SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

Content Area	Fourth Grade Mathematics
Recommended Days of Instruction	**Fourth Nine Weeks**

Standards/Indicators Addressed:

Standard 4-5: The student will demonstrate through the mathematical processes an understanding of elapsed time; conversions within the U.S. Customary system; and accurate, efficient, and generalizable methods of determining area.

4-5.6* Apply strategies and procedures to determine the amount of elapsed time in hours and minutes within a 12-hour period, either a.m. or p.m. (C3)

4-5.7* Use Celsius and Fahrenheit thermometers to determine temperature changes during intervals. (C3)

Standard 4-6: The student will demonstrate through the mathematical processes an understanding of the impact of data-collection methods, the appropriate graph for categorical or numerical data, and the analysis of possible outcomes for a simple event.

4-6.1* Compare how data-collection methods impact survey results. (B2)

4-6.2* Interpret data in tables, line graphs, bar graphs, and double bar graphs whose scale increments are greater than or equal to 1. (B2)

4-6.3* Organize data in tables, line graphs, and bar graphs whose scale increments are greater than or equal to 1. (B4)

4-6.4* Distinguish between categorical and numerical data. (A4)

4-6.5* Match categorical and numerical data to appropriate graphs. (B2)

4-6.6* Predict on the basis of data whether events are *likely, unlikely, certain, impossible*, or *equally likely* to occur. (B2)

4-6.7* Analyze possible outcomes for a simple event. (B4)

* These indicators are covered in the following 5 Modules for this Nine Weeks Period.
Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.

Module 4-1 - Time			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**

Module 4-1 Lesson A 4-5.6 Apply strategies and procedures to determine the amount of elapsed time in hours and minutes within a 12-hour period, either a.m. or p.m. (C3)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics	See Instructional Planning Guide Module 4-1 Introductory Lesson A	See Instructional Planning Guide Module 4-1 Lesson A Assessing the Lesson

	<u>Assessment Sampler</u>: Grades 3-5 ETA Cuisenaire, <u>Hands-On Standards</u>: Grades 3-4		

Module 4-2 - Temperature			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**

Module 4-2 Lesson A 4-5.7 Use Celsius and Fahrenheit thermometers to determine temperature change during intervals. (C3)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics	See Instructional Planning Guide Module 4-2 Introductory Lesson A	See Instructional Planning Guide Module 4-2 Lesson A Assessing the Lesson

	<u>Assessment Sampler</u>: Grades 3-5 ETA Cuisenaire, <u>Hands-On Standards</u>: Grades 3-4		

Module 4-3 - Collection and Representation			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 4-3 Lesson A 4-6.1 Compare how data-collection methods impact survey results. (B2) 4-6.3 Organize data in tables, line graphs, and bar graphs whose scale increments are greater than or equal to 1. (B4)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered	See Instructional Planning Guide Module 4-3 Introductory Lesson A	See Instructional Planning Guide Module 4-3 Lesson A Assessing the Lesson

Module 4-3 Lesson B 4-6.4 Distinguish between categorical and numerical data. (A4) 4-6.5 Match categorical and numerical data to appropriate graphs. (B2)	Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4	See Instructional Planning Guide Module 4-3 Introductory Lesson B	See Instructional Planning Guide Module 4-3 Lesson B Assessing the Lesson

Module 4-4 - Data Analysis			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**

Module 4-4 Lesson A 4-6.2 Interpret data in tables, line graphs, bar graphs, and double bar graphs whose scale increments are greater than or equal to 1. (B2)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics	See Instructional Planning Guide Module 4-4 Introductory Lesson A	See Instructional Planning Guide Module 4-4 Lesson A Assessing the Lesson

	Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4		

Module 4-5 - Probability			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 4-5 Lesson A 4-6.6 Predict on the basis of data whether events are *likely, unlikely, certain, impossible,* or *equally likely* to occur. (B2)	**STANDARD SUPPORT DOCUMENT** http://ed.sc.gov/agency/Standards-and-Learning/Academic-Standards/old/cso/standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5	See Instructional Planning Guide Module 4-5 Introductory Lesson A See Instructional Planning Guide Module 4-5, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 4-5 Lesson A Assessing the Lesson

Module 4-5 Lesson B 4-6.7 Analyze possible outcomes for a simple event. (B4)	Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 3-4	See Instructional Planning Guide Module 4-5 Introductory Lesson B	See Instructional Planning Guide Module 4-5 Lesson B Assessing the Lesson

MODULE

4-1

Time

This module addresses the following indicators:

4-5.6 Apply strategies and procedures to determine the amount of elapsed time in hours and minutes within a 12-hour period, either *a.m.* or *p.m.* (C3)

* This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

In third grade, student used analog and digital clocks to tell time to the nearest minute (3-5.6) and recalled equivalencies associated with time and length: 60 seconds = 1 minute and 36 inches = 1 yard (3-5.7).
In fourth grade, students apply strategies and procedures to determine the amount of elapsed time in hours and minutes within a 12-hour period, either a.m. or p.m. (4-5.6). This is the first time students are introduced to the concept of elapsed time.

