SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

Content Area	Fourth Grade Mathematics
Recommended Days of Instruction	**Third Nine Weeks**

Standards/Indicators Addressed:

Standard 4-4: The student will demonstrate through the mathematical processes an understanding of the relationship between two- and three-dimensional shapes, the use of transformations to determine congruency, and the representation of location and movement within the first quadrant of a coordinate system.

4-4.3* Predict the results of multiple transformations of the same type—translation, reflection, or rotation—on a two-dimensional geometric shape. (B2)

4-4.5* Use transformation(s) to prove congruency. (B3)

4-4.6* Represent points, lines, line segments, rays, angles, and polygons. (C2)

4-4.7* Represent with ordered pairs of whole numbers the location of points in the first quadrant of a coordinate grid. (C2)

4-4.8* Illustrate possible paths from one point to another along vertical and

Standard 4-5: The student will demonstrate through the mathematical processes an understanding of elapsed time; conversions within the U.S. Customary system; and accurate, efficient, and generalizable methods of determining area.

4-5.1* Use appropriate tools to measure objects to the nearest unit: measuring length in quarter inches, centimeters, and millimeters; measuring liquid volume in cups, quarts, and liters; and measuring weight and mass in pounds, milligrams, and kilograms. (C3)

4-5.2* Compare angle measures with referent angles of 45 degrees, 90 degrees, and 180 degrees to estimate angle measures. (B2)

4-5.3* Use equivalencies to convert units of measure within the U.S. Customary System: converting length in inches, feet, yards, and miles: converting weight in ounces, pounds, and tons: converting liquid volume in cups, pints, quarts, and gallons: and converting time in years, months, weeks, days, hours, minutes, and seconds. (C3)

4-5.4* Analyze the perimeter of a polygon. (B4)

4-5.5* Generate strategies to determine the area of rectangles and triangles. (B6)

4-5.8* Recall equivalencies associated with liquid volume, time, weight, and length: 8 liquid ounces = 1 cup, 2 cups = 1 pint, 2 pints = 1 quart, 4 quarts = 1 gallon; 365 days = 1 year, 52 weeks = 1 year; 16 ounces= 1 pound, 2,000 pounds = 1

ton; and 5,280 feet = 1 mile. (A1)
4-5.9* Exemplify situations in which highly accurate measurements are required. (B2)
* These indicators are covered in the following 5 Modules for this Nine Weeks Period.
Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.

Module 3-1 Perimeter and Area			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 3-1 Lesson A 4-5.4 Analyze the perimeter of a polygon. (B4)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/van	See Instructional Planning Guide Module 3-1 Introductory Lesson A	See Instructional Planning Guide Module 3-1 Lesson A Assessing the Lesson
Module 3-1 Lesson B 4-5.5 Generate Strategies to determine the area of rectangles and triangles. (B6)		See Instructional Planning Guide Module 3-1 Introductory Lesson B See Instructional Planning Guide Module 3-1, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 3-1 Lesson B Assessing the Lesson

	dewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4		

Module 3-2 Plane/Spatial Relationships			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 3-2 Lesson A 4-4.6 Represent points, lines, line segments, rays, angles and polygons.	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/van	See Instructional Planning Guide Module 3-2 Introductory Lesson A See Instructional Planning Guide Module 3-2, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 3-2 Lesson A Assessing the Lesson
Module 3-2 Lesson B 4-4.7 Represent with ordered pairs of whole numbers the location of points in the first quadrant of a coordinate grid. (C2) 4-4.8 Illustrate possible paths from one point to another along vertical and horizontal grid lines in the first quadrant of the coordinate plane. (C2)		See Instructional Planning Guide Module 3-2 Introductory Lesson B See Instructional Planning Guide Module 3-2, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 3-2 Lesson B Assessing the Lesson

	dewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4		

Module 3-3 Transformational Geometry			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**

Module 3-3 Lesson A 4-4.3 Predict the results of multiple transformations of the same type – translation, reflection or rotation – on a two-dimensional geometric shape. (B2) 4-4.5 Use transformation(s) to prove congruency. (B3)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/Standards-and-Learning/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries	See Instructional Planning Guide Module 3-3 Introductory Lesson A	See Instructional Planning Guide Module 3-3 Lesson A Assessing the Lesson

	NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4		

Module 3-4 Length, Liquid Volume and Mass			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 3-4 Lesson A 4-5.1 Use appropriate tools to measure objects to the nearest unit: measuring length in quarter inches, centimeters, and millimeters; measuring liquid volume in cups, quarts, and liters; and measuring weight and mass in pounds, milligrams, and kilograms. (C3) 4-5.9 Exemplify situations in which highly accurate measurements are required. (B2)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th	See Instructional Planning Guide Module 3-4 Introductory Lesson A	See Instructional Planning Guide Module 3-4 Lesson A Assessing the Lesson

Module 3-4 Lesson B 4-5.2 Compare angle measures with referent angles of 45 degrees, 90 degrees, and 180 degrees to estimate angle measures. (B2)	Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4	See Instructional Planning Guide Module 3-4 Introductory Lesson B	See Instructional Planning Guide Module 3-4 Lesson B Assessing the Lesson

Module 3-5 Equivalencies and Conversions			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**

Module 3-5 Lesson A 4-5.8 Recall equivalencies associated with liquid volume, time, weight, and length: 8 liquid ounces = 1 cup, 2 cups = 1 pint, 2 pints = 1 quart, 4 quarts = 1 gallon; 365 days = 1 year, 52 weeks = 1 year; 16 ounces = 1 pound, 2,000 pounds = 1 ton; and 5, 280 feet = 1 mile. (A1) 4-5.3 Use equivalencies to convert units of measure within the U.S. Customary System: converting length in inches, feet, yards, and miles: converting weight in ounces, pounds, and tons: converting liquid volume in cups, pints, quarts, and gallons: and converting time in years, months, weeks, days, hours, minutes, and seconds. (C3)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries	See Instructional Planning Guide Module 3-5 Introductory Lesson A See Instructional Planning Guide Module 3-5, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 3-5 Lesson A Assessing the Lesson

	NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4		

MODULE

3-1

Perimeter and Area

This module addresses the following indicators:

4-5.4 Analyze the perimeter of a polygon. (B4)
4-5.5 Generate strategies to determine the area of rectangles and triangles. (B6)

* This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. ***Planning the Module***

- ***Continuum of Knowledge***

  Third grade was the first time students are introduced to the concept of perimeter. They generate strategies to determine the perimeters of polygons (3-5.5).

  In fourth grade, students analyze the perimeter of polygon (4-5.4).

  In fifth grade, students apply formulas to determine the perimeters and areas of triangles, rectangles and parallelograms (5-5.4).

- **Key Concepts/Key Terms**

  These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

  * perimeter
  * polygon
  * measure
  * area
  * side length

II. Teaching the Lesson(s)

1.Teaching Lesson A: Never Changing Perimeter

In third grade students generated strategies to determine the perimeters of polygons. They have an understanding that perimeter is the measure of the distance around a closed figure and is an extension of length measurement.

Fourth graders will analyze the perimeter of polygons. In other words fourth grade students should begin to break the concept down and determine how length and width relate to each other and to the object with which they are associated. Classroom experiences should include, but not be limited to, regular polygons.

Fourth grade is the first time students are introduced to the concept of area. They are expected to generate strategies to determine area of rectangles and triangles. Students will apply formulas for determine perimeter and area in fifth grade.

Area and perimeter can be a source of confusion for students. It may be because both involve regions to be measured or because students are taught formulas for both concepts and tend to get formulas confused. These two concepts are often presented together, but they are markedly different and need to be developed carefully. Students should have ample opportunities to solve problems with real-world settings. Such lessons and problems will help them distinguish between the two concepts. Intuitive concepts of area are developed when children cover index cards or book covers with color tiles. Students can build triangles on geoboards. It will help them focus on base length and height rather than shape or appearance when considering the area of a triangle. Measuring the areas of surfaces using nonstandard measures such as index cards, hexagons, and pizza box lids leads to the use of standard units. Students should see that each length has a corresponding area unit: inch to square inch, foot to square foot, yard to square yard, centimeter to square centimeter, and meter to square meter. This can be an extremely complex concept. Therefore, it is important for students to see that one unit on the perimeter corresponds to one square unit of area.

4-5.4

For this indicator, it is **essential** for students to:

- Recall their generated strategy for finding perimeter
- Understand the properties of polygons they have learned
- Understand the meaning of perimeter
- Find missing values in order to compute the perimeter. For example, students may be given a rectangle where only the length of one side and width of the other side are given. They will need to use find the lengths of the two other sides before finding the perimeter.
- Analyze the perimeters of polygons where the lengths of the sides are given in pictorial or word form

For this indicator, it is **not essential** for students to:

- Use traditional formulas to compute the perimeter. For example, $P = 2L + 2W$

4-5.5

For this indicator, it is **essential** for students to:

- Recall their generated strategy for finding perimeter
- Understand the properties of polygons they have learned
- Understand the meaning of perimeter

- Find missing values in order to compute the perimeter. For example, students may be given a rectangle where only the length of one side and width of the other side are given. They will need to use find the lengths of the two other sides before finding the perimeter.
- Analyze the perimeters of polygons where the lengths of the sides are given in pictorial or word form

For this indicator, it is **not essential** for students to:

- Use traditional formulas to compute the perimeter. For example, P = 2L + 2W

As students measure rectangles and squares, they often relate them to their knowledge of the area concept of multiplication. (Area of rectangle=base x height) Teacher-led lessons provide the applied instruction for children to begin understanding formulas for parallelograms and triangles. Once students see the area relationship between a rectangle and a triangle of the same height and base, the formula for the area of a triangle can be generated. (Area of triangle = ½ x base x height)

It is essential that students have the opportunity to engage in many activities in which they determine area and perimeter of real objects by direct measurement. Students will apply formulas for determine perimeter and area in fifth grade.

a. Indicators with Taxonomy

Indicator → 4-5.4 Analyze the perimeter of a polygon. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual

b. Introductory Lesson:

Materials Needed

- Strings (non-stretchy) cut to lengths of 24 cm or 24 inches – see lesson to decide which measure you will use.
- Centimeter grid paper OR inch grid paper
- Strings (non-stretchy) cut to other lengths that can be made into rectangles such as 18 units, 20 units, 36 units, etc.

Give students a string (non-stretchy) with length of 24 centimeters with centimeter grid paper, or 24 inches with inch grid paper. Ask students to use the string to create as many

rectangles as they can with different dimensions and/or different areas. This will allow them to see that by fixing perimeter, area can change. Students will see that perimeter is the distance around a figure and area is the space inside by finding it first hand. Have students use a different length of string such as 12 inches or 36 inches to do the activity and share their findings.

c. Misconceptions/Common Errors

Students will also be learning about area for the first time in fourth grade. Students tend to confuse these two concepts; therefore, concrete and/or pictorial models demonstrating the difference may need to be used.

When given a pictorial model where only some of the side measures are given, students tend to find the sum of only the given measures and forget to find the missing values before computing the perimeter.

d. Additional Instructional Strategies/Differentiation

To deepen conceptual understanding, classroom experiences should include but not be limited to regular polygons.

Navigating Through Measurement in Grades 3 through 5, "Changing Garden," p. 62. Students find various gardens with the same perimeter and can also find areas of those rectangular gardens.

Navigating Through Measurement in Grades 3 through 5, "Geo-Exploration – Triangles," p. 71.

e. Technology

Geoboards from the National Library of Virtual Manipulatives:

http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html?open=activities&from=category_g_2_t_3.html

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of the learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Have students respond to the following task and collect their responses, not for a grade, but for formative assessment only.

How many rectangles can you make with a string of 20 inches? What are their dimensions? Draw them on grid paper and label the sides. What do you notice?

2. Teaching Lesson B: Generating Area of Rectangles and Triangles

a. Indicators with Taxonomy

Indicator → 4-5.5 Generate strategies to determine the area of rectangles and triangles. (B6)
Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual

b. Introductory Lesson
Materials Needed

- Rectangle pairs cut and color coded as per Part A and Part B below.
- Other Part A materials → beans, pennies, counters, scissors, tape….
- Other Part B materials → single square units (cm or inches whichever is used)

Lesson – these focus on rectangles, but could be transferred to triangles easily.

Part A
Comparing Rectangles, Part A → Give students rectangles cut out on paper with no grids. Introduce them in pairs.
The task is to find the rectangle in each pair with the greatest area.
Pair A: 2 x 9 and 3 x 6 (centimeters or inches will work)
Pair B: 1 x 10 and 3 x 5
Pair C: 3 x 8 and 4 x 5
It may be helpful to color code the sets for each pair, trio or quad.

Provide students with materials such as beans, pennies, etc. Scissors could also be used to cut and compare… these are just a few possibilities. **NO rulers or grid paper!!**

Part B

Comparing Rectangles, Part B → Students will work with centimeters or inches in this activity, whichever is appropriate. Students are given the following:

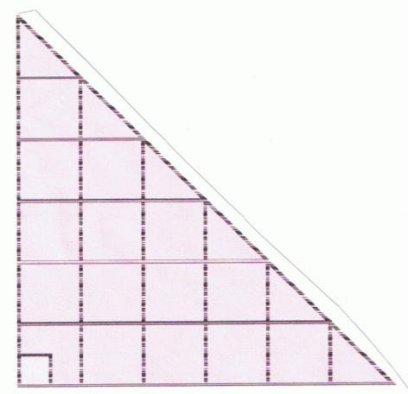
Pair A: rectangles measuring 4 x 10 and 5 x 8 units
Pair B: rectangles measuring 5 x 10 and 7 x 7
Pair C: rectangles measuring 4 x 6 and 5 x 5
A single square unit of whichever measure you found appropriate (cm or inches)

Determine which rectangle is larger or if the areas are the same – they can draw on the rectangles but cannot cut them out. They may choose to make notes of words, drawings, and even numbers to explain their conclusions.

Teacher Note: the goal is NOT to develop a formula, but to apply developing concepts of multiplication to the area of rectangles. NOT all students will make the connection to multiplicative relationship and **that's okay!**

PART C: Triangles

Pose the following triangle to the class…ask them to find the area.



c. Misconceptions/Common Errors

Area and perimeter is a source of confusion for students and they mistakenly use the wrong process for finding each.

Students also confused the concepts of height and side. Part of this confusion may stem from the fact that in some of the shapes students have encountered, the height of the shape is the same as one of the sides.

Students tend to get perimeter confused with area because they are typically taught formulas for both. By focusing on conceptual understanding and hands on learning, these misconceptions can hopefully be addressed.

d. Additional Instructional Strategies/Differentiation

Allow students to use grid paper when appropriate or needed.

The focus of the indicator is for students to generate strategies; therefore, the role of the teachers is to provide materials, facilitate discussion and encourage critical thinking through questioning. One possible teacher strategy is to explain to students the concept of area then give student square grid or geoboards. Draw a rectangle on the board that has whole number dimensions and challenge students to find the area of the rectangle using the grid paper or geoboard. Students should explore this process using a variety of examples.

Once students have an understanding of the process for computing the area of rectangles, ask students to use that process to compute the area of a triangle.

Submitted from A5

Navigating Through Measurement in Grades 3 through 5, "Changing Garden," p. 62. Students find various gardens with the same perimeter and can also find areas of those rectangular gardens.

Navigating Through Measurement in Grades 3 through 5, "Geo-Exploration – Triangles," p. 71.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reaches, you may move to pictorial

representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Give students rectangles A, B, C and D, with the following dimensions: A = 2 x 10, B = 4 x 4, C = 3 x 8, D = 5 x 6. Use centimeters or inches, whichever has been used in class.

Order the rectangles in order of smallest area to largest. Use manipulatives, unit squares or scissors to compare. After making your decisions, write your order using letters A-D on white boards.

Have students hold up their white boards on signal and compare results. *Answer: B, A, C, D*

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

4-5.4
The objective of this indicator is to analyze which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To analyze means to break material down (side lengths) and determine how the parts relate to one another and to an overall purpose (perimeter). The learning progression to **analyze** requires students to recall the concept of polygons and their properties. Students also understand the concept of perimeter. They use their understanding of these concepts to analyze information (4-1.1) given in pictorial and word forms. They use concrete representations to support understanding where appropriate. As students compute the perimeter, they explain and justify their answers (4-1.3) using correct, complete and clearly written and oral mathematical language (4-1.5).

4-5.5
The objective of this indicator is to generate which is in the "create conceptual" knowledge cell of the Revised Taxonomy. To create means to put ideas together into a new structure; therefore, students use prior knowledge to generate new strategies. The learning

progression to **generate** requires students to recall the properties of rectangles and triangles. They have a conceptual understanding of the meaning of area. Using concrete and/or pictorial models, students apply their understanding of these properties to determine how to compute the area. As students analyze information (4-1.1) from these experiences, they generate mathematical statements (4-1.4) about the relationships they observe then explain and justify their strategies (4-1.3) to their classmates and their teachers. They then generalize connections (4-1.6) between these statements and the concepts of "length, width, base, height and area". Students recognize the limitations of various strategies and representations (4-1.8) and use correct, complete and clearly written and oral language to communicate their ideas (4-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Draw 4 different rectangles each with a perimeter of 16 inches. (Note: The rectangles should have different areas). Use grid paper to create your drawings. You may choose to use color tiles or other manipulatives before you create your drawing.

Possible Answers: 1 x 7, 2 x 6, 3 x 5, and 4 x 4.

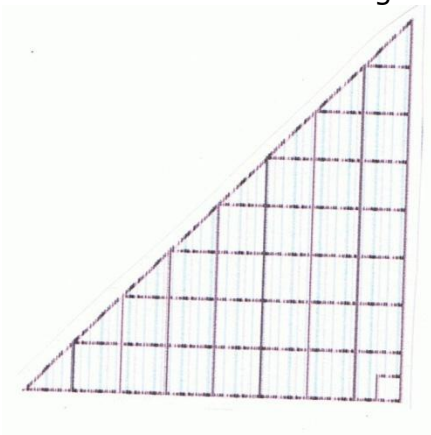
a. Consider the areas of each of the rectangles. What is the area of each?

