SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

Content Area	Fourth Grade Mathematics
Recommended Days of Instruction	**Second Nine Weeks**

Standards/Indicators Addressed:

Standard 4-2: The student will demonstrate through the mathematical processes an understanding of decimal notation as an extension of the place-value system; the relationship between fractions and decimals; the multiplication of whole numbers; and accurate, efficient, and generalizable methods of dividing whole numbers, adding decimals, and subtracting decimals.

4-2.2* Apply divisibility rules for 2, 5, and 10. (C3)

4-2.5* Generate strategies to divide whole numbers by single-digit divisors. (B6)

Standard 4-3: The student will demonstrate through the mathematical processes an understanding of numeric and nonnumeric patterns, the representation of simple mathematical relationships, and the application of procedures to find the value of an unknown.

4-3.1* Analyze numeric, nonnumeric, and repeating patterns involving all operations and decimal patterns through hundredths. (B4)

4-3.2* Generalize a rule for numeric, nonnumeric, and repeating patterns involving all operations. (B2)

4-3.3* Use a rule to complete a sequence or a table. (C3)

4-3.4* Translate among letters, symbols, and words to represent quantities in simple mathematical expressions or equations. (B2)

4-3.5* Apply procedures to find the value of an unknown letter or symbol in a whole-number equation. (C3)

4-3.6* Illustrate situations that show change over time as either increasing, decreasing, or varying. (B2)

Standard 4-4: The student will demonstrate through the mathematical processes an understanding of the relationship between two- and three-dimensional shapes, the use of transformations to determine congruency, and the representation of location and movement within the first quadrant of a coordinate system.

4-4.1* Analyze the quadrilaterals squares, rectangles, trapezoids, rhombuses, and parallelograms according to their properties. (B4)

4-4.2* Analyze the relationship between three-dimensional geometric shapes in the form of cubes, rectangular prisms, and cylinders and their two-dimensional nets. (B4)

4-4.4* Represent the two-dimensional shapes trapezoids, rhombuses, and parallelograms and the three-dimensional shapes cubes, rectangular prisms, and cylinders. (B2)

* These indicators are covered in the following 6 Modules for this Nine Weeks Period.
Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.

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4-2.5 Generate strategies to divide whole numbers by single-digit divisors. (B6)</td><td rowspan="2">STANDARD SUPPORT DOCUMENT
http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html

NCTM's Online Illuminations
http://illuminations.nctm.org

NCTM's Navigations Series 3-5

Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle

Blackline Masters for Van de Walle Series
www.ablongman.com/vandewalleseries

NCTM's Principals and Standards for School Mathematics (PSSM)</td><td>See Instructional Planning Guide Module 2-1 Introductory Lesson A

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4-2.2 Apply divisibility rules for 2, 5, and 10. (C3)</td><td>See Instructional Planning Guide Module 2-1 Introductory Lesson B

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	NCTM, <u>Mathematics Assessment Sampler</u>: Grades 3-5 ETA Cuisenaire, <u>Hands-On Standards</u>: Grades 3-4		

Module 2-2 - Patterns, Relationships and Functions			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-2 Lesson A 4-3.1 Analyze numeric, nonnumeric, and repeating patterns involving all operations and decimal patterns through hundredths. (B4) 4-3.2 Generalize a rule for numeric, nonnumeric, and repeating patterns involving all operations. (B2)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries	See Instructional Planning Guide Module 2-2 Introductory Lesson A	See Instructional Planning Guide Module 2-2 Lesson A Assessing the Lesson
Module 2-2 Lesson B 4-3.3 Use a rule to complete a sequence or a table. (C3)		See Instructional Planning Guide Module 2-2 Introductory Lesson B See Instructional Planning Guide Module 2-2, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 2-2 Lesson B Assessing the Lesson

	NCTM's <u>Principals and Standards for School Mathematics</u> (PSSM) NCTM, <u>Mathematics Assessment Sampler</u>: Grades 3-5 ETA Cuisenaire, <u>Hands-On Standards</u>: Grades 3-4		

Module 2-3 - Representations, Properties, and Proportional Reasoning			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-3 Lesson A 4-3.4 Translate among letters, symbols, and words to represent quantities in simple mathematical expressions or equations.	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries	See Instructional Planning Guide Module 2-3 Introductory Lesson A	See Instructional Planning Guide Module 2-3 Lesson A Assessing the Lesson

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Module 2-4 - Solve Mathematical Situations			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-4 Lesson A 4-3.5 Apply procedures to find the value of an unknown letter or symbol in a whole-number equation. (C3)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries	See Instructional Planning Guide Module 2-4 Introductory Lesson A See Instructional Planning Guide Module 2-4, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 2-4 Lesson A Assessing the Lesson

	NCTM's <u>Principals and Standards for School Mathematics</u> (PSSM) NCTM, <u>Mathematics Assessment Sampler</u>: Grades 3-5 ETA Cuisenaire, <u>Hands-On Standards</u>: Grades 3-4		

Module 2-5 - Change in Various Contexts			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-5 Lesson A 4-3.6 Illustrate situations that show change over time as either increasing, decreasing or varying. (B2)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler:	See Instructional Planning Guide Module 2-5 Introductory Lesson A See Instructional Planning Guide Module 2-5, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 2-5 Lesson A Assessing the Lesson

	Grades 3-5 ETA Cuisenaire, <u>Hands-On Standards</u>: Grades 3-4		

Module 2-6 - Dimensionality			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-6 Lesson A 4-4.1 Analyze the quadrilaterals squares, rectangles, trapezoids, rhombuses, and parallelograms according to their properties.	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries	See Instructional Planning Guide Module 2-6 Introductory Lesson A See Instructional Planning Guide Module 2-6, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 2-6 Lesson A Assessing the Lesson
Module 2-6 Lesson B 4-4.2 Analyze the relationship between three-dimensional geometric shapes in the form of cubes, rectangular prisms, and cylinders and their two-dimensional nets. (B4)		See Instructional Planning Guide Module 2-6 Introductory Lesson B	See Instructional Planning Guide Module 2-6 Lesson B Assessing the Lesson

Module 2-6 Lesson C 4-4.4 Represent the two-dimensional shapes trapezoids, rhombuses, and parallelograms and the three-dimensional shapes cubes, rectangular prisms, and cylinders. (B2)	NCTM's <u>Principals and Standards for School Mathematics</u> (PSSM) NCTM, <u>Mathematics Assessment Sampler</u>: Grades 3-5 ETA Cuisenaire, <u>Hands-On Standards</u>: Grades 3-4	See Instructional Planning Guide Module 2-6 <u>Introductory Lesson C</u>	See Instructional Planning Guide Module 2-6 <u>Lesson C Assessing the Lesson</u>

MODULE

2-1

Operations –Division

This module addresses the following indicators:

4-2.5 Generate strategies to divide whole numbers by single-digit divisors. (B6)
4-2.2 Apply divisibility rules for 2, 5, and 10. (C3)

* This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Module

- ***Continuum of Knowledge***

4-2.2
Fourth grade is the first year students are formally introduced to the concept of division. Students apply divisibility rules for 2, 5 and 10 and generate strategies to divide whole numbers by single-digit divisors.

In fifth grade, students apply an algorithm to divide whole numbers fluently (5-2.2) and understand the relationship among the divisor, dividend and quotient (5-2.3)

4 -2.5
In second grade focused on interpreting models of sharing equally (division) as repeated subtraction (2-2.6).

In third grade, students compared the inverse relationship between multiplication and division (3-2.8).

Fourth grade is the first time students are formally introduced to the concept of division. In fourth grade, students generate strategies to divide whole numbers by single-digit divisors (4-2.5) and they apply divisibility rules for 2, 5, and 10 (4-2.2).

In fifth grade, students apply an algorithm to divide whole numbers fluently (5-2.2) and understand the relationship among the divisor, dividend and quotient (5-2.3).

Second grade focused on interpreting models of sharing equally (division) as repeated subtraction while third grade compared the inverse relationship between multiplication and division. In second grade, the concept of division as repeated subtraction begins with a whole set of objects that must be shared equally (no remainders for this experience). For example, in the story "And the Doorbell Rang" 12 cookies are to be shared by friends. If there were two friends the cookies could be shared as a 2 by 6 array. If there were 3 friends, then a 3 by 4 array would result. The concept of "subtraction" comes in as student remove cookies from the original pile to form the array. Again, the concept of division is NOT introduced, but students have informally investigated the concept as sharing equally and arrays. The emphasis in second grade is on building the conceptual knowledge and understanding so that in later grades they can link repeated addition to multiplication and repeated subtraction to division. In third grade, students will be formally introduced to the concept of multiplication and in fourth grade the concept of division will be formally introduced.

Using the appropriate terminology is important. State the division problem as dividend ÷ by divisor regardless of the format of the problem. This will help to eliminate confusion around the order of the numbers. For example, given $6\overline{)24}$, it should be read as 24 divided by 6 not 6 into 24. Although the second statement is true, students become confused and think that 6÷24 is the same thing and 24 ÷6.

It is important that students understand what the remainder means. They do not have to write the remainder in fractional form but they should be able to explain that the reminder is leftover from being unable to create equal groupings.

4-2.2
For this indicator, it is **essential** for students to:

- Be able to divide whole numbers with a single-digit divisor in order to apply the divisibility rules
- Understand if the number ends in an even digit, the number is evenly divisible by 2.
- Understand if the number ends in 0 or 5, then the number is evenly divisible by 5.
- Understand if the number ends in 0, then the number is evenly divisible by 10.

For this indicator, it is **not essential** for students to:
None noted

4-2.5
For this indicator, it is **essential** for students to:

- Understand the inverse relationship between multiplication and division
- Recall basic multiplication facts
- Explore division in story problems situations
- Generate their own strategy for division
- Recognize the remainder in story problem situations and with concrete models
- Give meaning to the remainder
- Perform division where the dividend has more than four digits

For this indicator, it is **not essential** for students to:

- Divide numbers by more than one single-digit divisor
- Gain computational fluency in dividing by one single-digit divisor
- Write the remainder in fractional form
- Learn the long division algorithm

- **Key Concepts/Key Terms**
  *These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.
  * divide
  * divisor
  * dividend
  * quotient
  * remainder
  * divisible
  * division
  * evenly divisible
  * even
  * odd

II. Teaching the Lesson(s)

1. Teaching Lesson A: Generating Strategies to Divide

a. Indicators with Taxonomy

4-2.5 Generate strategies to divide whole numbers by single-digit divisors. → B6

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual

b. Introductory Lesson A

Materials Needed
Various manipulatives such as grid paper, two colored counters, base ten blocks, unifix cubes, small cups/containers, etc.

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. Teaching Student Centered Mathematics: Grades 3-5

A significant method of developing meaning for division is to have students solve contextual problems or story problems. However, there is more to think about than simply giving students' word problems to solve. Consider the following problem: For a class project, we want to recover all of the bulletin boards in the school for our Fall Festival. If it takes 7 yards to cover a bulletin board, how many bulletin boards can be covered if there are 175 yards of bulletin board paper in the teacher workroom? "Contextual problems are connected as closely as possible to children's lives. They are designed to anticipate and to develop children's mathematical modeling of the real world." Contextual problems might derive from recent experiences in the classroom (a field trip, a discussion, or from children's literature). Students are more likely to exhibit their most spontaneous and meaningful approaches when solving contextual problems because they have a connection to it.

Good lessons built around contextual problems will involve more than just students solving problems but also using words, pictures, and manipulatives to explain how they went about solving the problem and justifying their answers. Students should be able to use whatever physical materials they feel they need to help them, or they can simply draw pictures. A complete lesson will often revolve around one or two problems and the related discussion.

More often than not, division does not result in a simple whole number. Students should not just think of remainders as "R" or "left over." Remainders should be put in context and dealt with accordingly.

Sample Contextual Problems:

- John's puppy, Petey, loves treats. John bought a bag of 134 treats. If John gives Petey 3 treats per day, how many days will the treats last?
- Nicole has a bag with 783 jelly beans. Nicole and her four friends want to share them equally. How many jelly beans will Nicole and each of her friends get?
- Tori can put 6 pictures on one page of her photo album. If she has 82 pictures, how many pages will she need?

c. *Misconceptions/Common Errors*

The quotient 24 divided by 6 is represented in three different ways: $24 \div 6$, $6\overline{)24}$, and $\frac{24}{6}$. The computational form $6\overline{)24}$ might not exist if it were not for the standard pencil-and-paper procedure that utilizes it. Students have a tendency to read this as "6 divided by 24" due to the left-right order of the numerals. Another issue with division notation is the phrase "six goes into twenty-four." This phrase carries little meaning about division especially in connection with a fair-sharing or partitioning context. The "goes into" (or "guzinta") phrase should be abandoned.

