to fully develop the concepts.

## I. Planning the Module

## - Continuum of Knowledge

## 3-2.3

In second grade, students generated strategies to add and subtract pairs of two-digit whole numbers with regrouping (2-2.7) and generated addition and subtraction strategies to find missing addends and subtrahends in number combination through 20 (2-2.8).
In third grade, students apply an algorithm to add and subtract whole numbers fluently. They also analyze the effect that adding, subtracting or multiplying odd and/or even numbers has on the outcome (3-2.9). In fourth grade, students will apply an algorithm to multiply whole numbers fluently (4-2.3) and generate strategies to divide whole numbers by single-digit divisors (4-2.5)

## 3-2.4

In second grade, students generated strategies to round numbers through 90 to the nearest 10 (2-2.9) and analyzed the magnitude of digits through 9,999 on the basis of their place values (2-2.10)
In third grade, students apply procedures to round any whole number to the nearest 10,100 , or 1,000 (3-2.4)

In fourth grade, students will use these rounding strategies to estimate answers to whole number multiplication.

3-2.9
In second grade, students explore even and odd numbers through patterns as they analyzed numeric patterns in skip counting that uses the numerals 1 through 10 (2-3.1)
In third grade, students analyze the effect that adding, subtracting, or multiplying odd and/or even numbers has on the outcome (3-2.9)

Students use these effects explore operations with whole numbers.

## - Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.
- *Addend
- *Sum
- *Difference
- *Estimate
- *Trade
- *Regroup

South Carolina $S^{3}$ Mathematics Curriculum

- *Odd
- *Even
- *Round
- *Tens
- *Hundreds
- *Thousands
- *Place value


## II. Teaching the Lesson(s)

## 1. Teaching Lesson A: Rounding on a Line

3-2.4 Apply procedures to round any whole number to the nearest 10,100 , or 1,000 . (C3)

## Teacher Note:

Students should build on those previously generated strategies and apply them to round whole numbers to the nearest 10,100 , or 1,000 . Since students use their knowledge of place value to add and subtract, that should be connected to rounding as a means of determining the reasonableness of answers. Rounding should include both front end estimation (rounding before completing an operation) and end estimation (performing an operation and then rounding). Students should compare the results of both techniques to determine which technique would be most appropriate for the situation.

For this indicator, it is essential for students to:

- Locate the specified place value
- Round any whole number to the nearest ten, hundred or thousand
- Understand that rounding is not replacing the value but just substituting it with a nice numbers that can be used more easily during computation or problem solving
- Determine between which to numbers the value is located

For this indicator, it is not essential for students to:

- Round to the nearest 10,000 or 100,000 . Although students have compared and written numbers 999,999, they do not need to explore rounding to those place values.


## a. Indicators with Taxonomy

3-2.4 Apply procedures to round any whole number to the nearest 10, 100, or 1,000. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

## b. Introductory lesson

Materials Needed:
5 Meter sticks
Masking tape
Marker
5 sets of Index cards labeled 100, 200, 300, 400, 500
Blank index cards
$* * *$ Prior to the lesson, place a length of masking tape that is one meter long in 5 different locations on the classroom floor.

Begin the lesson by asking students "About how many students are in the third grade at $\qquad$ School? We don't have time to count the students in each class, so how might you estimate the number of students?" Have students share their thinking.
Share with students how you use estimation when you go to the grocery store or when purchasing a group of items and only want to spend a certain amount. Knowing how to estimate or round numbers lets you use numbers that are easy to work with when you need to add or subtract numbers mentally.

Divide students into 5 groups. Assign each group to one of the 5 pieces of masking tape on the classroom floor. Let each group measure the length of tape with the meter stick. Ask "What number is in the middle of the meter stick?" Have students make a vertical mark on the tape to indicate the center of the number line.


Give each group a set of index cards (100-500). Explain to the students that you will say a 3 digit number between 100 and 500 that they will write on one of the blank index cards. Example: 274. They are to decide which hundred comes before and after that number (200 and 300) and place those index cards at each end of the number line. Assess whether groups are able to identify the hundreds that the given number is
between and whether the hundreds are placed in the correct order on the number line. Ask students to identify the number that would be in the center of the number line. Write that number on a blank card and place it on the midpoint. Next have a student in each group hold the number card (274) and stand on the number line to show which hundred it is closest to. Have students share why they decided to stand in that particular part of the number line.
Call out a few more numbers for students to practice placing the hundred cards on the endpoints and the center number and then standing in the correct half of the number line. Suggested numbers: 139, 405, 361, 458

Have students return to their desks to work with a partner. Each pair should draw an open-ended number line at the top of a piece of paper. Tell students they are going to explore numbers that are between 900 and 1000 . Students should write those numbers at each end of their number line. Have students write the number that would be the midpoint. Each student will take turns saying a 3-digit number that falls between 900 and 1000. The other student in the pair will write the number below the number line in the correct half. (See example)

***The emphasis is not on placing the numbers in the exact spot on the number line, but on whether the number is greater than or less than 950.