Areas: 7 square inches, 12 square inches, 15 square inches and 16 square inches (respectively)

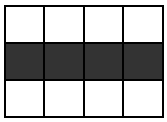
b. Which has the greatest area? What do you notice about the shape?

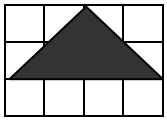
Answer: Greatest area is the square.

2. Find the area of the right triangle.

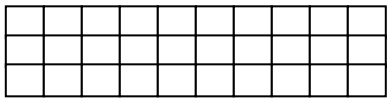


3. Explain in words or pictures how you know the shaded areas are congruent.





4. What is the area of the following?



= 1 square centimeter

A. 30 square centimeters
B. 24 square centimeters
C. 25 square centimeters
D. 27 square centimeters

MODULE

3-2

Plane/Spatial Relationships

This module addresses the following indicators:

4-4.6 Represent points, lines, line segments, rays, angles, and polygons. (C2)
4-4.7 Represent with ordered pairs of whole numbers the location of points in the first quadrant of a coordinate grid. (C2)
4-4.8 Illustrate possible paths from one point to another along vertical and horizontal grid lines in the first quadrant of the coordinate plane. (C2)

* This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Module

- ***Continuum of Knowledge***

4-4.6
In third grade, students classified lines and line segments as either parallel, perpendicular, or intersecting (3-4.3), exemplified points, lines, line segments, rays, and angles (3-4.6), and classified polygons as either triangles, quadrilaterals, pentagons, hexagons, or octagons according to the number of their sides (3-4.2).

In fourth grade, students represent points, lines, line segments, rays, angles, and polygons (4-4.6) using symbolic notation.

In fifth grade, students compare the angles, side lengths, and perimeters of congruent shapes (5-4.2)

4-4.7
In fourth grade, students represent with ordered pairs of whole numbers the location of points in the first quadrant of a coordinate grid (4-4.7)

In sixth grade, students represent with ordered pairs of integers the location of points in a coordinate grid (6-4.1).

4-4.8
In fourth grade, students illustrate possible paths from one point to another along vertical and horizontal grid lines in the first quadrant of the coordinate plane (4-4.8), and represent with ordered pairs of whole numbers the location of points in the first quadrant of a coordinate grid (4-4.7).

In sixth grade, students represent with ordered pairs of integers the location of points in a coordinate grid (6-4.1), and apply strategies and procedures to find the coordinates of the missing vertex of a square, rectangle, or right triangle when given the coordinates of the polygons other vertices (6-4.2)

- ***Key Concepts/Key Terms***

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

* angle
* line
* line segment
* vertical
* point
* polygons
* ray
* horizontal

* mathematical notation for vocabulary
  - Ray $\overrightarrow{AB}$
  - Segment $\overline{AB}$
  - Line $\overleftrightarrow{AB}$

* coordinate grid
* x axis
* y axis
* path

II. Teaching the Lesson(s)

1. *Teaching Lesson A: Hoop Web*

In 3rd grade, students exemplified points, lines, line segments, rays, and angles by giving an example or illustration that shows their understanding concretely. In 4th grade, students learn to represent points, lines, line segments, rays, angles, and polygons using symbolic notation. Although students may have encountered coordinate grids in other subject areas, this is the first time in math that they will plot points in the first quadrant of the coordinate plane and illustrate the paths from one point to another.

In 4th grade, students illustrate possible paths from one point to another along vertical and horizontal grid lines in the first quadrant of the coordinate plane. When students illustrate the possible paths, it is important for them to see that there is often more than one path. Sometimes a path is chosen because it is the shortest, sometimes for the least turns, or other criteria. Teachers should incorporate maps to allow students to make connections to real-world applications.

Teacher Note: The terminology *vertical* and *horizontal* is beneficial and necessary for student understanding.

One critical skill for the students to understand is that the location of the numbers in an ordered pair corresponds to the x- and y-axis of the coordinate plane, and that movement begins from the origin (0, 0). The first number in the ordered pair indicates how far to move across (horizontal). The second number in the ordered pair indicates how far to move up (vertical).

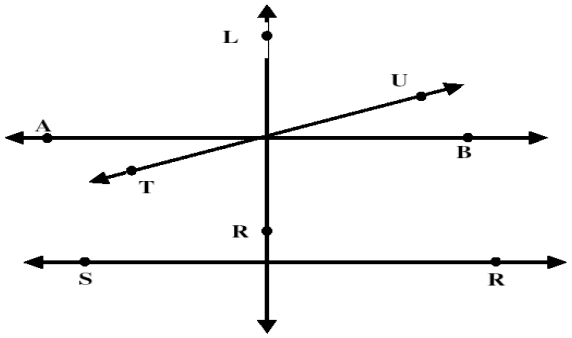
4-4.6

For this indicator, it is **essential** for students to:

- Understand that an angle is where two rays meet with a common endpoint.
- Understand that a line goes on forever in both directions

- Understand that a line segment is part of a line and has two endpoints
- Understand that intersecting lines cross or meet each other at a point
- Represent points, lines, line segments, rays, angles, and polygons using appropriate notation

*** Which two line segments appear to be parallel? B.**



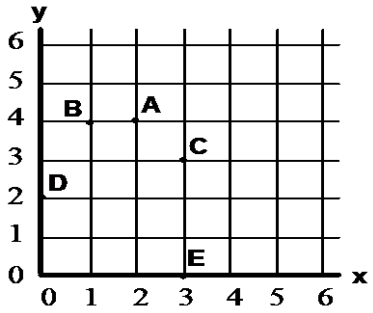
A $\overline{LR}$ and $\overline{AB}$
B $\overline{AB}$ and $\overline{SR}$
C. $\overline{AB}$ and $\overline{TU}$
D. $\overline{SR}$ and $\overline{LR}$

For this indicator, it is **not essential** for student to:
None noted

4-4.7
For this indicator, it is **essential** for students to:

- Understand that the first number in the ordered pair indicates how far to move across (horizontal)
- Understand that the second number in the ordered pair indicates how far to move up (vertical).
- Understand the structure of the coordinate grid
- Represent with ordered pairs of whole numbers the location of points in the first quadrant of a coordinate grid such as:

* Which ordered pair locates point B? (1,4)



For this indicator, it is **not essential** for students to:

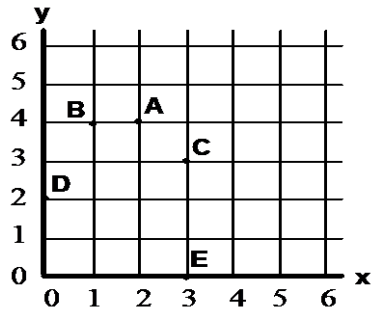
- Locate points beyond the first quadrant of a coordinate plane.

4-4.8

For this indicator, it is **<u>essential</u>** for students to:

- Understand that the first number in the ordered pair indicates how far to move across (horizontal)
- Understand that the second number in the ordered pair indicates how far to move up (vertical).
- Understand the structure of the coordinate grid
- Represent with ordered pairs of whole numbers the location of points in the first quadrant of a coordinate grid such as:

  * Which ordered pair locates point B? (1,4)



For this indicator, it is **not essential** for students to:

- Locate points beyond the first quadrant of a coordinate plane.

a. *Indicators with Taxonomy*

4-4.6 Represent points, lines, line segments, rays, angles, and polygons. (C2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

b. *Introductory Lesson A:*

Materials Needed:

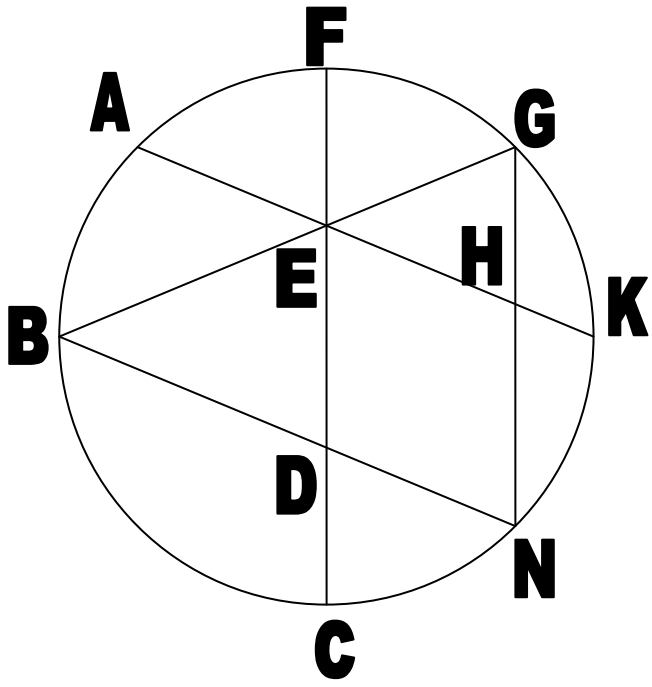
- yarn
- Hula hoop ***(You may need two if your class is large.)***
- Math notebooks (students need these)
- letters of the alphabet cut out, placed on card stock, and laminated-attach dots to the letters to represent points
- sheet of light colored poster board or butcher paper, large enough to encompass the circumference of the hoop

You should prepare the Hula hoop web designs ahead of time. The directions and a picture follow.

- Tie one end of a LONG piece of yarn to a point on the hoop. You may want to just pull from the skein until you are finished.
- Run the yarn across to another point on the hoop and loop it around the edge, working to create a "web design."
- Continue running the yarns to different points along the edge of the hoop until you are satisfied with your web. Use strips of masking tape along the outside of the hoop to hold the yarn in place.
- Place points on the web creating line segments, lines, angles, rays, and polygons.

NOTE: If you have to make two hoop webs, they don't have to be identical. In fact, two different webs provide more "shapes" for students to identify.

There is a hoop web design handout included.



<u>Directions for teaching the lesson:</u>

Arrange the room so that one end is clear. Cover a desk (or desks, depending on the size of your hoop web) with light colored poster board or butcher paper. Set the hoop web on the paper so that the lines in the design are clearly visible. Have students form a horseshoe around the design. You may also hang the hoop on the wall, if you are able to do so.

Students were introduced to the vocabulary in third grade. The object now is to focus on mathematical notation. A brief review may be necessary.

Use popsicle sticks with the students' names on them to randomly choose students to take turns naming line segments, lines, rays, angles and polygons. As the children name them, draw the representations using mathematical notation on the board/overhead/chart paper.

After several practice problems, the students find the figures in the web and write out the figures that they find using correct form and correct mathematical labeling. If your class is large, you may put up two webs and divide the students between them. Students record the figures in their math notebooks, while you monitor students' work, check for accuracy, and discuss errors with students.

Follow up with an assignment on drawing and labeling these figures using mathematical notation.

c. Misconceptions/Common Errors

Students tend to confuse the notation for lines, line segments and rays.
When writing the notation for rays, students may draw their arrow in the opposite direction. They may not recognize that unlike with line and line segments, the order matters.

d. Additional Instructional Strategies/Differentiation

Hoop Web Game

Use the same webs, but change the positions of the points. Put students in teams of four. Randomly choose the team that will go first. Challenge teams to find figures in the web, giving points for each figure.

Suggested point values:
Line segments, lines, and rays = 10 points
Angles = 15, with 2 extra points if they can accurately identify the angle as obtuse or acute

Polygons = 20 points, with 2 extra points for each different polygon named
First team to 100 points wins

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reaches, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

As outlined in the lesson above: Students record the figures in their math notebooks, while you monitor students' work, check for accuracy, and discuss errors with students.

Follow up with an assignment on drawing and labeling these figures using mathematical notation.

1. ***Students should use the scoring guide below to find at least:***
   - Two line segments (20 points total)
   - Two rays (20 points total)
   - 3 angles classified and named (45 points total)+ extra points available
   - 2 polygons (40 points total) + extra points available

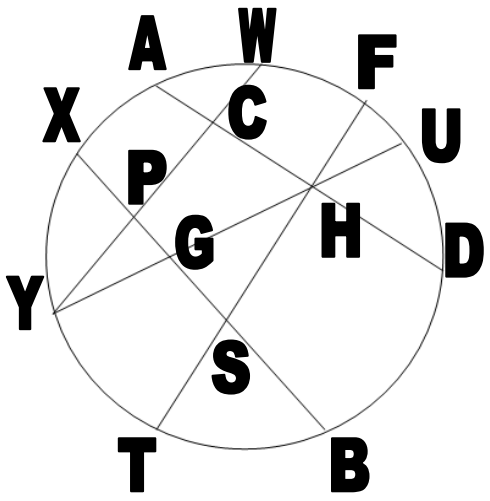
   In the figure below:

Possible Points:

Line segments, lines, and rays = 10 points

Angles = 15, with 2 extra points if they can accurately identify the angle as obtuse or acute

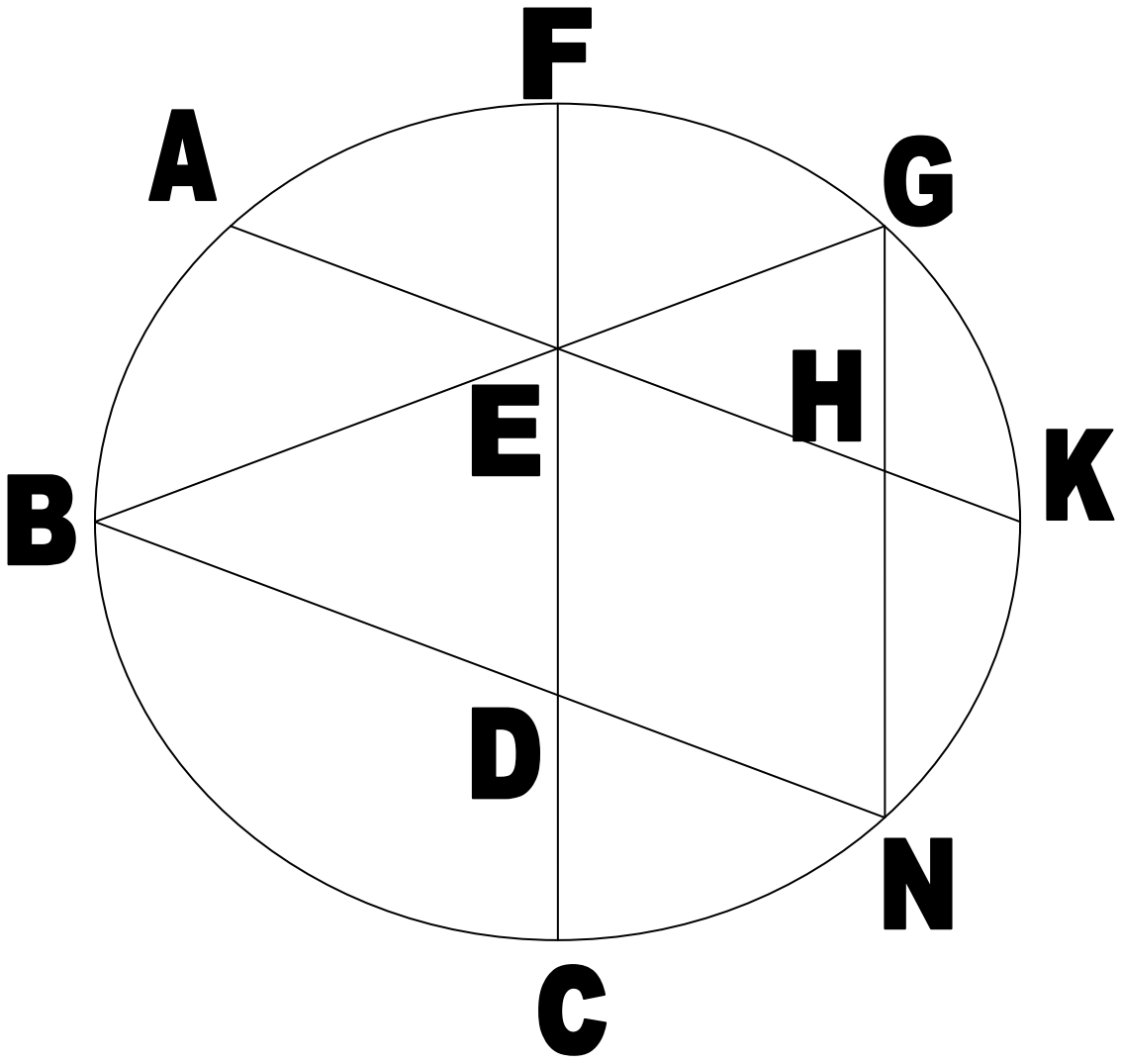
Polygons = 20 points, with 2 extra points for each different polygon named



The score should come from the ratio of number of points earned to total possible points… then add in the bonus points.
For example:

$$\frac{point s...earned}{125} \quad + \text{ bonus points = grade on this task}$$

HANDOUT



2. Introductory Lesson B: Locating Points/path in the first quadrant of a coordinate grid

a. Indicators with Taxonomy

4-4.7 Represent with ordered pairs of whole numbers the location of points in the first quadrant of a coordinate grid. (C2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

4-4.8 Illustrate possible paths from one point to another along vertical and horizontal grid lines in the first quadrant of the coordinate plane. (C2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

b. Introductory Lesson

Materials Needed:

- Large open space
- index cards with ordered pairs (use the numbers 0-10 in any ordered pair combination)
- 2 ten-feet strips of cash register tape, numbered 0 – 20 at 6-inch intervals (each strip will be used to represent either the x- or y-axis framing the first quadrant of the coordinate plane)

Use appropriate vocabulary as you work through this activity. Require that students do the same when they respond.

