## SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

## Content Area $3^{\text {rd }}$ Grade Math <br> Recommended Days of Instruction $\quad$ Fourth Nine Weeks

Standard 3-4: The student will demonstrate through the mathematical processes an understanding of the connection between the identification of basic attributes and the classification of two-dimensional shapes.
3-4.8 Predict the results of one transformation-either slide, flip, or turn-of a
geometric shape. (B2)
Standard 3-5: The student will demonstrate through the mathematical process an understanding of length, time, weight, and liquid volume measurements; the relationships between systems of measure; accurate, efficient, and generalizable methods of determining the perimeters of polygons; and the values and combinations of coins required to make change.
3-5.5 Generate strategies to determine the perimeters of polygons. (B6)
Standard 3-2: The student will demonstrate through the mathematical processes an understanding of the representation of whole numbers and fractional parts; the addition and subtraction of whole numbers; accurate, efficient, and generalizable methods of multiplying whole numbers; and the relationships among multiplication, division, and related basic facts.
3-2.10 Generate strategies to multiply whole numbers by using one single -digit factor and one multi-digit factor. (B6)
3-2.11 Use basic number combinations to compute related multiplication problems that involve multiples of 10 . (C3)
Standard 3-6: The student will demonstrate through the mathematical processes an understanding of organizing, interpreting, analyzing and making predictions about data, the benefits of multiple representations of a data set, and the basic concepts of probability.
3-6.1 Apply a procedure to find the range of a data set. (C3)
3-6.2 Organize data in tables, bar graphs, and dot plots. (B4)
3-6.3 Interpret data in tables, bar graphs, pictographs, and dot plots. (B2)
3-6.4 Analyze dot plots and bar graphs to make predictions about populations. (B4)
3-6.5 Compare the benefits of using tables, bar graphs, and dot plots as representations of a given data set. (B2)

3-6.6 Predict on the basis of data whether events are likely, unlikely, certain, or impossible to occur. (B2)
3-6.7 Understand when the probability of an event is 0 or 1 . ( $B 2$ )

* These indicators are covered in the following 5 Modules for this Nine Weeks Period.

Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.

## Module 4-1 Transformations

Indicator
Recommended
Resources
Suggested Instructional Strategies
Assessment
Guidelines

| Module 4-1 Lesson A <br> 3-4.8 Predict the results of one transformationeither slide, flip, or turn-of a geometric shape. (B2) | STANDARD SUPPORT DOCUMENT <br> http//:www.ed.sc.gov/apps/c so/standards <br> NCTM's Online <br> Illuminations <br> http://illuminations.nctm.o <br> rg <br> NCTM's Navigations Series <br> Teaching StudentCentered Mathematics Grades K-3 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle <br> NCTM's Principals and Standards for School Mathematics (PSSM) | See Instructional Planning Guide Module 4-1 Introductory Lesson A | See Instructional Planning Guide Module 4-1 Lesson A Assessing the Lesson |
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## Module 4-2 Perimeter

| Module 4-2 Lesson A <br> 3-5.5 Generate strategies to determine the perimeters of polygons. (B6) | STANDARD SUPPORT DOCUMENT <br> http//:www.ed.sc.gov/apps/c <br> so/standards <br> NCTM's Online <br> Illuminations <br> http://illuminations.nctm.o <br> rg <br> NCTM's Navigations Series <br> Teaching StudentCentered Mathematics Grades K-3 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle <br> NCTM's Principals and Standards for School Mathematics (PSSM) | See Instructional Planning Guide Module 4-2 Introductory Lesson A | See Instructional Planning Guide Module 4-2 Lesson A Assessing the Lesson |
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## Module 4-3 Multiplication

| Indicator | Recommended Resources | Suggested Instructional Strategies | Assessment Guidelines |
| :---: | :---: | :---: | :---: |
| Module 4-3 Lesson A <br> 3-2.10 Generate strategies to multiply whole numbers by using one single -digit factor and one multi-digit factor. (B6) | STANDARD SUPPORT DOCUMENT <br> http//:www.ed.sc.gov/apps/c so/standards <br> NCTM's Online <br> Illuminations <br> http://illuminations.nctm.o <br> rg <br> NCTM's Navigations Series <br> Teaching StudentCentered Mathematics Grades K-3 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle <br> NCTM's Principals and Standards for School Mathematics (PSSM) | See Instructional Planning Guide Module 4-3 Introductory Lesson A | See Instructional Planning Guide Module 4-3 Lesson A Assessing the Lesson |


| Module 4-3 Lesson B <br> 3-2.11 Use basic <br> number combinations to <br> compute related <br> multiplication <br> problems that involve <br> multiples of 10. (C3) |  | See Instructional Planning Guide Module 4-3 <br> Introductory Lesson B | See Instructional <br> Planning Guide <br> Module 4-3 Lesson B <br> Assessing the Lesson |
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Module 4-4 Lesson A
3-6.1 Apply a
procedure to find the range of a data set. (C3)

3-6.2 Organize data in tables, bar graphs, and dot plots. (B4)

3-6.3 Interpret data in tables, bar graphs, pictographs, and dot plots. (B2)

3-6.4 Analyze dot plots and bar graphs to make predictions about populations. (B4)

3-6.5 Compare the benefits of using tables, bar graphs, and dot plots as representations of a given data set. (B2)

| STANDARD SUPPORT DOCUMENT <br> http//:www.ed.sc.gov/apps/c so/standards <br> NCTM's Online <br> Illuminations <br> http://illuminations.nctm.o rg <br> NCTM's Navigations Series <br> Teaching StudentCentered Mathematics Grades K-3 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle <br> NCTM's Principals and Standards for School Mathematics (PSSM) | See Instructional Planning Guide Module 4-4 Introductory Lesson A <br> See Instructional Planning Guide Module 4-4, Lesson A Additional Instructional Strategies | See Instructional Planning Guide Module 4-4 Lesson A Assessing the Lesson |
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$\left.\left.\begin{array}{|l|l|l|l|}\hline \begin{array}{l}\text { Module 4-4 Lesson B } \\ \text { 3-3.4 Illustrate } \\ \text { situations that show } \\ \text { change over time as } \\ \text { increasing (B2) }\end{array} & & \begin{array}{l}\text { See Instructional Planning Guide Module 4-4 } \\ \text { Introductory Lesson B }\end{array} & \begin{array}{l}\text { See Instructional } \\ \text { Planning Guide } \\ \text { Module 4-4 Lesson B }\end{array} \\ \text { See Instructional Planning Guide Module 4-4, } \\ \text { Lesson B Additional Instructional Strategies }\end{array}\right] \begin{array}{l}\text { Assessing the Lesson }\end{array}\right]$

| Module 4-5 Lesson A <br> 3-6.6 Predict on the basis of data whether events are likely, unlikely, certain, or impossible to occur. (B2) <br> 3-6.7 Understand when the probability of an event is 0 or 1 . (B2) | STANDARD SUPPORT DOCUMENT <br> http//:www.ed.sc.gov/apps/c so/standards <br> NCTM's Online <br> Illuminations <br> http://illuminations.nctm.o <br> rg <br> NCTM's Navigations Series <br> Teaching StudentCentered Mathematics Grades K-3 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle <br> NCTM's Principals and Standards for School Mathematics (PSSM) | See Instructional Planning Guide Module 4-5 Introductory Lesson A <br> See Instructional Planning Guide Module 4-5, Lesson A Additional Instructional Strategies | See Instructional Planning Guide Module 4-5 Lesson A Assessing the Lesson |
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## MODULE

## 4-1

## Transformations

This module addresses the following indicators:

3-4.8 Predict the results of one transformation—either slide, flip, or turn-of a geometric shape. (B2)

This module contains 1 lesson. This lesson is INTRODUCTORY ONLY. Lessons in $\mathrm{S}^{3}$ begin to build the conceptual foundation students need.
ADDITONAL LESSONS will be required to fully develop the concepts.

## I. Planning the Module

## Continuum of Knowledge:

In third grade, students predict the results of one transformationeither slide, flip, or turn-of a geometric shape (3-4.7). This is their first encounter with transformations (slide, flip, or turn) of shapes.

In fourth grade, students predict the results of multiple transformations of the same type- translation, reflection, or rotationon a two-dimensional geometric shape (4-4.3). In fifth grade, students predict the results of multiple transformations on a geometric shape when combinations of translations, reflections, and rotations are used (5-4.5).

## Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.
- *Slide (Translation)
- *flip (Reflection)
- *turn (Rotation)
- *Predict
- *Transformation


## II. Teaching the Lesson(s)

## 1. Teaching Lesson A-Transformations

Using geometric shaped manipulatives for the students to perform a transformation such as a slide, flip, or turn will help them conceptually understand the results. After many opportunities, the students should visualize the changes and be able to predict the result of one transformation.

For this indicator, it is essential for students to:

- Understand that a turn is when a figure is turned a certain angle and direction around a point
- Understand that a flip is creating a mirror image of a figure on the opposite side of a line (line of reflection)
- Understand that a slide is a transformation that slides a figure a given distance in a given direction
- Understand the difference between a flip, slide, and turn.
- Predict the results of one transformation-either slide, flip, or turn-of a geometric shape such as:

Which picture represents a flip?

Figure $A$


Figure $B$


Figure C


Figure D


For this indicator, it is not essential for students to:

- Students are not required to predict the results of multiple transformations


## a. Indicators with Taxonomy

3-4.8 Predict the results of one transformation-either slide, flip, or turn-of a geometric shape. (B2)

Cognitive Process Dimension: Understand Knowledge Dimension: Conceptual Knowledge

## b. Introductory Lesson

## Materials:

Color tiles
1 inch grid paper
Color pencils or crayons

## Lesson:

Have students work in pairs for this lesson. Give each pair of students some color tiles. Challenge them to make as many different shapes that they can with 4 color tiles. The shapes must have one side touching. They must be laid flat. These are called tetrominoes. When they make one, have them draw it on the 1 inch grid paper. After they have made several, ask: Can you make any more? If they make some that are the same, only turned around, ask: How are these two different? If you cut one out and turn it or flip it, will it fit on top of the other one? They should come up with 5 unique tetrominoes.

Have them cut out their tetrominoes and check to make sure they are all different by trying to place them on top of each other. When they are convinced that they are all different, display all of the different shapes. Then give each pair of students a blank 1 inch grid paper. The students take turn choosing a shape and placing it on the grid paper. The shape must touch the bottom of the grid or the side of another shape. They may turn, slide or flip the shape to make it fit. Once they have placed the
tetrominoe, they pick it up and color in where they had placed it. The challenge is to have as few empty spaces as they can when they can't place any more shapes on the grid paper. When they have finished, challenge them to try again to see if they can play again and have fewer empty spaces when they finish.

## c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

## d. Additional Instructional Strategies/Differentiation

- Give students a sheet of paper that is divided into 4 parts. Give each student a pattern block. Students trace and color the pattern block in the top left quadrant. Then ask the students to do the following:
- Slide the block to the right into the next quadrant and trace.
- Flip the block into the quadrant below it and trace.
- Turn the block 90 degrees and slide it to the left into the last quadrant and trace.
- Give each student a sheet of paper, a shape, and a brad fastener. Fasten the shape to the piece of paper. Trace the shape. Now, turn it 90 degrees. Trace it again. This represents a 90 degree turn. Now, turn it 90 degrees again and trace it. This represents another 90 degree turn.
- Make sure that the students realize that shape does not change during a slide, flip, or turn. Use capital letters cut from a die cut machine to demonstrate slides, flips, and turns. They can trace them in their original position and then again after the letter has completed a slide, flip, or turn.



## e. Technology

- Icy Slides, Flips, Turns (Students click on either slide, flip, or turn according to the movement of the picture.) http://www.harcourtschool.com/activity/icy slides flips turns/
- Slides, Flips, and Turns Lesson Plan http://www.uen.org/Lessonplan/preview.cgi?LPid=16273
- Slide, Flip and Turn Activity (Illuminations activity) http://illuminations.nctm.org/lessons/developgeometric/tangramactivit ysheet.pdf
- SMART
http://technology.usd259.org/resources/whiteboards/smartlessons/notebook lessons/2nd3rdGradeFlipsSlidesRotationsWithSenteoQuiz.notebook
- Slides, Flips, and Turns for Two-Dimensional Figures (SMART Board activity) http://education.smarttech.com/ste/en-US/Ed+Resource/Lesson+activities/Senteo/Canada/Elementary/K3/Math/Slides+Flips+and+Turns+for+Two+Dimensional+Figures+Que stion+set.htm
- Transformation Games (Print out the cards to play.) http://www.ntcschool.com/sec/math/t resources/gamezone/pdfs/mac3 04/class ch0 6.pdf
- Shape Cutter http://illuminations.nctm.org/ActivityDetail.aspx?ID=72
- Tangram Puzzles (students use a flip, slide, or turn to place pieces) http://illuminations.nctm.org/LessonDetail.aspx?id=L168
- SMART Board Interactive Whiteboard Lessons and Resources http://www.scholastic.com/interactivewhiteboards/
- Tangram Puzzles (students use a flip, slide, or turn to place pieces) http://illuminations.nctm.org/LessonDetail.aspx?id=L168

Students decide if shapes have performed a flip, slide, or turn

- http://www.harcourtschool.com/activity/icy slides flips turns/


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

The questions included while the students are making the tetrominoes will help you determine if they are having trouble seeing the transformations.

## III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The objective of this indicator is predict, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Conceptual knowledge
is not bound by specific examples; therefore, the student's conceptual knowledge of predicting the results of one transformation-either slide, flip, or turn-of a geometric shape should be explored using a variety of examples. The learning progression to predict requires students to recall the meaning of flip (reflection), slide (translation) and turn (rotation). Student use concrete models to visualize and create transformations of their own. They construct arguments (3-1.2) about what will be the result of a transformation. They explain and justify their answers to their classmates and their teacher (3-1.3) using correct, complete, and clearly written and oral mathematical language communicate their ideas.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. A student created this quadrilateral on his geoboard. Draw what the quadrilateral will look like after it is flipped over line segment AB.

2. What will this arrow will look like after you have turned it--- one turn to the right.

a.

b.

c.

d.


# MODULE 

## 4-2

## Perimeter

## This module addresses the following indicators:

3-5.5 Generate strategies to determine the perimeters of polygons. (B6)

This module contains 1 lesson. This lesson is INTRODUCTORY ONLY. Lessons in $\mathrm{S}^{3}$ begin to build the conceptual foundation students need. ADDITONAL LESSONS will be required to fully develop the concepts.

## 1. Planning the Module

## Continuum of Knowledge:

Third grade is the first time students are introduced to the concept of perimeter.

In fourth grade, students analyze the perimeter of polygon (4-5.4).
In fifth grade, students apply formulas to determine the perimeters and areas of triangles, rectangles and parallelograms (5-5.4).

## Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the $*$ are additional terms for teacher awareness, knowledge and use in conversation with students.
- *Perimeter
- *sides
- *polygons


## II. Teaching the Lesson(s)

## 1. Teaching Lesson A-Perimeter

Third grade is the first time students are formally introduced to the concept of perimeter. As a result students should generate strategies to determine the perimeters of polygons. That means that students should not be simply told some formula or procedure for finding perimeter but should be given a problem in context and then determine a method to solve the problem. Perimeter is the measure of the distance around a closed figure and is an extension of length measurement. Problems dealing with realistic situations give children practical experiences that help them to determine perimeters and to distinguish perimeter from the measures of area with which they will deal in later grades.

Teachers should use Cuisenaire rods, trains of Unifix cubes, Color tiles, or centimeter rulers to find perimeters of such things as pictures and window frames, greeting card borders, and other objects that have trims. They can also use tape measures or trundle wheels to determine perimeters of classrooms, area rugs, playground areas, and larger regions. Real-life connections are made to mathematics as children engage in projects in which they determine the perimeter of a
picture in order to put a border around it and investigate the cost of the chain-link fence that encloses a play area at school.

As the term "generate strategies" indicates, students should "figure out" how to find the perimeter of areas/objects. It is not appropriate for students to simply memorize that perimeter is the distance around an area/object and can be determined by adding the length of all sides or using a formula such as $2 \mathrm{~L}+2 \mathrm{~W}$. While they may start with the definition/information, it is important to provide students with learning experiences that enable them to determine how to find the perimeter of a specified area/object. Geoboards are tools students may use to determine the perimeter of various polygons and how those perimeters differ based on the size of the polygon. In fourth grade students will apply formulas to determine the perimeter of polygons. Therefore, in third grade it is extremely important that students have multiple opportunities to develop a conceptual understanding that they can transfer to symbolic manipulation in fourth grade.