Avoid the key word strategy! In contrast to common practice, researchers and mathematics educators have long cautioned against the strategy for key words for the following reasons:

- Key words are misleading. Often the key word or phrase in a problem suggests an operation that is incorrect.
- Many problems have not key words.
- The key word strategy sends a terribly wrong message about doing mathematics. The most important approach to solving any contextual problem is to analyze its structure—to make sense of it. The key word approach encourages students to ignore the meaning and structure of the problem and look for an easy way out. Mathematics is about reasoning and making sense of situations. A sense-making strategy will always work.
- Student sometimes think that the remainder isn't important and they do not understand how it fits into the problem so they disregard it. Exploring remainders in story problems, gives the remainder meaning and helps with their understanding.

d. *Additional Instructional Strategies/Differentiation*

For additional instructional strategies, see NCTM, Navigating through Number and Operations: 3-5, 2006.

e. *Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

There is no specific technology recommended for this lesson at this time.

f. *Assessing the <u>Lesson</u>*

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed. See appendix A.

Use students solving problems and using words, pictures, and manipulatives to explain how they went about solving the problem and justifying their answers as formative assessment.

2. Teaching Lesson B: Divisibility Lesson

a. Indicators with Taxonomy

4-2.2 Apply divisibility rules for 2, 5, and 10. → C3

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural

b. Introductory Lesson B

Materials Needed:

An index card with the following set of division problems – one card per student pair:

1 card:

434 ÷ 2 =

673 ÷ 2 =

765 ÷ 2 =

348 ÷ 2 =

236 ÷ 2 =

Tell students that mathematicians have found ways to tell by looking at a number whether or not it will come out even when it is divided by certain digits. Challenge the students to be mathematicians and see if they can "figure" out the rule.

Then give each student pair an index card with the above division problems written on it. Ask the students, "By looking at the problems on the card, can you tell me which digit we are looking for a rule for first?" (2) "So, your challenge is to see if you can tell how you know which numbers are evenly divisible by three without doing the division."

Allow the student pairs time to work. Afterwards, allow students to share their strategies.

Repeat this process by making up division problems for 5 and for 10.

NOTE: The following divisibility rules are provided for teacher information and should be discovered by students – not memorized:

- If the number ends in an even digit, the number is evenly divisible by 2.
- If the number ends in 0 or 5, then the number is evenly divisible by 5.
- If the number ends in 0, then the number is evenly divisible by 10.

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. **Additional Instructional Strategies/Differentiation**

Materials Needed:

Index cards with division problems that are divisible by

2, 5 OR 10.

Divisibility War is a game used to practice and review divisibility rules.

a. The game is played with two to six players.

b. Cards are shuffled and dealt face down to all players. Any extra cards will be set aside.

c. Players take turns turning over cards and then stating whether the number on the top card is divisible by 2, 5, or 10.

d. Players may challenge a statement of divisibility.

e. The player with the most correct answers wins the round.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:
http://mathforum.org/dr.math/faq/faq.divisibility.html

f. Assessing the __Lesson__

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Use an exit ticket to formatively assess this indicator. A possible question might be: Write a number that is divisible by 2, 5, and 10 and explain your thinking.

III. Assessing the __Module__

4-2.2
The objective of this indicator is to apply which is in the apply procedural knowledge cell of the Revised Taxonomy. To apply means to carry out a procedure in a familiar or unfamiliar situation; therefore, students should be able to use divisibility rules in a variety of situations. The learning progression to **apply** requires students to be able to divide whole numbers by a single digit divisor. This will be a context in which students analyze problems (4-1.1) and generalize the connection (4-1.6) between the ending digit in the number and divisibility of the number. As students discover these connections, they explain and justify their mathematical ideas (4-1.3) to their classmates and their teacher using correct, complete and clearly written and oral mathematical language (4-1.5).

4-2.5
The objective of this indicator is to generate which is in the "create conceptual" knowledge cell of the Revised Taxonomy. To create means to put elements together to form a new structure; therefore, students use their prior knowledge to create their own strategy for division. The learning progression to generate requires students to **recall** basic multiplication and division facts and understand place value. Using concrete and/or pictorial models, students apply their understanding of number relationships to determine how to break down problems. As students analyze information (4-1.1) from these experiences, they generate mathematical statements (4-1.4) about the relationships they observe then explain and justify their strategies (4-1.3) to their classmates and their teachers. Students recognize the limitations of various strategies and representations (4-1.8) and use correct, complete and clearly written and oral language to communicate their ideas (4-1.5).

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Shawn has 42 pictures to put in his photo album. He wants to put 3 pictures on each page. Draw a picture that shows your thinking.

How many pages does Shawn need for his album?

A. 6
B. 7
C. 14
D. 45

2. Write a number that is divisible by 2, 5, and 10 and explain your thinking.

3. The number 2,345 is divisible by

A 2
B 5
C 10
D 2, 5 and 10

4. The students in a class made a total of 112 cookies. They divided the cookies equally into 8 bags. How many cookies were in each bag? Draw a picture to show your thinking.

MODULE

2-2

Patterns, Relationships, and Functions

This module addresses the following indicators:

4-3.1 Analyze numeric, nonnumeric, and repeating patterns involving all operations and decimal patterns through hundredths. (B4)
4-3.2 Generalize a rule for numeric, nonnumeric, and repeating patterns involving all operations. (B2)
4-3.3 Use a rule to complete a sequence or a table. (C3)

* This module contains 3 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Module

- Continuum of Knowledge

4-3.1
The objective of this indicator is to generate which is in the "create conceptual" knowledge cell of the Revised Taxonomy. To create means to put elements together to form a new structure; therefore, students use their prior knowledge to create their own strategy for division. The learning progression to generate requires students to **recall** basic multiplication and division facts and understand place value. Using concrete and/or pictorial models, students apply their understanding of number relationships to determine how to break down problems. As students analyze information (4-1.1) from these experiences, they generate mathematical statements (4-1.4) about the relationships they observe then explain and justify their strategies (4-1.3) to their classmates and their teachers. Students recognize the limitations of various strategies and representations (4-1.8) and use correct, complete and clearly written and oral language to communicate their ideas (4-1.5).

4-3.2
Students have been analyzing and extending patterns since kindergarten. In second and third grades, students analyzed (2-3.1) and created (3-3.1) numeric patterns involving whole number operations.

In fourth grade, students analyze numeric, nonnumeric, and repeating patterns including decimal patterns through hundredths (4-3.1). This grade is the first time students are introduced to decimals. They generalize (4-3.2) and use (4-3.2) a rule to complete a sequence or table.

In fifth grade, students represent numeric, nonnumeric, and geometric patterns in words, symbols, algebraic expressions, and algebraic expressions (5-3.1). In this grade, students also analyze patterns and functions with words, tables, and graphs (5-3.2).

4-3.3
Students have been analyzing and extending patterns since kindergarten. In second and third grades, students analyzed (2-3.1) and created (3-3.1) numeric patterns involving whole number operations.

In fourth grade, students analyze numeric, nonnumeric, and repeating patterns including decimal patterns through hundredths (4-3.1). This grade is the first time students are introduced to decimals. They generalize (4-3.2) and use (4-3.2) a rule to complete a sequence or table.

In fifth grade, students represent numeric, nonnumeric, and geometric patterns in words, symbols, algebraic expressions, and algebraic expressions (5-3.1). In this grade, students also analyze patterns and functions with words, tables, and graphs (5-3.2).

- **Key Concepts/Key Terms**
  *These are vocabulary terms that are reasonable for students to know and be able to use. Terms without * are additional terms for teacher awareness, knowledge and use in conversation with students.
  * pattern
  * rule
  * table
  * sequence
  * numeric
  * non-numeric patterns
  * repeating
  * decimal
  * tenths
  * hundredths
  * stem   Added from A5
  * core

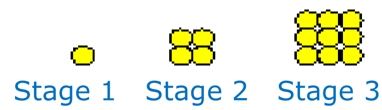
II. Teaching the Lesson(s)

1. *Teaching Lesson A*: What Comes Next?

4-3.1
For this indicator, it is **essential** for students to:

- Understand place value
- Compare decimals through hundredths
- Add, subtract, and multiply whole numbers
- Recall multiplication facts through 12 x 12 and corresponding division facts
- Analyze decimal patterns through hundredths using models, counting, or skip counting such as
  - 0.10, 0.15, 0.20, ____ because it applies a familiar whole number pattern that involves skip counting (counting by 5's).
  - 0.2, 0.4, 0.6, ____ because it applies a familiar whole number pattern that involves skip counting (counting by 2's).
- Analyze information to solve increasingly more sophisticated problems such as
  - 3, 5, 4, 6, ___ (add 2 then subtract 1)
  - 1, 4, 13, 40, ___ (multiply by 3 then add 1)

- Analyze information to solve nonnumeric patterns involving manipulative, pictures or symbols.
  - Determine the number of dots in Stage Four of the pattern.



For this indicator, it is **<u>not essential</u>** for students to:

- Analyze patterns that involve formal algorithms with decimals such as
  - 0.2, 0.5, 0.8, ___ because this involves adding decimals
  - 4.7, 3.5, 2.3, ___ because this involves subtracting decimals

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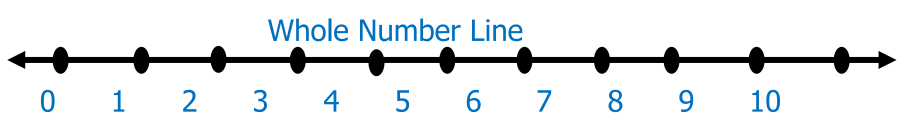
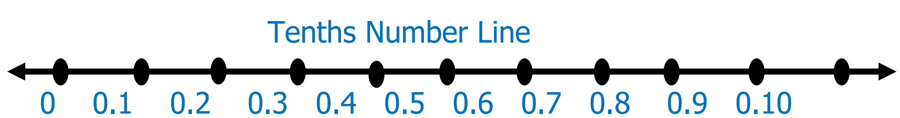
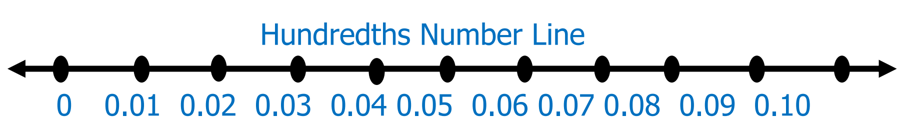
Students need experiences analyzing whole number patterns and decimal patterns using tenths and hundredth. Students need to know that between zero and one, there are ten tenths and one hundred-hundredths. See below. **These Number Lines are not drawn to scale. They are just examples and are not related to each other.**







a. *Indicators with Taxonomy*

Indicator → 4-3.1 Analyze numeric, nonnumeric, and repeating patterns involving all operations and decimal patterns through hundredths. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual

b. Introductory Lesson A

Materials Needed

Handout of pattern on next page

Suggested literature connection:

Anno's Magic Seeds by Mitsumasa Anno
At each of the three stages of this story, students can develop a table. They can make generalizations about the data and extend the current pattern into the future. They can describe each pattern (repeating, numeric, non-numeric). Several operations are necessary to complete the tables.

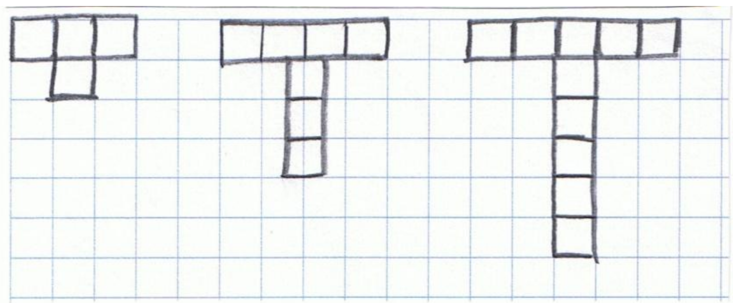
Introductory Lesson A

Given the first 3 or 4 steps of a pattern:

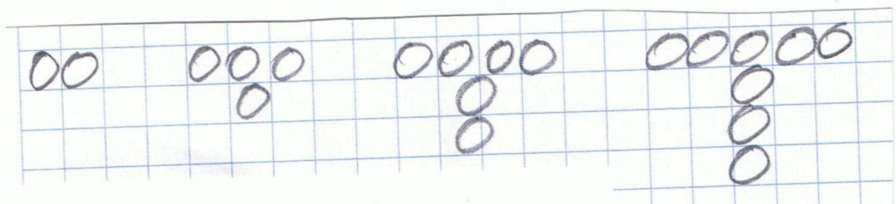
- Build or draw (or both) the next couple steps
- Create a table with one column being the step number and the next column the number of items in the step.
- Write a rule for the pattern

HANDOUT

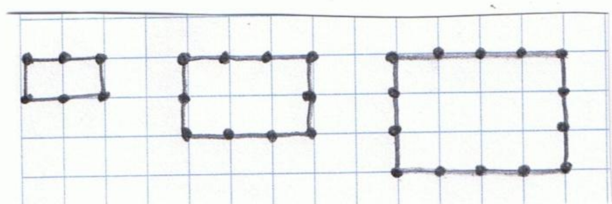
1. pattern: number of squares



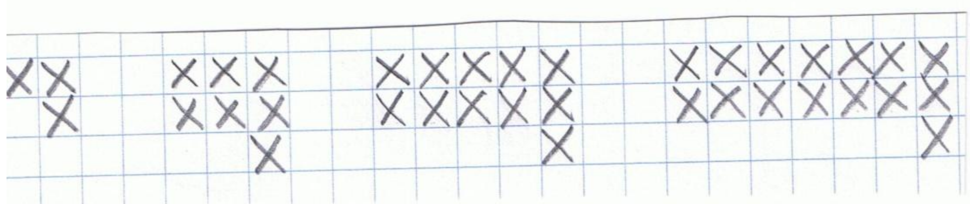
2. pattern: number of circles



3. pattern: number of dots



4. pattern: number of x's



ANSWERS….