In fifth grade, students apply strategies and procedures to determine the amount of elapsed time in hours, minutes and seconds within a 24-hour period, either a.m. or p.m. (5-5.6)

- **Key Concepts/Key Terms**

  - Time periods*
  - Elapsed Time*
  - a.m.*
  - p.m.*
  - 12 hour period*

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

II. Teaching the Lesson(s)

1. Teaching Lesson A-Elapsed Time

Please note that the twelve-hour period is within either a.m. or p.m. – not between the two periods.
Ask questions to help students bring to mind prior knowledge of telling time.

- What do the hands on the clock represent? How do we use them to tell time?
- How do we count around the face of the clock? How many minutes does each number on the face of the clock represent?

Discuss that moving from number to number on an analog clock is in five-minute intervals and from mark to mark is an interval of one minute. For example, put the hour hand on eight and the minute hand on the three. Review counting by fives from number to number until they reach fifteen minutes. Have students move the minute hand to two lines past the number three on the clock face. Discuss counting minutes by five and then by one-minute intervals to develop understanding of time. Practice reading minutes after and before the hour as a large group and then work in pairs. Go through your school day's schedule and have the students show you on their clocks the times for different events during the day.

For this indicator, it is **essential** for students to:

- Understand the meaning of elapsed time
- Understand the difference between a.m. and p.m.
- Understand the meaning of 12-hour period
- Understand past and future time
- Determine elapsed time when the information in given in word or pictorial form
- Find the elapsed time when given the start and end time
- Find the end time when given the start time and elapsed time
- Find the start time when given the end time and elapsed time

For this indicator, it is **not essential** for students to:

- Determine elapsed time during a 24 hour period (5th grade)

a. Indicators with Taxonomy

4-5.6 Apply strategies and procedures to determine the amount of elapsed time in hours and minutes within a 12-hour period, either *a.m.* or *p.m. (C3)*
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson

Materials Needed:

- One large analog and digital clock
- Make a Clock handout
- Paper fasteners (small brads)
- Index cards
- What's My Schedule? Handout (Students should complete this before the day you plan to teach this lesson.)
- School day schedule

Pair students so that they may work together and talk about the math they are doing.

In this lesson, students will construct a clock and use it to count backward and forward within a 12 hour period to find elapsed time.

Have students follow the directions on the Make a Clock handout to construct a personal clock. Gluing the clock on the paper plate helps discourage students from removing the paper fastener that holds the hands on the face of the clock. Trim the edge of the plate to match the size of the clock.

Once clocks are constructed, begin with a quick review of telling time.

Ask questions to help students bring to mind prior knowledge of telling time.

- What do the hands on the clock represent? How do we use them to tell time?
- How do we count around the face of the clock? How many minutes does each number on the face of the clock represent?

Discuss that moving from number to number on an analog clock is in five-minute intervals and from mark to mark is an interval of one minute. For example, put the hour hand on eight and the minute hand on the three. Review counting by fives from number to number until they reach fifteen minutes. Have students move the minute hand to two lines past the number three on the clock face. Discuss counting minutes by five and then by one-minute intervals to develop understanding of time. Practice reading minutes after and before the hour as a large group and then work in pairs. Go through your school day's schedule and have the students show you on their clocks the times for different events during the day.

Progress to finding the elapsed time between two events. Pose questions like the following:

- If I leave for school at __________ and arrive at ___________, how long did it take me to drive to school?
- If school begins at ___________ and lunch is at _________, how much time passes between school starting and lunchtime?

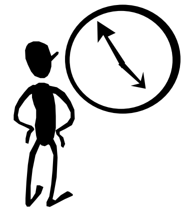
Have students use their What's My Schedule? handout to make up their own problems and solve them with their partners. Students may also write their problems on index cards that you can collect and use at different times.

Students will need to practice this important life skill **in context** as much as possible. Take advantage of every opportunity you find to present real situations for them to show understanding of time measurement in their daily schedules.

A lesson plan that uses digital and analog clocks to teach understanding is "How Much Time Has Passed?" by Melissa Harrison (harrm650@elon.edu). This lesson was taken from the Educator's Reference Desk website.