3-5.5 Generate strategies to determine the perimeters of polygons. (B6)

For this indicator, it is essential for students to:

- Recall the types of polygons they have learned (triangles, quadrilaterals, pentagons, etc..)
- Understand that perimeter is the distance around a closed figure
- Understand that perimeter is related to length
- Generate their own strategies
- Use appropriate units

For this indicator, it is not for students to:

- Develop a formula using variables such as $P=2 L+2 W$
- Examine perimeter of circles (circumference)


## a. Indicators with Taxonomy

3-5.5 Generate strategies to determine the perimeters of polygons. (B6)

Cognitive Process Dimension: Create Knowledge Dimension: Conceptual Knowledge

## b. Introductory Lesson

## Materials:

Measuring tools such as rulers, measuring tape, string, color tiles, etc.
Grid paper
Large paper polygons

## Lesson:

Explain the definition of perimeter: the distance around a shape. Ask them how they might find the distance around their desk. Let them work together to find the perimeter of their desk. Ask: What strategy did you use to find the perimeter of your desk? Let them share their strategies. Keep a record of strategies that the students like and want to remember. Ask: Why do you think people came up with different answers? (Some students might not put the measuring devices one right after the other, therefore not measuring the entire desk.) What might we do to make sure that our answers are closer? Ask: Find two other shapes in the room and find the perimeter of each. Provide materials for them to use. Have them share their strategies. Ask: What strategies work best for you? Did someone use a different strategy than you that you might like to try?

Give each student a paper polygon and grid paper. Ask them to find the perimeter of the polygon. They may trace it on the grid paper and count the sides, trace it with string and then measure the string, or use a ruler or measuring tape to find the perimeter. After they have found the perimeter, ask them to pair up and explain how they found the perimeter to each other. Then have the pairs find another pair and form quartets. Have them explain their methods to each other.

## c. Misconceptions/Common Errors

Although students are not formally introduced to the concept of area, students still think that the perimeter is related to how much space is covered by the shape.

## d. Additional Instructional Strategies/Differentiation

- Have the students use Unifix cubes to determine the perimeter of their math books, board erasers, and various sized objects.
- Use Geoboards to demonstrate and determine various perimeters. For example, have the students create a small square with a large square around it. Find the perimeter of each square and discuss the relationship between them.
- The teacher will create various straight edged polygons on centimeter graph paper. The students will find the perimeters using various strategies (count side squares, rulers, etc).

- Pose question: How could you use a ruler to help find the perimeter? Give students a worksheet with various polygons and allow them to use rulers to find perimeters.
- Demonstrate real life examples of finding perimeter, using trundle wheel to measure the perimeter of rugs, playgrounds, etc. Discuss how you would estimate the cost of a fence around the playground.


## e. Technology

- Perimeter Explorer (Use this online activity for students to generate the perimeter of shapes without the use of a "formula.") http://www.shodor.org/interactivate/lessons/PerimeterElem/
- Adam Ant (Adam walks the perimeters and teaches how to find the perimeter.)
http://www.beaconlearningcenter.com/WebLessons/AdamAnt/page 1.htm
- Geoboard (Use this online geoboard to help determine perimeter.) http://nlvm.usu.edu/en/nav/frames asid 281 g 2 t 4.html?open= activities\&from=category g 2 t 4.html
- SMART Board Interactive Whiteboard Lessons and Resources http://www.scholastic.com/interactivewhiteboards/


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

Exit ticket: How might you find the perimeter of the Cafeteria?
Poster: Design and draw a dog pen for a Labrador retriever. How much fencing will you need to build this pen?

## III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The objective of this indicator is to generate which is in the "create conceptual" knowledge cell of the Revised Taxonomy. To create means to put ideas together into a new structure; therefore, students use prior knowledge to generate new strategies. The learning progression to generate requires students to recall polygons and their properties. Using concrete and/or pictorial models, students apply their understanding of polygons to determine how to compute the distance around the shape. As students analyze information (3-1.1) from these experiences, they generate mathematical statements (3-1.4) about the relationships they observe then explain and justify their strategies (3-1.3) to their classmates and their teachers. They then generalize connections (3-1.6) between these statements and the concept of "perimeter'. Students recognize the limitations of various strategies and representations (3-1.8) and use correct, complete and clearly written and oral language to communicate their ideas (3-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Katie has a blanket that is 7 feet long and 4 feet wide. She wants to put a blue ribbon around the edge of her blanket. How many feet of ribbon will Katie need?


# MODULE 

## 4-3

## Multiplication

## This module addresses the following indicators:

3-2.10 Generate strategies to multiply whole numbers by using one single -digit factor and one multi-digit factor. (B6)
3-2.11 Use basic number combinations to compute related multiplication problems that involve multiples of 10 . (C3)

This module contains 2 lessons. These lessons are INTRODUCTORY ONLY. Lessons in $\mathrm{S}^{3}$ begin to build the conceptual foundation students need. ADDITONAL LESSONS will be required to fully develop the concepts.

## I. Planning the module

## Continuum of Knowledge

## 3-2.10

In second grade, students interpret models of equal grouping (multiplication) as repeated addition and array (2-2.5).

In third grade, students generate strategies to multiply whole numbers by using one single-digit factor and one multi-digit factor (3-2.10). Student use basic number combinations to compute related multiplication problems that involve multiples of ten. (3-2.11).

In fourth grade, students apply an algorithm to multiply whole numbers fluently (4-2.3) and explain the effect on the product when one of the factors is changed (4-2.4).

## 3-2.11

In second grade, students interpret models of equal grouping (multiplication) as repeated addition and array (2-2.5).

In third grade, students use basic number combinations to compute related multiplication problems that involve multiples of ten (3-2.11) and generate strategies to multiply whole numbers by using one single-digit factor and one multi-digit factor (3-2.10).

In fourth grade, students apply an algorithm to multiply whole numbers fluently (4-2.3) and explain the effect on the product when one of the factors is changed $(4-2.4)$.

## Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.
- *Factor
- *Multiple
- *product
- combinations


## II. Teaching the Lesson(s)

## 1. Teaching Lesson A-Multiplying by multi-digit numbers

The focus is for students to build conceptual knowledge of operations by generating their own strategies; therefore, the emphasis in not on computational fluency. That will be addressed in fourth grade.
Since students are to generate their own strategies, questions should promote and encourage students to think about how they can use prior knowledge to create their own process. Samples questions are "How is this problem different from multiplication problems we have done before? How can we decompose this number to make it easier to work with? How can we use what we know about tens and ones to break down this problem?"

Most of the generated strategies will be rooted in the traditional algorithm to some degree. Having students compare strategies and discuss similarities will bring out some of the commonalities.

For this indicator, it is essential for students to:

- develop their own strategies
- understand place value
- compose and decompose numbers
- use concrete and pictorial models to generate strategies

For this indicator, it is not essential for students to:

- Gain computational fluency
- Generate these strategies without concrete or pictorial models
- Use multi-digits beyond four digits


## a. Indicators with Taxonomy

3-2.10 Generate strategies to multiply whole numbers by using one single -digit factor and one multi-digit factor. (B6)

Cognitive Process Dimension: Create Knowledge Dimension: Conceptual Knowledge
b. Introductory Lesson

Materials:
Base ten blocks
Counters

## Lesson:

Tell this story to the class. Mrs. Brown hired 4 people to clear out all of the weeds in her yard. She agreed to pay them each $\$ 15$. How much did she pay to have the weeds cleared out of her yard?

Ask the students to talk with a partner about how they would solve this problem. Then ask them to solve the problem and represent how they solved the problem either with manipulatives or with a picture. Have the students do a gallery walk to see how everyone else solved the problem. When they finish the gallery walk, ask: Is there any problem that you would like further explanation as to how they solved it? Encourage them to share their ideas and question their classmates when they don't understand.

Repeat this process with other similar word problems.

## c. Misconceptions/Common Errors

Although students have decomposed numbers (tens and ones) they may not connect this idea and will believe that they can only find the answer using repeated addition.

## d. Additional Instructional Strategies/Differentiation

Have the students create word problems that involve one digit by multi digit numbers. Then solve each others' problems explaining how they did it.