Display an overhead of a coordinate graph that has certain locations pictured – for example, school, park, etc. Point to a location and ask students to write two numbers which they think describe the location. Ask student volunteers to share the numbers they wrote. (There will be discrepancies in the order of the numbers used) Ask students to share how they determined the location. Use student thinking and apply the ordered pairs listed on the board. This will likely reveal different locations other than desired. Ask students what could be done to ensure that everyone can find the same spot if given two numbers. Students should come to the conclusion that the order of movement should be consistent. Point out that just like the agreed to standard units of measure, there is an agreed to order for locating points on the coordinate grid. Then as a whole class practice locating ordered pairs. Explain to the students that because coordinates are always written in the same order, we call them ordered pairs.

On an open area, horizontally lay out a 10-ft strip of cash register paper. Lay another 10 ft. strip of register paper at the end of that strip, extending vertically. Ask students what the two strips of paper represent. Pass out to each student the previously prepared index cards with an ordered pair. Have each student come to the coordinate plane and stand at his/her ordered pair. Have another student check to see that he/she is correct. Continue until each student has had a chance to locate his/her point. To progress to a pictorial model, tell students we need to look for a Hidden Picture.
To find the hidden picture: Use centimeter grid paper. Have students label the bottom (x or horizontal) axis starting at 0 and numbering to 20. They should also label the vertical (y) axis starting at 0 and going up to 20 as well. List the following ordered pairs: (4,2), (10,2), (12,7), (10, 12), (4,12), (2,7).

Instruct the students to locate the ordered pairs on the sheet and to connect the points around the outside. It is self-checking because if the points are placed correctly and the "points connected around the outside" a hexagon should be formed.

NOTE: To form the quadrant in which students will be physically locating points, you may choose to use sidewalk chalk if you have a paved area you can use to work. Otherwise, you will need to have the strips of paper weighted with something to keep them from blowing away if you have to work outside. If you're in a gym or large room, you may tape the strips down. Do what works best for you!

Extending the lesson to include indicator 4-4.8.

Materials:

- Large open space
- index cards with ordered pairs (use the numbers 0-10 in any ordered pair combination)
- 2 ten-feet strips of cash register tape, numbered 0 – 20 at 6-inch intervals (each strip will be used to represent either the x- or y-axis framing the first quadrant of the coordinate plane)
- Object to represent obstacles for students to navigate around (Frisbees, paper cut-outs)

Once students are familiar with plotting points in the first quadrant, they should progress to moving from one point to another along horizontal and vertical lines in the first quadrant.

Use the same set-up as you did for identifying ordered pairs in the first quadrant. You may even use the same ordered pairs.

Have a student stand at a point represented by the ordered pair on one of the index cards. Place the obstacles at different points in the first quadrant. Another student or pair of students then navigates from the origin to the stationed student using ordered pairs to define the path. Navigators may only move horizontally or vertically to reach their destination. Students take turns being stationed and navigating.

c. Misconceptions/Common Errors

Students may move up or vertically first, and then move across or horizontally.

Students may start at the wrong letter or place to begin their path. Many examples and practice should be given.

d. Additional Instructional Strategies/Differentiation

Although students may have encountered coordinate grids in other subject areas, this is the first time in math that they will plot points in the first quadrant of the coordinate plane. One critical skill for the students to understand is that the location of the numbers in an ordered pair corresponds to the x- and y-axis of the coordinate plane, and that movement begins from the origin (0, 0). Again, the first number in the ordered pair indicates how far to move across (horizontal), and the second number in the ordered pair indicates how far to move up (vertical).

Creating a large grid on the floor and having students move to an indicated point is an engaging and fund way to explore plotting coordinates.

When students illustrate the possible paths, it is important for them to see that there is often more than one path. Sometimes a path is chosen because it is the shortest, sometimes for the least turns, or other criteria. Teachers should incorporate maps to allow students to make connections to real-world applications.

Activity: Given students a coordinate grid like the one from the example above. Tell the students that you want them to get from one point to another using only a certain amount of steps. For example, can you find a path from point C to point D using only two steps? Three steps?

Turtle Pond (following a path on a coordinate grid) http://illuminations.nctm.org/ActivityDetail.aspx?ID=83

Give students grid paper. Have them randomly draw two objects in different places (e.g., rabbit, carrot). Have them draw the shortest route from the rabbit to the carrot and describe their path.

Write directions to tell how to get from the rabbit to the carrot on the shortest path, the longest path, and other paths.

See technology link and description below

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Simple Maze Game.

In this game, students navigate a robot around obstacles to reach a target

www.shodor.org/interactivate/activities/SimpleMazeGame

If a computer is not available for each student or student pair, this could serve as a center activity.

Materials Needed:

- Computer for each student or pair of students
- Pencil
- Graph paper handout with a couple of "windows"

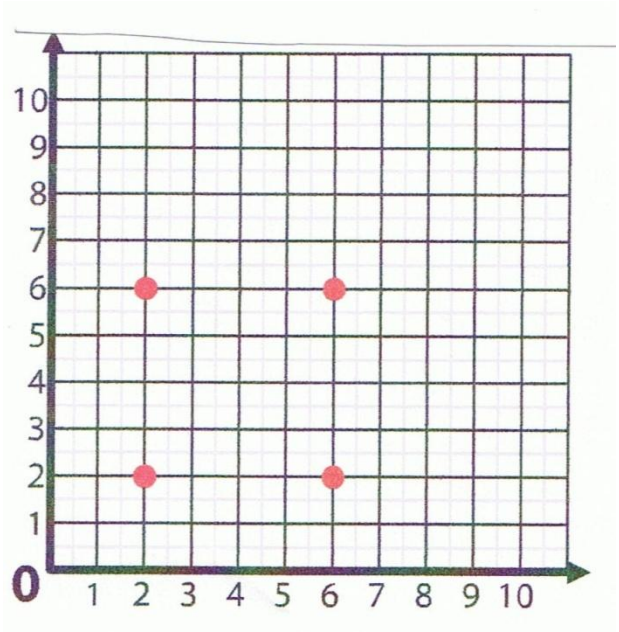
As students play the Simple Maze Game, they should copy the positions of the robot, mines, and target onto their graph paper. They should trace the moves they make and identify the ordered pairs that mark positions on their path from start to target.

f. Assessing the Lesson

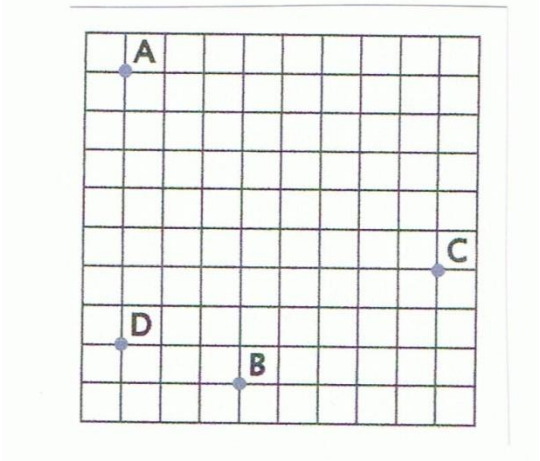
Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Students should respond in their journals/notebooks to the following prompts. Take up notebooks for review, not grades!

1. Using the provided grid, identify the point that represents the coordinates (6,2), and label the point with the given coordinates.



2. Choose two points on the grid and explain how to get from one to the other while staying on the grid lines.



III. Assessing the Module

4-4.6
The objective of this indicator is to represent which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To represent means to translate from one form to another; therefore, students use their understanding of these concepts to represent them using appropriate notation or drawings. The learning progression to **represent** requires students to recall the names and characteristics of lines, segments, angles, ray and polygons. Students analyze each representation (4-1.1) and determine similarities and differences. Students demonstrate flexibility in the use of mathematical representations (4-1.7) by creating their own representations of these concepts using concrete and pictorial models. Students explore these representations with their classmates and generate mathematical statements summarizing the mathematical processes they used to construct their representation or drawing (4-1.4). They use correct, complete and clearly written and oral language and notations to communicate their ideas (4-1.5).

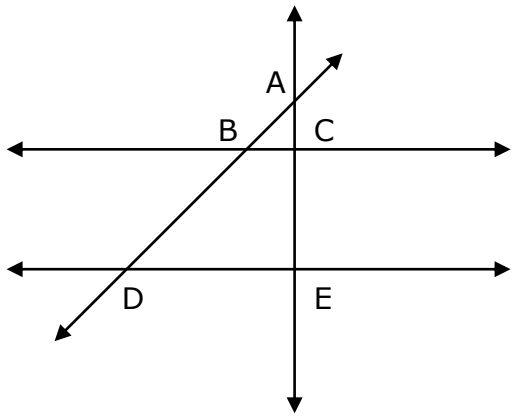
4-4.7
The objective of this indicator is to represent which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To represent means to translate from one form to another; therefore, students use their understanding of these concepts to represent numerical coordinate in graphical form. The learning progression to **represent** requires students to understand the structure of coordinate notation and the coordinate grid. Students recall that the first number tells us how far to move on the x-axis (horizontal) and the second number tells us how far to move on the y-axis (vertical). Students demonstrate flexibility in the use of mathematical representations (4-1.7) by plotting the coordinates on a coordinate grid and by writing coordinate notation from a given graph. Students explore these representations with their classmates and generate mathematical statements summarizing the mathematical processes they used to plot their points (4-1.4). They use correct, complete and clearly written and oral language to communicate their ideas (4-1.5).

4-4.8
The objective of this indicator is to illustrate which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Illustrate is to find specific examples. The learning progression to **illustrate** requires students to recall the structure of coordinate grid. Students also recall the meaning of terms such as north, south, east, or west, and up, down, left, or right. Students use flexibility in mathematical representations by exploring multiple paths from one point to another. They explain and justify their paths (4-1.3) to their

classmates and teacher using correct, complete, and clearly written and oral mathematical language (4-1.5).

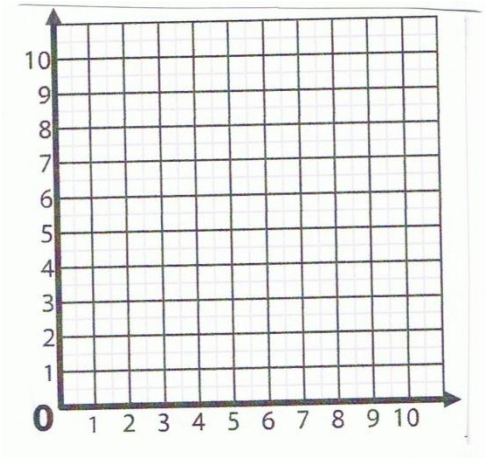
At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.
The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Use the figure to answer the questions below:

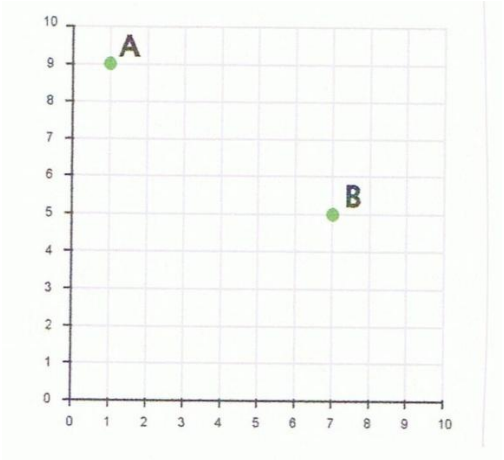


a. Name two line segments
b. Name two rays
c. Name 3 angles – classify them as well
d. Name 2 polygons – classify them as well

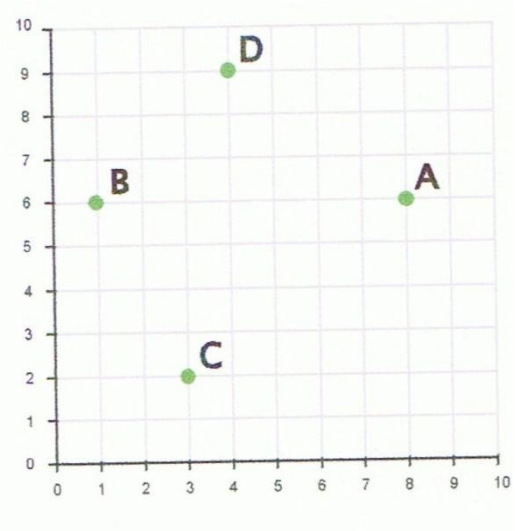
2. Lucas was working on a math problem involving ordered pairs. The problem said to plot the point located at (3,2). Lucas said that in order to plot the points, go up three units, and then over two units from the origin. What would you tell Lucas about plotting (3,2) correctly.



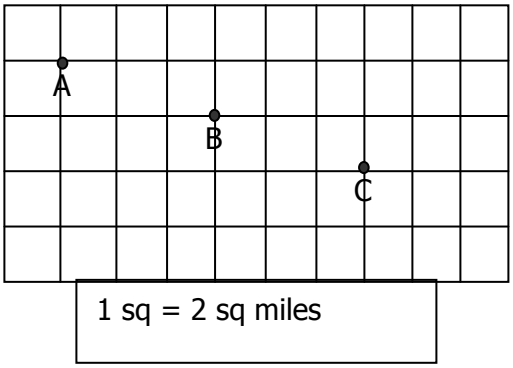
3. Explain how to get from Point A to Point B, staying on the grid lines. Use your directional words (North, South, East, West).



4. Choose two of the points on the grid and explain how to get from one point to the other using directional words (Left, Right, Up and Down).

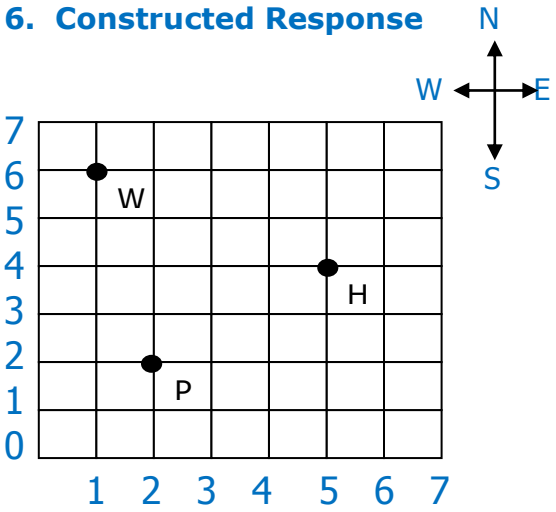


5. Point A is Bill's house. Point B is the school. Point C is the YMCA. Describe a path from school to the YMCA.



A. Three blocks left and one block up.
B. Three blocks down and two blocks to the right
C. Three blocks to the right and one block down.
D. Three blocks to the right and two blocks down

6. Constructed Response



Shannon wants to go from Westside (W) to Hanna (H). On her way, she will pick up a pizza at The Pizza Place (P). Give the directions she should follow.

MODULE

3-3

Transformational Geometry

This module addresses the following indicators:

4-4.3 Predict the results of multiple transformations of the same type—translation, reflection, or rotation—on a two-dimensional geometric shape. (B2)

4-4.5 Use transformation(s) to prove congruency. (B3)

* This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

4-4.3

In first grade, students use the positional and directional terms north, south, east, and west to describe location and movement (1-4.5). In second grade, students predict the results of combining and subdividing polygons and circles (2-4.3). In the third grade, students will predict the results of one transformation-either slide, flip, or turn-of a geometric shape (3-4.8).

In fourth grade, students predict the results of multiple transformations of the same type-translation, reflection, or rotation-on a two-dimensional geometric shape. (4-4.3)

In fifth grade, students predict the results of multiple transformations on a geometric shape when combinations of translation, reflections, and rotation are used. (5-4.5)

4-4.5

In third grade, students predicted the results of one transformation of a geometric shape using slides, flips, and turns (3-4.8).

In fourth grade, students use transformation(s) to prove congruency (4-4.5). These transformations are now referred to as transformations, reflections, and rotations. Students will also predict the results of multiple transformations of the same type—translation, reflection, or rotation—on a two-dimensional geometric shape (4-4.3).

In fifth grade, students classify shapes as congruent (5-4.3) and predict the results of multiple transformations on a geometric shape when combinations of translation, reflection, and rotation are used (5-4.5) In sixth grade, students identify the transformation(s) used to move a polygon from one location to another in the coordinate plane (6-4.5).

- **Key Concepts/Key Terms**

*These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

*translation (slide)
*reflection (flip)
*rotation (turn)
*transformation
*congruency
degrees

*horizontal line
*vertical line

II. Teaching the Lesson(s)

1. Teaching Lesson A: From Here to There

Students can apply this knowledge to use transformations to prove congruency. Congruency means that two figures are the same size and same shape. To prove congruency, students can apply transformations of the same type to a two-dimensional geometric shape so that it will can be positioned on top of the other shape to demonstrate they are the same size and same shape.

In 3rd grade, students predicted the results of one transformation of a geometric shape using slides, flips, and turns. In 4th grade, these applications are now referred to as translations, reflections, and rotations. Not only will students use this new terminology, they will now explore the results of more than one transformation **of the same type**. After sufficient exploration, students will be able to predict the results. It is not until 5th grade that students predict the results of multiple transformations involving combinations of translations, reflections, and rotations.

a. Indicators with Taxonomy

4-4.3 Predict the results of multiple transformations of the same type—translation, reflection, or rotation—on a two-dimensional geometric shape. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

b. Introductory Lesson

Part A

Materials Needed

- Math notebooks for recording work
- Colored pencils
- *two* right triangle cut outs for each student (pattern included-run on cardstock)

Use appropriate vocabulary as you work through this activity. Require that students do the same when they respond.