1.

Add 3	
1	*4*
2	*7*
3	*10*
4	*13*
5	*16*
6	*19*
7	*22*

2.

Add 2	
1	*2*
2	*4*
3	*6*
4	*8*
5	*10*
6	*12*
7	*14*

3.

Add 4	
1	*6*
2	*10*
3	*14*
4	*18*
5	*22*
6	*26*
7	*30*

4.

Add 4	
1	*3*
2	*7*
3	*11*
4	*15*
5	*19*
6	*23*
7	*27*

c. Misconceptions/Common Errors

When dealing with patterns involving decimals, students tend to initially believe that .14 is larger than .5 because 14 is greater than 5. Place value needs to be emphasized when comparing decimals.

d. Additional Instructional Strategies/Differentiation

- Use a hundred board. Use rules to find patterns within the numbers (multiples, factors, digits with a difference of 1). Students use different colors to show rules on their own hundred board.

- Use triangles to create a day old "worm." Follow with a 2-day and 3-day-old worm. Find the pattern to create a ten-day-old worm.

1 day 2 days 3 days 4 days

- Use Navigating Through Algebra in Grades 3 through 5, "Hundred-Board Wonders." p. 9. Students explore number patterns on a hundred board.

- Use Navigating Through Algebra in Grades 3 through 5, "Watch Them Grow," p. 12. Students construct patterns with pattern blocks, record patterns in a table, and state rules for extending patterns.

- Use Navigating Through Algebra in Grades 3 through 5, "Calculator Patterns," p. 15. Students explore patterns using a calculator and translate patterns into numerical patterns.

e. Technology

This link is from the National Library of Virtual Manipulatives. Students will be able to work on their patterning.

http://nlvm.usu.edu/en/nav/frames_asid_271_g_2_t_3.html?open=instructions&from=category_g_2_t_3.html

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

These are questions you would want the students to be able to answer during or after the lesson:

- How did you determine your pattern or what was the basis for your determination?

Students could respond in their math notebooks/journals one of their examples from their notes/work/homework AND answer the above question using their work to support their answer.

2. Teaching Lesson B: Pattern Rules

For this indicator (4-3.2), it is **essential** for students to:
- Determine the pattern
- Write the rule in words. For example, multiply by 2 add 3

For this indicator, it is **not essential** for students to:
- Find additional terms in the pattern

a. Indicators with Taxonomy

Indicator → 4-3.2 Generalize a rule for numeric, non-numeric and repeating patterns involving all operations. (B2).
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

***b*. Introductory Lesson B**

Materials Needed
Pattern blocks
Recording sheet – 1 per student

Display a pattern using the pattern blocks and have the students copy the sequence.

Have students extend the pattern and create their own patterns.

△ △ □ △ △ □

1 1 2 1 1 2

Both the shape and number patterns follow an AAB pattern. The stem is △ △ □ or 1, 1, 2

c. Misconceptions/Common Errors

None noted

***d*. Additional Instructional Strategies/Differentiation**

While additional learning opportunities are needed, no suggestions are included at this time.

***e*. Technology**

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Ask students to describe how they created their own block patterns. See Appendix A for other forms of formative assessment that can be used with this lesson.

3. Teaching Lesson C: What Comes Next Again!

For this indicator, it is **essential** for students to:
- Determine the pattern
- Interpret a rule
- Use the rule to complete a table or sequence
- Perform operations fluently

For this indicator, it is **not essential** for students to:
None noted

a. **Indicators with taxonomy**

4-3.3 Use a rule to complete a sequence or table. (C3)
Cognitive Process: Apply
Knowledge Dimension: Conceptual

b. Introductory Lesson C:

Materials Needed

Handout of Patterns below.

Lesson

Given the first 3 or 4 steps of a pattern:
- *Build or draw (or both) the next couple steps*
- *Create a table with one column being the step number and the next column the number of items in the step.*
- *Write a rule for the pattern*

Materials Needed

Handout with tables and sequences below.

Part A

(Adapted From Teaching Student Centered Mathematics Grades 3-5, Van de Walle, John A. and Lovin, LouAnn, 2006, page 299)

<u>What comes next</u>? → Show students several numbers in pattern or sequence in a list form or table form. The students should extend the pattern, write a rule for the pattern (in words), then use the rule to extend the pattern through the 15^{th} entry.

<u>Examples of sequences that can be used are</u>:

(***TEACHER NOTE***) – *possible* ***<u>answers are listed below</u>****, students should discover these themselves… do NOT give them to the students ☺)*

A. (each digit repeats according to its value)
B. (even numbers – skip counting by 2)
C. (double the previous number)
D. (double the previous number and add 1)
E. (successively add 1, then 2, then 3, then 4…)
F. (add the preceding two numbers)

HANDOUT

A. 1,2,2,3,3,3,…

B.

Step Number	1	2	3	4	5
Pattern	2	4	6	8	10

C. 1,2,4,8,16,…

D.

Step Number	Pattern
1	2
2	5
3	11
4	23

E. 1,2,4,7,11,16,…

F. 2,2,4,6,10,16,…

Part B

Complete the sequence or table by first determining the rule.

A. 2, 11, 4, 15, 6, 19, _____, 23, 10, _____, _____

B. _____, 6629, 6604, 6579, 6554, _____, _____

C.

Step Number	Pattern
1	1
2	
3	9
4	16
5	25
6	36
7	49
8	
9	

D. 1, 2, 4, 7, 11, _____, 22, 29, _____, _____

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

Challenge students to create their own sequence and rule…. They can write these on index cards and trade with a partner.

e. Technology

There is no specific technology recommended for this lesson at this time.

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulative should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

*f. Assessing the **Lesson***

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

These are questions you would want the students to be able to answer during or after the lesson:

Given the following pattern, have students answer the bulleted questions below on an Exit Slip.

Step Number	Pattern
1	4
2	6
3	8
4	10
5	12
6	14

- How did you determine the pattern?
- Is there a rule for the pattern? How do you know?

*III. Assessing the **Module***

4-3.1

The objective of this indicator is to analyze, which is in the "understand conceptual" knowledge cell of the Revised Bloom's Taxonomy. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge of numeric, non-numeric and repeating patterns should be explored using a variety of examples. The learning progression to **analyze** requires students to recall basic skip counting patterns and apply that understanding to decimal patterns. Students should be able to compare the magnitude of numbers and the relationship among objects in order to predict the next element in a pattern. They should use these processes to analyze information to solve increasingly more sophisticated problems (4 – 1.1). Students should also explain and justify answers (4 – 1.3) using correct, complete and clearly written and oral mathematical language to pose questions, communicate ideas, and extend problem situations (4 – 1.5). As illustrated by the examples, it is not essential for students to add and subtract decimals to meet this indicator.

4-3.2

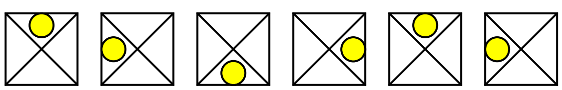
The objective of this indicator is to generalize which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Understand conceptual knowledge is to construct meaning using a variety of examples. The learning progression to **generalize** requires student to analyze patterns and determine the relationship between terms within the sequence. Students generate a mathematical description or mathematical statement about these relationships (4-1.4) and explain and justify their answers (4-1.3) to their classmates and their teacher. Students translate these statements into mathematical rules using correct, complete, and clearly written and oral mathematical language (4-1.5).

4-3.3
The objective of this indicator is to use which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Procedural knowledge is knowledge of how to do something. The learning progression to **use** requires students to understand whole number operations. Students analyze a rule (4-1.1) to determine the operations involved. Students generate a verbal or written description of the pattern using their own words and use that description to complete a sequence or table. They explain and justify their answers (4-1.3) to their classmates and their teacher.

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Based on the pattern, draw the 10th figure in the pattern.



2. Find the rule and complete the sequences:

103.16, 97.16, 91.16, _____, 79.16, 73.16, _____, _____

256, 128, 64, _____, 16, _____, _____

61.50, 61.25, 61, 60.75, _____, 60.25, _____, _____

2, 5, 11, 23, _____, 95, 191, _____, _____

3. What is the core (stem) of the pattern?

△ △ □ ○ △ △ □ ○ △

A. △ △ □ ○

B. △ □ △ □

C. △ □ ○ □

D. △ □ □ ○

MODULE

2-3

Representations, Properties, and Proportional Reasoning

This module addresses the following indicators:

4-3.4 Translate among letters, symbols, and words to represent quantities in simple mathematical expressions or equations. (B2)

* This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Lesson

- ***Continuum of Knowledge***

In third grade, students used symbols to represent unknown quantities for addition, subtraction and multiplication equations (3-3.3).

In fourth grade, students will translate simple mathematical expressions or equations with letters, symbols, and words to represent quantities (4-3.4). This will be the first time students are formally introduced to a variable and its use.

- ***Key Concepts/Key Terms***

*These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

* translate
* expression
* equation
* multiple representations
* relationship
* represent
* variable

II. Teaching the Lesson(s)

1. ***Teaching Lesson A***: ***Writing missing information as a variable***

Students in third grade used symbols to represent unknown quantities for addition, subtraction and multiplication equations. Fourth grade students will translate simple mathematical expressions or equations with letters, symbols, and words to represent quantities. This will be the first time students are formally introduced to a variable and its use. Students need to be provided with many examples of situations that can be represented as simple mathematical expressions or equations. For example: How many legs do 4 chairs have? Students can use symbols such as 4x4 = ___ OR 4x4 = (box) or letters such as 4x4 = n.

Students should also represent simple mathematical relationships as number sentences. Through examples, students should see that an equation is a strategy that can be used to translate "word problems" into "symbols and letters for solving." As the students' understanding grows, so can the complexity of the problems that are presented.

These variables and the understanding they bring are an essential tool for the students as they progress into middle school algebra and more complex equations. The emphasis here is on understanding the relationship between and being able to translate among letters, symbols, and words to represent quantities in simple mathematical expressions or equations.

In fourth grade, students will use variables to represent an unknown quantity (for example: 5 x F = 20, F = 4). In order for students to do this, they must understand that the symbol is a placeholder for an unknown number and that for each equation or problem solving situation, the variable represents a different answer. This can be confusing to students at first – they must understand that F does not always equal 4, that F is a placeholder in that equation. They also need to understand that 4 + y = 6 and 4 + M = 6 will have the same solution. This understanding comes through many, different and varied experiences, beginning with simple problems to ensure understanding.

Also, in fourth grade, students will use equations to represent relationships. Many students may have only seen an equation with an expression on the left side and a place for the answer on the right side (for example 15 – 6 = G). Students need to experience equations in many different ways. One way is simply reversing it (for example G = 15 – 6). Another way is for students to see expressions on both sides of the equals sign (for example 4 x 5 = 15 + 5). Students will also experience equations that represent relationships that are not equal (for example 8 + 5 > 20 – 9).

Fourth grade students will need many examples of using equations to represent relationships. For example: Harry sells ham sandwiches for $2.00. If Harry earned $10.00, how many sandwiches did he sell? How can we write an equation to show this? ($2.00 x S = $10.00) Through examples, the students will see that an equation can help in solving the problem.

For this indicator, it is **essential** for students to:

- Understand the relationship between unknown quantities and symbols and letters
- Interpret simple word problems
- Understand the difference between an expression and an equation
- Understand the relationship between the words form and the expression or equation
- Write number sentences
- Understand whole number operations

For this indicator, it is **<u>not essential</u>** for students to:

- Solve equations

a. Indicators with Taxonomy

Indicator → 4-3.4 Translate among letters, symbols, and words to represent quantities in simple mathematical expressions or equations. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Procedural

b. Introductory Lesson A:
Materials Needed

- manipulatives (counting disks, paper clips…)
- story problems (written on an overhead, board or SmartBoard)

Introductory Lesson

<u>Story Translations</u> → Introduce using equations to represent relationships in a simple problem that most students can answer easily. For example: show the students a set of 5 counting disks (or any manipulative) on an overhead, board or SmartBoard. Tell them that you want to add some more and now you have a total of 11. Write an equation to show this 5 + K = 11. Ask the students how many more counting disks you added (6). So, K=6.