## e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- While there are many multiplication games and activities online, for this indicator, focus on using manipulatives (such as base 10 blocks or Cuisenaire rods) in the classroom for children to generate strategies to multiply.
- Batter's Up Baseball (Select "double" level for two-digit multiplication. Have manipulatives available for students to use to help them solve the problems.) http://www.prongo.com/math/multiplication.html
- Base

10
Blocks
Online http://nlvm.usu.edu/en/nav/frames asid 152 g 2 t 1.html?from= category g 2 t 1.html

- Multiplication Activity (Have manipulatives available for students to use to help them solve the problems. Use as practice.) http://www.numbernut.com/basic/activities/mult quiz $2 \times 1$ nocarryv.shtml
- Ghost Blasters (Set unfriendly ghosts as multiples of 10.) http://resources.oswego.org/games/Ghostblasters1/gbcd.html


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

Use the gallery walk as a way to assess the students' understanding of multiplication.

## 2. Teaching Lesson B-multiplying by multiples of 10

Although the focus of the indicator is to use procedures, students should explore the concept of multiply with multiples of ten through inquiry. Students build conceptual knowledge of these relationships through the use of concrete models and generating and analyzing patterns.

Students may need access to a calculator to explore patterns. The calculator is simply used to quickly compute answers so that students can explore and analyze the patterns.

For this indicator, it is essential for students to:

- Recall basic facts
- Skip count by tens
- Understand that skip counting by tens is creating multiples of ten
- Solve problems where only one factor is a power of ten up to the thousands place. For example, $12 \times 7000$. Students use their multiplication fact $12 \times 7$ and their understanding of multiples of ten to solve this problem.

For this indicator, it is not essential for students to:

- Use basic combinations with multiples of ten beyond 1000
- Use basic combination where both factor are powers of ten such as $30 \times 400$. Students have only multiplied single-digit factor by one multi-digit factor.


## a. Indicators with Taxonomy

3-2.11 Use basic number combinations to compute related multiplication problems that involve multiples of 10 . (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge
b. Introductory Lesson

## Materials:

Calculators
Base ten rods

## Lesson:

Working in pairs and with base ten rods, have students model these problems and write the problems on paper with the answer.
$3 \times 10$
$6 \times 10$
Ask: What do you notice about these problems? (The second answer is double the first one.)
$4 \times 10$
$8 \times 10$
Ask: What do you notice about these problems? (The second answer is double the first one.)
$3 \times 10$
$3 \times 20$
Ask: What do you notice about these problems? (The second answer is double the first one.)
$4 \times 10$
$4 \times 20$
Ask: What do you notice about these problems? (The second answer is double the first one.)
They may notice that when one of the factors has been doubled, the answer is doubled, or that all of the answers end in zero.

Ask: Do you think this will be true for all problems? Let them experiment with the calculators to check their theory.
Next, have them use the calculator to see what happens when they multiply one of the factors by 10 . Have them pick a multiplication fact, write it out: $3 \times 6=18$. Then have them multiply (let them use the calculators) $3 \times 60$ and $30 \times 6$ and write the equation on their paper. Now ask: What happens to the product when one of the factors is multiplied by 10 ? Will this hold true all the time? Have them experiment with their calculators to see if it will always be true. Ask: How will knowing this help you solve other problems when one of the factors is a multiple of 10 ?

## c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

## d. Additional Instructional Strategies/Differentiation

Have students write examples of related number combinations on a T chart in their math journals.

Ask: What pattern do you notice with zeros in the factors and in the products?

| $3 \times 5=15$ | 15 ones |
| :---: | :--- |
| $3 \underline{0} \times 5=15 \underline{0}$ | 15 tens |
| $300 \times 5=15 \underline{000}$ | 15 hundreds |
| $3 \underline{000} \times 5=15, \underline{000}$ | 15 thousand |

e. Technology

- While there are many multiplication games and activities online, for this indicator, focus on using manipulatives (such as base 10 blocks or Cuisenaire rods) in the classroom for children to generate strategies to multiply.
- Batter's Up Baseball (Select "double" level for two-digit multiplication. Have manipulatives available for students to use to help them solve the problems.) http://www.prongo.com/math/multiplication.html
- Base 10 Blocks Online http://nlvm.usu.edu/en/nav/frames asid 152 g 2 t 1.html?from= category g 2 t 1.html
- Multiplication Activity (Have manipulatives available for students to use to help them solve the problems. Use as practice.) http://www.numbernut.com/basic/activities/mult quiz $2 \times 1$ nocarryv.shtml
- Ghost Blasters (Set unfriendly ghosts as multiples of 10.) http://resources.oswego.org/games/Ghostblasters1/gbcd.html


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

Exit slip question: What did you discover today about multiplying by a multiple of 10 ?

## III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The objective of this indicator is to use which is in the "apply procedural" knowledge cell of the Revised Taxonomy. To use is to carry out a procedure to a given situation (familiar or unfamiliar); therefore, students should understand how to use their procedure in a variety of situation. The learning progression to use requires students to recall basic multiplication facts and understand multiples of ten. Students explore number patterns and construct arguments (3-1.2) about what happens as the multiples of 10 increase. Students generalize these connections (3-1.6) and use correct, complete and clearly written and oral language to communicate their ideas (3-1.5). Student use this understanding to develop procedures that can be used to compute number combinations.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

Teacher Notes: Have students solve the problems by using a strategy that is meaningful to them. If manipulatives are used, have students draw a model that shows how they were used to solve the problems.

1. Six classes visited the computer lab on Tuesday. Each class had 15 students. Which model should you use to find the number of students that visited the lab?
2. Each bookshelf in the media center has 27 books. How many books will be on 3 shelves?
3. Each soccer team had 20 water bottles in a cooler to drink after the game. If there were 6 teams, how many water bottles were in all the coolers? Name the basic fact that will help you solve this problem.
4. Jerry had 30 dimes in each bag. He had 7 bags. How many dimes did he have?
Larry had 70 dimes in each bag. He had 3 bags. How many dimes did he have?
Write one or two sentences that explain the relationship between the two problems.

# MODULE 

## 4-4

## Data Analysis

## This module addresses the following indicators:

3-6.1 Apply a procedure to find the range of a data set. (C3)
3-6.2 Organize data in tables, bar graphs, and dot plots. (B4)
3-6.3 Interpret data in tables, bar graphs, pictographs, and dot plots. (B2)
3-6.4 Analyze dot plots and bar graphs to make predictions about populations. (B4)
3-6.5 Compare the benefits of using tables, bar graphs, and dot plots as representations of a given data set. (B2)
3-3.4 Illustrate situations that show change over time as increasing

This module contains 2 lessons. This lesson is INTRODUCTORY ONLY. Lessons in $\mathrm{S}^{3}$ begin to build the conceptual foundation students need.
ADDITONAL LESSONS will be required to fully develop the concepts.

## I. Planning the Module

## Continuum of Knowledge:

## 3-6.1

In third grade, students apply a procedure to find the range of a data set (36.1). This is the first time students are introduced to the concept of range of a data set.

In fifth grade, students calculate the measures of central tendency (mean, median, and mode) (5-6.3) and interpret the meaning of the measures of central tendency (5-6.4)

## 3-6.2; 3-6.3; 3-6.4; and 3-6.5

In first grade, students organize data in picture graphs, object graphs, bar graphs, and tables (1-6.2).

In second grade, students organize data in charts, pictographs and tables (26.2)

In third grade, students analyze dot plots and bar graphs to make predictions about populations (3-6.4). They compare the benefits of using tables, bar graphs, and dot plots as representations of a given data set (3-6.5).
In third grade, students organize data in tables, bar graphs, and dot plots (36.2). They also interpret data in tables, bar graphs, pictographs, and dot plots (3-6.3). Students also illustrate situations that show change over time as increasing (3-3.4).

In fourth grade, students organize data in tables, line graphs, and bar graphs whose scale increments are greater than or equal to $1(4-6.3)$. They also interpret data in tables, line graphs, and bar graphs whose scale increments are greater than or equal to $1(4-6.2)$.

## 3-3.4

First grade students classified change over time as qualitative or quantitative (1-3.6). Second grade students identified (2-3.4) and analyzed (2-3.5) qualitative and quantitative change over time.

In third grade, students illustrate situation that show change over time as increasing (3-3.4). Students also interpret data in tables, bar graph, pictographs and dot plots (3-6.3).