The website address is http://eduref.org/cgi-bin/printlessons.cgi/Virtual/Lessons/Mathematics/Measurement/MEA0202.html

What's My Schedule?



Name ______________________________

Fill in the times below. It's okay to estimate.

Before school (in the morning – A.M.)

I get up at ______________________________

I leave for school/
get on the bus at ______________________________

After school (in the afternoon – P.M.)

List at least 3 things you do after school (You may list more if you like. ☺)	Tell what time this "thing" starts.

c. *Misconceptions/Common Errors*

No typical student misconceptions noted at this time.

d. *Additional Instructional Strategies/Differentiation*

While additional learning opportunities are needed, no suggestions are included at this time.

e. *Technology*

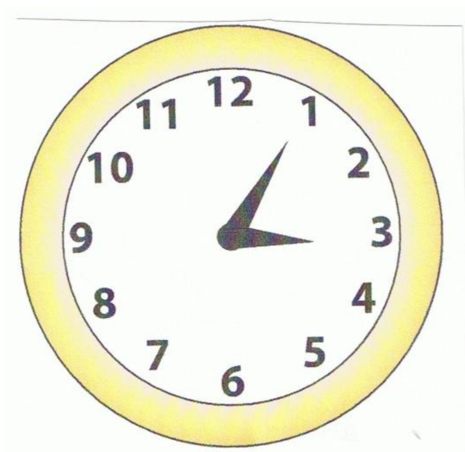
These were stated in the lesson write up as well:

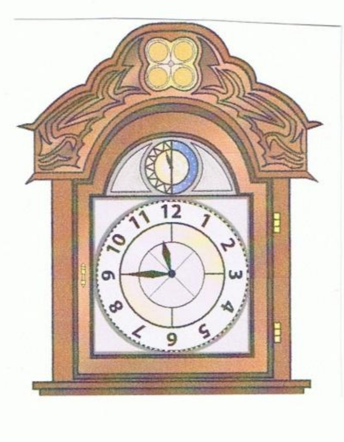
A lesson plan that uses digital and analog clocks to teach understanding is "How Much Time Has Passed?" by Melissa Harrison (harrm650@elon.edu). This lesson was taken from the Educator's Reference Desk website.

The website address is http://eduref.org/cgi-bin/printlessons.cgi/Virtual/Lessons/Mathematics/Measurement/MEA0202.html

f. *Assessing the Lesson*

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.





The clock on the left shows what time Sally got home from school. A friend came home with her to spend the night and they stayed up late. The clock on the right shows what time they finally went to bed. How long were they up before they went to bed?

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The objective of this indicator is to apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is to apply a procedure, the learning progression should integrate activities that build both conceptual and procedural knowledge. The learning progression to **apply** requires students to recall the meaning of a.m. and p.m. They also understand the meaning of past and future dates and the meaning of elapsed time. Students explore a variety of real world examples given in word, pictorial and concrete form and use their understanding to determine either the elapsed time, start time or end time. Students explain and justify their answers (4-1.3) using correct, complete and clearly written and oral language (4-1.5)

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. John got up at 8:10 Saturday morning. John read a book, watched some TV with his parents, then had lunch. If he had lunch 3 hours and 15 minutes after he got up, what time did he eat lunch?

2. A skating rink opens at 1:30pm on Saturday. By the time the staff closes and cleans up, it is 11:15pm. If a worker came in at 1:30pm and got off at 11:15pm, how long did they work?

3. If Barbara finished her morning run at 8:15am and she ran for 1 hour and 10 minutes, what time did she start her run?

MODULE

4-2

Temperature

This module addresses the following indicators:

4-5.7 Use Celsius and Fahrenheit thermometers to determine temperature changes during intervals. (C3)

* This module contains 1 lesson. This lessons is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge:**

In first grade, students used Celsius and Fahrenheit thermometers to measure temperature (1-5.11).
In fourth grade, students use Celsius and Fahrenheit thermometers to determine temperature changes during time intervals (4-5.7).
In fifth grade, students understand the relationship between the Celsius and Fahrenheit temperature scales (5-5.7).