Use at least 2 more examples (one might be subtraction) for the students to solve. Focus on writing an equation to match the problem.

Once the students seem to understand the concept, have them write an equation for a word problem that you have created. Show the problem: *A group of 25 fourth grade students went to the movies. 16 students were boys and the rest were girls. How many fourth grade girls went to the movies?* Give the students enough time to discuss the problem, use manipulatives if they need them, and write an equation for the problem. Then have them solve the problem, being ready to explain how they solved it. When the groups have finished, ask a group to share their equation and solution. Allow other groups to share if they have a different equation – for example, one group may write 16 + g = 25 and another may write 25 – g = 16. Discuss any differences.

Another problem may be as follows: There are 3 full boxes of balls and 5 extra balls. There are 41 balls in all. Write an equation that means the same thing. Students may write,
3 x b + 5 = 41.

Another one to pose is: Al has 3 times as many baseball cards as Mark. Mark has 75 cards, how many does Al have? Students may write 3 x 75 = A or A ÷ 3 = 75. Discuss what makes these alike and why each works. This will result in better understanding of inverse operations.

Try another one or two in this format. Then as a challenge, reverse the activity by giving students an equation with an unknown and asking them for a story.

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

Submitted from A5

Make 10 to 12 sets of cards with missing addends on them represented by letters and the letter and answer on the other. Distribute them at random face up and call on one student to find the matching card by looking on the other students' desk. Continue until all matches have been found.

67 + s = 84	**s = 17**

Use Navigating Through Algebra in Grades 3 through 5, "The Variable Machine." Students explore variables as symbols and substitute numbers for variables to find unknown values.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Examples of effective questions were stated in the lesson write up.
ALSO… Students can respond in their math notebooks to the following:
"Barbara is baking cookies for a school function. She has baked twice as many as Bill. Bill has baked 100. How many has Barbara baked?" One equation to reach a solution would be 2 x 100 = C. Another equation would be C ÷ 2 = 100. What makes both of these correct equations?

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The objective of this indicator is to translate which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Translate means to change from one form to another. The learning progression to **translate** requires students to analyze simple problem situations and determine known and unknown quantities. Students understand the relationship between the unknown quantity, a letter and/or symbol and whole number operations. They generate mathematical statements about these relationships (4-1.4) and explain and justify their answers (4-1.3) to their classmates and teacher. As students generalize connections (4-1.6) among letter, symbols and words, they use that understanding to translate a variety of problems.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Kris left for school with 4 boxes of pencils. Each box had 6 pencils. At school, she gave away 4 pencils from one box. Which number sentence can be used to find the number of pencils that were left.
   Answer: 4 x 6 – 4
   (Adapted from Mathematics Assessment Sampler Grades 3-5, National Council of Teachers of Mathematics, page 59.)

2. Which of the following situations can be modeled with A – 5?
   a. Janis is 5 years older than Seth. If A is Seth's age, how old is Janis?
   b. Todd is 5 years younger than Amelia. If A is Amelia's age in years, how old is Todd?
   c. Isaac is 5 times old as Bert. If A is Bert's age in years, how old is Isaac?
   d. Nathan is one-fifth as old as Leslie. If A is Nathan's age in years, how old is Leslie?
   Answer: b
   (Adapted from Mathematics Assessment Sampler Grades 3-5, National Council of Teachers of Mathematics, 2005, page 61.)

3. Joanna's mother gave her $15.00 to spend on her class field trip. Her father gave her some money too. She has $25.00 to spend all together. Write an equation that represents the situation.

MODULE

2-4

Solve Mathematical Situations

This module addresses the following indicators:

4-3.5 Apply procedures to find the value of an unknown letter or symbol in a whole-number equation. (C3)

* This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Lesson

- ***Continuum of Knowledge***

In third grade, students used symbols to represent unknown quantities for addition, subtraction and multiplication equations (3-3.3).

In fourth grade, students will translate simple mathematical expressions or equations with letters, symbols, and words to represent quantities (4-3.4). This will be the first time students are formally introduced to a variable and its use.

- **Key Concepts/Key Terms**

*These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are terms for teacher awareness, knowledge and use in conversation with students.

* expression
* equation
* equivalency
* known/unknown
* variable

II. Teaching the Lesson (s)

1. Teaching Lesson A: Finding the value of an unknown

Students in third grade used symbols in whole-number equations and applied procedures to find missing numbers. Indicator 4-3.4 requires fourth grade students to translate among letters, symbols, and words to represent quantities in simple mathematical expressions or equations. When it is clear that students are comfortable doing so, students should begin to apply procedures to find the value of the unknown in simple whole-number equations. The focus for fourth graders is to become efficient in applying procedures to find the value of an unknown letter or symbol in a whole-number equation – not on formal equation solving.

(Adapted From Guiding Children's Learning of Mathematics, Kennedy, Leonard M., Tipps, Steve, and Johnson, Art, page 153.)

While solving problems, students communicate and represent thinking through modeling, drawing and writing.

Several algebraic concepts are introduced as students write number sentences: variables, constants, functions, equations. When first writing number sentences, students think about numbers in a very specific situation. To move to a more generalized understanding of number sentences and equations, they need many activities that are expressed in number sentences.

Another problem students have with number sentences and equations is the idea of equality. Referring to the equal sign as a balance point may help with clarity. Understanding the concept of equivalency is important. Student should begin to move away from the idea that the equals sign means to do something. Manipulatives such as balance pans should be used to help students apply the procedures for finding an unknown. This experience will assist students in discovering that what is done to one side of the equal sign in an equation must be done to the other.

For this indicator, it is **essential** for students to:

- Find the value when the symbol representing the unknown quantity in different positions. For example,
  - □ x 3 = 6
  - 3 + □ = 5
  - 7 – 2 = G

For this indicator, it is **not essential** for students to:

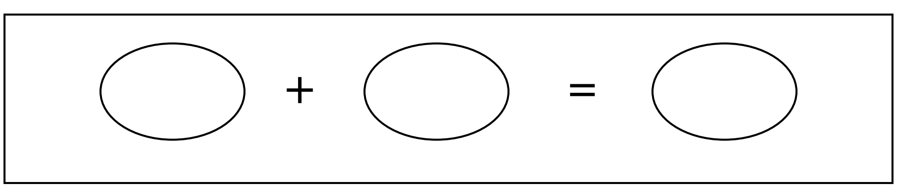
- Use formal algebraic method to solve problems

a. *Indicators with Taxonomy*

Indicator → 4-3.5 Apply procedures to find the value of an unknown letter or symbol in a whole-number equation. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedure

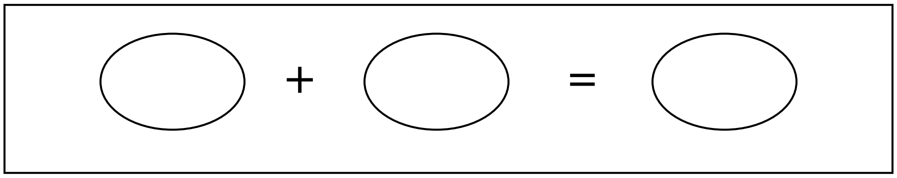
b. *Introductory Lesson*

Materials Needed
Manipulatives (counters, chips, cubes)
Equation Mat one per student

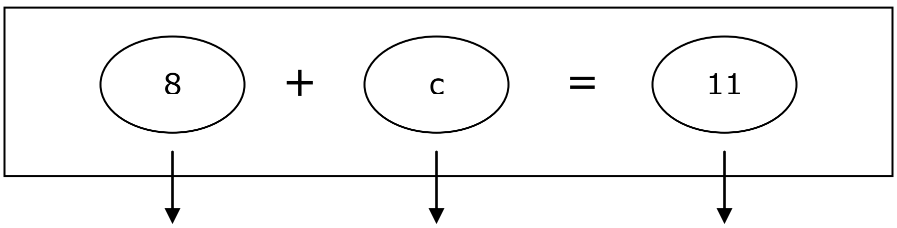


Lesson A
Complete the model below to set up the equation in each of the following problem situations. Complete the model with the knowns, fill in the unknowns with variables and then **USING**

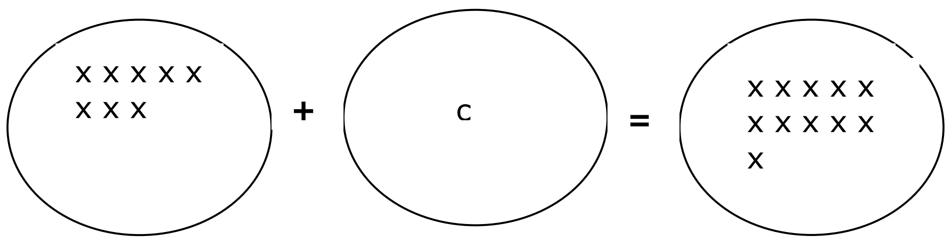
MANIPULATIVES, solve for the unknown. It is important for students to physically "move the equation" around.



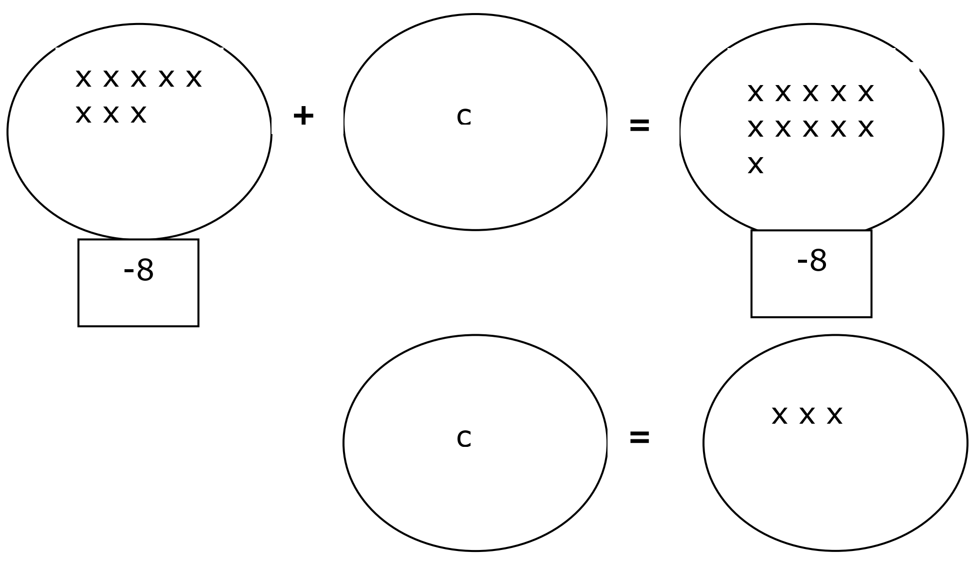
(1) There are 11 pieces of candy. Daisy has 8, how many does Betty have if Betty has the rest of them?



With Manipulatives



Students may remove 8 from each circle, leaving them with 3 for the value of the unknown.



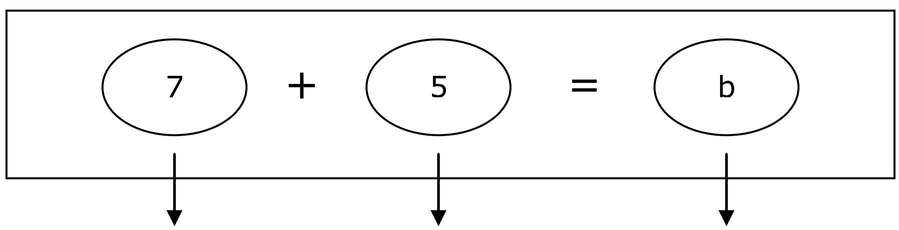
Write out the following equation: $8 + c = 11$, then show

$$\begin{array}{rcl} -8 & & -8 \\ \hline & c & = 3 \end{array}$$

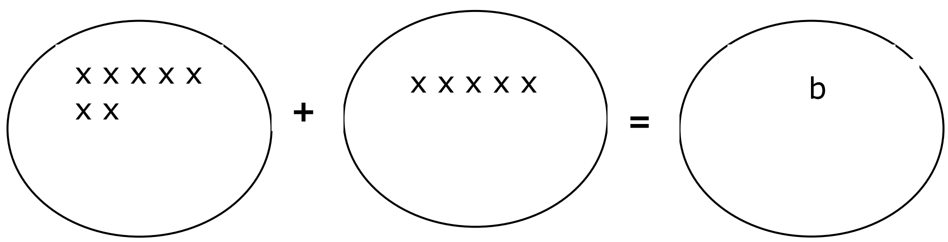
Discuss where the parts for this were in the manipulatives work… you may even want to show this step by step beside the manipulative work.

Solution: Betty has 3 pieces of candy.