In fourth grade, students illustrate situation that show change over time as either increasing, decreasing or varying (4-3.6)

## Key Concepts/ Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the $*$ are additional terms for teacher awareness, knowledge and use in conversation with students.
- *Range
- *data sets
- *tables
- *bar graphs
- *pictographs
- *dot plots
- *Label
- *Scales
- *Title
- Horizontal axis
- Vertical axis
- *Key
- *Symbols
- Population
- increasing


## II. Teaching the Lesson(s)

## 1. Teaching Lesson A-Working with Data

Third grade is the first time students are introduced to the concept of range of a data set. Therefore, students will need many learning experiences on that topic. Determining range (distance between the highest and lowest data values) helps students recognize how spread out the data are. Students can use this information to make conjectures about the data.

A dot plot is made by making a horizontal line and placing an " $x$ " or "dot" above the corresponding value on the line for every data element (ex. Color of eyes, favorite food items, etc). Every piece of data is thus shown on a dot plot. When organizing data in tables, bar graphs, and dot plots, the scale should be limited to one. Students will progress to scales greater than one in fourth grade.

To reinforce understanding, students may examine tables and bar graphs from the newspaper or other media. Students should examine the scale, key, label, axes, title, etc...

Third grade students should analyze dot plots and bar graphs to make predictions about populations. Analyzing data from a representative
sample of a population allows students to make predictions about that population.

When comparing the benefits of using different forms of representation, students should recognize that the various forms of representation give different levels of the depth of information about the data set. Therefore, students should not only deal with tables, bar graphs and dot plots they create but should examine those forms of representation in newspapers, magazine, etc. in order to experience how representation impacts interpretation and can thus be misleading.

For this indicator (3-6.1), it is essential for students to:

- Understand that range is the difference between the highest number and the lowest number in a set of data
- Subtract fluently
- Find the greatest and least value in a set of data
- Apply a procedure to find the range of a data set such as:

Kelly made the following grades on Math Tests during the nine weeks: $90,85,80,100,75,80,90,80$. What is the range for this set of test scores?
A 25
C 80
B 75
D 100

- Apply a procedure to find the range of a data set in more sophisticated problems such as:

What is the range of temperatures during the week in this dot plot?


For this indicator (3-6.1), it is not essential for students to:

- Calculate the mean, median, and the mode. That will be dealt with in fifth grade.

For this indicator (3-6.2), it is essential for students to:

- Understand that a bar graphs use bars to show data and have titles, labels, scales, and bars
- Understand that a dot plot is a graph for displaying data, and each numerical value is represented by an x or dot placed over a horizontal number line
- Understand that data can be used to create tables, bar graphs, and dot plots
- Organize data in tables, bar graphs, and dot plots such as:

Look at table below and answer the following question.

| Favorite Ice Cream Flavors of $3^{\text {rd }}$ Graders |  |
| :---: | :---: |
| Favorite Ice Cream Flavor | Number of <br> Students |
| Chocolate | 4 |
| Vanilla | 5 |
| Banana | 2 |
| Butterscotch | 2 |

Which bar graph shows the information in the table above?
A.

B.

Favorite Ice Cream


For this indicator (3-6.2), it is not essential for students to:

- Use scale increments greater than 1 at this level.

For this indicator (3-6.3), it is essential for students to:

- Understand that a bar graphs use bars to show data and have titles, labels, scales, and bars
- Understand that a dot plot is a graph for displaying data, and each numerical value is represented by an $x$ or dot placed over a horizontal number line
- Understand that a pictograph is a graph that uses pictures or symbols to show data and has a title, symbols, key, and labels
- Analyze data to determine patterns
- Recognize change in the data
- Interpret data in tables, bar graphs, pictographs, and dot plots such as:

Use the bar graph below to answer the next question.


Which restaurant had the most votes?
A Big Burger
B Tasty Tacos
C Paul's Pizza
D Sub Shoppe
Ms. Collier's class caught butterflies for their class butterfly farm. The graph represents the butterflies four students caught from the farm. How many more butterflies did Ashley and Teresa catch than Rolando and James?


For this indicator (3-6.4), it is essential for students to:

- Understand that a bar graphs use bars to show data and have titles, labels, scales, and bars
- Understand that a dot plot is a graph for displaying data, and each numerical value is represented by an $x$ or dot placed over a horizontal number line
- Understand that they are analyzing a representative sample
- Analyze dot plots and bar graphs to make predictions about populations such as:

Here are the scores from Ken's last six spelling tests.

| X |  |  |  |
| :---: | :---: | :---: | :---: |
|  | X |  |  |
| X | X | X | X |
| $80^{111} 85$ | 90 | 95 | 100 |

What is the best prediction for the score for Ken's next spelling test?
A. Ken is definitely going to get a 100.
B. Ken will make less than an 80 .
C. Ken most likely make 90 or above.
D. Ken will get either an 80 or 85 .

For this indicator(3-6.5), it is essential for students to:

- Understand that a bar graphs use bars to show data and have titles, labels, scales, and bars
- Understand that a dot plot is a graph for displaying data, and each numerical value is represented by an $x$ or dot placed over a horizontal number line
- Understand that a table can be used to gather and organize data
- Compare the benefits of using tables, bar graphs, and dot plots as representations of a given data set such as:

When is it better to use a bar graph than a dot plot to show data?
A. When showing a small amount of data
B. When showing a large amount of data
C. When collecting numerical data
D. When showing an increase over time

## a. Indicators with Taxonomy

3-6.1 Apply a procedure to find the range of a data set. (C3) Cognitive Process Dimension: Apply Knowledge Dimension: Procedural Knowledge

3-6.2 Organize data in tables, bar graphs, and dot plots. (B4) Cognitive Process Dimension: Analyze Knowledge Dimension: Conceptual Knowledge

3-6.3 Interpret data in tables, bar graphs, pictographs, and dot plots. (B2)

Cognitive Process Dimension: Understand Knowledge Dimension: Conceptual Knowledge

3-6.4 Analyze dot plots and bar graphs to make predictions about populations. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

# 3-6.5 Compare the benefits of using tables, bar graphs, and dot plots as representations of a given data set. (B2) <br> Cognitive Process Dimension: Understand <br> Knowledge Dimension: Conceptual Knowledge 

## b. Introductory Lesson

## Materials:

## Suggested Literature Connection:

Lemonade for Sale by Stuart Murphy: Students can create a bar graph to match the data while the teacher reads the story. The students can describe what is happening with the data (more Tuesday than Monday, etc.), why it is important to increase the vertical axis by tens instead of ones, and how the bar graph informed the kids about their goal. This allows students to understand the benefits of organizing data.

If You Hopped Like a Frog by David M. Schwartz
Graph paper
Several rolls of adding-machine tap or thick string or yarn
Rulers or measuring tapes
Meterstick

## Lesson:

(If you don't have access to If You Hopped Like a Frog, then start the lesson with the question: How far do you think you can jump from a standing position.) Introduce the lesson by reading "If You Hopped Like a Frog" by David M. Schwartz. After reading the story, ask: how far do you think you can jump from a standing position? Have a student demonstrate a standing jump before they guess. Have them put their guess on a sticky note and place their guess on the board. Have someone arrange the guesses from least to greatest. Ask: What is the least guess? What is the most? Explain that the range is the least guess to the highest guess?

Ask: What is another way we can display this data? Have them work in pairs or trios to represent the data. If they don't come up with a dot plot, show them what a dot plot looks like by drawing a number line on the board and placing the sticky notes over the correct number.

Now, tell them that they are going to collect some data on standing jumps. Have students work in pairs or trios. Give them adding machine tape and measuring tape. Have them make a starting point, (masking tape will work) and let one student stand and jump. The other student will put adding machine tape from the start point to the back of the closest heel. Have them cut the tape and then measure it. Write the length of the adding machine tape on the tape. Let them take turns until all students have jumped and measured their jump. Have them line up the adding machine tape representing the length they jumped on the floor. Discuss how this is a bar graph.
Ask:

- How might we display these so it will be easier to read? (put in order from least to greatest)
- What is the range of our data?
- How close are our predictions to the actual distance?
- If we had a jumping contest with another $3^{\text {rd }}$ grade class, who would you want to represent our class?
- How far do you think the other $3^{\text {rd }}$ grade students can jump? Why?

Have them transfer this data to a dot plot. Ask: Which representation of the data is easier to understand? Why? How might you represent this data in a different way?

## c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

## d. Additional Instructional Strategies/Differentiation

Lemonade for Sale by Stuart Murphy
Students can create a bar graph to match the data while the teacher reads the story. The students can describe what is happening with the data (more Tuesday than Monday, etc.), why it is important to increase the vertical axis by tens instead of ones, and how the bar graph informed the kids about their goal. This allows students to understand the benefits of organizing data.