After students have 10-15 numbers on their chart, ask students to describe the numbers that are closest to 900 or about 900. What do they notice about the numbers that are closest to 1000 ? Would the same thing happen with another number line?

On a class chart, draw a 600-700 number line. What number is in the center? How do you know? Call out a number and have students show where it would fall on the number line. Write the number in the correct place. After several numbers are labeled, have students compare these numbers to their "900" number line. Do they notice the same pattern? Ask students if there is a way they could decide which hundred a
number is closest to without using a number line? Example: 834. Is it closer to 800 or 900? How did you decide? Students should be able to say that if it is less than 850, it is closer to 800 , if it is greater than 850, it is closer to 900.
Students may ask about 850 and which number it would be closer to. Explain that mathematicians have decided that if a number is in the middle that we will round the number to the higher hundred.

Students will need additional lessons on applying what was learned in this lesson to rounding to the nearest 1000. Also, third grade students should be able to round any whole number to the nearest ten, hundred, or thousand. Example: 2,467 is about 2,470 or 2,500 , or 2,000 .

## c. Misconceptions/Common Errors

Students are often taught "rules" for rounding numbers to the nearest 10, 100, or 1000. "Five and above, give it a shove. Four and below, leave it alone." Students should work with numbers in a context, such as a number line, so that they develop an understanding of why they would round up. Students with number sense are able to visualize where a number would be located in a given range.

There are several common errors students make when rounding due to teaching "tricks" or "rules" for rounding.

- Errors related to identifying the hundreds or thousands a number is located between. Example: A student may round 429 to 200 because they know to look at the 2 and think 200.
- When rounding a 4-digit number to the nearest hundred, students sometimes forget to write the thousands place. Example: A student may round 3,749 to 700 . Since their focus is on the hundred and tens place, they ignore the digit that is in the thousands place.
Each of these errors can be avoided by using number lines so that students are basing their estimations on the relationships between numbers.


## d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

## e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial
representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- Round Numbers to Nearest Thousand (An interactive math tutorial, practice, and games) http://www.aaamath.com/est41c-round1000.html
- Rounding for $3^{\text {rd }}$ Graders (Register for free to use this activity.)
http://www.prometheanplanet.com/server.php?show=ConRe source. 10258
- Rounding

Memory
Game http://www.numbernut.com/advanced/activities/estimate m em20 round1000.shtml

- Rounding to Nearest Hundred Memory Game http://www.numbernut.com/advanced/activities/estimate m em20 round100.shtml
- Seashell Rounding Activity Page http://www.janbrett.com/piggybacks/rounding.htm
- Glowla's Estimation Contraption (Students round numbers and add.) http://pbskids.org/cyberchase/games/ballparkestimation/ind ex.html
- Estimation Valley Golf (Students solve math problems by rounding to find an estimate. Levels of difficulty increase as answers are correct.) http://www.mrnussbaum.com/estimationvalley.htm
- Estimator Four (Similar to connect 4. Choose addition level 1. Students round numbers and add the two.) http://www.shodor.org/interactivate/activities/EstimatorFour/ ?version=1.6.0 02\&browser=MSIE\&vendor=Sun Microsyste ms Inc.
- Front End Estimation http://www.321know.com/est73ax2.htm
- SMART Notebook Lessons/Activities (This site offers choices of grade levels, subject, and teachers can choose state standard correlations. Browse Educator Resources, Lesson Activities for more than Notebook Lessons. This site also offers SMART Response question sets, teacher-created lessons and activities, SMART sync collaboration activities, and SMART Ideas Software activities. Browse by curriculum standards and the website will find correlated activities for the standard you choose.) http://education.smarttech.com/ste/en-
US/Ed+Resource/Lesson+activities/Notebook+Activities/Corr elated+Search+us.htm
- SMART Board Interactive Whiteboard Lessons and Resources http://www.scholastic.com/interactivewhiteboards/


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

## 2. Teaching Lesson B

3-2.3 Apply an algorithm to add and subtract whole numbers
fluently. (C3)

## Teacher Note:

In third grade the emphasis is on applying an algorithm. Students should take the strategies developed using concrete/pictorial models in second grade and connect them to algorithms in $3^{\text {rd }}$ grade. As a result, by the end of third grade students should exhibit fluency when solving a wide range of addition and subtraction problems involving whole numbers. If the students truly understand the relationship between place value, regrouping, addition, and subtraction, the "size of the numbers" used in problem solving is not important. In order to determine fluency with addition and subtraction, all the aspects of mathematics related to place value, addition, and subtraction must be taken into consideration. When adding and subtracting, students should be given, and should also create, a wide range of problems that show real world situations. Before computing, students should estimate the outcome. "Rounding is the most familiar form of estimation. Estimation based on rounding is a way of changing the problem to one that is easier to work with mentally." (Van de Walle, Pg. 234)