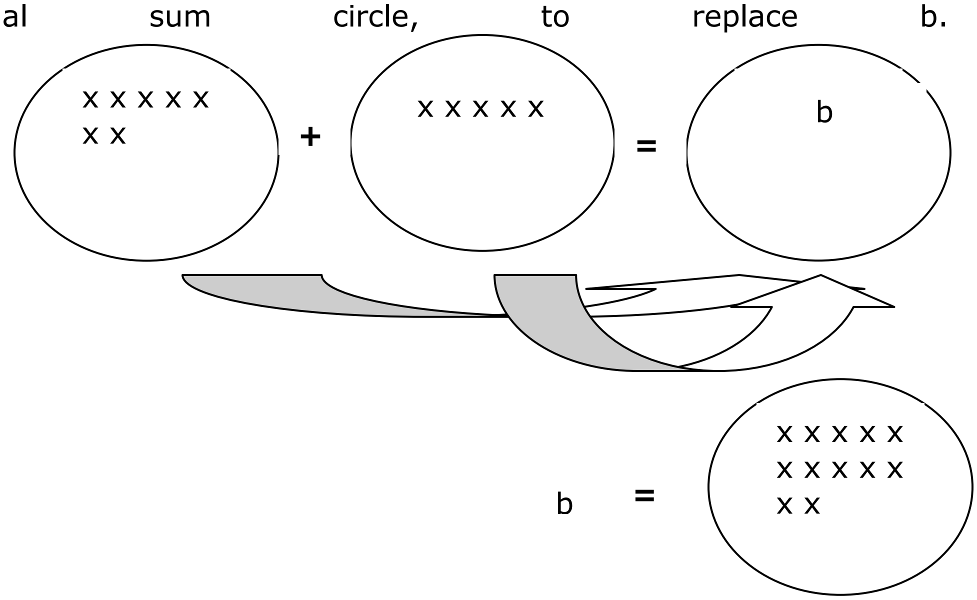
(1) Chris has 7 baseballs and Austin has 5. How many baseballs do they have all together?



With Manipulatives



Students may move 7 from the first circle to join 5 from the second circle in the final sum circle, to replace b.



Provide students opportunity to do several more examples with unknown addends (especially). They should use the manipulatives along with the symbolic and **NOT** just the symbolic. Once students can discuss and understand where the abstract symbolic method comes from, they may decrease their use of manipulatives. Also, as the equations use larger numbers the manipulatives are not as efficient.

It IS IMPORTANT that students build these equations to solve the situations. They should use the model to build.

$$\bigcirc + \bigcirc = \bigcirc$$

c. Misconceptions/Common Errors

Students struggle with the concept that same variable can represent a different quantity in a different problem.

Many students may have only seen an equation with an expression on the left side and a place for the answer on the right side (for example 15 – 6 = G). Students need to experience equations in many different ways.

d. Additional Instructional Strategies/Differentiation

Manipulatives such as balance pans should be used to help students apply the procedures for finding an unknown. This experience will assist students in discovering that what is done to one side of the equal sign in an equation must be done to the other.

For students in need of more of a challenge, give them the following type problems:

Given the following, find the value of the square and the triangle.

$$\square + \square = \mathbf{6}$$

$$\triangle + \square = \mathbf{12}$$

Answer: the square must be 3, making the triangle 9.

e. *Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. *Assessing the Lesson*

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

These are questions you would want the students to be able to answer during or after the lesson:

- What is known?
- What is unknown?
- What helps you determine knowns and unknowns?

ALSO…
Students should be encouraged to represent solutions (using drawings) to the following and write the algebraic solutions as well: (They may choose to draw in Base 10 blocks as the numbers are larger than those in the examples of the introductory lesson.) This should be a homework assignment that will be turned in the following day for teachers to assess where students are in their thinking and explanations.

1) 61 + B = 102
2) Ralph went to the store and bought more balloons than Mary. All together they had 36 balloons. If Mary bought 12, how many did Ralph buy? *Answer: R + 12 = 36.*
3) There is $1020 worth of school picture money collected by the PTA for 4th grade. If pictures packages cost $20 each, how many packages were sold to 4th grade students? *Answer: $20P = $1020 OR $1020 ÷ $20 = P.*

III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The objective of this indicator is to <u>apply</u> which is in the "apply procedural" knowledge of cell of the Revised Taxonomy. Although the focus of the indicator is procedural, the learning progression should integrate the student conceptual understanding of unknown quantities and equivalency. The learning progression to **apply** requires students to <u>understand</u> the concept of a unknown quantity and equivalency and <u>generalize</u> the connection between the two concepts (4-1.6). Students <u>analyze</u> the equation (4-1.1) to determine known and unknown quantities and <u>generate</u> strategies to find the value of the unknown. Students <u>explore</u> their strategies through the use of concrete models and <u>explain</u> and <u>justify</u> their reasoning (4-1.3) to their classmates and their teacher. They <u>use</u> their understanding of inverse relationships to check their answers.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Joanna's mother gave her $15 to spend on her class' field trip. Her father gave her some money too. She has $25 to spend all together. Write an equation that represents the situation. How much money did her father give her? Illustrate the solution using manipulatives/drawings. *(Teacher note: This is one of the assessment items in the previous module. The students are asked to write and solve now.)*

2. Gregory spent $12 of the money he had. He is left with $30. Write an equation to represent how much money he started with. How much money did he start with?

3. Taro has *m* amount of money in his piggy bank. If he can triple it by the end of the year, he will have $300. Write an equation you can use to determine how much money he has in his piggy bank now. How much does he have now?

MODULE

2-5

Change in Various Contexts

This module addresses the following indicators:

4-3.6 Illustrate situations that show change over time as either; increasing, decreasing, or varying. (B2)

* This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Module

- ***Continuum of Knowledge***

First grade students classified change over time as qualitative or quantitative (1-3.6). Second grade students identified (2-3.4) and analyzed (2-3.5) qualitative and quantitative change over time.

In third grade, students illustrate situation that show change over time as increasing (3-3.4). Students also interpret data in tables, bar graph, pictographs and dot plots (3-6.3).

In fourth grade, students illustrate situation that show change over time as either increasing, decreasing or varying (4-3.6)
In fifth grade, students analyze situations that show change over time (5-3.5).

- **Key Concepts/Key Terms**

*These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

* change
* line graph
* increasing
* decreasing
* varying

II. Teaching the Lesson(s)

1. ***Teaching Lesson A: The Ups and Downs of Patterns***

Be sure to point out to students when reading a line graph that it is read from left to right. If not, students will misinterpret the increasing value for decreasing if read backwards.

Also, when looking at relationships, usually you look at y-values as x-values increase. As x-values increase, y-values will increase, decrease or vary (both increase and decrease on the same graph).

They should have multiple opportunities to examine data to determine what kind of change occurs. For example: Students might create a line graph of plant growth and recognize that the plant growth is increasing as time increases.

Although the indicator does not reference change that stays the same, part of recognizing a situation is recognizing counterexamples i.e. situations where the characteristics are not present. For example, when given a situation where the change stays the same, students should be able to explain that it is not increasing because the values do not get bigger; it is not decreasing because the graph is not going down, etc….

For this indicator, it is **essential** for students to:

- Understand change over time
- Determine if change has occurred
- Understand the concepts of increasing, decreasing and varying
- Use their understanding of change over time to find examples increasing, decreasing and varying change
- Describe observed change in words
- Recognize counter-examples. For example, when change stays the same.

For this indicator, it is **not essential** for students to:

- Indicate by how much the data is increasing or decreasing. For example, the data is increasing by five.

a. Indicators with Taxonomy

Indicator → 4-3.6 Illustrate situations that show change over time as either increasing, decreasing, or varying. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

b. Introductory Lesson A

(Adapted from: Navigating through Algebra in Grades 3-5, NCTM, 2001, page 27.)

Materials Needed

- Worksheet – next page

Students will examine data to answer questions dealing with changes in patterns. Present the three situations orally to the students. Have them discuss the situations in small groups. They should appoint one student to record their answers on the worksheet.

Situation One: The Car Trip

The Thompson family is going on a trip to visit friends. The table below shows the number of miles they drove and the amount of gasoline left in the car's tank as they traveled.

Distance Driven (Miles)	Gas Left (Gallons)
0	14
50	12
100	10
150	8
200	6
250	4
300	2

To make the situation appropriate for the grade level, we assume a constant rate of speed throughout the trip. Ask the students the following questions;

- What patterns do you see?
- What rule could you use to describe the number of gallons of gas left as the miles traveled increases?

Situation Two: Rub-a-Tub

The table below shows the level of water in a bathtub from shortly after you begin to fill it with water until it is completely full. Ask the following questions;

- What pattern do you see?
- Write a rule that tells how fast the bathtub is filling up.

Time in Minutes	Level of Water in Inches
3	2
6	4
9	6
12	8
15	10
18	12

Situation Three: Road Race

- What pattern do you see?
- How can you describe Ted's Race?

Ted's Road Race Results

Time (Seconds)	Distance (Meters)
3	4
6	5
9	8
12	9
15	10
18	13
21	14
24	17

From the data in each of the situations have the students graph and label each story. Ask them to describe and compare the resulting graphs. Students should respond that the Car Trip "goes down", The Rub-a-Tub "goes up" and the Road Race "varies".

Extension: Students could write a story for Ted's road race.

HANDOUT

Students will illustrate situations that show change over time as increasing, decreasing, or varying.

Graph and label each situation. Describe and compare the resulting graphs.

Situation One: The Car Trip

Distance Driven (Miles)	Gas Left (Gallons)
0	14
50	12
100	10
150	8
200	6
250	4
300	2

Situation Two: Rub-a-Tub

Time in Minutes	Level of Water in Inches
3	2
6	4
9	6
12	8
15	10
18	12
21	14

Situation Three: Road Race

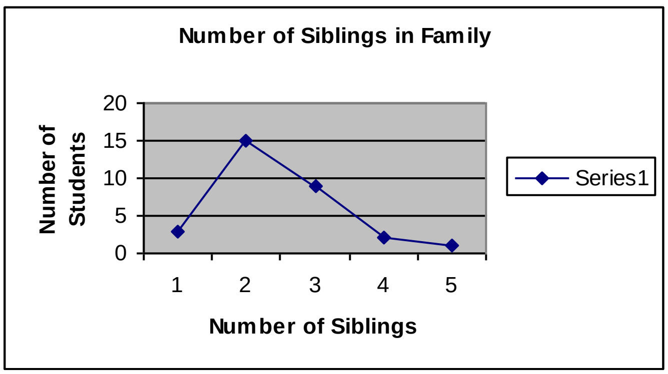
Ted's Road Race Results

Time (Seconds)	Distance (Meters)
3	4
6	5
9	8
12	9
15	10
18	13
21	14
24	17

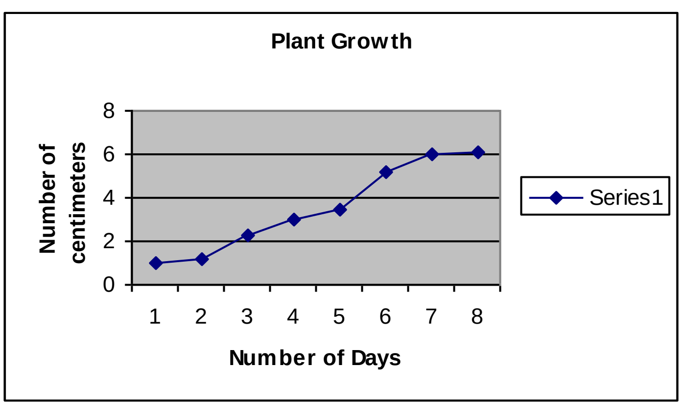
c. Misconceptions/Common Errors

Students may read the graph from right to left instead of left to right. This is especially important when students are reading a line graph. If not, students will misinterpret the increasing value for decreasing if read backwards.

Students sometimes tend to graph discrete data using a line graph which is for continuous data. Remember in a line graph, EVERY point on the line should have a value. Don't use a line graph when a bar graph may be a better choice. **Consider the graph below (it is an INCORRECT use of a line graph**. For example, is there a value at 1½ siblings? NO, but it looks like there is on the graph. (van de walle, page 334)



The following is a correct use of a line graph. Notice how it grows every day and in between whole number of days, as plants do – they continually grow.



d. Additional Instructional Strategies/Differentiation

Students could grow plants and graph growth to show change over time. They can graph data from tables or charts containing information on time and temperature.