## e. Technology

- Every Breath You Take (In this real-life activity, students estimate, experiment, and collect data to represent graphically.) http://illuminations.nctm.org/LessonDetail.aspx?ID=L243
- Sticks and Stones (Students will collect data, investigate the likelihood of various moves, and use basic ideas of expected value
to determine the average number of turns needed to win a game.) http://illuminations.nctm.org/LessonDetail.aspx?id=L585
- As People Get Older, They Get Taller (In this two-lesson unit, students compare the heights of friends and classmates at different ages. Through the course of the lessons, students are exposed to algebra, measurement, and data analysis concepts. A major theme of the unit is analyzing change.) http://illuminations.nctm.org/LessonDetail.aspx?id=U171
- Introduction to Bar Graphs (This lesson allows students to learn what bar graphs are used for, how to interpret the data presented, and how to organize their own data using bar graphs.) http://www.shodor.org/interactivate/lessons/IntroBarGraphs/
- Online Bar Graph (Use to create a bar graph.) http://nlvm.usu.edu/en/nav/frames asid 323 g 2 t 5.html?from =category g 2 t 5.html
- Create a Graph (Choose bar graphs for $3^{\text {rd }}$ graders.) http://nces.ed.gov/nceskids/createagraph/default.aspx
- Bar

Grapher
http://www.amblesideprimary.com/ambleweb/mentalmaths/graphe r.html

- Bar Graphs and Dot Plots lesson (Only use $1^{\text {st }}$ lesson to cover $3^{\text {rd }}$ grade indicators.) http://www.keymath.com/documents/da1/CondensedLessonPlans/ DA CLP 01.pdf
- SMART Notebook Lessons/Activities (This site offers choices of grade levels, subject, and teachers can choose state standard correlations. Browse Educator Resources, Lesson Activities for more than Notebook Lessons. This site also offers SMART Response question sets, teacher-created lessons and activities, SMART sync collaboration activities, and SMART Ideas Software activities. Browse by curriculum standards and the website will find correlated activities for the standard you choose.) http://education.smarttech.com/ste/enUS/Ed+Resource/Lesson+activities/Notebook+Activities/Correlated +Search+us.htm
- SMART Board Interactive Whiteboard Lessons and Resources http://www.scholastic.com/interactivewhiteboards/
- Interactivate Bar graph (enter data to create a bar graph.) http://www.shodor.org/interactivate/activities/bargraph/
- Data Analysis and Probability Games http://www.mathwire.com/games/datagames.html
- Understanding Graphs (Students complete a frequency table, create a bar graph and pictograph.) http://www.bbc.co.uk/schools/ks2bitesize/maths/activities/interpre tingdata.shtml
- Slug Ball (Print this activity from Cyberchase. Students collect data and use the data to make predictions.) http://pbskids.org/cyberchase/parentsteachers/show/episodes/609 html
- Exploring the Range of Data (Students find the range in a set of data.)
http://www.harcourtschool.com/activity/elab2004/gr4/25 b.html
- Predicting Outcomes (Students play a game in which they can use number cube, coins, and/or spinner in order to predict outcomes.) http://www.harcourtschool.com/activity/elab2004/gr4/28.html
- Data Analysis and Probability Promethean Board activities http://www.prometheanplanet.com/server.php?ResourceSearch\%5 Bsearch text\%5D=data+and+probability\&ResourceSearch\%5Bsubj ect\%5D=00200n009002002002\&ResourceSearch\%5Bgrade\%5D=0 0200n009002003002\&display=006007001\&ResourceSearch\%5Bact ion\%5D=advanced\&change=ResourceSearchResults\&catMatchType =includeChildren\&searchType=basic\&x=40\&y=9
- http://www.bbc.co.uk/schools/ks2bitesize/maths/activities/interpre tingdata.shtml students organize data into graphs


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

Journal entry: Now that you have collected data on the standing long jump, how might you collect data on another activity?

## 2. Teaching Lesson B-Change over time

The emphasis here is on understanding that the change is increasing over the time period. For example, students may record the temperature when they arrive at school and again at two-hour intervals during the day. The next day students could examine the data in light of the increase of temperature (change) over time.

An important strategy to support students as they find examples is providing non examples. Although students do not have label other types of change using mathematical terminology, they should be able to state that a given example is not showing increasing change and why.

For this indicator (3-3.4), it is essential for students to:

- Understand change over time
- Determine if change has occurred
- Understand the concept of increasing change
- Use their understanding of change over time to find example of increasing change
- Describe observed change in words
- Recognize counter examples (change that does not increase)

For this indicator, it is not essential for students to:
None noted

## a. Indicators with Taxonomy

3-3.4 Illustrate situations that show change over time as increasing (B2)

Cognitive Dimension: Understand Knowledge Dimension: Conceptual

## b. Introductory Lesson

## Materials:

thermometers
Lesson:
Working in trios, have the students place a thermometer in different parts of the school. Make sure several of them are placed in different places outside. The students check the temperature every 2 hours during the day. They record the time and temperature on a table. At the end of the day, have the students compare all of the different tables.
Ask:

- What are you noticing about the tables?
- Which ones are similar? How are they similar?
- Whose thermometer increased in temperature the most during the day? What might be the reason for this?
- Whose thermometer didn't change much during the day? What might be the reason for this?
c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

## d. Additional Instructional Strategies/Differentiation

Collect data on other things that change over time: plant growth, student height, weight, etc.

## e. Technology

There is no specific technology recommended for this lesson at this time.

## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. However, other formative assessment strategies should be used.

Journal entry: Name something that increases over time? What are your hunches about why this increases over time?

## III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

3-6.1
The objective of this indicator is to apply, which is in the "apply procedural " knowledge cell of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem. The learning progression to apply requires students to understand the concept of range. Students analyze the data (3-1.1) to determine the greatest and least value. They subtract these values to determine the range. They explain and justify their answers (3-3.3), and use correct, complete, and clearly written and oral mathematical language to their communicate ideas to their classmates and teacher.

## 3-6.2

The objective of this indicator is to organize which is in the "analyze conceptual " knowledge cell of the Revised Taxonomy. Conceptual knowledge is not bound by specific examples. Therefore, the student's conceptual knowledge of organizing data in tables, bar graphs, and dot plots should be explored using a variety of examples. The learning progression to organize requires students to understand the characteristics of tables, bar graphs, and dot plots. Students analyze data and determine how the data should be categorized or sorted. Students should recognize the limitations of each type of representation (3-1.8) and use that understanding to select the most appropriate method for organizing the data. They explain and
justify answers on the basis of mathematical properties, structures, and relationships (3-3.3), and use correct, complete, and clearly written and oral mathematical language to pose questions, communicate ideas, and extend problem situations (3-1.5) with their classmates and teacher.

3-6.3
The objective of this indicator is to interpret, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge of interpreting data in tables, bar graphs, pictographs and dot plots should be explored using a variety of examples. The learning progression to interpret requires students to understand the characteristics of each type of representation and recognize the limitations of each (3-1.8). Students analyze the representation (3-3.1) to detect changes in the data. Students generate descriptions and mathematical statements based on their observations and explain and justify their interpretation (3-3.3) using correct, complete, and clearly written and oral mathematical language (31.5).

3-6.4
The objective of this indicator is to analyze, which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. Analyze conceptual break down data into its parts and determine how the parts relate to one another and to the overall structure (population); therefore, the student's conceptual knowledge of analyzing dot plots and bar graphs to make predictions about populations should be explored using a variety of examples. The learning progression to analyze requires students to understand the characteristics of each type of representation. Students interpret the data and generate descriptions and mathematical statements about relationships they observe (3-1.4). They explain and justify their observations (3-1.3) to their classmates and their teacher. They use this information make predictions about given populations.

3-6.5
The objective of this indicator is to compare, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To compare means to determine relationship between and among ideas. Conceptual knowledge is not bound by specific examples; therefore, the student's understanding of the benefits of each representation should be explored using a variety of examples. The learning progression to compare requires students to understand the characteristics of each type of representations. Students should use that understanding to create different representations of the same data and discuss the advantages and/or disadvantages of using one representation over another. Students generate descriptions and mathematical statements about relationships between and among the representations (3-1.4) and recognize the limitations of each representation (3-1.8). They explain and justify answers on the basis of mathematical properties, structures, and relationships (3-3.3), and use correct, complete,
and clearly written and oral mathematical language to pose questions, communicate ideas, and extend problem situations (3-1.5) with their classmates and teacher.