For this indicator, it is essential for students to:

- Understand place value
- Connect concrete and pictorial models from second grade with symbolic (number only) strategies
- Understand the process of regrouping
- Add and subtract number with and without regrouping
- Using rounding strategies to estimate their answers
- Use an appropriate strategy to verify their answers

For this indicator, it is not essential for students to: None noted

## a. Indicators with Taxonomy

3-2.3 Apply an algorithm to add and subtract whole numbers fluently. (C3)

Cognitive Process Dimension: Apply

Knowledge Dimension: Procedural Knowledge

## b. Introductory Lesson

## Materials needed:

A variety of manipulatives that could be used with addition and subtraction.
Suggestions: base ten blocks, counters, mini ten frames, unifix cubes, hundreds charts
"The primary goal for all computation should be students' ability to compute in an efficient manner - not what algorithms are used. That is, the method of computing is not the objective; the ability to compute is the goal." (John A. Van de Walle Teaching Student-Centered Mathematics p. 161)

In order to facilitate the use of the computation strategies students generated in second grade, use story problems to introduce the addition and subtraction of larger numbers.
Example:

- The third grade sold 264 tickets to the Fall Festival. The second grade sold 129 tickets. How many tickets did they sell altogether?

Have manipulatives available for students to use if necessary. Let students work in pairs to solve the problem. Before students begin to solve the problem, have them discuss whether they should add or subtract and explain why. They should use their rounding skills to estimate what a reasonable answer would be.
Allow the students to share their estimates and the strategies used to solve the problem. This is the most important part of the lesson. Encourage students to ask questions of their classmates. Have students compare different strategies. Record different methods on a class strategies chart.

Pose another story problem to the students. (See the list below that describes different types of problems) Follow the same procedure described above. Do not force students to use the traditional algorithms for addition and subtraction, but when a student does share it with the class, make sure the student understands why it works and can explain it to the class.

There are four basic structures for addition and subtraction story problems, join, separate, part-part-whole and compare. It is important that students be exposed to all forms within these structures. (Van de Walle)
The problems below are examples of the different types. Third grade problems should use 3-digit to 6-digit numbers.
Word problem examples:
Join: result unknown- Susie had 3 cookies. Linda gave her 5 more. How many cookies did Susie have altogether?
Join: change unknown- Susie had 3 cookies. Linda gave her some more. Now, Susie has 8 cookies. How many cookies did Linda give her?
Join: initial unknown- Susie had some cookies. Linda gave her 5 more. Now, Susie has 8 cookies. How many cookies did Susie start with?
Separate: result unknown-Susie had 8 cookies. She gave 5 to Linda. How many cookies does she have now?
Separate: change unknown- Susie has 8 cookies. She gave some to Linda. Now, she has 3 cookies. How many did she give to Linda?
Separate: Initial unknown- Susie had some cookies. She gave 5 to Linda. Now she has 3 left. How many cookies did she start with?
Part-part-whole: whole unknown- Teresa has 6 green jelly beans and 8 red jelly beans. How many jelly beans does she have?
Part-part-whole: part unknown- Teresa has 14 jelly beans. 6 of them are green and the rest are red. How many are red?
Compare: difference unknown- Susie has 6 jelly beans and Teresa has 8 jelly beans. How many more jelly beans does Teresa have than Susie?
Compare: Larger Unknown- Teresa has 2 more jelly beans than Susie. Susie has 6 jelly beans. How many jelly beans does Teresa have?
Compare: Smaller Unknown- Teresa has 2 more jelly beans than Susie. Teresa has 8 jelly beans. How many jelly beans does Susie have?

## c. Misconceptions/Common Errors

A misconception for teachers is that it is best for students to use the traditional algorithm. Traditional algorithms tend to make students think in terms of digits rather than the composite number that the digits make up. Rather than rely on a single method, the most appropriate method can and should change flexibly as the numbers and context change. The development over time of an assortment of flexible skills, including the ability to compute mentally, will best serve students in the real world. (Van de Walle p. 216)

A common error associated with the traditional algorithm is for students to ignore the place value of digits and line up numbers from the left to the right instead of ones to the greater place value position. Example:


This error can be avoided by having students estimate the sum or difference before solving the problem and thinking about the composite number instead of digits as they solve the problem.

A common error with the subtraction algorithm occurs when zeros are part of the minuend.


Provide the students with base ten blocks so they can develop an understanding of trading hundreds, tens, and ones.