- **Key Concepts/Key Terms**

  - Fahrenheit*
  - Celsius*
  - Thermometer*
  - Scale*
  - Temperature change*

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lesson(s)

Although the focus of the indicator is procedural, students should explore these concepts in context to build conceptual understanding and to support the retention of strategies. One possible activity: A large demonstration thermometer can be used to model how liquid in the thermometer moves up and down and is read on the Celsius and Fahrenheit scales. Various temperatures can be set for class readings. Daily temperature records can be kept on large demonstration thermometers and digital weather thermometers. Teachers may want to hang real thermometers throughout the classroom in order for students to periodically read the temperatures throughout the day. Temperature changes can be observed. A table, chart, and/or line graph of daily temperatures provides a record for discussions of temperatures increasing and decreasing over time. It connects mathematics and science in a realistic context. It is important to note that the concept of line graphs is introduced in fourth grade for the first time. Therefore, students will need additional work on line graphs other than simply recording temperature change over time. The important focus here is not on the fact that temperature changes but that there is a relationship between temperature and time.

For this indicator, it is **essential** for students to:

- Understand how to read a thermometer given in concrete and pictorial form

- Understand that temperature changes over time
- Understand if the temperature is increasing or decreasing
- Understand how subtract and add whole numbers fluently
- Recognize the notation for Celsius and Fahrenheit

For this indicator, it is **not essential** for students to:

- Use the Celsius and Fahrenheit formulas

1. Teaching Lesson A-Temperature Change

a. Indicators with Taxonomy

4-5.7 Use Celsius and Fahrenheit thermometers to determine temperature changes during intervals. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson

Materials Needed:

- Thermometers with suction cups to place on the outside of the classroom window
- Indoor thermometers to record the temperature inside
- Overhead Projector, Smart Board, white/chalkboard, chart paper (any one of these will suffice)
- Colored pencils (preferably erasable ones)
- Hot Topics handout
- Hot Topics transparency
- Thermometer handouts (*graphics included* – Use them to make handouts that suit your needs.)
- A weeks' worth of high and low temperatures (collected from newspaper or a website like the Weather Channel website – www.weather.com or the WISTV website – www.wistv.com) NOTE: It may be helpful to create a handout with this information on it.

Lesson

TEACHER NOTE: Take time before teaching this unit to explore the Weather Channel website. There are some very interesting displays and graphs there that might be of use to you in teaching this material. If you're able to view the website with your students, they would be able to practice finding change in temperature without having to use worksheets.

After the material is introduced, set up the thermometers (both indoor and outdoor) at different places in the classroom so that they are accessible to students. Leave them in place even after this material is taught so that students can continue to practice the skill.

Put students in groups of four so they may work together and talk about the math they are learning. Each group should have at least one real thermometer to examine.

Give each student the Hot Topics handout. Lead a discussion focusing on the questions/topics on the handout. This will serve as their "notes" for this material and should be kept in their math notebooks for later reference. The benchmarks do not need to be memorized; they are offered to give students reference points and an opportunity to mark temperatures on the thermometer. Use the transparency to provide them a check for their work.

Move on to finding the change in temperature for a given period of time. The sample handout provided could serve as a pattern for worksheets that fit the data you collected prior to beginning teaching this series of lessons.

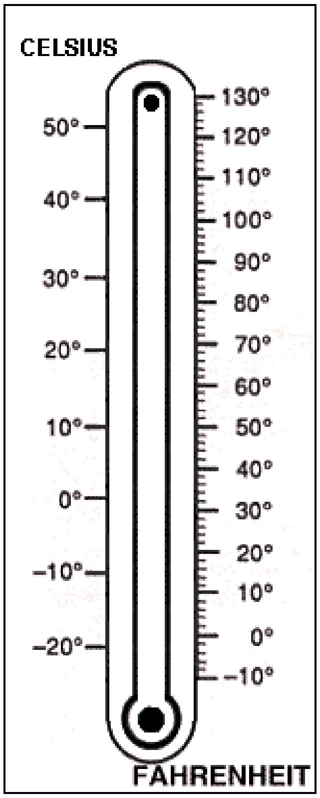
Ask the students for suggestions on how to find the difference between the low and high temperatures for Monday. Previous experiences with subtraction problems should help them find the difference. Besides simply subtracting the lower temp from the higher, students might also mark each temp on the thermometer provided and count the difference, using the thermometer as a number line.

Provide further opportunities for students to practice.

Hot Topics

Name:

Group Members:



Fahrenheit – The scale we use to measure temperature in the United States. It's written in °F and read "degrees Fahrenheit."