## 3-3.4

The objective of this indicator is to illustrate which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To illustrate means to find specific examples of a concept; therefore, students should explore a variety of example to build understanding of the concept of increasing change. The learning progression to illustrate requires students to understand change over time and meaning of increasing. Students explore teacher generated examples and analyze information (3-1.1) from those examples to determine if change has occurred. They generate descriptions (3-1.4) of the observed change then explain and justify their answer on the basis of mathematical relationships (3-1.3). Student use this understanding to find other examples of increasing change and analyze non examples to support conceptual understanding.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Sam created a dot plot to show the number of videos that his friends rented during the month of March.


Number of Videos Rented in March
a. How many friends rented 5 videos in March?
b. What is the range in the data shown in the dot plot?
c. If Fred is one of Sam's friends, how many videos did he most likely rent?
2. Here are the scores from Ken's last six spelling tests.

| X |  |  |  |
| :---: | :---: | :---: | :---: |
| X | X |  |  |
|  | X | X | X |
| $80^{111} 85$ |  | 95 | 100 |

What is the best prediction for the score for Ken's next spelling test?
E. Ken is definitely going to get a 100 .
F. Ken will make less than an 80.
G. Ken most likely make 90 or above.
H. Ken will get either an 80 or 85 .
3. Kelly made the following grades on Math Tests during the nine weeks: $90,85,80,100,75,80,90,80$. What is the range for this set of test
scores?
A. 25
B. 75
C. 80
D. 100

The assessment item \#4 is adapted from Grades 3-5 Mathematics Assessment Sampler, Jane D. Gawronski, editor. 2005. Pages 157.
4. The students in Mr. Kirby's class voted for their favorite book of the past 3 months.
The table shows the votes each book received:

| Title of Book | Number of Votes |
| :--- | :--- |
| Babe, the Gallant Pig | 7 |
| Sarah, Plain and Tall | 6 |
| Stone Fox | 11 |

Make a bar graph, pictograph and dot plot using the information in the table.

The assessment item \#5 is adapted from SC Jury Exemplar Samples, Sample ID 2721, page 79.
5. The table below shows the number of rainy days each month in Columbia, SC for part of last year.

| Month | Number of Rainy Days |
| :--- | :--- |
| January | 7 |
| February | 9 |
| March | 9 |
| April | 10 |
| May | 5 |
| June | 8 |
| July | 9 |
| August | 8 |

Create both a bar graph and a dot plot to display the data in the table. Compare the table, bar graph, and dot plot.
Which display do you think best represents the data? Share reasons for your decision. (see graph templates on next page)


These assessment items (\#6 and \#7) came from the website: Study Zone http://www.studyzone.org/testprep/math4/d/changeovertimep.cfm
6. Look at the table below. What can you predict about the growth of the plants in week 5 ?


The Carthage Fifth Graders are planting plants in their classroom. Each week they measured how high the plant had grown. The students recorded the data below in a chart and line graph.

| Height of Plants |  |
| :---: | :---: |
| Number of Weeks | Height in Inches |
| Week 1 | $\underline{0.5}$ |
| Week 2 | $\underline{1}$ |
| Week 3 | $\underline{3}$ |
| Week 4 | $\underline{\underline{5}}$ |


|  |  |
| :---: | :---: |
| Number of Weeks |  |
| Week 1 | Weight |
| Week 2 | 8 pounds 3 ounces |
| Week 3 | 8 pounds 10 ounces |
| Week 4 | 9 pounds 5 ounces |
| Week 5 | 9 pounds 11 ounces |

7a.. What is approximately the weight of the newborn baby in Week 5 ?
A. 10 pounds 3 ounces
B. 8 pounds 6 ounces
C. 20 pounds 6 ounces

7b. Do you think that in Week 6 the baby will weigh more or less than Week 5 ?

# MODULE 

## 4-5

## Probability

This module addresses the following indicators:
3-6.6 Predict on the basis of data whether events are likely, unlikely, certain, or impossible to occur. (B2)
3-6.7 Understand when the probability of an event is 0 or 1 . (B2)

This module contains 1 lesson. This lesson is INTRODUCTORY ONLY. Lessons in $\mathrm{S}^{3}$ begin to build the conceptual foundation students need.
ADDITONAL LESSONS will be required to fully develop the concepts.

## I. Planning the Module

## Continuum of Knowledge

In first grade, students predict on the basis of data whether events are likely or unlikely to occur (1-6.4). In second grade, students predict on the basis of data whether events are more likely or less likely to occur (2-6.4)

In third grade, students predict on the basis of data whether events are likely, unlikely, certain, or impossible to occur (3-6.6). Third grade students also understand when the probability of an event is 0 or 1(36.7).

In fourth grade, students predict on the basis of data whether events are likely, equally likely, unlikely, certain, or impossible to occur (4$6.6)$, and analyze possible outcomes for a simple event (4-6.7). In fifth grade, students represent the probability of a single-stage event in words and fractions (5-6.1) and conclude why the sum of the probabilities of the outcomes of an experiment must equal 1(5-6.6).

## Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.
- *Certain
- *Event
- *Experiment
- *Likely
- *Impossible
- *Predict
- *Probability
- *Unlikely


## II. Teaching the Lesson

## 1. Teaching Lesson A-Probability

Third grade students generally grasp the concept of certain and impossible quickly, but have trouble understanding the difference between likely and unlikely. Therefore, when introducing the four terms, begin with outcomes that are certain (the sun will rise everyday, I will pull a piece of gum out of a package of bubble gum) or impossible (pigs will fly, I will pull a piece of candy out of a package of
South Carolina $S^{3}$ Mathematics Curriculum
bubble gum). Afterwards, the focus should move toward outcomes that are likely (I will eat sometime today, I will pull out a green marble out of my pocket filled with nine green and two yellow marbles) and unlikely (It will snow in July in South Carolina, I will pull out a yellow marble out of my pocket filled with nine green and two yellow marbles).

Students need to understand when the probability of an event is 1 or 0 . The requirements of this indicator mean more than simply reciting that "1 means certain" and "0 means impossible". As the verb "Understand" implies, students should be able to explain and give examples of events that fit each probability. This is laying a foundation for their work with probability and the fact that the sum of event must be one.

For this indicator (3-6.6), it is essential for students to:

- Understand that probability is the chance that a given event will occur
- Understand that an event is something that might happen
- Understand that an outcome is a possible result of an experiment
- Understand that an event that is likely has a good chance of happening
- Understand that an event that is unlikely does not have a good chance of happening
- Understand that an event that is impossible will never happen
- Understand that an event that is certain will always happen
- Predict on the basis of data whether events are likely, unlikely, certain, or impossible to occur such as:

Kim has 5 cards face-down on a table. She is going to pick one card.


What is the probability that she will select a card with a 3 on it? Certain

- Predict on the basis of data whether events are likely, unlikely, certain, or impossible to occur to solve increasingly more sophisticated problems such as:

Create an event where you are likely to choose a heart:


For this indicator, it is not essential for students to:

- Predict events that are equally likely to occur.

For this indicator (3-6.7), it is essential for students to:

- Understand that probability is the chance that a given event will occur
- Recall the definition of event
- Understand that an outcome is a possible result of an experiment
- Understand that an event with the probability of 0 is impossible to happen
- Understand that an event with the probability of 1 is certain to happen
- Understand when the probability of an event is 0 or 1 such as:

Which is an example of an event with a probability of 1 ?
A. Tossing a coin heads up
B. Rolling a 6 on a dice numbered 1-5
C. Pulling a blue sock from a drawer with white, black, and blue socks
D. Pulling a red crayon from a box of red crayons

## a. Indicators with Taxonomy

3-6.6 Predict on the basis of data whether events are likely, unlikely, certain, or impossible to occur. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge
3-6.7 Understand when the probability of an event is 0 or 1 . (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

## b. Introductory Lesson

This lesson is adapted from NCTM Navigations Series: Navigating through Data Analysis and Probability in Grades 3-5, "How Likely Is It to Land in the Trash Can?" pp. 62-67

## Materials:

Sheets of scrap paper or recycled paper-all the same size Trash cans of other large containers, such as buckets

Metric tape measures or meter sticks
Masking tape
Pad of sticky notes

## Lesson:

Ask:

- Have you ever crumpled paper and thrown it into the trashcan?
- Did it go in?
- Can we predict if it will go in?
- Why might some go in and others miss?