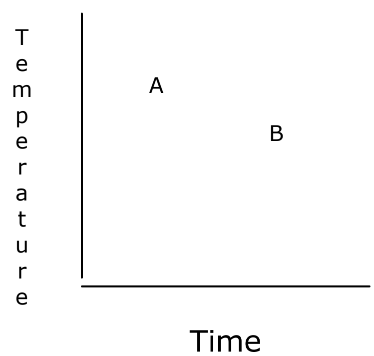
e. Technology

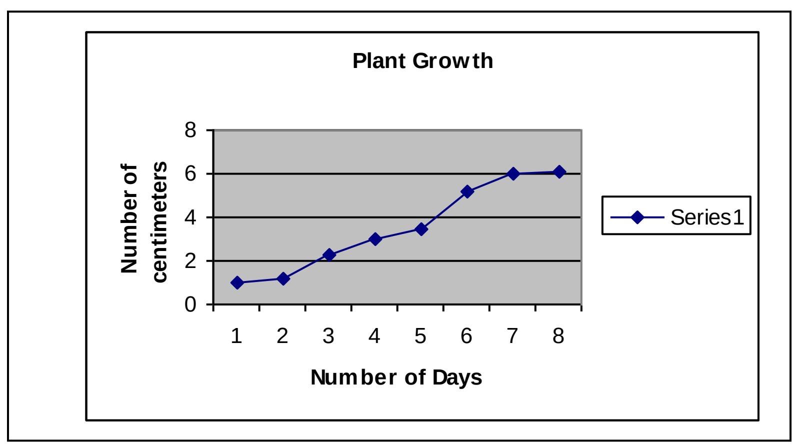
Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation: however, other formative assessment strategies should be employed.

To formatively assess, give students two graphical representations such as:





Have them respond to the following questions concerning the graphical representations.

- As one axis increases, what does the other one do?
- What relationship is there between the two points/the two axes?

- What is happening over time? What change is occurring?

You may want to assign this as homework or an exit slip… either way, it's not for a grade but for a way for the teacher to gain some insight.

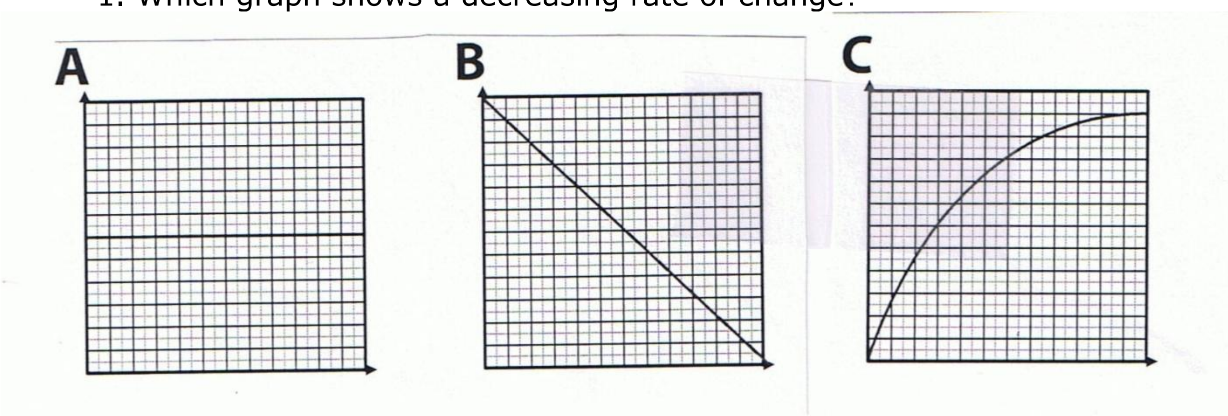
III. Assessing the Module

The objective of this indicator is to illustrate which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To illustrate means to find specific examples of a concept; therefore, students should explore a variety of examples to build understanding of the concept of increasing, decreasing and varying change. The learning progression to **illustrate** requires students to understand change over time and characteristics of increasing, decreasing and varying data. Students explore teacher generated examples and analyze information (4-1.1) from those examples to determine if change has occurred. They also explore counter-examples and explain why the data is not increasing, decreasing or varying. They generate descriptions (4-1.4) of the observed change then explain and justify their answer on the basis of mathematical relationships (4-1.3). Students use this understanding to find other examples of increasing, decreasing and varying change and analyze non examples to support conceptual understanding.

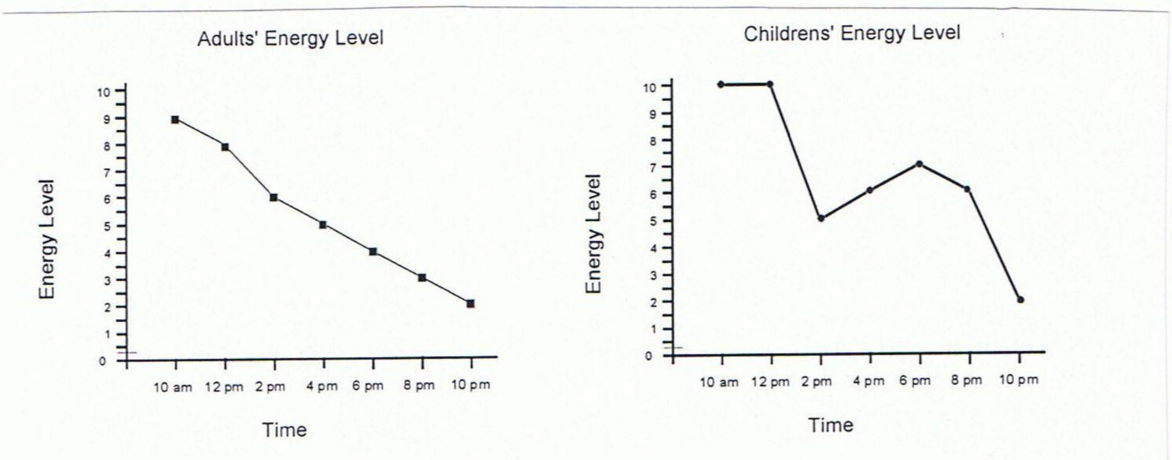
At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Which graph shows a decreasing rate of change?



2. The two graphs below show kids' and adults' energy level on a scale of 1 to 10 recorded throughout the day at Seven Flags Magic Molehill. Compare and contrast the general shape of the graphs. How are they the same? How are they different?



MODULE

2-6

Dimensionality

This module addresses the following indicators:

4-4.1 Analyze the quadrilaterals squares, rectangles, trapezoids, rhombuses, and parallelograms according to their properties. (B4)
4-4.2 Analyze the relationship between three-dimensional geometric shapes in the form of cubes, rectangular prisms, and cylinders and their two-dimensional nets. (B4)
4-4.4 Represent the two-dimensional shapes trapezoids, rhombuses, and parallelograms and the three-dimensional shapes cubes, rectangular prisms, and cylinders. (B2)

* This module contains 3 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning for the Module

- ***Continuum of Knowledge***

4-4.1

In first grade, students analyze the two dimensional shapes circles, square, triangle, and rectangle (1-4.2). In third grade, students classify polygons as triangles, quadrilaterals, pentagons, hexagons, or octagons according by the number of their sides and angle attributes. (3-4.2) Students also analyzed the results of combining and subdividing quadrilaterals (3-4.7).

In fourth grade, students analyze the quadrilaterals squares, rectangles, trapezoids, rhombuses, and parallelograms according to their properties (4-4.1)

In fifth grade, students apply the relationships of quadrilaterals to make logical arguments about their properties. (5-3.2)

4-4.2

In first grade, students identify the three dimensional geometric shapes prisms, pyramid, and cone (1-4.1). In second grade, students identified and analyzed three-dimensional shapes spheres, cubes, cylinders, prisms, pyramids, and cones according to the number and shape of the faces, edges, corners, and bases of each (2-4.1). In third grade, the concept of three-dimensional geometric shapes in the form of cubes, rectangular prisms, and cylinders and their two-dimensional nets is not addressed.

In fourth grade, students use those attributes and their knowledge of two-dimensional shapes to analyze the relationship between-three dimensional shapes and their nets (4-4.2).

In fifth grade, students translate between two-dimensional representations and three-dimensional objects. (5-4.4)

4-4.4

In first grade, students analyze the two-dimensional shapes circle, square, triangle, and rectangle (1-4.2). In second grade, students identified and analyzed three-dimensional shapes spheres, cubes, cylinders, prisms, pyramids, and cones according to the number and shape of the faces, edges, corners, and bases of each (2-4.1)
In fourth grade, students represent the two-dimensional shapes trapezoid, rhombuses, and parallelograms and the three-dimensional shapes cubes, rectangular prisms, and cylinder (4-4.4) .

In fifth grade, students translate between two-dimensional representations and three-dimensional objects (5-4.4).

- **Key Concepts/Key Terms**

  *These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

** squares*
** rectangles*
** trapezoids*
** rhombuses (rhombi)*
** parallelograms*
** symmetry*
** diagonal*
** right/acute/obtuse angle*
** perpendicular*
** sides*
** cubes*
** rectangular Prism*
** cylinder*
** net*
** two-Dimensional*
** three-Dimensional*
** parallel*
** midpoint*

II. Teaching the Lesson(s)

1. **Teaching Lesson A: *Shape Properties***

It is important that students see many different examples/models of each type of quadrilateral (squares, rectangles, trapezoids, rhombuses, and parallelograms) so they will learn the unique properties of each. Students should understand the relationship among the various quadrilaterals based on the number of sides, opposite sides, side lengths, and angle measures. By providing a variety of examples, the students are able to classify these quadrilaterals by their properties instead of a memorized picture of one example. It is also important for the students to verbalize correct vocabulary (squares, rectangles, trapezoids, rhombuses, and parallelograms) as they explore these different quadrilaterals. Sorting examples into their group as they name the quadrilateral is a good way for the students to practice this skill.

An understanding of these quadrilateral properties is essential as students' progress to representing the two dimensional shapes trapezoids, rhombuses, and parallelograms and the three –dimensional shapes cubes, rectangular prisms, and cylinders. Initial representations should include concrete models (straws, sticks, Geoboards, etc.), then students can transition to the use of graph paper or unlined paper and a straight-edge.

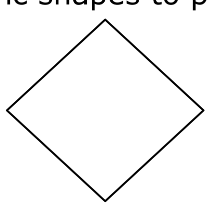
A net is a two-dimensional representation of a three-dimensional shape. It is best if the students can begin with concrete three-dimensional shapes so they can touch and see the two-dimensional shapes that can be created from a net. These models can be made of paper or manipulatives (such as Polydrons). First, students must identify the two-dimensional shapes that make the three-dimensional shape. Then they can explore different ways that those two-dimensional faces/bases can be placed to create the three-dimensional shape. Using graph paper is an excellent way to guide students in illustrating the different nets for the three-dimensional shapes. It is important for students to see that there are a variety of ways to create a net for each three-dimensional shape, but to also see the common characteristics. Students should also to be able to represent cubes, rectangular prisms, and cylinders and in order to do so must know the properties of each. Students can represent these first using models or manipulatives, and then graph paper or unlined paper and a straight-edge.

In 3^{rd} grade, students classified polygons by the number of sides, and classified triangles by their side length and angle attributes. Students have already classified polygons as quadrilaterals and have analyzed the results of combining and subdividing quadrilaterals. In 4^{th} grade, students use these skills to analyze the quadrilaterals squares, rectangles, trapezoids, rhombuses, and parallelograms according to their properties.

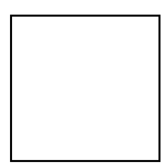
In 2^{nd} grade, students identified and analyzed three-dimensional shapes spheres, cubes, cylinders, prisms, pyramids, and cones according to the number and shape of the faces, edges, corners, and bases of each. Now, students use those attributes and their knowledge of two-dimensional shapes to see their relationship to three-dimensional shapes and their nets.

4-4.1

Teachers should occasionally draw these shapes upside down, facing different directions, or just tilted over, to force students to look at the essential properties. The best ways to clarify this misconception is to allow students to draw geometric shapes in any direction, provide examples of shapes in a variety of directions, rearrange displays of geometric shapes to point in different directions.



rhombus

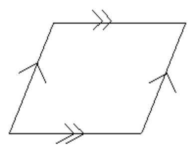


square

- The use of a Venn diagram is a good way for students to sort quadrilaterals according to their properties.

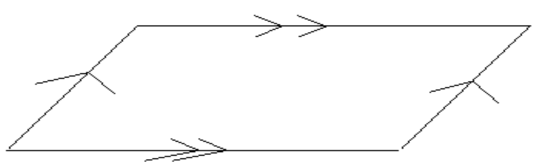
- Students may use a geoboard to make a square, copy it on geoboard dot paper, then turn the geoboard and copy the square again. This should be done several times to help students explore the properties and articulate their mathematical thinking with classmates and teacher in order to analyze the different type of quadrilaterals (i.e. squares, rectangles, trapezoids, rhombuses, and parallelograms.)

- A rhombus has 4 sides that are congruent, opposite sides that are parallel, and angles that are equal. Teacher note to help students visualize this particular concept (It looks like a square that is being pushed over).



The arrows show parallel sides.

- Opposite sides are the same length and they are parallel. Teacher note: (A parallelogram looks like a rectangle that has been pushed over.)



The arrows show parallel sides.