As an introduction to vocabulary, ask the students to stand by their desk or table. Connect the words with which they are already familiar (turn, flip, and slide) to the formal vocabulary of transformations: rotate, reflect, and translate. Ask them to *turn* to face the board and

then *rotate* to face the windows. Try asking them to *rotate* 90 degrees or 180 degrees to their left or right. Have the students take a book and gently *flip* it over on their desk. Ask them to *reflect* the book again. Instruct them to *slide* their left or right foot out to the side or *slide* their chair front to back and then *translate* it side to side.

TEACHER NOTE: If you have a magnetic board or an Interactive White Board, your movements will mirror the students. If you use an overhead, and you're facing your students, your movements will be opposite your students' movements. Keep this in mind.

Use the examples to get students started thinking about what translation it takes to get the shape from the starting position (here) to the ending position (there). The shapes with dotted outlines are the intermediate moves. Don't show those to the students until they have explored the moves on their own. Using dotted lines or a colored pencil to draw intermediate steps is an effective way for students to record their work as they experiment with the examples.

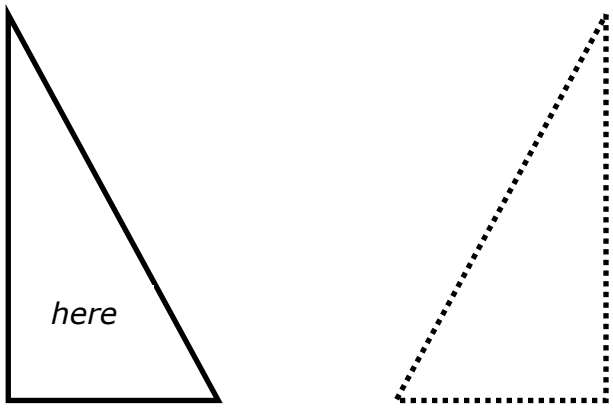
- To clarify the meaning of horizontal and vertical, it may be helpful to provide students with mnemonic strategies to ensure that they develop mathematical fluency when using one or more terms. It is important for teachers to emphasize both "horizontal" and "vertical" lines when helping students predict the results of multiple transformations.

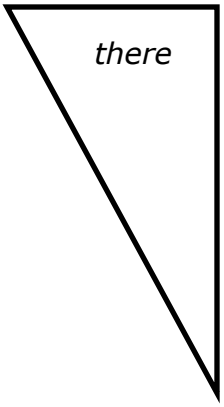
vertical lines (up and down) horizontal lines (side to side)

- The tracing paper approach to explore the idea of rotation, reflection and translation may be helpful to promote meaningful discussion when predicting the results of multiple transformation.

- When discussing and explaining the concepts of translation, reflection, and rotation, students may choose to physically act out the movements in order to understand the concept better. Have students make 90° and 180° turns using body movement to help them conceptualize 90° = 1/4 turn and 180°=1/2 turn.

- Use concrete materials such as tangrams, pattern blocks, or cut out shapes from graph paper to help students develop conceptual knowledge of how to analyze the results of multiple transformations.



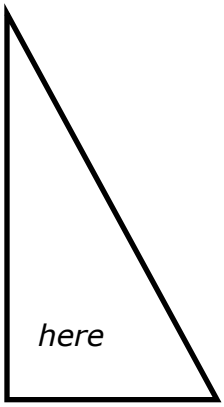


Reflecting Example

How might I get from here to there by *reflecting* two times*?*

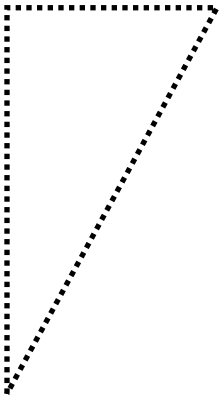
One possibility:

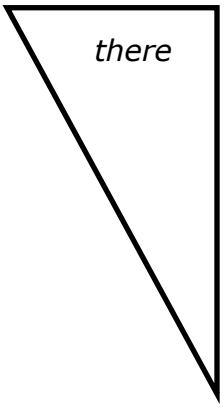
Horizontal reflection then a vertical reflection

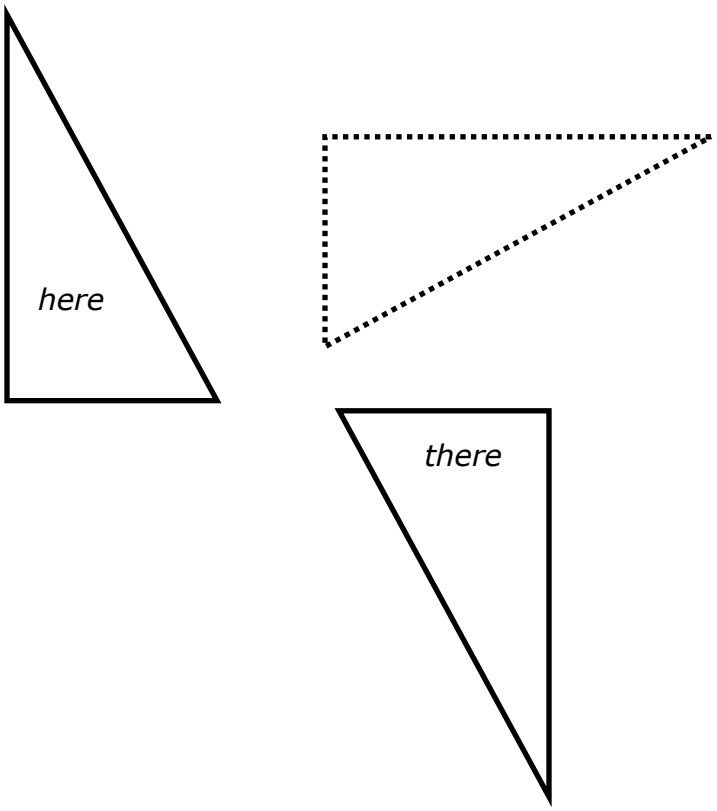


Another possibility:

Vertical reflection then a horizontal reflection







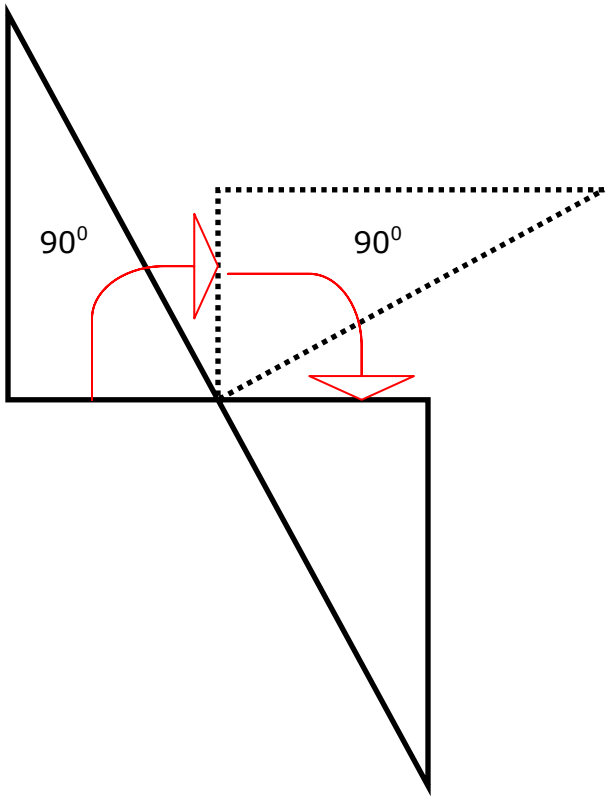
Rotating example-part 1

How might I get from here to there by *rotating* two times?

One possibility

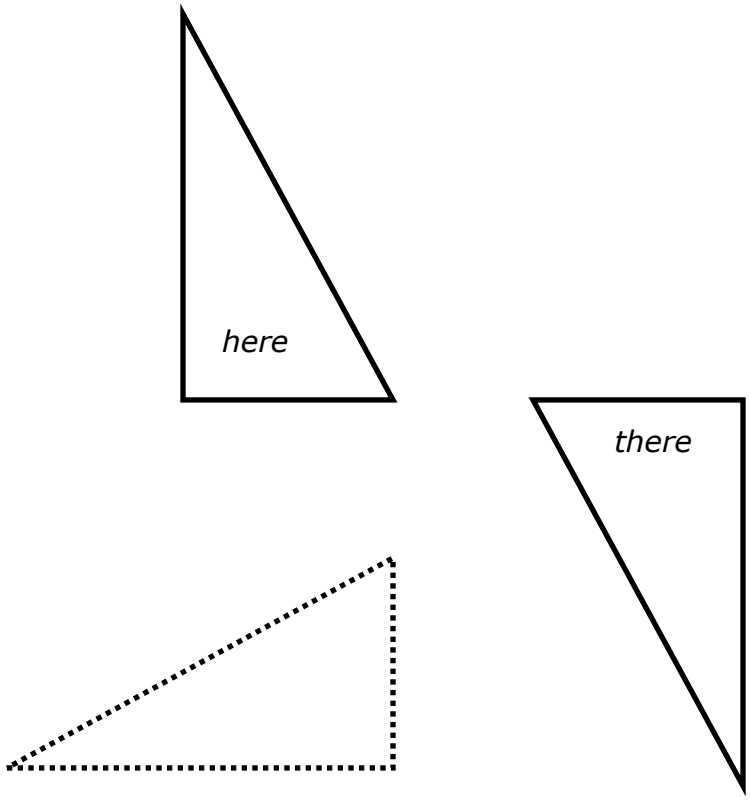
Rotate right (clockwise) 90^0

Repeat – rotate right (clockwise) 90^0



Rotating example-part 2

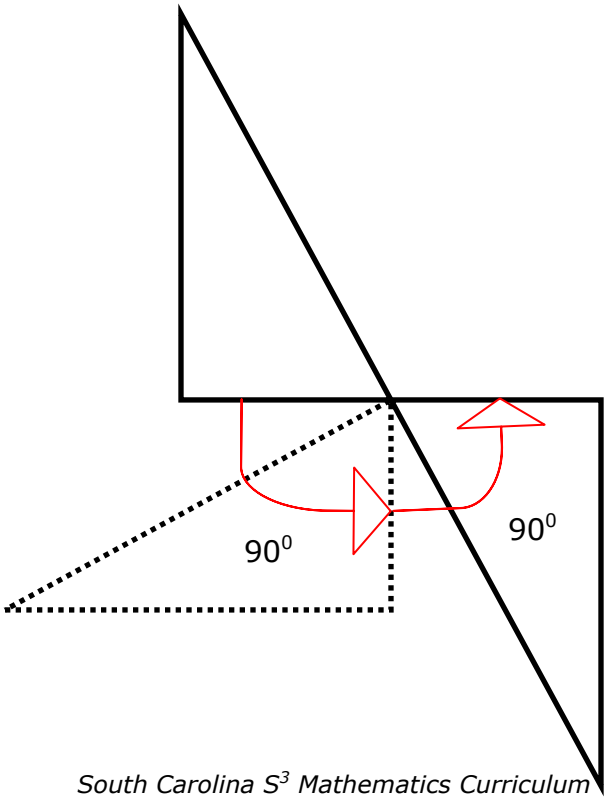
How might I get from here to there by *rotating* two times?

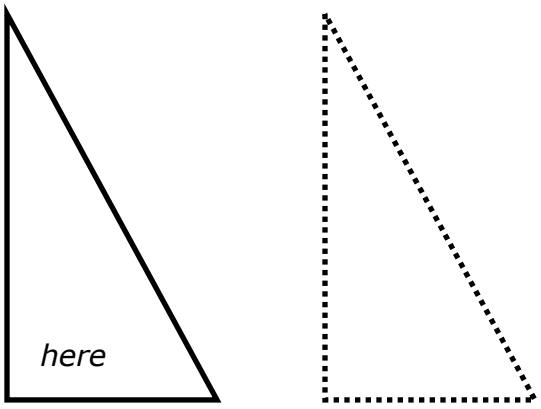


Another possibility

Rotate left (counterclockwise) 90^0

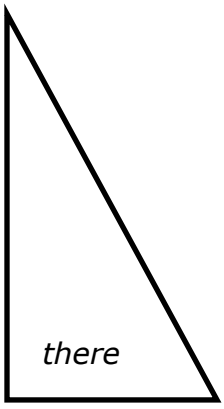
Repeat – rotate left (counterclockwise) 90^0





Translating example

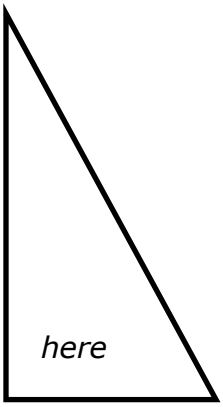
How might I get from here to there by *translating* two times*?*