Introduce the idea that some tosses are certain to go in, some are likely to go in, some are unlikely to go in and some that can't possibly go in. What factors might determine which kind of toss that will be?

Write certain, likely, equally likely to occur and not occur, unlikely, and impossible on the board. Ask: What do these words mean to you? Make sure you get several responses to get a feel for their understanding.

Tell the class that we are going to do an experiment to see what the probability of tossing a piece of paper into the trashcan is with different situations. Have them work in trios and keep their own data. Each group should have masking tape, measuring tape or meterstick, scrap paper, and a bucket. After they place the bucket, have them place masking tape these distances from the bucket: 3 $\mathrm{cm}, 150 \mathrm{~cm}, 250 \mathrm{~cm}, 350 \mathrm{~cm}$, and 6 m . Tell them to toss the paper (made into a ball) from each distance 3 times and record whether or not it went in. When they toss from the 6 m line, they must close their eyes.

When everyone has finished tossing and recording their data, make a class table with the labels: Distance, Number of Tosses That Went in, and Number of Tosses, Tosses In/Total Tosses. Collect all of the data to fill in the chart.
Ask:

- What are your hunches about the classes tossing ability?
- Is there a distance that we can say our class is "certain" to make it in?
- Is there a distance that we can say our class is "impossible" to make it in?
- Which descriptor would you use for each distance?

Draw a long line segment on the board and label it with a "0" and the word "impossible" at one end and a "1" and the work "certain" at the other end. Explain that this is a probability line and that we
can use it to quantify the likelihood of events. Probability is a number that describes the likelihood of an event occurring.

Ask: Where do you think we would be equally likely to occur and not occur on this continuum? Likely? Unlikely? Have them place sticky notes with these labels on the continuum. They should place "Likely" somewhere from the middle towards "1" and "Unlikely" somewhere between the middle and " 0 ". Have them write events on sticky notes and place them on the continuum.
Examples:

- The sun will rise.
- The sun will set.
- We will have recess today.
- We will go to lunch today.
- Santa Claus will visit the class.
- It will rain today.
- My dog will learn to fly.


## c. Misconceptions/Common Errors

No noted misconceptions at this time.
d. Additional Instructional Strategies/Differentiation

- Put twenty color tiles, candy pieces, or marbles in a paper sandwich sacks. Divide the students into groups. Have each group look at the objects, record the colors of them and place them back in the sack. Have students record the probability of selecting a particular color as likely, unlikely, certain, or impossible. Then give each group their own bag and have them take turns picking one object out of the bag, recording it, and then putting it bag in the bag. After they have done this about 30 times, have them compare their results with their predictions.
- Put clear chips of different colors on the overhead and explain the probability of picking each color. Have students record them as likely, unlikely, certain, or impossible. Put all of the same color (red) on the overhead. What is the probability of picking a red one? A blue one? Explain why you chose this answer? How might you change the chips so that the probability of picking a green one is likely?

Literature connection: Probably Pistachio by Stuart Murphy, HarperCollins Publishers; 2001

## e. Technology

- Sticks and Stones (Students will collect data, investigate the likelihood of various moves, and use basic ideas of expected value to determine the average number of turns needed to win a game.)
http://illuminations.nctm.org/LessonDetail.aspx?id=L585
- Spinner (Use online for activities. Can be used to explore 0 and 1 also.)
http://nlvm.usu.edu/en/nav/frames asid 186 g 2 t 5.html?op en=activities\&from=category g 2 t 5.html
- Certain, Probable, Unlikely, Impossible (Students choose from these. Enter site as guest.) http://www.ixl.com/math/practice/grade-3-certain-probable-unlikely-impossible
- Is It Likely or Is It Unlikely (Online lesson that is written for Grade One in another state but can be a great review for $3^{\text {rd }}$ grade or can easily be modified to include certain and impossible.) http://www.genevaschools.org/standards/m11.pdf
- Likely or Unlikely (Online lesson from Marilyn Burns.) http://www.uen.org/Lessonplan/preview.cgi?LPid=6180
- Probability

Games
http://www.betweenwaters.com/probab/probab.html

- Data Analysis and Probability Games http://www.mathwire.com/games/datagames.html
- Four Great Math Games (Favorite activities from Marilyn Burns to try in your classroom.) http://teacher.scholastic.com/lessonrepro/lessonplans/grmagam .htm
- SMART Notebook Lessons/Activities (This site offers choices of grade levels, subject, and teachers can choose state standard correlations. Browse Educator Resources, Lesson Activities for more than Notebook Lessons. This site also offers SMART Response question sets, teacher-created lessons and activities, SMART sync collaboration activities, and SMART Ideas Software activities. Browse by curriculum standards and the website will find correlated activities for the standard you choose.) http://education.smarttech.com/ste/enUS/Ed+Resource/Lesson+activities/Notebook+Activities/Correla ted+Search+us.htm
- SMART Board Interactive Whiteboard Lessons and Resources http://www.scholastic.com/interactivewhiteboards/
- Data Analysis and Probability Promethean Board activities http://www.prometheanplanet.com/server.php?ResourceSearch \%5Bsearch text\%5D=data+and+probability\&ResourceSearch\% 5Bsubject\%5D=00200n009002002002\&ResourceSearch\%5Bgra de\%5D=00200n009002003002\&display=006007001\&Resource Search\%5Baction\%5D=advanced\&change=ResourceSearchResu Its\&catMatchType=includeChildren\&searchType=basic\&x=40\&y三9


## f. Assessing the Lesson

Formative assessment is embedded within the lesson through questions and observation. The ability to choose events and place them correctly on the continuum will let you know if the students understand the concepts.

## III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

3-6.6
The objective of this indicator is to predict, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To predict is to draw conclusions from presented information. Conceptual knowledge is not bound by specific examples; therefore, students use a variety of examples to make predications. The learning progression to predict requires students to recall the meaning of likely, unlikely, certain, and impossible. Students explore teacher generated examples and construct arguments (3-1.2) about whether the event is likely, unlikely, certain or impossible. They explain and justify their arguments (3-1.3) with their classmates and their teacher. Students generalize connections among examples of each event and deepen their conceptual understanding by generating examples of events that are likely, unlikely, certain or impossible. Students use correct, complete, and clearly written and oral mathematical language to explain and justify their examples.

3-6.7
The objective of this indicator is to understand, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning; therefore, students understanding of the concepts should go beyond rote memorization and extend to a variety of examples. The learning progression to understand requires students to recall the characteristics of certain and impossible events. Students connect 0 and 1 with their understanding of these events. Students analyze a variety of problems situations and generalize connections among characteristics of each problem. They use this understanding to find examples of events that have a probability of 0 or 1 . They explain and justify their examples (3-1.3) using correct, complete, and clearly written and oral mathematical language (31.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

This assessment item \#1 is adapted from Grades 3-5 Mathematics Assessment Sampler, Jane D. Gawronski, editor. 2005, page 191.

1. The gum ball machine has 100 gum balls: 20 are yellow, 30 are blue, and 50 are red. The gum balls are well mixed inside the machine. What color is likely to come out of the machine when you turn the knob?
Explain why you chose that color.

This assessment item \#2 is adapted from Grades 3-5 Mathematics Assessment Sampler, Jane D. Gawronski, editor. 2005, page 195.

2a. Consider the following list of events. Describe each of the following as certain, likely, unlikely, or impossible.
a. Take 2 cubes, each with the numbers 1, 2, 3, 4, 5, and 6 written on each of its six faces. Throw them at random, and the sum of the numbers on the top face is less than or equal to 12.
b. New Year's Day falls on January 1.
c. You will be younger tomorrow.
d. There are 6 wooden cubes in a bag: 4 are yellow, 1 is green, and 1 is blue. You pull a red cube from the bag.
e. A spinner has 6 sections. Each section has a number. The numbers on the spinner are $2,4,5,6,8,9$. The likelihood that you will spin an even number.

2b. Which of the events have a probability equal to 0 ?
2c. Which of the events have a probability equal to 1 ?

3a. Design each of the spinners below so that the following will happen when you spin:

Spinner A: It is impossible to spin red.
Spinner B: You are unlikely to spin red.
Spinner C: You are likely to spin red.
Spinner D: You are certain to spin red.


3b. Which of the spinners has a probability equal to 1 that you will spin red? Explain your answer.