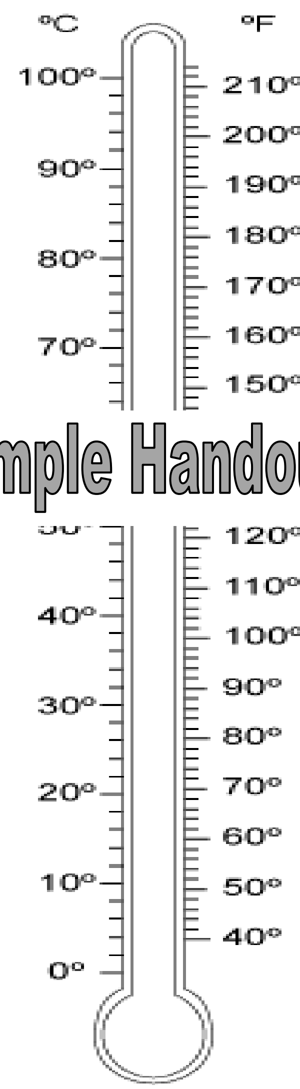
Celsius – The Metric scale that most of the rest of the world uses to measure temperature. It's written in °C and read "degrees Celsius."

Look carefully at the thermometer.

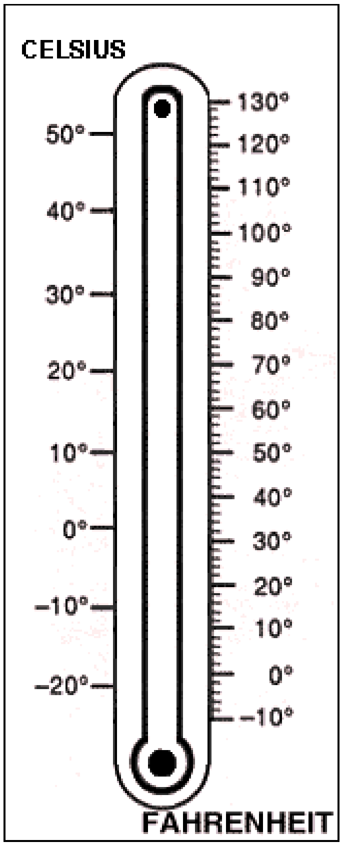
- What is the scale on the Fahrenheit side of the thermometer?
- What is the scale on the Celsius side of the thermometer?

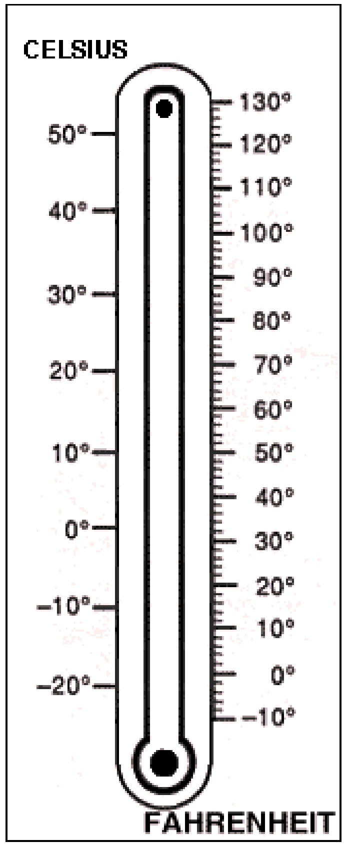
Compare the scales on the paper thermometer to the scales on the real thermometer.

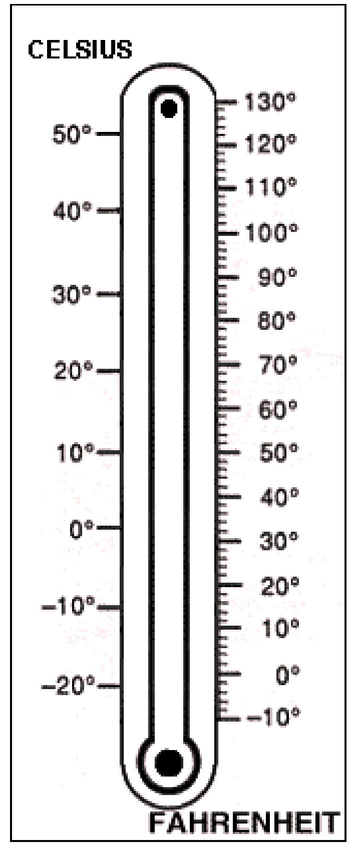
- What do you notice?



IMPORTANT TEMPERATURE BENCHMARKS Find each temperature on the thermometer at the right and mark it with the color indicated.			
water boils	212°F	100°C	red
water freezes	32°F	0°C	blue
normal human body temperature	98.6°F	37°C	green
comfortable room temperature	68°F	20°C	orange







c. Misconceptions/Common Errors

Students may not pay attention to the increments used on the thermometer.

d. Additional Instructional Strategies/Differentiation

These activities are adapted from Anderson 5

Make up situation cards as follows, "At 9:00 a.m. it was 72 degrees Fahrenheit. At 10:00 a.m. it was 52 degrees Fahrenheit. Describe what the weather might feel like at each time of day. What are you noticing about the temperature? What do you predict will happen with the temperature the rest of the day? What might be some reasons you think that way?