## d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

## e. Technology

- Space Shuttle Launch (Choose "hard" for the level more appropriate for $3^{\text {rd }}$ grade in addition and subtraction. This can be used for all operations.) http://www.playkidsgames.com/games/shuttleLaunch/shuttleLa unch.htm
- Save the Apples (Choose "hard" for the level more appropriate for $3^{\text {rd }}$ grade in addition and subtraction. This can be used for all operations. The easier levels may be useful for students who continue to struggle with basic addition or subtraction.)
http://www.playkidsgames.com/games/apples/savetheApples.ht m
- Math Baseball (Choose addition or subtraction and the level. Once $3^{\text {rd }}$ graders are more comfortable with addition/subtraction algorithms, the "superbrain" level will be a great challenge!) http://www.funbrain.com/math/index.html
- UFO Addition and Subtraction Mystery Game (Students perform operations with addition and subtraction and match the answers in a puzzle. A great game for those who reach fluency with the algorithms and need that extra challenge.) http://www.dositey.com/addsub/mystery2AS.htm
- Addition and Subtraction Practice (Choose from adding three digit numbers, more than two numbers, word problems, etc. Enter site as guest.) http://www.ixl.com/math/grade/third/
- SMART Notebook Lessons/Activities (This site offers choices of grade levels, subject, and teachers can choose state standard correlations. Browse Educator Resources, Lesson Activities for more than Notebook Lessons. This site also offers SMART Response question sets, teacher-created lessons and activities, SMART sync collaboration activities, and SMART Ideas Software activities. Browse by curriculum standards and the website will find correlated activities for the standard you choose.) http://education.smarttech.com/ste/enUS/Ed+Resource/Lesson+activities/Notebook+Activities/Correlat ed+Search+us.htm
- SMART Board Interactive Whiteboard Lessons and Resources http://www.scholastic.com/interactivewhiteboards/


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

## 3. Teaching Lesson C: What's do You Notice?

3-2.9 Analyze the effect that adding, subtracting, or multiplying odd and/or even numbers has on the outcome. (This is also repeated under "Operations - Multiplication" in Module 2-1) (B4)

## Teacher Note:

Students should be given opportunities to come to the conclusion that the sum of two odd or two even numbers will be even; while the sum of one odd and one even will be odd. The same is true for subtraction. So, for example, if they are adding two even numbers and get an odd
number answer, they may conclude that the answer is incorrect. It is extremely important that students be provided experiences that enable them to come to those conclusions rather than being provided facts that they must memorize.

For this indicator, it is essential for students to:

- Understand the characteristics of even numbers
- Understand the characteristics of odd numbers
- Explore the following odd-even relationships

| even + even = even | even - even $=$ even | even ${ }^{\times}$even $=$even |
| :---: | :---: | :---: |
| odd + odd $=$ even | odd - odd = even | odd $\times$ odd = odd |
| even + odd = odd | even - odd = odd | $\text { even }{ }^{\mathrm{X}} \text { odd }=\text { even }$ |
| odd + even $=$ odd | odd - even $=$ oddd | odd even $=$ even |

For this indicator, it is not essential for students to:

- Recite the odd-even relationships


## a. Indicators with Taxonomy

3-2.9 Analyze the effect that adding, subtracting, or multiplying odd and/or even numbers has on the outcome. (This is also repeated under "Operations - Multiplication" in Module 2-1) (B4)

Cognitive Process Dimension: Analyze Knowledge Dimension: Conceptual Knowledge

## b. Introductory lesson

Materials needed:

## 2-colored counters

Review the meaning of odd and even numbers with students. List two different groups of numbers on the board or a chart. One group is odd and the other has even numbers. Make sure there are different sized numbers (2-digit, 3-digit, etc) Do not label the groups. Have students guess the rule for each group of numbers.

Tell students they are going to investigate odd and even numbers. Ask them what they think the sum of 2 odd numbers will be? Have students predict and then investigate. Let students record their trials on a three-column chart.

| Odd + Odd | Even + Even | Odd + Even |
| :--- | :--- | :--- |
|  |  |  |

Continue with Even + Even and Odd + Even. Record findings in the other two columns.

Next, have students model two one-digit odd numbers with their counters. Example: $5+3$
Questions to ask:
How might you group the counters to prove the sum is even?
What happens when you add 2 odd numbers?
Why do you get an even sum instead of an odd sum?
Continue with students modeling even + even and odd + even.
Conclude by having students share how they might use this information as they solve addition and subtraction problems.

## c. Misconceptions/Common Errors

Students may be confused when exploring these even-odd relationships; therefore, students should be give sufficient opportunities to explore these relationships over time.
d. Additional Instructional Strategies/Differentiation

None noted at this time.

## e. Technology

There is no specific technology recommended for this lesson at this time.

## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

## III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

## Assessment Guidelines

## 3-2.3

The focus of the indicator is for students apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Although, the focus of the indicator is procedural, the learning progression should integrate strategies that encourage students to balance procedural knowledge with their conceptual knowledge. The learning progression to apply requires student to recall and understand their generated strategy for addition and subtraction. Student should use estimation strategies prior to applying their chosen strategy. Students use their understanding of place value and regrouping to find sum and difference. They should use correct, complete and clearly spoken or written language (3-1.5) to explain and justify their answers to their classmates and teacher on the basis of properties and relationships (3-1.3).

## 3-2.4

The objective of this indicator is to apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Although the focus of indicator is to apply, the learning progression should include experiences that balance conceptual and procedural knowledge in order to support retention. The learning progression to apply requires students to locate the specified place value. Students use an appropriate rounding strategy to find the answer. Students generalize the connection (3-1.6) between the rounded value and the actual number. Students use correct, complete and clearly written and oral language (3-1.5) to explain and justify their answers on the basis of mathematical relationships (3-1.3) such as determining two numbers that the value is between.

3-2.9
The objective of the indicator is to analyze which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To analyze means to determine relevant features and relationships. The learning progression to analyze requires students to recall the characteristics of even and odd numbers. Students explore even-odd relationships by examining these relationships in contexts. They analyze information (3-1.1) from these explorations, generalize connections (3-1.6) between even and odd numbers then explain and justify their answers (3-1.3) to their classmates and their teachers. They use correct, complete and clearly written and oral language to communicate their ideas (3-1.5) and generate mathematical statements (3-1.4) about these even-odd relationships.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. A builder bought 21,508 bricks from a local business. He has to buy 9,689 more bricks. How many bricks does the builder need for the house he is building?
2. Use the Populations of SC Counties Chart to answer the following questions.

| Populations of South Carolina Counties |  |
| :--- | :--- |
| County | Population |
| Richland County | 320,677 |
| Lexington County | 216,014 |
| Charleston County | 309,909 |
| Greenville County | 379,616 |
| Horry County | 196,629 |

a. About how many more people live in Charleston County than in Lexington County? Estimate the difference to the nearest thousand.
b. Michelle said that the sum of Richland and Horry County would be an odd number. Do you agree? Explain your reason.
3. Margaret brought an odd number of cookies in a bag. Sandra also brought an odd number of cookies in a bag. They decide to put all of their cookies together and share them. Will they be able to share the cookies evenly? Explain your answer.
4. There were 25,738 people at the football game on Saturday. Last week there were 1,000 fewer people at the game. How many people were at the game last week?

## MODULE

## 1-3

## Patterns, Relationships, and Functions

## \&

## Representations, Properties, and Proportional Reasoning

This module addresses the following indicators:
3-3.1 Create numeric patterns that involve whole-number operations. (B6)
3-3.2 Apply procedures to find missing numbers in numeric patterns that involve whole-number operations. (C3)
3-3.3 Use symbols to represent an unknown quantity in a simple addition, subtraction, or multiplication equation. (B3)

This module contains 2 lessons. These lessons are INTRODUCTORY ONLY. Lessons in S3 begin to build the conceptual foundation students need.
ADDITIONAL LESSONS will be required to fully develop the concepts.

## I. Planning the Module

## Continuum of Knowledge

## 3-3.1 and 3-3.2

In second grade, students analyzed numeric patterns in skip counting that used the numerals 1 through 10 (2-3.1) and analyzed relationships to complete and extend growing and repeating patterns with numbers, symbols and objects (2-3.3)

Third grade is the first time students are formally introduced to patterns involving operations. They create numeric pattern involving whole number operations (3-3.1) and apply procedures to find the missing number in numeric patterns that involve whole number operations (33.2).

In fourth grade, students analyze numeric, nonnumeric and repeating patterns involving all operations and decimal patterns through hundredths (4-3.1). They also generalize a rule for numeric, nonnumeric and repeating patterns involving all operations.

## 3-3.3

In second grade, students had experiences generating strategies for addition and subtraction pairs of two-digit whole numbers with regrouping.

In third grade, students use symbols to represent unknown quantities in addition, subtraction, and multiplication equations.

In fourth grade, students translate among letters, symbols and words to represent quantities in simple mathematical expression or equations (4$3.4)$. They also apply procedures to find the value of an unknown letter or symbol in a whole number equation (4-3.5)

## Key Concepts/Key Terms

- *Pattern
- Whole number operation
- *Missing term
- *Numeric pattern
- *Symbol
- *Equation
- *Expression
- *Rule

South Carolina $S^{3}$ Mathematics Curriculum

## - *Balance

- *Equivalency
* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.


## II. Teaching the Lesson(s)

## 1. Teaching Lesson $A$

Teacher Notes:
Third grade is the first time students are formally introduced to patterns involving operations. Third grade students are expected to create patterns that involve the whole-number operations of addition and subtraction and basic multiplication. This should include situations where elements of the pattern are missing either within the sequence or at the end of the sequence.