One possibility

Translate horizontally right

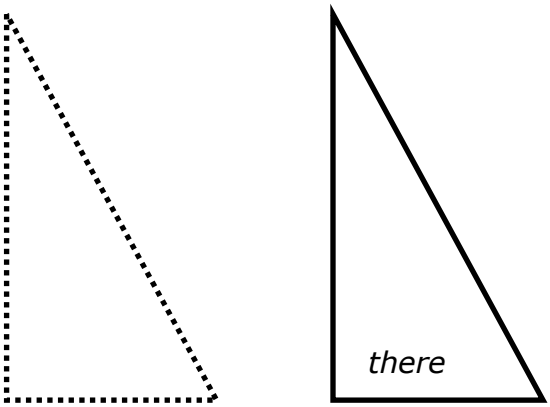
Then translate vertically down

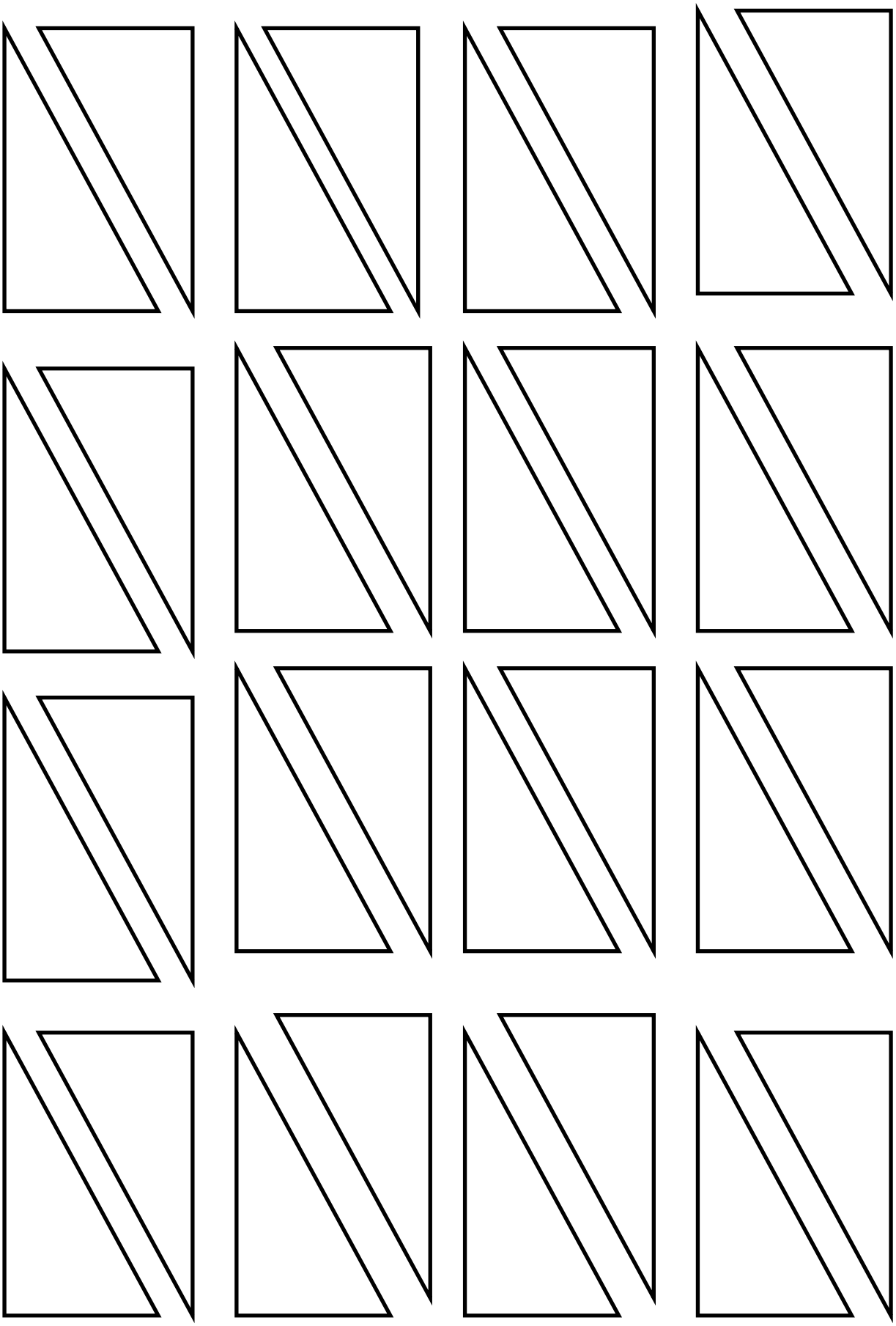


Another possibility

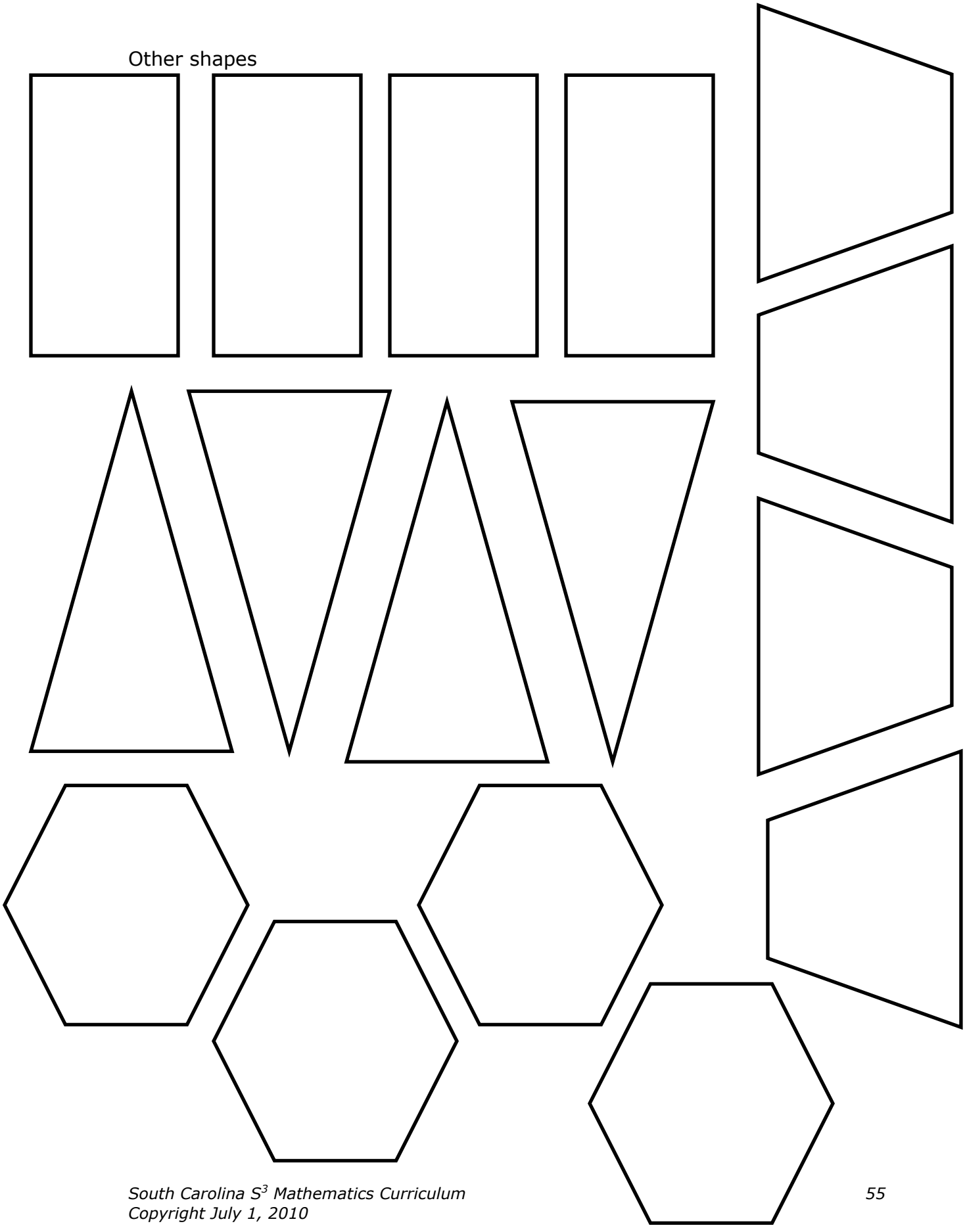
Translate vertically down

Then translate horizontally right





Other shapes



Part B

Lead students in the following discussion… they should link prior knowledge of congruent shapes to the lesson on transformations.

Look back at the lesson and the formative assessment samples. When a figure is transformed by translation, reflection or rotation, do the actual side lengths or angle measures change? *Answer: No*

Why or why not? *Consider the coordinate points.*

If angle measures and side lengths don't change, we can call those figures what/if they remain identical except for orientation? *Answer: congruent.*

Could you prove congruency? *Answer: yes by measuring the sides and angle measures.*

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

Isometric Drawing Tool
http://illuminations.nctm.org/ActivityDetail.aspx?ID=125

Alphabet Geometry-This interactive website helps students develop a conceptual understanding of rotation, reflection, and translation.
http://www.misterteacher.com/abc.html

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build

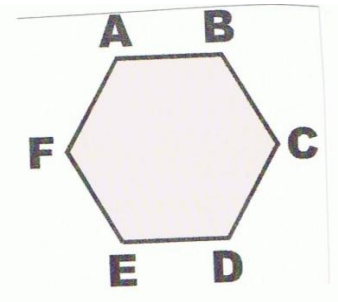
conceptual understanding. Real life situations/representations are critical for conceptual.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Have students respond to the following problem on their white boards/dry erase boards. When prompted, have them hold them up and show their responses. At a glance, the teacher can see who has misconceptions.

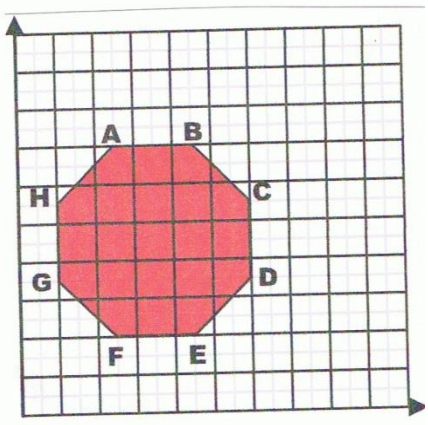
1. Look at the hexagon below. Which description matches what it would look like if it was translated down.



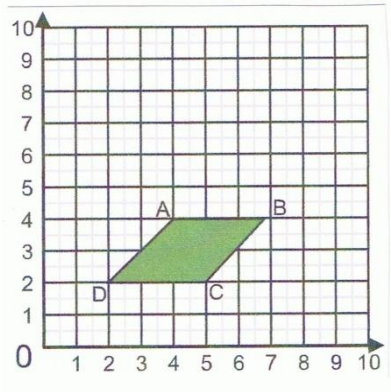
a. Side AB is on the bottom
b. Side AB is still on the top
c. Side AB is on the far right

For 2 and 3, have students turn in their answers on an exit slip. This way, the teacher can look over them before class the next day and know how to do some differentiation for the class based on needs and understanding.

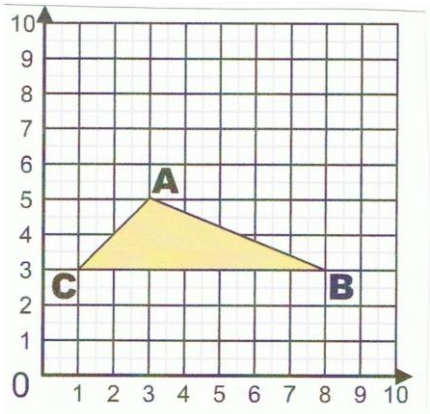
2. Describe the octagon below after it has been translated to the right 4 units. If the scale increments are 1 whole on both axes, what ordered pair would represent Point A? Point E.



3. Look at the shape below and describe its appearance after being rotated in a counter-clockwise direction ¼ turn.



4. Describe the relative positions of the points A, B and C after the triangle has undergone a reflection to the left – which point will be on the top, which point will be on the left and which point will be on the right? Will the two shapes be congruent? Explain your reasoning.



III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

4-4.3
The objective of this indicator is predict, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge of predicting the results of multiple transformations—either slide, flip, or turn—of a geometric shape should be explored using a variety of examples. The learning progression to **predict** requires students to recall the meaning of flip (reflection), slide (translation) and turn (rotation). Student use concrete models to visualize and create transformations of their own. They construct arguments (4-1.2) about what will be the result of a series of transformations. They explain and justify their answers to their classmates and their teacher (4-1.3) using correct, complete, and clearly written and oral mathematical language communicate their ideas. Student analyze this information to solve increasingly more difficult problems (4-1.1) without the use of concrete models.

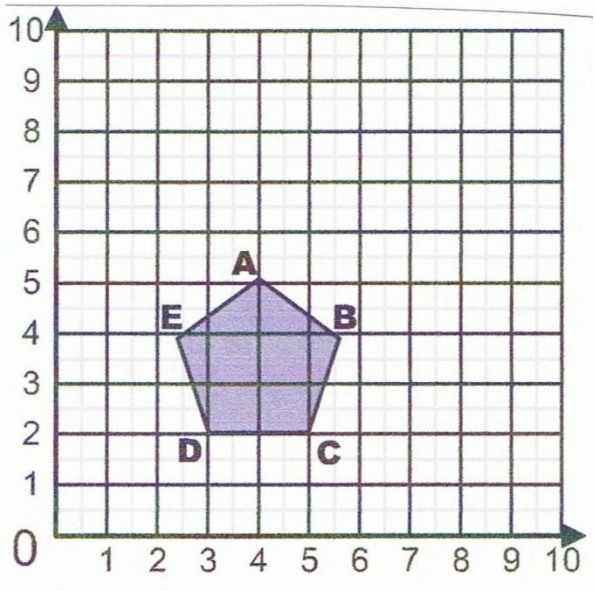
4-4.5
The objective of this indicator is use, which is in the "apply procedural" knowledge cell of the Revised Taxonomy. In this indicator, student use the process of transformation to build a deeper conceptual understanding of congruency. The learning progression to **use** requires students to recall the meaning of reflections, translations, and rotation and perform those transformations fluently. Students explore the effect of transformations on two dimensional shapes using concrete and/or pictorial models. They explain and justify their observations (4-1.3) to their classmates and their teacher on the basis of mathematical properties, structures, and relationships they have learned. Students then use correct, complete, and clearly written and oral mathematical language (4-1.5) to generalize connections between congruency and transformations (4-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

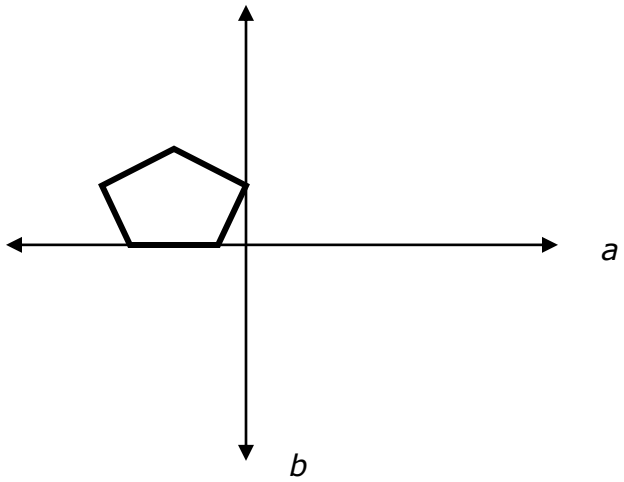
1. Look at the shape below. Where would the new coordinates for vertex D be if the shape were translated up 4 units and to the right 2 units?

Draw the resulting shape in red.

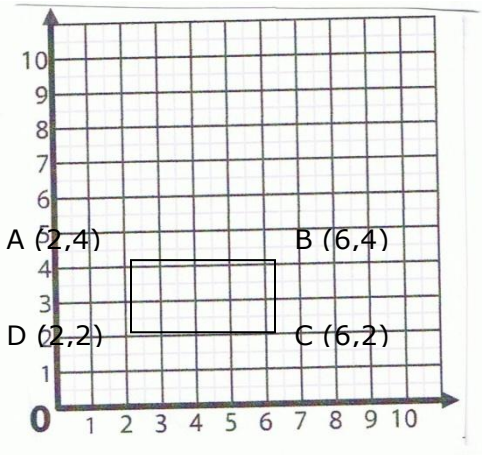
Are the figures congruent? How do you know?



2. Look at the figure below. Describe what it would look like after undergoing a reflection over line *a* then a reflection over line *b*.



3. Jack is asked to draw a rectangle on the coordinate grid. He draws rectangle, ABCD. What would the rectangle look like if it were rotated in a clockwise direction ¼ turn around point B, then rotated another ¼ turn around point B?



MODULE

3–4

Length, Liquid Volume, Mass

This module addresses the following indicators:

4-5.1 Use appropriate tools to measure objects to the nearest unit: measuring length in quarter inches, centimeters, and millimeters; measuring liquid volume in cups, quarts, and liters; and measuring weight and mass in pounds, milligrams, and kilograms. (C3)
4-5.2 Compare angle measures with referent angles of 45 degrees, 90 degrees, and 180 degrees to estimate angle measures. (B2)
4-5.9 Exemplify situations in which highly accurate measurements are required. (B2)

* This module contains 2 lessons. These lesson are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Module

- *Continuum of Knowledge*

  4-5.1

  In second grade, students use appropriate tools to measure objects to the nearest whole unit: measuring length in centimeters, feet, and yards; measuring liquid volume in cups, quarts, and gallons; measuring weight in ounces and pounds; and measuring temperature on Celsius and Fahrenheit thermometers (2-5.3). They also generate common measurement referents for feet, yards, and centimeters (2-5.4) and use common measurement referent to make estimates in feet, yards and centimeters (2-5.5).In third grade, students use appropriate tools to measure objects to the nearest unit: measuring length in meters and half inches, measuring liquid volume in fluid ounces, pints and liters; and measuring mass in grams (3-5.2).

  In fourth grade, students use appropriate tools to measure objects to the nearest unit; measuring length in quarter inches, centimeters and millimeters; measuring liquid volume in cups, quarts and liters; and measuring weight and mass in pounds, milligrams and kilograms (4-5.1). They also use equivalencies to convert units to measure within the US Customary System (4-5.3).

  In fifth grade, students use appropriate tools and units to measure objects to the precision of one-eighth inch (5-5.1).

  4-5.2

  In third grade, students classified angles as either right, acute or obtuse (3-4.4). They also classified triangles by the length of their sides and by the size of their angles (3-4.5).

  In fourth grade, students compare angle measures with referent angles of 45 degrees, 90 degrees and 180 degrees to estimate angle measures (4-5.2).

  In fifth grade, students compare the angles, side lengths and perimeters of congruent shapes (5-4.2) and use a protractor to measure angles from 0 to 180 degrees (5-5.2).

- **Key Concepts/Key Terms**

  *These are vocabulary terms that are responsible for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

*inch	*quarter-inch	*half-inch	*referent
*centimeter	*millimeter	*volume	*degree
*cup	*quart	*liter	*45 degrees

*weight	*pounds	*milligrams	*90 degrees
*kilograms	*accuracy	*angle referents	*180 degrees
*estimate	*nearest (closest)	*accurate	

II. Teaching the Lesson(s)

1. Teaching Lesson A:

The purpose of angle referents is so that students will have a basis for estimating the measure of other angles. While on the surface this seems to be an unimportant concept, quite the contrary is true. Without achieving a comfort level with the specified benchmarks students will have no basis on which to judge the reasonableness of their answers when they actually measure angles in fifth grade. One way to informally introduce the measuring process and to compare benchmark angles is to have children make folds in wax paper to compare angles. Children will realize that a new sort of unit, the degree, is needed to measure angles.

The term "exemplify" in Indicator 4-5.9 means that students should find examples of situation where highly accurate measurements are required. By discussing the examples, students will begin to have a clearer understanding of the need for precision in measurement.

4-5.1
For this indicator, it is **essential** for students to:

- Understand which unit of measure is most appropriate for length, volume and mass
- Locate the nearest unit
- Use other words synonymous with nearest such as "closest to"
- Understand that their measurement is an approximation in some cases
- Understand quarter inches
- Use appropriate abbreviations for measurements (meters is m, pounds is lb, etc..)
- Measure using actual tools
- Read a measurement from a pictorial representation

For this indicator, it is **not essential** for students to:
None noted

4-5.2
For this indicator, it is **essential** for students to:

- Recognize angles that measure 45, 90 and 180
- Understand the relationship between these angles.
- Understand the meaning of referent (benchmark)
- Judge the reasonable of measurement based on these angle measurements

- State whether or not the given angle is between, greater than or less than the benchmark angles

For this indicator, it is **not essential** for students to:

- Measure these angles using protractor
- State the exact angle measure

4-5.9
In fourth grade, students exemplify situations in which highly accurate measurements are required (4-5.9).

In eighth grade, students analyze a variety of situations to determine the necessary level of accuracy and precision (8-5.6).

a. Indicators with Taxonomy

4-5.1→ Use appropriate tools to measure objects to the nearest unit: measuring length in quarter inches, centimeters, and millimeters; measuring liquid volume in cups, quarts, and liters; and measuring weight and mass in pounds, milligrams, and kilograms. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural

4-5.2 → Compare angle measures with referent angles of 45 degrees, 90 degrees, and 180 degrees to estimate angle measures. (B2)

Cognitive Dimension: Understand
Knowledge Dimension: Conceptual

4-5.9 → Exemplify situations in which highly accurate measurements are required. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

b. Introductory Lesson A: Part 1

Materials Needed

- Super Inch handouts
- Super Centimeter handouts
- transparencies of the handouts so that you may demonstrate for the students
- standard and cm measuring tapes real or paper (template for paper tapes included ~ **You will need to put these together for them before teaching this lesson.**)
- colored pencils (2 colors for each student)
- common objects for groups to measure

Students should be paired so that they may talk about the math they are doing.

Give each student a copy of the Super Inch handout. Tell them that you're going to focus in on the parts of an inch. Be sure they understand that the inch starts at the first mark on the ruler, not the end of the ruler. This is a common mistake that students make.

The big picture, Super Inch, is the actual inch enlarged by a factor of four. Students don't necessarily need to know this, they just need to have a clear understanding that we don't use Super Inch to measure. We're just using it to take a closer look at the parts of the real inch. Ask students to use their pencils to draw dashed lines from the beginning mark to the bottom edge of the ruler. Ask them to do the same thing for the 1-inch mark. Demonstrate this on your Super Inch transparency.

Ask students to find the half –inch mark on Super Inch. If they seem unsure, ask them what it means for something to be half of something else. They should be able to tell you that dividing something into two equal parts makes halves. Students should use a colored pencil to mark the half-inch on Super Inch and label it ½. Demonstrate this on your Super Inch transparency.

Ask students to consider the first half of Super Inch. Ask them to find the half-way point between the beginning of Super Inch and the ½ inch mark they've already identified. Move around the room observing and listening as students decide where that half-way point is. Once they decide, have them draw a dotted line from the mark to the bottom edge of the ruler with a different colored pencil than the one they used for the ½ inch mark. Demonstrate this on your Super Inch transparency.