4-4.1

For this indicator, it is **essential** for students to:

- Explore many different examples/models of each type of quadrilateral (squares, rectangles, trapezoids, rhombuses, and parallelograms) so they will learn the unique properties of each.
- Understand the relationship among the various quadrilaterals based on the number of sides, opposite sides, side lengths, and angle measures.
- Classify these quadrilaterals by their properties instead of a memorized picture of one example.
- Analyze quadrilaterals that are oriented in position other than the upright position
- Use appropriate vocabulary
- Sort examples into categories

For this indicator, it is **not essential** for students to:

- Measure angles to determine properties of each type of quadrilateral (squares, rectangles, trapezoids, rhombuses, and parallelograms)

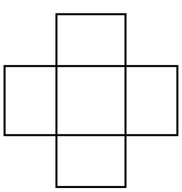
4-4.2

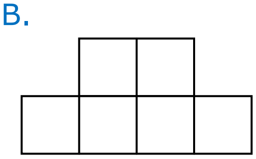
- Pre-assessing student's generalization about quadrilaterals may be done using KWL graphic organizer. This information can be used to design learning experience centered around what students need to know.

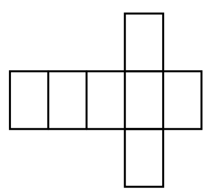
- It may be helpful to have students connect this mathematical concept using materials found in the students' environment such as a shoe box, a Pringle container, and a tissue box. Using these real world materials will provide concrete materials to help students analyze the relationship between three-dimensional geometric shapes in the form of cubes, rectangular prisms, and cylinders and their two-dimensional nets to help them formulate meaningful connections to learning.

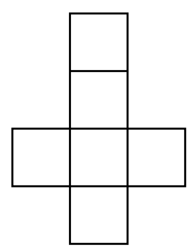
- In this example, teachers should help students identify the two-dimensional shapes that make the three-dimensional shape. The two dimensional shapes that creates a cube is a square. Also, it helps students develop a deeper conceptual understanding of the properties that represent a specific geometric cube. A cube has 6 square faces. Therefore, the answer should be D.

Which figure can be folded to make a cube?

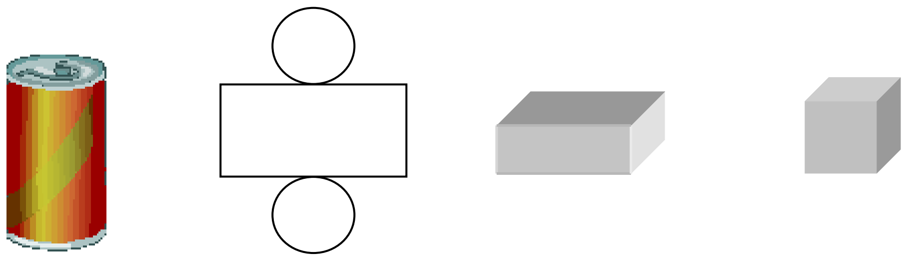
A.


B.


C.


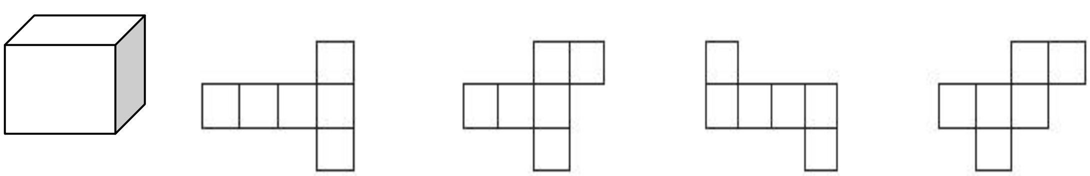
D.


- Explore Nets for Cubes
  http://illuminations.nctm.org/ActivityDetail.aspx?ID=84



For this indicator, it is **essential** for students to:

- Understand that a net is a two-dimensional representation of a three-dimensional shape.
- Identify the two-dimensional shapes that make the three-dimensional shape.
- Explore different ways that those shapes can be placed to create the three-dimensional shape.
- Comprehend the properties that represent cubes, rectangular prisms, and cylinders.
- Construct three-dimensional shapes from nets.
- Understand that there are a variety of ways to create a net for each three-dimensional shape, but to also see the common characteristics as shown below.



These are possible nets for a cube.

For this indicator, it is **not essential** for students to:
None noted

4-4.4

The teacher should use Geoboards and dot paper to help students develop a conceptual understanding to assist them as they represent trapezoids, rhombuses, parallelograms, cubes, rectangular prisms.

For this indicator, it is **essential** for students to:

- Recall the properties of each shape
- Represent shapes using models, manipulatives
- Represent shapes using graph paper or unlined paper.

For this indicator, it is **not essential** for students to:

- Classify trapezoid, rhombuses, parallelograms, cubes, rectangular prisms, and cylinders.

a. *Indicators with Taxonomy*

Indicator → 4-4.1 Analyze the quadrilaterals squares, rectangles, trapezoids, rhombuses, and parallelograms according to their properties. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual

b. *Introductory Lesson A*

Materials Needed

- *shape sheets (in lesson)*
- *index cards (to check for right angles, mark side lengths, draw straight lines, etc.*
- *tracing paper (to check for angle congruence and rotational symmetry)*
- *mirrors (to check for line symmetry)*

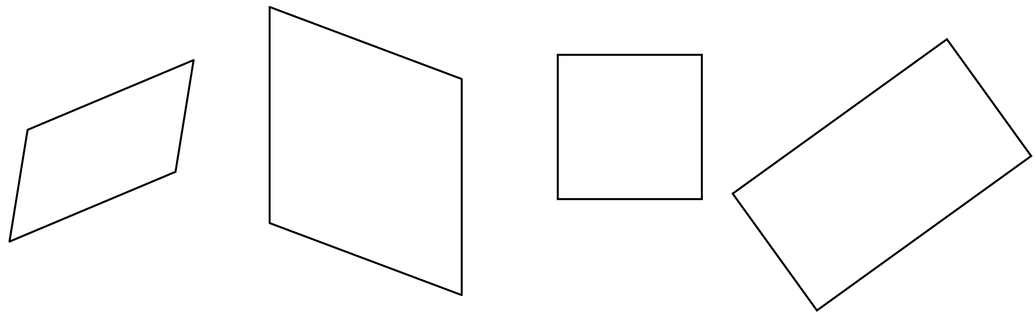
Property Listings → *Give students worksheets of shapes and all materials. Students should organize their notes as a table to record their findings. See "notes table" example in this lesson. Assign students working in trios or quads to one type of quadrilateral. Their task is to list as many properties as they can in their notes table. Each property listed must be applicable to all shapes on their sheet. Encourage them to use all the materials available. Encourage them to be specific in their property listings.*

This activity may take longer than one day to complete.

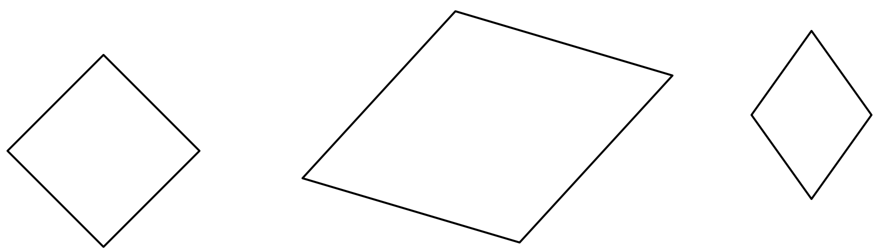
For share out, have students share properties from the parallelograms first, then the rhombi, rectangles, squares and last, the trapezoid. Have one parallelogram group present first, then the other group of parallelograms (if any) add to or dispute properties given.

As the teacher, clarify any terms needed to be formally introduced such as right/acute/obtuse angle, parallel, perpendicular, midpoint, etc.

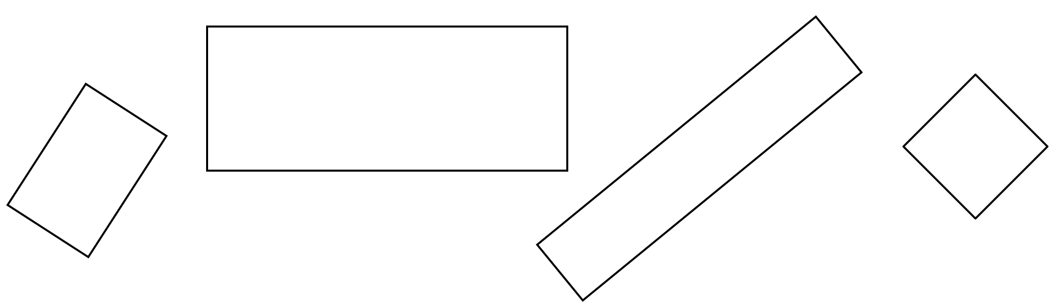
Sheet A: Parallelogram



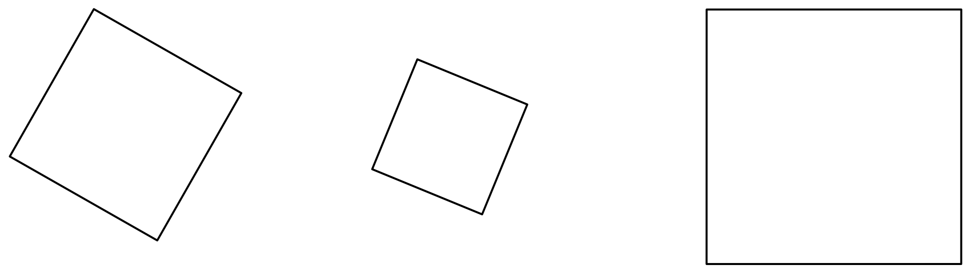
Sheet B: Rhombuses (Rhombi)



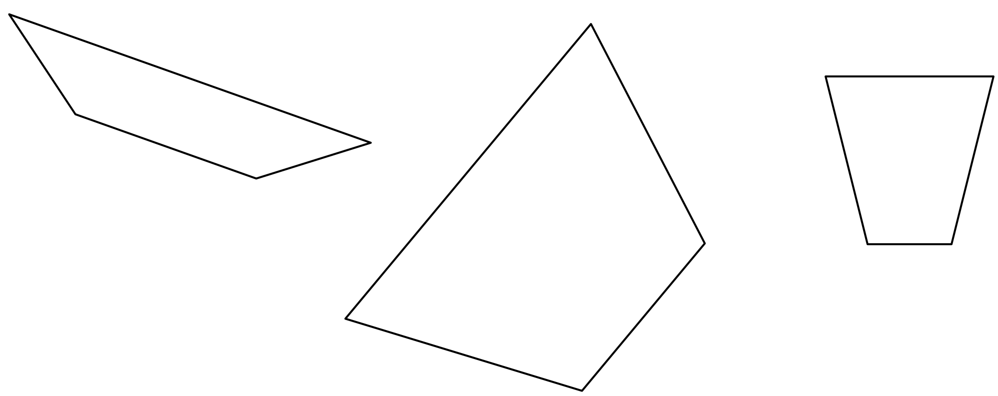
Sheet C: Rectangles



Sheet D: Squares



Sheet E: Trapezoid



Notes Pages Set Up

Shape	Sides	Angles	Diagonals	Symmetry
Parallelogram				
Rhombuses (Rhombi)				
Rectangles				
Squares				
Trapezoids				

c. Misconceptions/Common Errors

Students have a hard time understanding that a square is a rectangle, but a rectangle is not a square. Focus on the properties of the two so that students will understand the difference.

Students often over-generalize during the learning process and often refer to square as a diamond. However, in math, there's no such shape as a diamond. It's either a square or a rhombus. This misconception is because students usually see or draw a square, right-angled in the upright position.

It is also sometimes hard for students to understand how the many shapes fall under the umbrella of quadrilaterals. How they are all quadrilaterals, but have different names based on properties.

d. Additional Instructional Strategies/Differentiation

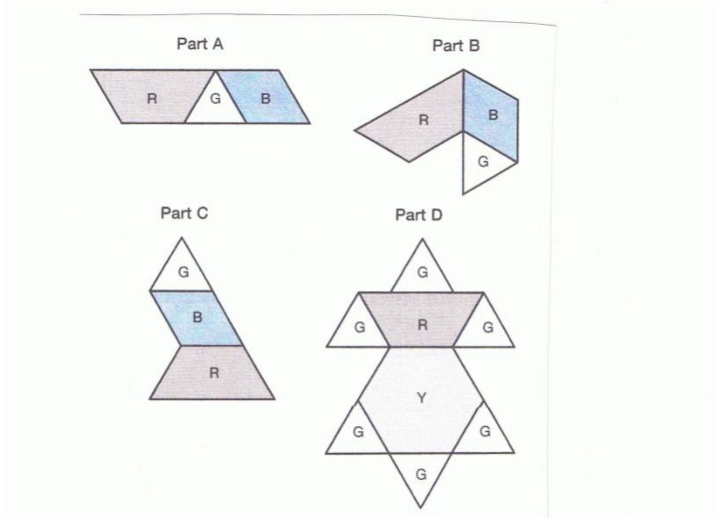
For more challenge, have students complete the following task.