## 3-3.1

For this indicator, it is essential for students to:

- Create numeric patterns involving addition and subtraction. An example is $5,8,7,10,9$, $\qquad$ . Students add 3 for the next term then subtract 1 for the next term then they repeat.
- Create numeric pattern involving basic multiplication (see 3-2.7)
- Communicate their pattern in written and oral form
- Use concrete and/or pictorial models to explore patterns For this indicator, it is not essential for students to:
- Combine operations to get the next term. An example is 5, 11, 23, $\qquad$ . The pattern is to multiply by 2 and add 1 to get to the next term then they repeat.


## 3-3.2

For this indicator, it is essential for students to:

- Analyze relationships between and among numbers to determine a pattern
- Determine the missing terms within a sequence of numbers
- Determine the missing term at the end of a sequence of numbers
- Be fluent in addition and subtraction
- Recall basic multiplication facts

For this indicator, it is not essential for students to:

- To apply procedures that involve the multiplication of numbers beyond the basic multiplication facts
- Combine operations to get the next term. An example is 5, 11, 23, $\qquad$ . The pattern is to multiply by 2 and add 1 to get to the next term then they repeat.


## a. Indicators with Taxonomy

3-3.1 Create numeric patterns that involve whole-number operations. (B6)

Cognitive Process Dimension: Create. Knowledge Dimension: Conceptual Knowledge

3-3.2 Apply procedures to find missing numbers in numeric patterns that involve whole-number operations. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

## b. Introductory Lesson A: Start and Jump

Lesson adapted from Teaching Student Centered Mathematics, Grades K-3 by John A Van de Walle

## Materials:

Student Copies of Hundreds Charts
Colored pencils or crayons

## Lesson:

"Start and Jump Numbers" (p.286)
Have students make a list of numbers that begins with 3 and adds 5 to each successive number. The 3 is called the "start number" and 5 is the "jump number." It is helpful to make the list in a vertical column (see below). Have students examine the list of numbers (about 10) and find as many patterns as possible. Allow time for students to share with the class.
Ask students: "What do you think would happen if you changed the start number but kept the same jump number?"
Have students try different start numbers (1-10) and compare the different lists.

- Do any new patterns appear?
- What is different?
- What is the same?

| Start with 3 <br> Jump by 5 | Start with <br> Jump by 5 | Start with <br> Jump by 5 | Start with <br> Jump by 5 |
| :--- | :--- | :--- | :--- |
| 3 |  |  |  |
| 8 |  |  |  |
| 13 |  |  |  |
| 18 |  |  |  |
| 23 |  |  |  |

Provide students with several hundreds charts. Have them color a different chart for each of the columns. Label their charts with the Start and Jump Numbers.

- What patterns appear on the hundreds chart that they did not see before?
- What do they notice when they compare the different charts?
- How do the patterns change when only the start number changes?
- Which skip counts or jumps make diagonal patterns on the hundreds chart and which make column patterns?
- Pick any number between 1 and 100. How can you tell if your pattern will land on that number?

Extension: Have students generate lists of numbers that have the same start number, but the jump number changes. Encourage students to use two-digit numbers as start numbers. Let students share their observations about these patterns.

- How do the patterns differ in each column?
- What would happen if they began with a different start number?
- What if they used a three-digit number as the start or jump number?
- Do the same patterns appear if you begin with a start number between 90 and 100 and subtract during your jumps?

As students work with patterns, monitor how individual students are able to reason with the patterns they explore.

## c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

## d. Additional Instructional Strategies/Differentiation

Having student create pictorial models not only incorporates the multiple representations but gives an additional entry point for students who may struggle with addition, subtraction and multiplication.