Ask students what that mark is called. Some students may quickly recall that "half of a half is a fourth." Others may not see that as quickly. Don't try to convince students who are a little slower in their understanding that is ¼, and don't label the quarter-inch yet.

Ask students to consider the second half of Super Inch. Ask them to find the half-way point between the ½ inch mark and the 1-inch mark. Move around the room observing and listening as students decide where that half-way point is. Once they decide, have them draw a dotted line from the mark to the bottom edge of the ruler with the same colored pencil they used to mark the other quarter inch mark. Demonstrate this on your Super Inch transparency.

Ask students how many sections Super Inch has been divided into. Now students who weren't convinced about the ¼ a little while ago can see that Super Inch has been divided into 4 equal parts. That makes each section ¼ of the whole. Have students label the quarter-inch marks. Bring to their attention that ½ is also 2/4. Demonstrate this on your Super Inch transparency.

Ask students to label their super inch along the clean space at the top of the ruler. This will make it easier to see the fractions.

Give students their paper tape measures or real ones. Paper tape measures can be found at: www.scribd.com/doc/7090539/Tape-Measure. This will give you an opportunity to download what is needed. Point out that the tape measure can be used for measuring inches and cm. They need to know that they should choose the side they want to use. In this case, it's the side that measures inches. Ask students to find the ¼, ½ and ¾ marks in the first inch on their tape measure. They may want to use the colored pencils they used to mark super inch to mark that first inch. They also need a piece of plain paper to write on.

Give students the following task (this should be given both visually and verbally):

1. Measure the short edge of your math book cover to the nearest quarter-inch.
2. Measure the long edge of your math book cover to the nearest quarter-inch.
3. Measure the long edge of your desk to the nearest quarter-inch.
4. Measure the short edge of your desk to the nearest quarter-inch.
5. Measure the top of the back of your chair to the nearest quarter-inch.

Students should record these measurements on their papers. You should have measured each of these items ahead of time and have a copy of the measurements for yourself as you talk to students. You should also model the correct notation for students. For example, "The edge of my computer keyboard is about 15 ¼ inches wide."

Form quads from the pairs of students that have been working together. Have them compare their measurements. While they are working, move around the room observing and listening as students work to reconcile their findings with one another. Be especially aware of students who are really off base with their

measurements. After a suitable period of time (use your professional judgment), share your findings with the students and lead a class discussion.

c. Misconceptions/Common Errors

Student may have difficulty understanding the concept of nearest or closest.

When using rulers to measure length, students need to be careful to notice where the first tick mark is located on different rulers.

d. Additional Instructional Strategies/Differentiation

Super Centimeter

This is essentially an extension of the Super Inch lesson. Follow the same steps as for Super Inch, concentrating on metric rather than standard. When it comes to notation, connect the metric measurement to decimal place value. For example, each mm is 1/10 (0.1) of each cm. So, 10 cm and 6 mm would be written as 10.6 cm.

Further measurement opportunities for both standard (to the nearest quarter-inch) and metric (to the nearest mm).

Sets of stuff

Keep students in their quads. Have sets of common objects for students to measure. The groups should create their own data collection sheets by using plain paper to record their measurements. Of course, you should measure each of the items so that you can check student work. Suggestions for sets of objects are listed below.

• 2 liter drink bottle • old (clean) shoe • book (students will need to specify what part they measured) • width of door	• aluminum drink can • width of classroom window • a different book • height of desk	• family-sized soup can • width of book case • note book • width of the hall	• Kool Aid can • poster on wall • height of book case (if it's short enough) • distance between two desks

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.
Have the students review their work with their groups and submit their papers to you. Review them for accuracy and information on what students are having difficulty with. No grades for formative purposes.

Introductory Lesson A: Part 2: Liquid Volume

a. Indicators with Taxonomy

4-5.1→ Use appropriate tools to measure objects to the nearest unit: measuring length in quarter inches, centimeters, and millimeters; measuring liquid volume in cups, quarts, and liters; and measuring weight and mass in pounds, milligrams, and kilograms. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural

Note: Rice or small dried beans are used as the materials to measure so that students are not pouring and repouring water throughout the course of the activity.

b. Introductory Lesson

Materials Needed:

- 1-cup measuring cups
- 1-quart containers
- 1-liter containers
- bins to hold the rice (number each bin)
- same number of empty bins (to pour rice into as it's measured)

- rice or some kind of small dried beans (to measure)
    - ***There should be a different amount of rice in each bin.***
- **How Many?** handouts
- vacuum cleaner (☺)

Students should be grouped in quads so that they may work together and talk about the math they are doing. If you have a number of students that isn't divisible by four, trios will work. Five students in a group isn't the best way to go, because someone will check out if he doesn't have a partner.

Set up the bins at stations around the room. Each station needs a measuring cup, a quart container, and a liter container; a bin of rice; and an empty bin to pour rice into as they measure. If your measuring tools aren't identical, that's okay. As a matter of fact, that's a good thing. Students should realize that not every container that measures the same amount looks exactly the same.

Show students the materials they will be using. Let them know that measuring liquid volume is a little different than measuring the rice, but that they're close enough to help them understand how to measure to the nearest cup, quart, or liter. Ask the students to tell you which one of those measures is "different" from the other two. They should be able to tell you from past experience that the liter is different because it is a metric measurement.

The object of this lesson is for students to measure their rice to the nearest cup, to the nearest quart, and to the nearest liter. Student groups will move from station to station during the course of the class period. Each student needs a record sheet and should fill out his own sheet as the group works.

Keep the groups moving through the stations. Give them a 3-minute warning and a 1-minute warning. Use your professional judgment to decide how long the total time spent at each station should be.

At each station:

Before they begin measuring, they need to note the bin/station number and write it on their record sheets.

Have them start with the 1-cup measure and keep track of the number of cups as they scoop from one bin and pour into the other.

Move onto the 1-quart measure. Rather than returning the rice to the original bin (as some kids may be inclined to do), just have them measure back into the first bin, keeping track again of the number of times they scoop and pour.

Finally, have them use the 1-liter measure.

While students are working, move around the room observing and listening to the students.

Options for class discussion of their findings:

- Ask one group of students to report out for Bin#1. Let other groups check to see if their findings were similar. Discuss any differences and possible reasons for them. Repeat with other Bin#s, letting different groups be the Reporters.
- Use chart paper to post your measurements at each station. Allow students to move through the stations, checking their work against yours. Again, discuss any differences and possible reasons for them.

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Formative assessment is embedded with the lesson through questioning and observation; however, other formative assessment strategies should be employed. See Appendix A

Have the students review their work with their groups and submit their papers to you. Review them for accuracy and information on what students are having difficulty with.

Introductory Lesson A: Part 3: Weight and Mass

a. Indicators with Taxonomy

4-5.1→ Use appropriate tools to measure objects to the nearest unit: measuring length in quarter inches, centimeters, and millimeters; measuring liquid volume in cups, quarts, and liters; and measuring weight and mass in pounds, milligrams, and kilograms. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural

b. Introductory Lesson

Materials Needed:

- bathroom scale that measures both pounds and kilograms
- a record sheet (blank one included ~ "How much does it weigh?" - you'll have to fill it in)
- various objects to weigh
    - Suggestions: bookbags, kids, bowling ball, the principal, the asst principal, the teacher down the hall; anything that weighs at least a few pounds ~ use your imagination!

The idea here is for students to practice weighing objects in pounds and kilograms. Even though the indicator also says a milligram, which is not a practical measure of weight to use in a classroom. A couple of days before you get ready to teach this lesson, you could ask students to bring in some items they'd like to weigh.

This activity would be best accomplished in stations, very much like the rice bins activity. That would mean having a collection of objects for students to weigh at each station with a scale that measures both pounds and kilograms. Small groups of students (trios or quads) would move from station to station, filling in their record sheets with the names of the objects and their weights to the nearest pound and kilogram. You would only need 4 objects or so at each station, and students would need only a short amount of time to weigh those objects.

While students are working, move around the room observing and listening to the students.

Options for class discussion of their findings:

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

- Ask one group of students to report out for Station #1. Let other groups check to see if their findings were similar. Discuss any differences and possible reasons for them. Repeat with other Stations, letting different groups be the Reporters.
- Use chart paper to post your measurements at each station. Allow students to move through the stations, checking their work against yours. Again, discuss any differences and possible reasons for them.

Introductory Lesson A: Part 4: Precision in Measurement

a. Indicators with Taxonomy

4-5.1→ Use appropriate tools to measure objects to the nearest unit: measuring length in quarter inches, centimeters, and millimeters; measuring liquid volume in cups, quarts, and liters; and measuring weight and mass in pounds, milligrams, and kilograms. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural

4-5.9 → Exemplify situations in which highly accurate measurements are required. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

b. Introductory Lesson

Materials Needed:
- Handouts from the previous Introductory Lessons

After students understand how to measure to the units specified in indicator 4-5.1, lead the students in a class discussion about situations that require a great deal of accuracy when measuring.

For example:
When is it okay to measure to the nearest inch?
When should you measure to the nearest quarter-inch?
When should you measure to the nearest kilogram?
When should you measure to the nearest milligram?

c. Misconceptions/Common Errors

When using a ruler or tape measure, students often think the end of the tool is the beginning of the measurement. They need to learn to examine the ruler or tape measure and look for the actual beginning mark. Some rulers and tapes DO start at the very beginning; others have a "dead spot" from the end of the tool to the first mark.

d. Additional Instructional Strategies/Differentiation

Navigating Through Measurement in Grades 3 through 5, "How Precise Should My Measurement Be?" Page 43

Although the indicator does not reference precision, students may think that their definition of accuracy is correct but it is really the definition of precision. Also students should examine non examples – situations where accuracy is not required. Below is a summary of these two concepts: Students must distinguish between "accuracy," which means the closeness of a measurement to the exact value, and "precision," which means the claimed or implied closeness. For example, if I said my desk was 2 meters wide, and you said it was 2.345 meters wide, your answer would be more precise (claiming that you know it down to the millimeter); but if the desk is really 2.123 meters wide, then my answer is more accurate!

Accuracy is the difference between our own calculation and the accepted real value. For example, suppose your math textbook tells you that the value of Pi is 3.14. You do a careful measurement by drawing a circle and measuring the circumference and diameter, and then you divide the circumference by the diameter to get a value for Pi of 3.16. The accuracy of your answer is how much it differs from the accepted value. In this case, the accuracy is 3.16 - 3.14 = 0.02. The precision with which the accepted value has been measured is not important. All that matters is *how different* your measurement is from that value; the bigger the difference, the less accurate your measurement. Also, accuracy depends on the instrument you are measuring with. But as a general rule: The degree of accuracy is half a unit each side of the unit of measure.

The precision of a measurement describes the units you used to measure something. For example, you might describe your height as *'about 6 feet'*. That wouldn't be very precise. If however you said that you were *'74 inches tall'*, that would be more precise. The smaller the unit you use to measure with, the more precise the measurement is.

e. Technology

Virtual manipulation should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

There are no specific recommendations for technology for this lesson at this time.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Simply have them review their work with their groups and submit their papers to you. Review them for accuracy and information on what students are having difficulty with. No grades for formative purposes.

2. Teaching Lesson B: Measuring Angles

a. Indicators with Taxonomy

4-5.2 → Compare angle measures with referent angles of 45 degrees, 90 degrees, and 180 degrees to estimate angle measures. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

b. Introductory Lesson: Angles

Materials Needed

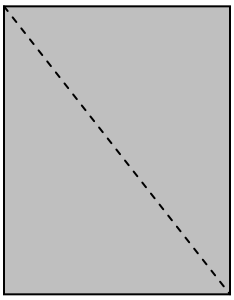
- Two squares of 4 in by 4 in paper for each student (The squares should be two different colors; it doesn't matter what they are, but for the rest of this lesson, I'm going to say gray and white.)
- straight edge for tracing straight lines
- paper and pencil for each student
- clear tape
- angles handout

Use appropriate angle vocabulary, such as the angle names and the angle types. Also, be sure students understand a straight angle measures 180 degrees – they may not be familiar with that term.

Students should be paired so that they may work together and talk about the math they are doing.

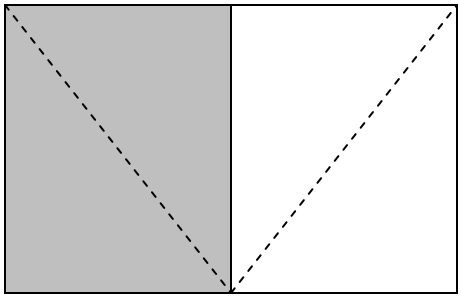
Begin by giving each student the two different colored squares. Ask them to tell you what the measure of each corner is. They should know from past experience that the corners are right angles and measure 90°.

Have the students take the gray square and fold it corner to corner and crease it well. When they open it, have them take the straight edge and trace the crease, using a dotted line.



Have students do the same thing to the white square. Ask students to place the two squares side-by-side as shown below. Depending on how they traced the line on the white square, they may have to trace it again. The creases/dotted lines need to meet in the two corners that are together at the bottom.

Students need to tape the squares on their sheet of paper, as shown below.



What students have now are the referents for 45°, 90°, 135°, and 180°. Rather than just telling the students what the angle measures are, challenge them to use what they know about 90° angles to find the other measures. Move around the room, observing and listening to students as they work. When they seem ready, give them the angles handout and let them estimate some angle measures.

c. Misconceptions/Common Errors

Students mistakenly relate the length of the sides of the angles to the measure of the angle i.e. the longer the length of the ray then the greater the measure of the angle.

d. Additional Instructional Strategies/Differentiation

One way to informally introduce the measuring process and to compare benchmarks angles is to have students make folds in wax paper to compare angles.

Students begin with a cease representing a straight angle; they label it. They then fold the paper in half to create a 90 degree angle and label. Then they fold the 90 degree angle along a diagonal (in half) to create a 45 degree angle and label. Having students shade in the regions that cover each angle will also build a conceptual understanding of spread of each benchmark. While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reaches, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build

conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Simply have them review their work with their groups and submit their papers to you. Review them for accuracy and information on what students are having difficulty with. No grades for formative purposes.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

4-5.1
The objective of the indicator is to use which is in the "apply procedural" knowledge cell of the Revised Taxonomy. To apply procedural knowledge is to know how to do something and the criteria for determining when to use those procedures. The learning progression to **use** requires students to understand the concepts of length, volume and mass. They use their understanding of these concepts to select appropriate measuring tools and units of measure. They explore a variety of real world situations to generalize connections between new mathematical ideas and other related measurements they learned in previous grades (4-1.6). To deepen conceptual understanding, they generate descriptions and mathematical statement about the relationship between measurements (4-1.4). Students estimate the measure using appropriate units. As students measure, they explain and justify their answers (4-1.3) using correct, complete and clearly written and oral mathematical language (4-1.5)
4-5.2
The objective of this indicator is to compare which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning; therefore, students gain an understanding of degrees by comparing angles to benchmarks. The learning progression to **compare** requires students to recall the meaning of degrees and understand how to classify angles as acute, right or obtuse. Students use hands-on experiences to create their benchmark angles. They

generalize connections (4-1.6) between these benchmarks and other angle measures. As they analyze these relationships, students explain and justify their answers (4-1.3) using correct, complete and clearly written and oral mathematical language (4-1.5).

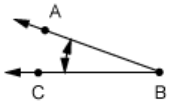
4-5.9

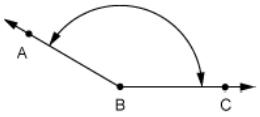
The objective of this indicator is to exemplify which is in "understand conceptual" knowledge cell of the Revised Taxonomy table. To exemplify is to find a specific examples. The learning progression to **exemplify** requires students to recall basic units of measurements. Students explore examples and non-examples of situations that require accuracy. Students analyze these situations (4-1.1) and make observations. They use these observations to generate descriptions and mathematical statements summarizing the concept of accuracy (4-1.4). They also generate examples and explain and justify their answers (4-1.3) using correct, complete and clearly written and oral mathematical language (4-1.5).

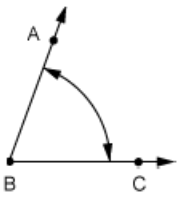
The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. For summative assessment of this lesson.. it should be more performance task based. For example, set up stations and using a checklist, determine how well students measure.. you should have a station or two for each of the unit measures.