Give the students a sheet with 'High and Low' for each day of the week. Have them listen to the weather for five days in a row and record the 'Highs and Lows' in Fahrenheit and Celsius. Display the results and discuss them. Based on the data you have collected, what do you predict will happen tomorrow with the weather?

e. *Technology*

These were mentioned in the lesson write up as well.

TEACHER NOTE: Take time before teaching this unit to explore the Weather Channel website. There are some very interesting displays and graphs there that might be of use to you in teaching this material. If you're able to view the website with your students, they would be able to practice finding change in temperature without having to use worksheets.

A weeks' worth of high and low temperatures (collected from newspaper or a website like the Weather Channel website – www.weather.com or the WISTV website – www.wistv.com) NOTE: It may be helpful to create a handout with this information on it.

The Weather Channel website has interesting tables and graphs that may be useful www.weather.com.

f. *Assessing the Lesson*

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Students should do these two problems over the course of 2 days. The teacher should call on students to share their responses on the second day. Use the popsicle sticks to randomly draw a student's name.

To turn in as an exit ticket (index card, slip of paper, etc.) students should write about what they notice about the temperature in degrees Fahrenheit and degrees Celsius since they were both outside temperatures.

1) Using the thermometer provided by your teacher, note the temperature right outside the classroom at the start of class. Record the results. ____°C

Wait one day before measuring the temperature again.

At the start of the next school day use the thermometer to measure the temperature right outside the classroom. Record the results. ____°C

How much has the temperature right outside the classroom changed from one day to the next? ____°C

2) How much does the temperature outside change every hour? A lot? A little? Let's find out! Use a Fahrenheit thermometer to determine the temperature outside. Write down the temperature in degrees Fahrenheit.

Wait an hour before taking a second reading of the temperature outside. Write this temperature in degrees Fahrenheit.

Now compare your findings. How much did the temperature outside change over the past hour? ____°F

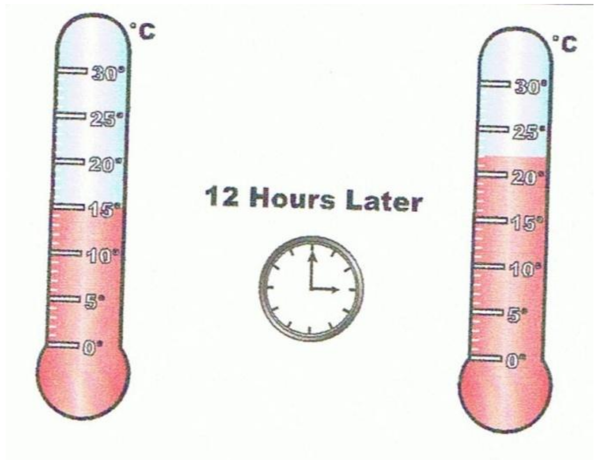
III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The objective of this indicator is to use which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Procedural knowledge is the knowledge of how steps and the criteria for when to using those steps. The learning progression to **use** requires students to recall the structure of a thermometer and the notation for Celsius and Fahrenheit. To demonstrate flexibility in mathematical representations (4-1.7), students explore problems in concrete, pictorial and word form. They analyze information (4-1.1) to determine if the temperature is increasing or decreasing and use that information to determine if they should add or subtract to find the change. Students explain and justify their answers (4-1.3) using correct, complete and clearly written and oral language (4-1.5).

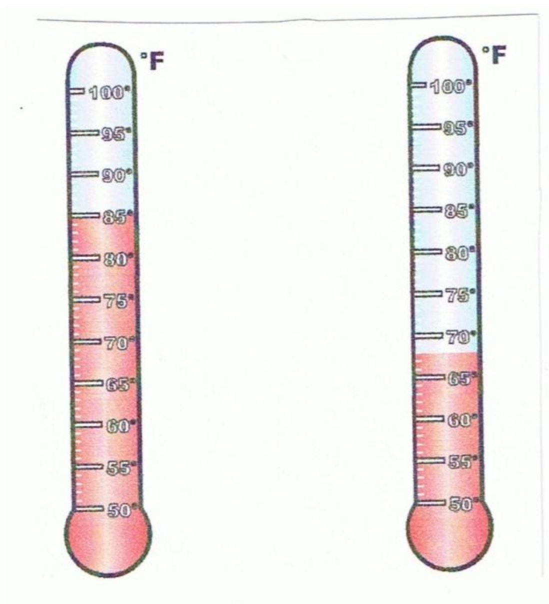
The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. What is the change in temperature? ____ degrees



2. At the beginning of the day, Delmer wants to go outside and play soccer, but the weather is too warm for him. He checks the temperature at 2pm using a Fahrenheit thermometer. Delmer decides to wait until the temperature drops 20 degrees before he will go out to play. At 6pm, he takes one last temperature reading. How much did the temperature change?