## e. Technology

- "What Comes Next_?" (The following activities deal with patterns and the importance of looking at data from more than a single perspective. Included are explorations designed to encourage open-mindedness and to help students develop alternative viewpoints. Students investigate a series of items that appear to be ordered in some "obvious" ways. Students also explore ways of sorting items into two disjoint groups on the basis of some definable characteristic. This lesson was adapted from "What Comes Next_?", written by William R. Speer \& Daniel J. Brahier. It appeared in the October, 1995 Teaching Children Mathematics Vol. 2, no. 2.) http://illuminations.nctm.org/LessonDetail.aspx?id=L286
- Patterns That Grow (Students use logical thinking to create, identify, extend, and translate patterns. They make patterns with numbers and shapes and explore patterns in a variety of mathematical contexts. These lessons give students an opportunity to create and analyze numeric and geometric patterns. Particular emphasis will be placed on growing patterns.)
http://illuminations.nctm.org/LessonDetail.aspx?id=U103
- Petals Around the Rose (A puzzle involving five dice and a nonstandard pattern is used to promote problem-solving skills. Prior to teaching this lesson, play this game yourself.) http://illuminations.nctm.org/LessonDetail.aspx?id=L576
- Number Patterns and Rules (Match patterns to their operation.) http://www.harcourtschool.com/activity/rubber number patter ns and rules/
- Patterns Video http://www.linkslearning.org/Kids/1 Math/2 Illustrated Lesson s/5 Patterns/index.html
- Crack Hacker's Safe (Extend patterns in shapes and numbers.) http://pbskids.org/cyberchase/webisode 1/web 1game.html
- Patterns to the Rescue (Lesson from Cyberchase) http://pbskids.org/cyberchase/parentsteachers/lessons/lessonpl ans/lesson4.html
- SMART Notebook Lessons/Activities (This site offers choices of grade levels, subject, and teachers can choose state standard correlations. Browse Educator Resources, Lesson Activities for more than Notebook Lessons. This site also offers SMART Response question sets, teacher-created lessons and activities, SMART sync collaboration activities, and SMART Ideas Software activities. Browse by curriculum standards and the website will find correlated activities for the standard you choose.) http://education.smarttech.com/ste/en-

US/Ed+Resource/Lesson+activities/Notebook+Activities/Correla ted+Search+us.htm

- SMART Board Interactive Whiteboard Lessons and Resources http://www.scholastic.com/interactivewhiteboards/
- Number Cracker (Guess the missing number from a series.) http://www.funbrain.com/cracker/index.html
- Missing Numbers in a Sequence http://www.beaconlearningcenter.com/WebLessons/MissingNum bers/default.htm
- Find Missing Numbers http://www.321know.com/g1fm by5.htm
- Pattern Lessons http://www.aaamath.com/pat.htm
- SMART Notebook Lessons/Activities (This site offers choices of grade levels, subject, and teachers can choose state standard correlations. Browse Educator Resources, Lesson Activities for more than Notebook Lessons. This site also offers SMART Response question sets, teacher-created lessons and activities, SMART sync collaboration activities, and SMART Ideas Software activities. Browse by curriculum standards and the website will find correlated activities for the standard you choose.) http://education.smarttech.com/ste/enUS/Ed+Resource/Lesson+activities/Notebook+Activities/Correla ted+Search+us.htm
- SMART Board Interactive Whiteboard Lessons and Resources http://www.scholastic.com/interactivewhiteboards/


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

## 2. Teaching Lesson B

## Teacher Notes

Third graders use symbols to represent unknown quantities in simple addition, subtraction, and multiplication equations. Students use symbols to represent specific unknown quantities such as $3 \times \quad{ }^{* *}=12$. Students need to understand that the symbol is a placeholder for an unknown quantity and that for each equation or problem-solving situation, the symbol may represent a different value. Students should start with simple number sentences to ensure understanding. This does NOT mean that third grade student solve equations. It simply means
that they begin to understand that a symbol can be used to represent a quantity in an equation and that the value of the symbol changes as the problem changes. The term "equation" is used to mean two mathematical expressions that equal (equation) each other.

For this indicator, it is essential for students to:

- Understand that the symbol is a placeholder for an unknown quantity
- Understand that the symbol represents a different number each equation or problem situation
- Understand that the equation sign represent balance between the two sides of the equation
- Explore these symbols in the context of story problems.
- Write number sentences
- Use the symbol to represent the unknown quantity in different positions. For example,

$$
\begin{aligned}
& \square \times 3=6 \\
& 3+\square=5 \\
& 7-2=
\end{aligned}
$$

For this indicator, it is not essential for students to:

- Use inverse operations to solve for the unknown quantity


## a. Indicators with Taxonomy

3-3.3 Use symbols to represent an unknown quantity in a simple addition, subtraction, or multiplication equation. (B3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Conceptual Knowledge

## b. Introductory Lesson

Write one of these statements on the board:

- Darren caught 15 fish but he threw some back in the lake. He took 12 fish home to cook.
- Darren caught 15 fish but he threw some back in the lake. He took 8 fish home to cook.

Tell them that mathematicians like to represent statements like this in mathematical terms. Sometimes, they use letters (also known as variables) to represent different people or objects.

Ask: How might mathematicians write this statement? One suggestion is:

- $15-F=12$

Another way to represent the unknown is with a symbol:


- $15-\quad=12$

What number would make the number sentence true? What strategy did you use to find your answer?
Would the same number be used for F and the diamond? Why?
Have a student write a number sentence for the second statement above.

- $15-F=8$

Ask "Does the variable $F$ equal the same thing in both equations? How can you justify your thinking? What is another symbol you could use to represent the unknown?

Repeat this activity with several more statements. Then have the students work in pairs or trios to create some of their own statements and mathematical equations that match.