2. For angles… have students describe the angles below in reference to 45 degrees, 90 degrees and 180 degrees.

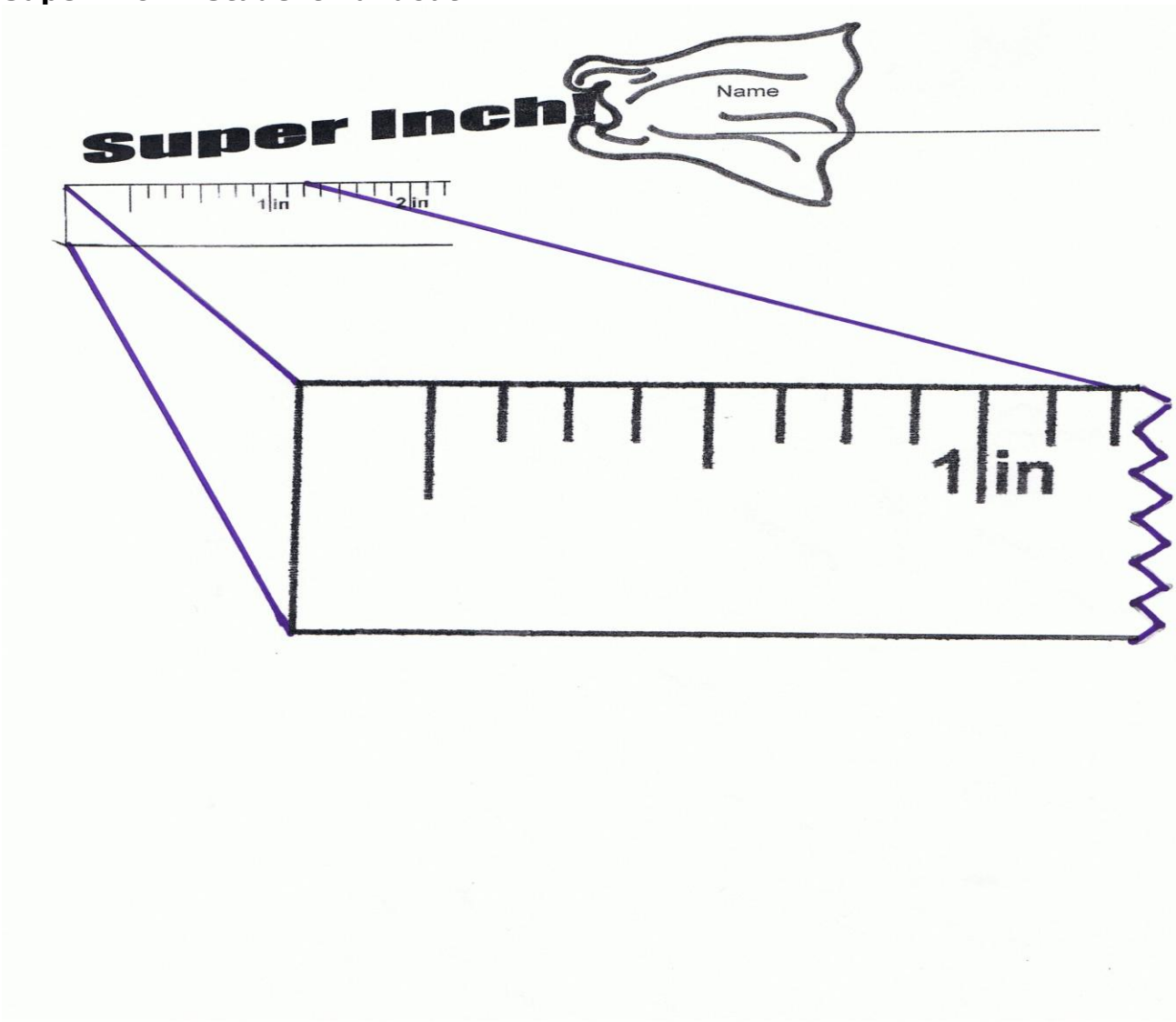




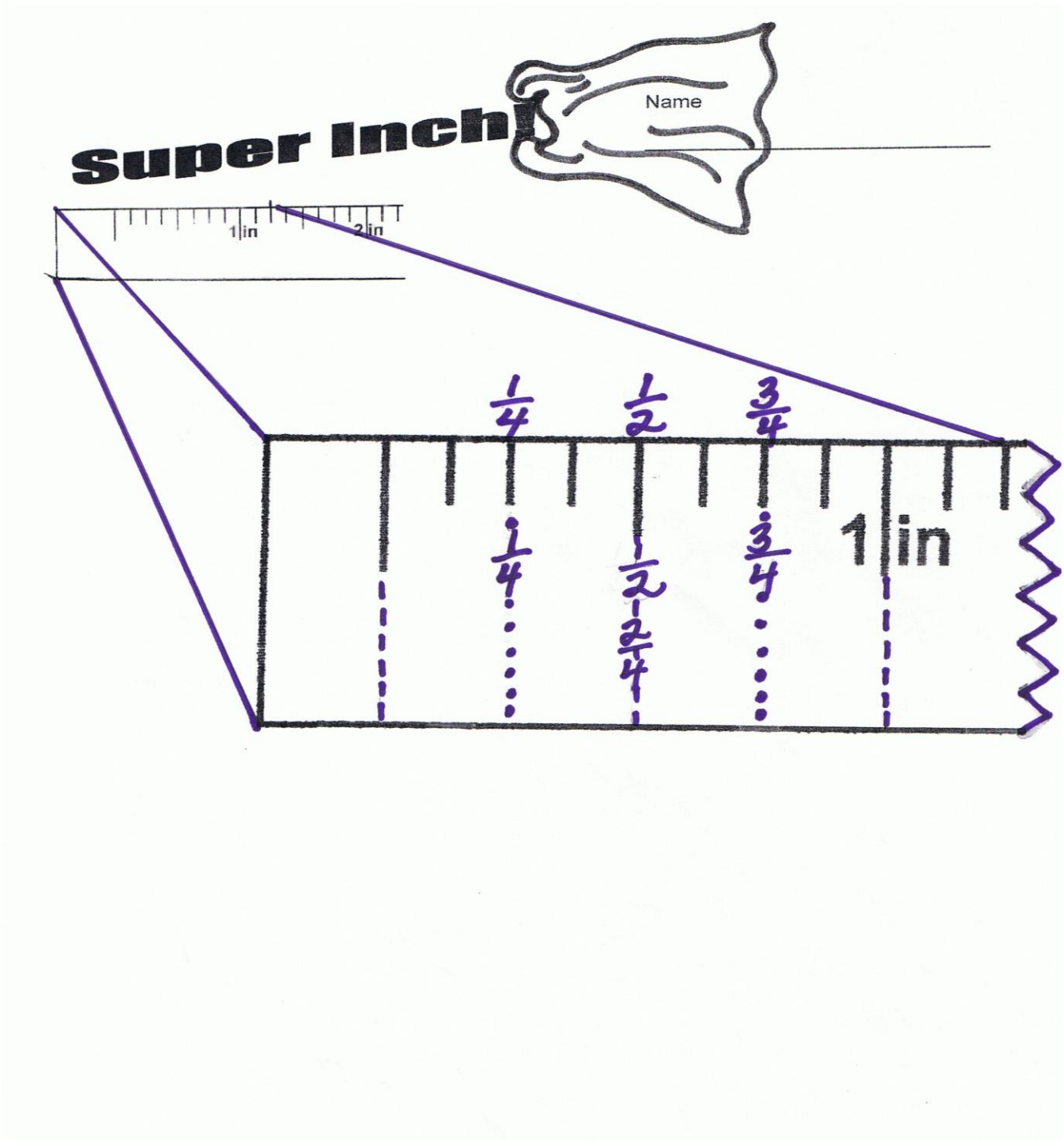


3. Which tool would you use to measure the:
   a. length of a school desk? (inches, centimeters or millimeters)
   b. volume of a bathtub of water? (cups, quarts or gallons)
   c. weight of a quarter? (pounds, milligrams or kilograms)

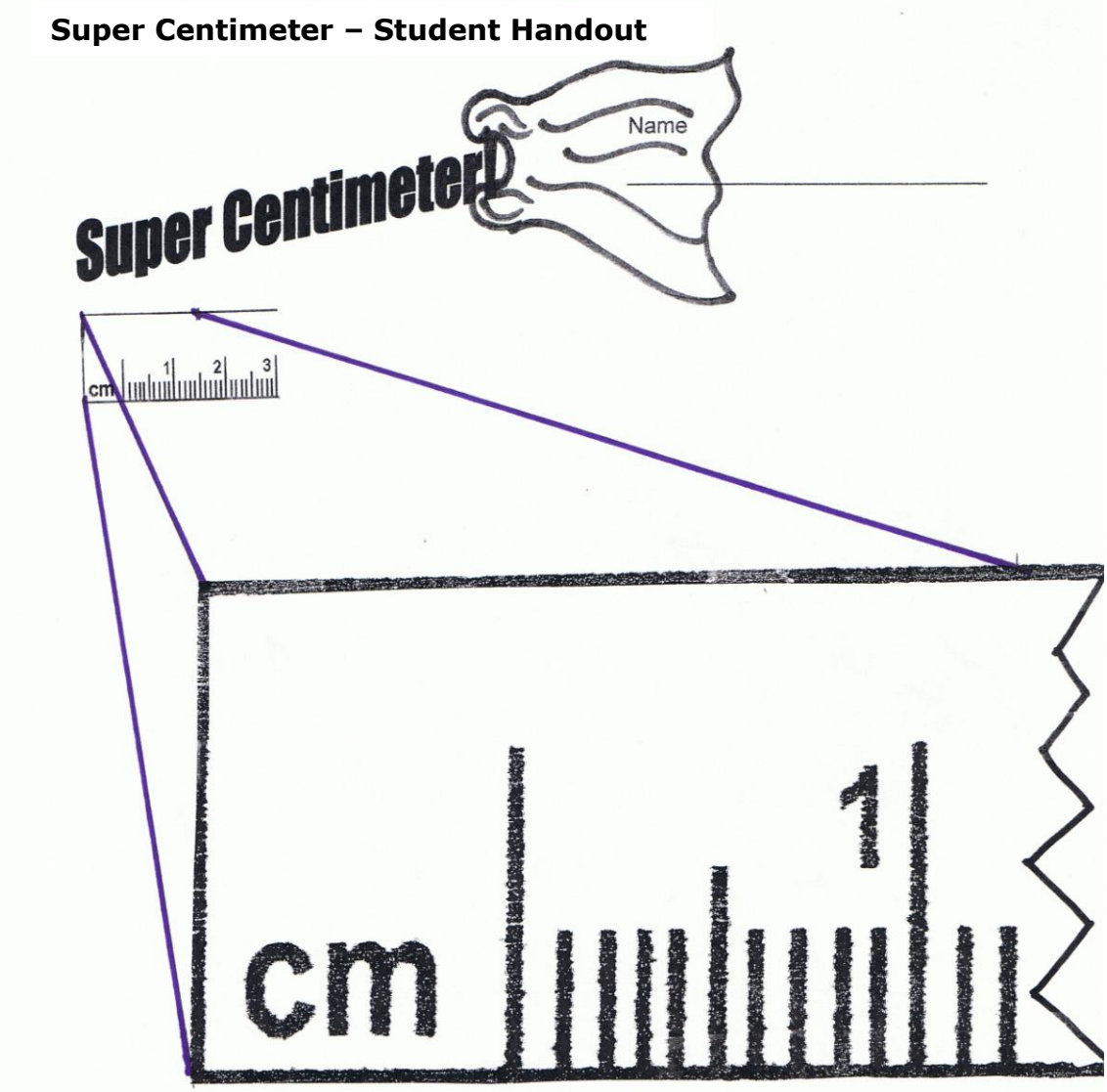
Super Inch – Student Handout



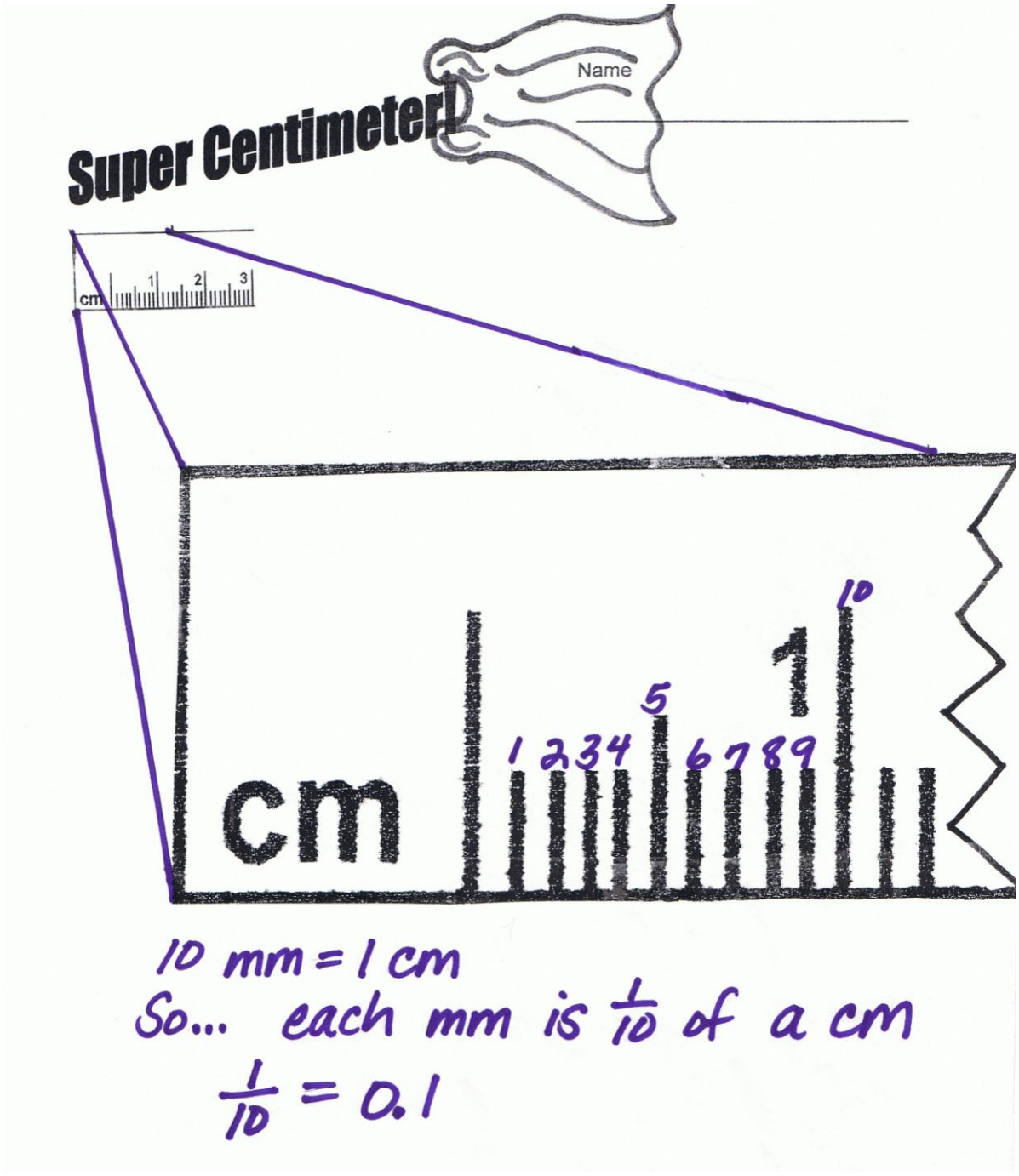
Super Inch –
Teacher Guide/Solution Sheet



Super Centimeter – Student Handout



Super Centimeter – Teacher Guide/Solution Sheet



How Many?

Bin #		
Measure	**# of**	**How much was left over?**
cups		
quarts		
liters		

Bin #		
Measure	**# of**	**How much was left over?**
cups		
quarts		
liters		

Bin #		
Measure	**# of**	**How much was left over?**
cups		
quarts		
liters		

Bin #		
Measure	**# of**	**How much was left over?**
cups		
quarts		
liters		

Bin #		
Measure	**# of**	**How much was left over?**
cups		
quarts		
liters		

Bin #		
Measure	**# of**	**How much was left over?**
cups		
quarts		
liters		

Bin #		
Measure	**# of**	**How much was left over?**
cups		
quarts		
liters		

How Much Does It Weigh?

Name_____

Group members_____

_____

Work with your group to measure the objects at each station.
Record the weight of each object to the nearest pound and kilogram.

Station # _____

object	pounds (lbs)	kilograms (kg)

Station # _____

object	pounds (lbs)	kilograms (kg)

Station # _____

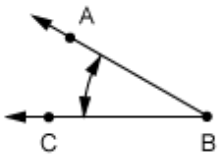
object	pounds (lbs)	kilograms (kg)

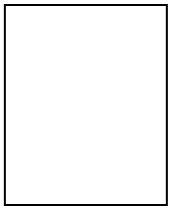
Station # _____

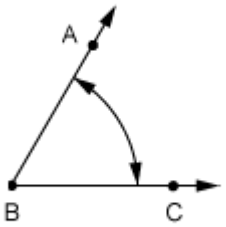
object	pounds (lbs)	kilograms (kg)

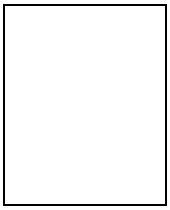
Angle Estimates

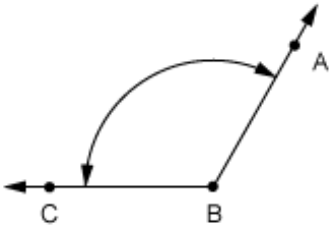
Use the work you did with your squares to estimate these angle measures.

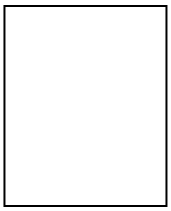


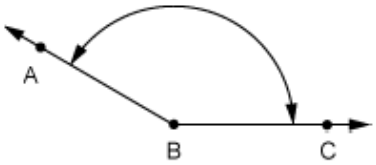


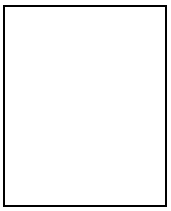












MODULE

3-5

Equivalencies and Conversions

This module addresses the following indicators:

4-5.8 Recall equivalencies associated with liquid volume, time, weight, and length: 8 liquid ounces = 1 cup, 2 cups = 1 pint, 2 pints = 1 quart, 4 quarts = 1 gallon; 365 days = 1 year, 52 weeks = 1 year; 16 ounces = 1 pound, 2,000 pounds = 1 ton; and 5, 280 feet = 1 mile. (A1)

4-5.3 Use equivalencies to convert units of measure within the U.S. Customary System: converting length in inches, feet, yards, and miles: converting weight in ounces, pounds, and tons: converting liquid volume in cups, pints, quarts, and gallons: and converting time in years, months, weeks, days, hours, minutes, and seconds. (C3)

* This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge:**

4-5.3

In second grade, students use appropriate tools to measure objects to the nearest whole unit: measuring length in centimeters, feet, and yards; measuring liquid volume in cups, quarts, and gallons; measuring weight in ounces and pounds; and measuring temperature on Celsius and Fahrenheit thermometers (2-5.3).