Create a table to show how what the temperature may have been every hour up to 6pm.



2pm 6pm

MODULE

4-3

Collection and Representation

This module addresses the following indicators:

4-6.1 Compare how data-collection methods impact survey results. (B2)
4-6.3 Organize data in tables, line graphs, and bar graphs whose scale increments are greater than or equal to 1. (B4)
4-6.4 Distinguish between categorical and numerical data. (A4)
4-6.5 Match categorical and numerical data to appropriate graphs. (B2)

* This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S[3] begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

4-6.1
Compare how data-collection methods impact survey results.

In first grade, students use survey questions to collect data (1-6.1). In second grade, students created survey questions to collect data (2-6.1). In third grade, students organized data in tables, bar graphs, and dot plots and compared the benefits of each form of representation. (3-6.2)
In fourth grade, will build on the knowledge they gained from comparing forms of representation to now compare how data collection methods impact survey results. (4-6.1)
In fifth grade, students analyze how data-collection methods affect the nature of the set. (5-6.2)

4-6.3
Organize data in tables, line graphs, and bar graphs whose scale increments are greater than or equal to 1.

In first grade, students use survey questions to collect data (1-6.1). In second grade students, create survey questions to collect data (2-6.1). Students collect data through surveys and organize data in charts, pictographs and tables and create survey questions to collect data. In third grade, students interpret data in tables, bar graphs, pictographs, and dot plots. (3-6.3) and analyze dot plots and bar graphs to make predictions about populations. (3-6.4)
In fourth grade, students organize data in tables, line graphs, and bar graphs whose increments are greater than or equal to 1(4-6.3).
In fifth grade, students do not get an opportunity to experience with organizing data in tables, line graphs, and bar graphs.

4-6.4
Distinguish between categorical and numerical data.

In third grade, students are not formally introduced to this concept. They engage in learning experiences to organize data in tables, bar graphs, and dot plots and compared the benefits of each form of representation. (3-6.2)

In fourth grade, is the first time students are formally introduced to the concepts of categorical and numerical data. (4-6.4)

In fifth grade, students are not exposed to this mathematical concept.

4-6.5
Match categorical and numerical data to appropriate graphs.

In third grade, students are not formally introduced to this concept. They engage in learning experiences to organize data in tables, bar graphs, and dot plots and compared the benefits of each form of representation. (3-6.2)

In fourth grade, is the first time students are formally introduced to the mathematical concept in which they match categorical and numerical data to appropriate graphs. (4-6.4)

In fifth grade, students are not exposed to this mathematical concept.

- **Key Concepts/Key Terms**
  - Categorical Data*
  - Numerical Data*
  - Graphical Representation
  - Survey*
  - Tally*
  - Set
  - Scale increments*
  - Tables*
  - Line graphs*
  - Bar graphs*
  - Double bar graph*

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lesson(s)

1. Teaching Lesson A-Data Collection

The teacher provide students with learning experiences to lead students to observe that small sample might yield different results than a large sample or that if a survey question deals with cosmetics and only men are surveyed, the results may be different than if women were survey. Students need sufficient learning experiences in order to compare and draw logical conclusions about how data-collection methods impact the results.

Real-world examples from the classroom learning environment may be used to help students understand how to organize data in tables, line graphs, and bar graphs using

For this indicator (4-6.1), it is **essential** for students to:

- Analyze a variety of data collection methods
- Compare and draw logical conclusions about how data-collection methods impact survey results.
- Explore data using real-world examples and its real world purpose
- Determine advantages and disadvantages of different methods

For this indicator, it is **not essential** for students to:

- Collect and organize data from a survey investigation.

For this indicator (4-6.3), it is **essential** for students to:

- Understand the structure of tables, line graphs, and bar graphs
- Determine which type of graph would best represent the data
- Engage in real-world experiences in which they organize data
- Create scales where the increments are greater than or equal to 1

For this indicator, it is **not essential** for students to:

- Organize data in a pictograph.

a. *Indicators with Taxonomy*

4-6.1 Compare how data-collection methods impact survey results. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

4-6.3 Organize data in tables, line graphs, and bar graphs whose scale increments are greater than or equal to 1. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

Suggested Literature Connection:

- *Lemonade for Sale*-Stuart J. Murphy
  Sheri keeps track on a bar graph, plotting the number of cups sold against the days of the week. Suddenly sales drop when Jed the Juggler comes to town. What will the Elm Street kids do?