## c. Misconceptions/Common Errors

Students often have a common misunderstanding with regard to the concept of equivalence. Prior to the third grade, students may have simply written an answer after the equal sign. Now, students must clearly understand that the equal sign does not mean "perform an operation". It means that there is a relationship of equivalence between the two expressions on either side of that equal sign.
Students typically view the equals sign as a symbol that separates the problem from the answer. It is important that students see and understand that there is a relationship between the expressions on each side of an equals sign. Instead of viewing $=$ as meaning an answer is coming, help students view the equals sign as meaning "is the same as." A good starting point to develop this understanding is to explore equations as true or false. Example: $4+1=6 \quad 8=10-1$ $5+4=9$

## d. Additional Instructional Strategies/Differentiation

Differentiation Strategies:

- For students who need more support with the unknown quantity, provide manipulatives such as unifix cubes or counters.
- Practice: Provide students with different equations that have variables. Have students write math stories to match the equation. Example: $6+s=25$
- Practice: If a pan balance or number balance is available, let students explore unknowns in an equation by using the balances with wooden cubes, color tiles, or any uniform manipulative that could be used with a pan balance.
- Students should start with simple story problems and number sentences to ensure understanding.


## e. Technology

- Completed Variable Machine (This online lesson provides an introduction to variables.) http://illuminations.nctm.org/LessonDetail.aspx?id=L291
- "The Mystery Number" (Students learn about variables by listening to this tune.) http://www.harcourtschool.com/jingles/jingles all/35mystery n umber.html


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

## III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

## Assessment Guidelines

## 3-3.1

The objective of this indicator is to create which is in the "create conceptual" knowledge cell of the Revised Taxonomy. To create means to put ideas together into a new structure; therefore, students use prior knowledge to create new numeric patterns. The learning progression to create requires students to recall basic addition, subtraction and multiplication facts and understand what a pattern is. Where appropriate, students use concrete and/or pictorial models to explore number relationships in order to develop a pattern. As students analyze information (3-1.1) from these experiences, they generate mathematical statements (3-1.4) about the relationships they observe then explain and justify their pattern (3-1.3) to their classmates and
their teachers. Students recognize the limitations of various strategies and representations (3-1.8) and use correct, complete and clearly written and oral language to communicate their ideas (3-1.5).

## 3-3.2

The focus of the indicator is for students apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Although, the focus of the indicator is procedural, the learning progression should integrate strategies that encourage students to balance procedural knowledge with their conceptual knowledge. The learning progression to apply requires student to recall and understand the meaning of pattern. Students analyze information (3-1.1) from the pattern and use their understanding of whole number operations to determine the relationship between numbers. They should use correct, complete and clearly spoken or written language (3-1.5) to explain and justify their answers to their classmates and teacher on the basis of properties and relationships (3-1.3).

## 3-3.3

The objective of the indicator is to use which is in the "apply conceptual" knowledge cell of the Revised Taxonomy. Applying conceptual goes beyond replacing a missing number with a symbol. Conceptual knowledge is not bound by specific examples; therefore, students should apply their understanding of unknown quantity in a variety of situations. The learning progression to use requires students to explore real world problems and analyze information (3-1.1) from the problem to determine known and unknown quantities in the problem. Students generate descriptions or mathematical statements (3-1.4) about the relationship between the known and unknown quantities. They explain and justify their answers (3-1.3) to their classmates and their teacher. Students explore the use of the same symbols in different problems to build a foundational understanding of the concept of variables.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Find the missing number in each of the patterns below.
A. 0, 2, 6, 12, 20, $\qquad$
B. $1,2,5,10,17$, 37
C. 25, 20, $\qquad$ $, 17,19,14,16,11,13,8$
2. The table shows the cost of hamburgers.

If the pattern continues, what will be the cost for 9 hamburgers?

| Number of Hamburgers | Total cost |
| :--- | :--- |
| 1 | $\$ 2.00$ |
| 2 | $\$ 4.00$ |
| 3 | $\$ 6.00$ |
| 4 | $\$ 8.00$ |
| 5 | $\$ 10.00$ |

3. The homes on the right side of Birch Street have the following address numbers:

$$
\begin{array}{llll}
1360 & 1363 & 1366 & 1369
\end{array}
$$

What will be the address number for the seventh house on right side of Birch Street?
4. Sally is 4 years younger than Kelly. Which expression would you use to find Sally's age, if $\square=$ Kelly's age? Justify your choice.
a.
$\square-4$
b.

x 4
C. 4 -

d. $4+$ $\square$
5. Mike had some baseball cards. Miguel had more baseball cards than Mike. Together they had 15 cards. What numbers could make the number sentence true?

$$
\square+\Delta=15
$$

$$
\square=\text { Mike's cards }
$$

$\triangle=$ Miguel's cards