In fourth grade, students use equivalencies to convert units of measure within the US Customary System: converting length in inches, feet, yards and mile; converting weight I ounces, pounds, and tons; converting liquid volume in cups, pints, quarts and gallons; and converting time in years, months, weeks, days, hours, minutes and seconds (4-5.3).

In fifth grade, students use equivalencies to convert units of measure within the metric system: converting length in millimeters, centimeters, meters an kilometers; converting liquid volume in milliliters, centiliters, liters and kiloliters; and converting mass in milligrams, centigrams, grams and kilograms (5-5.3).

4-5.8

In second grade, students recalled equivalencies associated with length and time (2-5.9). In third grade, students recall equivalencies associated with time and length: 60 seconds = 1 minute and 36 inches = 1 yard (3-5.7).

In fourth grade, students recall equivalencies associated with liquid volume, time, weight, and length: 8 liquid ounces = 1 cup, 2 cups = 1 pint, 2 pints = 1 quart, 4 quarts = 1 gallon; 365 days = 1 year, 52 weeks = 1 year; 16 ounces= 1 pound, 2,000 pounds = 1 ton; and 5,280 feet = 1 mile (4-5.8). They also use equivalencies to convert units of measure within the US Customary System: converting length in inches, feet, yards and mile; converting weight I ounces, pounds, and tons; converting liquid volume in cups, pints, quarts and gallons; and converting time in years, months, weeks, days, hours, minutes and seconds (4-5.3).

In fifth grade, students recall equivalencies associated with length, liquid volume, and mass: 10 millimeters = 1 centimeter, 100 centimeters = 1 meter, 1,000 meters = 1 kilometer; 10 milliliters = 1 centiliter, 100 centiliters = 1 liter, 1,000 liters = 1 kiloliter; and

10 milligrams = 1 centigram, 100 centigrams = 1 gram, 1,000 grams = 1 kilogram (5-5.8).

- **Key Concepts/Key Terms**

Ounce
Cup
Pint
Quart
Gallon
Equivalency
Convert
Centimeters
Feet
Yards
pounds

II. Teaching the Lesson(s)

1. Teaching Lesson A: Measuring Mania

Although the focus of the indicator (4-5.3) is to use equivalencies for conversions, students do not have fluency in division; therefore, the conversions do not require them to divide. Since students are fluent in multiplication, they should use multiplicative patterns to perform conversions.

Although the focus of this indicator (4-5.8) is on recalling equivalencies, retention of these facts is enhanced by concept building activities that enable students to discover these relationships.

For this indicator (4-5.3), it is **essential** for students to:

- Analyze concrete and or pictorial models to determine the relationships among these measures.
- Convert units using the basic relationships

For this indicator, it is **not essential** for students to:

- Divide fluently to make conversions

For this indicator (4-5.8), it is **essential** for students to:

- Recall the listed equivalencies

For this indicator, it is **not essential** for students to:

- Perform unit conversions

a. Indicators with Taxonomy

4-5.8 Recall equivalencies associated with liquid volume, time, weight, and length: 8 liquid ounces = 1 cup, 2 cups = 1 pint, 2 pints = 1 quart, 4 quarts = 1 gallon; 365 days = 1 year, 52 weeks = 1 year; 16 ounces = 1 pound, 2,000 pounds = 1 ton; and 5, 280 feet = 1 mile. (A1)

Cognitive Process Dimension: Remember
Knowledge Dimension: Factual Knowledge

4-5.3 Use equivalencies to convert units of measure within the U.S. Customary System: converting length in inches, feet, yards, and miles: converting weight in ounces, pounds, and tons: converting liquid volume in cups, pints, quarts, and gallons: and converting time in years, months, weeks, days, hours, minutes, and seconds. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson

LESSON NOTE:

This is a progression of parts that start with establishing and recalling equivalencies and then converting between units of measurement. It's likely this will take several days, depending on the length of time allotted for daily math instruction.

Part 1 – Liquid Volume

Materials Needed:

- ounce cups, cups, pints, quarts, gallons –containers (The dose cups that come with over the counter meds measure ounces. You will need to take a black permanent marker and mark the 1 oz line.)
- water
- paper/pencil
- teacher made cards
- Measuring Mania recording sheet

NOTE: If it's possible, you may want to arrange to be in a lab classroom for this lesson. That way you have water and sinks available, and it's likely the floor will be tiled instead of carpeted.

Put students in pairs. Each pair of students should have a set of measuring tools and a gallon of water. If you're able to be in the lab, kids can get water as they need it.

Students may work in any order they choose, but should establish the following:

8 oz = 1 cup
2 cups = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

As students work, they record their findings on the Measuring Mania sheet. Each child should have his own sheet. While students work, circulate and monitor their progress. Ask probing questions to be sure they understand what they're doing. For example, "How do you know 2 pints make a quart? How did you find out? What if you had 4 pints?"

When students get to the "How will I remember?" section at the bottom of the page, encourage them to create something of their own. You might suggest a concept map or pictures that represent each equivalency. For students who are just plain stuck, there is always "Gallon Guy" or "Gallon Girl." There's a copy of each included. Please start with the students' own representations – those will be more meaningful.

Part 2 – Length

Materials Needed:

- paper measuring tapes (copy included)
- clear tape
- paper/pencil
- Measuring Mania II record sheet (blank)
- original Measuring Mania record sheet (to use as a reference)

Put students in groups of four. Give each student a 12 inch paper ruler.

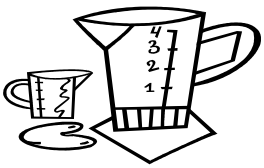
Guiding Questions:

- Do you remember the unit name/equivalency for 12 inches? *1 foot*
- What is the next larger unit using feet as the parts? *1 yard*
- How many feet are in a yard? *3 feet*
- How many inches are in a yard? 36 inches
    - Have students place 3 of their foot long rulers together to verify that there are indeed 36 inches in a yard.

Give students the other parts of the "yard stick" paper ruler and have them tape the pieces together to make a yard long measuring tape.

Students should now work on the Measuring Mania II record sheet. As they work, you should circulate to monitor their progress. The big idea is that as they create equivalency tables building on their original work, they'll see patterns emerging, multiplicative patterns that will help them make conversions from one unit to another.

By studying the tables, they should deduce that they multiply to convert larger units to smaller units and divide to convert smaller units to larger units. Two yards multiplied by three feet equals six feet. Students should have familiarity with measurement units, be able to select the appropriate unit for the attribute, and have knowledge of a few important relationships between units.



Measuring Mania

Name: _____

Partner: _____

Use your tools for measuring liquids to complete the table below.

☐ ounces	1 cup
☐ cups	1 pint
☐ pints	1 quart
☐ quarts	1 gallon

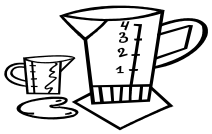
Think about it…

Now use your measuring work to find other equivalencies.

If I have 16 ounces…	I have ☐ cups.
If I have 6 cups…	I have ☐ pints.
If I have 12 pints…	I have ☐ quarts.
If I have 8 quarts…	I have ☐ gallons.

How will I remember?

On a separate sheet of paper, create a picture of some sort that will help you remember the original equivalencies you discovered in the first table..



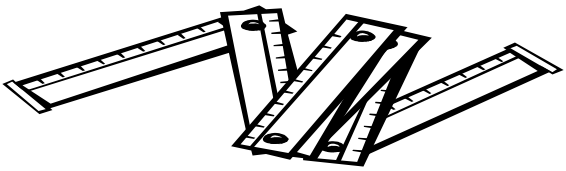
Measuring Mania II

Name: _____

Group Members: _____

_____

_____



"Found" Measurements ~ I found these using measuring tools!

☐ ounces	1 cup
☐ cups	1 pint
☐ pints	1 quart
☐ quarts	1 gallon

☐ inches	1 foot
☐ feet	1 yard

"Given" Measurements ~ These were given to me!

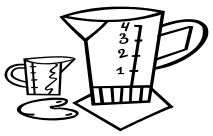
16 ounces	1 pound
2,000 pounds	1 ton

5,280 feet	1mile

"Remembered" Measurements ~ I remember these from working with them!

☐ seconds	1 minute
☐ minutes	1 hour
☐ hours	1 day

☐ days	1 week
☐ days	1 year
☐ weeks	1 year



ounces to cups

☐ ounces	1 cup
☐ ounces	2 cups
☐ ounces	3 cups
☐ ounces	4 cups

Describe a way to find out how many ounces are in any given number of cups.

cups to pints

☐ cups	1 pint
☐ cups	2 pints
☐ cups	3 pints
☐ cups	4 pints

Describe a way to find out how many cups are in any given number of pints.

pints to quarts

☐ pints	1 quart
☐ pints	2 quarts
☐ pints	3 quarts
☐ pints	4 quarts

Describe a way to find out how many pints are in any given number of quarts.

quarts to gallons

☐ quarts	1 gallon
☐ quarts	2 gallons
☐ quarts	3 gallons
☐ quarts	4 gallons

Describe a way to find out how many quarts are in any given number of gallons.

ounces to pounds

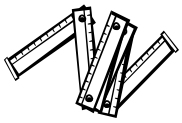
☐ ounces	1 pound
☐ ounces	2 pounds
☐ ounces	3 pounds
☐ ounces	4 pounds

Describe a way to find out how many ounces are in any given number of pounds.

pounds to tons

☐ pounds	1 ton
☐ pounds	2 tons
☐ pounds	3 tons
☐ pounds	4 tons

Describe a way to find out how many pounds are in any given number of tons.



inches to feet

☐ inches	1 foot
☐ inches	2 feet
☐ inches	3 feet
☐ inches	4 feet

Describe a way to find out how many inches are in any given number of feet.

feet to yards

☐ feet	1 yard
☐ feet	2 yards
☐ feet	3 yards
☐ feet	4 yards

Describe a way to find out how many feet are in any given number of yards.

feet to miles

☐ feet	1 mile
☐ feet	2 miles
☐ feet	3 miles
☐ feet	4 miles

Describe a way to find out how many feet are in any given number of miles.

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

Once in a Kingdom – at the end of this module – provides a mnemonic device for students.

Mr. G's parts – at the end of this module - another device for helping students see the relationship between the customary capacity units.

e. Technology

There are no specific recommended technology resources for this lesson at this time.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Students should respond to the following prompts in their binders/notebooks using the charts created in the lesson. When finished, use the popsicle sticks with student names on them to determine which student will share his/her answer with the class… be sure justifications are discussed!

1) How many inches are in 12 feet? How did you determine your answer? Include an explanation, drawing, or whatever method is appropriate.

2) How many ounces are in 10 pounds?

3) How many cups are in 12 pints? How did you determine your answer? Include an explanation, drawing, or whatever method is appropriate.

4) 12 hours is how many minutes? How did you determine your answer? Include an explanation, drawing, or whatever method is appropriate.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The objective of this indicator (4-5.3) is to use which is in the "apply procedural" knowledge cell of the Revised Taxonomy. To use is to have knowledge of steps and the criteria for using those steps. The learning progression to **use** requires students to understand the concepts of length, weight, volume and time. Students explore hands-on, concrete and/or pictorial models to discover the relationships between and among measures within these concepts. As they analyze information (4-1.1) from these experiences, students generalize connections (4-1.6) among measures and generate descriptions and mathematical statements (4-1.4) summarizing these relationships. They explain and justify their answers (4-1.3) using correct, complete and clearly written and oral mathematical language (4-1.5). Students use these statements and their understanding of multiplicative patterns to convert units of measure.

The objective of this indicator (4-5.8) is to recall which is in the "remember factual" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is to recall factual knowledge, learning experience should integrate both memorization and concept building strategies to support retention. The learning progression to **recall** requires student to explore these measurements in context with concrete and/or pictorial models, where appropriate. They analyze information (4-1.1) from these learning experiences to generate mathematical statements (4-1.4) about the relationship between and among these measures. Students use correct, complete and clearly written and oral language (4-1.5) to communicate their understanding of these relationships and the relationships between larger and smaller units of measure.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Beau and Hope each entered a caterpillar in the Annual Caterpillar Race. Beau's caterpillar is able to crawl forward 5 feet and Hope's caterpillar crawls ahead one yard by the end of the race. How many feet has Hope's caterpillar crawled at the end of the race?

2. How many yards are in 27 feet? How did you determine your answer? Include an explanation, drawing, or whatever method is appropriate.

3. Wolfgang is helping his father wash the family car. His father asks him to fill a bucket with soap and 12 pints of warm water. How many quarts of water should Wolfgang fill the bucket with?

4. It is Patrick's turn to supply a beverage for his soccer team after practice. He brings two gallons of lemonade for his teammates. How many quarts of lemonade does Patrick bring?

5. Madison wants to go to the mall to buy a gift for her friend. Her mother is too busy to drive her there and asks Madison to wait. She has to wait 240 seconds before her mother can take her. How many hours is this?

6. Kaleigh took her dog to the vet. When the vet weighed her dog, the scale read 8 kilograms. About how many pounds does Kaliegh's dog weigh?

A) 4 lb B) 8 lb C) 10 lb D) 16 lb

7. Dre competed in the 40 yard dash competition. About how many meters did Dre have to run?

A) 20 meters B) 40 meters C) 80 meters D) 100 meters

8. If you run a mile, about how many kilometers did you run?

A) 1 km B) 2 km C) 10 km D) 20km

9. About how many liters are in a gallon? (Remember: 1 gallon = 4 quarts)

10. Which is more? Circle your choice for each situation.

3 liters or 3 quarts 100 kilometers or 100 miles

50 yards or 50 meters 5 pounds or 5 kilograms

11. Randy picked a melon from the patch. He decided to weigh his melon and discovered it weighed 14 kilograms. About how many pounds does Randy's melon weigh?

A) 7 lb B) 14 lb C) 28 lb D) 50 lb

12. Anya measure the distance from the classroom to the cafeteria and discovered it was 18 meters. About how many yards is the classroom from the cafeteria?

A) 10 yards B) 20 yards C) 30 yards D) 40 yards

13. Keeli ran a 6 mile race. About how many kilometers did she run? Use the table to help you decide.

Number of km	Number of Miles
1	
2	
3	
4	
5	
6	

14. On a hot summer day when you are "super" thirsty, would you rather have a liter of water or a quart of water? Explain why.

15. Which is more? Circle your choice for each situation.

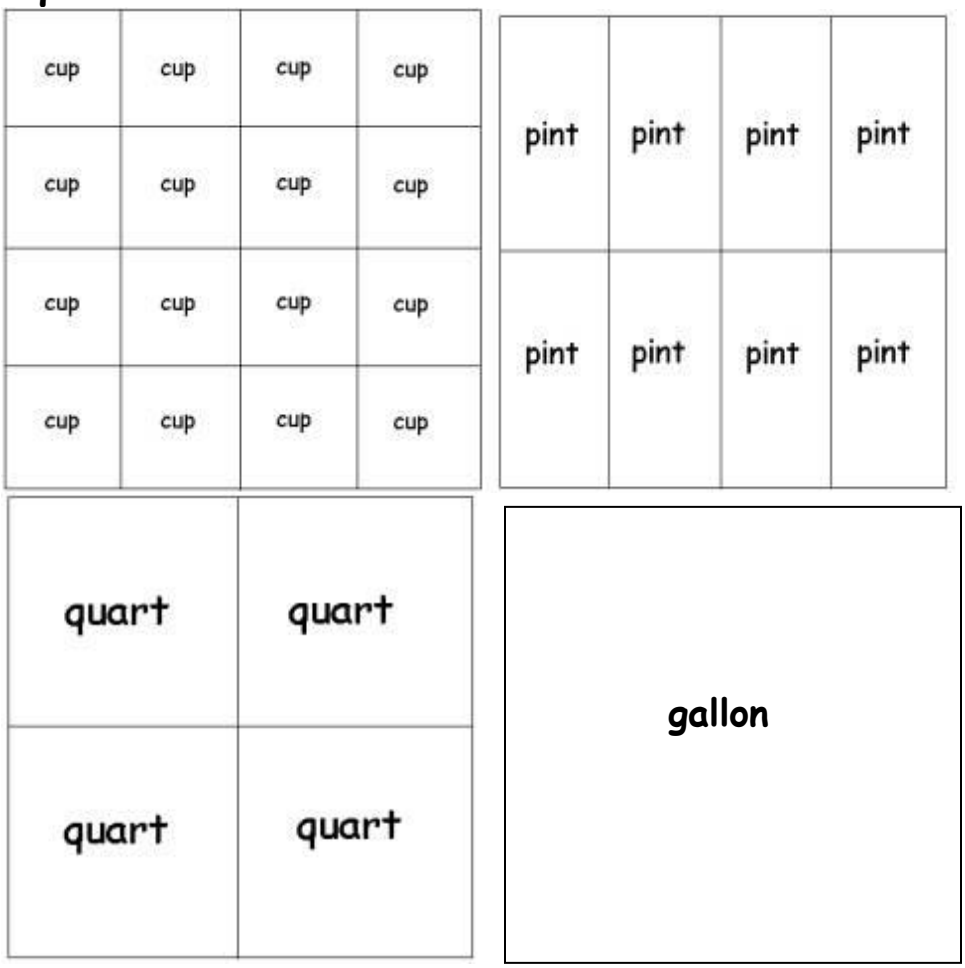
20 liters or 10 quarts 50 kilometers or 10 miles

100 yards or 50 meters 5 pounds or 3 kilograms

Materials Sheet

- Glue
- Markers
- 16 pieces of paper for the cups
- 8 pieces of paper for the pints
- 4 pieces of paper for the quarts
- 1 piece of paper for the gallon

Equal Measurements:



You may want to draw the gallon picture shown on the next page rather than using the square gallon on this page.