Part 1: Use the trapezoid (red), the triangle (green) and the rhombus (blue) pattern blocks. Complete parts A-C.

A. Create a parallelogram using all three blocks.
B. Create a concave polygon using all three blocks.
C. Create a pentagon using all three blocks.

Part 2: Use any pattern blocks you wish, create a polygon with the following conditions:

- the number of yellow blocks (hexagons) is equal to the number of red blocks (trapezoids). (there may be other colors/blocks in the figure)
- The final shape has only one reflective line of symmetry.
- 8 blocks in total must be used.



e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation: however, other formative assessment strategies should be employed. See Appendix A.

Students should use their properties charts to answer the following questions in their math notebooks/journals. Have them trade their paper with someone else and that person should add to or question what they've recorded. Pair them up and have them discuss the papers and turn in one consensus paper to the teacher.

- What do you notice about the sides, angles, diagonals and symmetry of the quadrilaterals given: square, rectangle, trapezoid, rhombus and parallelogram.

2. *Teaching Lesson B: Nothing but Nets*

a. Indicators with Taxonomy

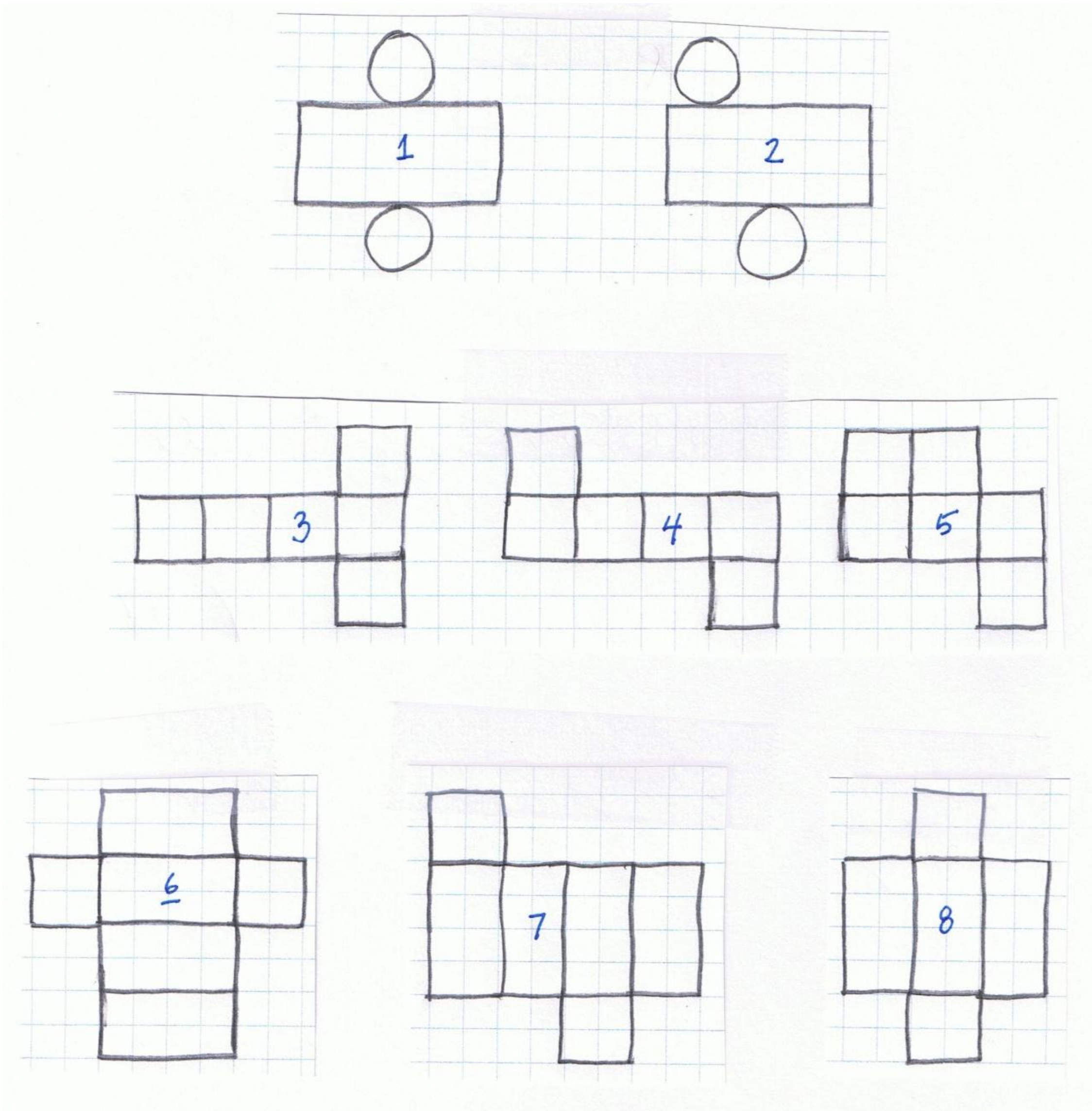
Indicator → 4-4.2 Analyze the relationship between three-dimensional geometric shapes in the form of cubes, rectangular prisms, and cylinders and their two-dimensional nets. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual

b. Introductory Lesson B

Materials Needed

- Students should be given two copies of the two-dimensional representations of rectangular prism and cylinder (net). One of each should be correct and the other should not. Each net example should be labeled with a number like 1,2,3,4 (each group should be given the same examples).

<u>Nothing but Nets</u> → Start the lesson with the discussion on cubes, rectangular prisms and cylinders. Students should share what they know about the attributes of these shapes. Students should also give examples on where in the "real world" you can find such shapes. After the discussion, students should look at the nets and decide what nets will or will not make the three-dimensional cube, rectangular prism or cylinder. Students should justify their decisions to the group and record their thoughts in their journals. After the students have decided, they should cut the nets out to prove if their conjectures were correct or incorrect. After students have tested all the nets, the class should come back together and share their findings.


1
2
3
4
5
6
7
8

c. *Misconceptions/Common Errors*

When students recall characteristics, they may connect a pattern with a misconception and learn erroneous properties based on their overgeneralization.

d. *Additional Instructional Strategies/Differentiation*

Submitted from A5

Have students unfold rectangular prisms, such as tissue boxes, cereal boxes, etc. to show the net. Have the students trace it and see if they can fold their representation to make the prism.

Have students collect tubes (cylinders) such as toilet paper rolls and trace the circular ends. Then have them cut out a straight line in the tube and unfold it. What shape is it? (rectangle)

Use Navigating Through Geometry in Grades 3 through 5, "Building Solids," p. 26. Students construct models of solids.

Use Navigating Through Geometry in Grades 3 through 5, "Exploring Packages," p. 80. Students compare shapes and draw a net for a package.

Use Navigating Through Geometry in Grades 3 through 5, "It's All in the Packaging." Students use geodot paper to create packages for objects.

e. *Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situation/representations are critical for conceptual understanding.

http://illuminations.nctm.org/LessonDetail.aspx?id=L570

f. *Assessing the Lesson*

Formative Assessment is embedded within the lesson through questioning and observation: however, other formative assessment strategies should be employed.

Students should be shown correct and incorrect two-dimensional nets of three-dimensional objects. As you flash one on the overhead or Smartboard, students should use their whiteboards to write yes or no: yes = correct net, no = incorrect net. Teachers can take a quick look around the room to see how many are correctly identifying the nets. Stop each time and let a student come up and explain how they knew.

3. Teaching Lesson C: Building two and three dimensional figures

a. Indicators with Taxonomy

Indicator → 4-4.4 Represent the two-dimensional shapes trapezoids, rhombuses, and parallelograms and the three-dimensional shapes cubes, rectangular prisms, and cylinders. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

b. Introductory Lesson C:

Materials Needed

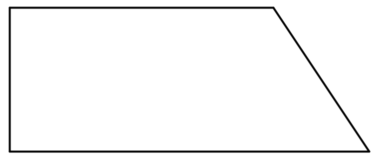
straws	*pipecleaners*
popsicle sticks	*tape*
glue	*geoboards & Rubberbands*
grid paper	*Other materials deemed appropriate by teacher…*

Students (individually) should use whatever manipulatives available to build a two-dimensional figure and a three-dimensional figure. Once the figures have been built, students should write instructions using mathematical language on how they constructed their figure and how they knew the characteristics. After writing their instructions and characteristics, they should turn in their papers and/or exchange with another student to follow their directions and build the figure.

For example, if I were to build a trapezoid… I would say the following:

- Cut two straws one longer than the other – and place them parallel to one another with about 1 inch or so between them.
- Cut a piece of straw that is 1 inch long and connect it to one endpoint of each of the other straw lengths. This straw will be perpendicular to both of the other straw lengths.

- You should cut the last straw length to connect the remaining endpoints of the original two straw lengths so that you have one acute angle and one obtuse angle formed.
- All together you will have two right angles, one obtuse angle and one acute angle.
- This is a right trapezoid as drawn below.



c. *Misconceptions/Common Errors*

Students often over generalize and represent parallelograms as a rhombuses or visa-versa.

Students often over generalize and represent parallelograms as a rhombuses or visa-versa.

d. *Additional Instructional Strategies/Differentiation*

The teacher should use Geoboards and dot paper to help students develop a conceptual understanding to assist them as they represent trapezoids, rhombuses, parallelograms, cubes, rectangular prisms.

e. *Technology*

Virtual manipulatives should NOT take the place of concrete maniputlation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Lesson on 3-d solids from Illuminations (NCTM):
http://illuminations.nctm.org/LessonDetail.aspx?id=U122

f. *Assessing the Lesson*

Formative Assessment is embedded within the lesson through questioning and observation: however, other formative assessment strategies should be employed.

Simply take up the work from the lesson and review it to see what terminology students are using… perhaps, make a tally chart of terminology (use correctly) and share different terms with students on the following day. This way they may learn some new terms and be more apt to make better descriptions the next time… Give them another opportunity to do this with the new list of terms.

III. Assessing the Module

4-4.1

The objective of this indicator is analyze, which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. Conceptual knowledge is the interrelationships among the basic elements (shapes) within the larger structure (quadrilaterals); therefore, students explore these relationships using a variety of examples to build conceptual understanding. The learning progression to **analyze** requires students to sort examples of quadrilaterals into groups based on their commonalities. Students then generate descriptions about relationships (4-1.4) between and among groups of quadrilaterals. Based on these descriptions, students relate their descriptions to the terms squares, rectangle, trapezoid, rhombus and parallelogram and generate mathematical statements (4-1.4) to describe each category. Students explain and justify answers on the basis of mathematical properties, structures, and relationships (4-3.3) they observe, and use correct, complete, and clearly written and oral mathematical language to pose questions and communicate ideas (4-1.5) with their classmates and teacher. Students then classify other examples of polygons.

4-4.2

The objective of this indicator is analyze, which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To analyze means to break material into its parts and determine how the parts relate to each other and the overall structure; therefore, student's conceptual understanding of three-dimensional shapes and their nets should be explored using a variety of examples. The learning progression to **analyze** requires students to recall the characteristics of cubes, rectangular prisms and cylinders. Students explore the shapes that are formed by folding a variety of two dimensional nets. Students then generate descriptions about relationships (4-1.4) between the structure of the net and the shape it forms. They explain and justify their answers to their classmates and teachers based mathematical properties and relationships (4-3.3). Based on these descriptions, students generalize the connection between three-dimensional shapes and their two dimensional nets using correct, complete, and clearly written and oral mathematical language to pose questions and communicate ideas (4-1.5). Students deepen their conceptual understanding by finding other real world examples of these shapes.

4-4.4
The objective of this indicator is to represent which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand means to construct meaning; therefore, the students' focus is on building conceptual knowledge of the relationships between the forms as opposed to reciting names and properties. The learning progression to **represent** requires students to recall the names and characteristics of two and three dimensional shapes. Students demonstrate flexibility in the use of mathematical representations (4-1.7) by creating their own representation of these shapes using concrete and pictorial models. Students explore these representations with their classmates and generate mathematical statements summarizing the mathematical processes they used to construct their shape (4-1.4). They use correct, complete and clearly written and oral language to communicate their ideas (4-1.5).

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Is a rhombus a parallelogram? Justify your answer.

2. Is a square a rectangle? Justify your answer.

3. Is a trapezoid a parallelogram? Justify your answer.

4. Draw the polygon that satisfies the following:
   * the polygon has only 2 right angles
   * the polygon has 1 set of parallel sides

5. Draw one correct net for a rectangular prism.

6. Could a rectangular prism have a square for a face? Explain in words or draw a picture.

7. Using your ruler, draw a polygon that has the following properties:
   - The polygon has only 2 acute angles
   - The polygon has only 1 pair of parallel lines.