This lesson will be handled as more of a class discussion approach. Pose the following situations and have students pair up to discuss answers/reasonings. Then hold whole group discussions.

1. Suppose you wanted to ask a survey question to your class. If you asked that question to all the boys only, would you get a good representation of your class? Explain your reasoning.

2. Miranda was concerned that she would one day need to wear glasses like her mother. She knew that imperfect vision was something she inherited from family, so when her family gathered for a reunion she decided to count the number of people who were wearing glasses. She observed 12 people wearing glasses, and assumed that 12 of her relatives had imperfect vision. When she shared this with her cousin, however, her cousin pointed out that a survey is also a good way of collecting information. Later, after surveying everyone at the reunion, Miranda learned that 20 people actually had imperfect vision. Did her data collection method affect her results? Why or why not?

3. Naoko asked her twin sons, Yuki and Seiko to find out the most popular ice cream flavor among their classmates so she would know what to bring to the class party for their birthday. Both boys conducted separate surveys. Seiko gave two different answer options for his survey, but Yuki gave no options, allowing the students to answer however they wanted. Examine their results recorded in the tables below, and explain how the presence of options affects the accuracy of the survey results.

Yuki's Results

Favorite Ice Cream Flavor	Number of Kids
Chocolate	6
Vanilla	5
Strawberry	3
Mint n' Chip	10
Chocolate Chip	4

Seiko's Results

Favorite Ice Cream Flavor	Number of Kids
Chocolate	16
Vanilla	12

4. For a class project, Mr. Farrington asked his students to conduct a survey in order to find out approximately how many kids in school had a computer at home. Since they only had time to survey a sample of the entire student body, Sylvia and Evan both chose a group of students to survey. Evan surveyed the 30 members of his after school computer club, while Sylvia gave the survey to 30 random students in the cafeteria at lunch time. Based on his results, Evan decided that about 25 out of 30 students have computers at home. Sylvia's results showed that 12 out of 30 had a home computer. Do you think that either Evan's or Sylvia's results exhibit bias? Why or why not?

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Data can be gathered through:
1) The U.S. Census Bureau at www.census.gov. This website contains statistical information by state, county or voting district.
2) Internet Movie Database at www.imdb.com. This website offers information about movies of all genres.

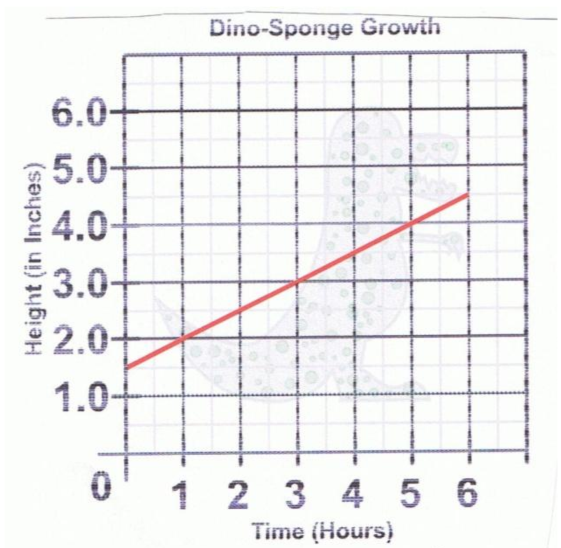
f. *Assessing the __Lesson__*

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

(1) Miranda wants to take a survey of how many of the 36 people in her class have her favorite video game, *Leonardo's Speedway*. What would be a good sample size that she could use to avoid a biased result?

(2) At Tom's school, the principal wants to select a new school mascot. There are 400 students at her school, and Gloria has surveyed all 70 of the 4th grade students. The principal refuses to name the mascot according to Gloria's survey results. Why is the principal correct?

(3)



Which table represents the graph most accurately?

Time (hours)	Height (inches)
0	1.5
1	2
2	2.5
3	3
4	3.5
5	4
6	4.5

Time (hours)	Height (inches)
1	1.5
2	2
3	2.5
4	3
5	3.5
6	4
7	4.5

2. *Teaching Lesson B-Categorical and Numerical Data*

Categorical data is data that can be categorized or placed in groups, such as different kinds of colors and numerical is data that can be ordered numerically, such as the heights of students in class or the age of students in class.

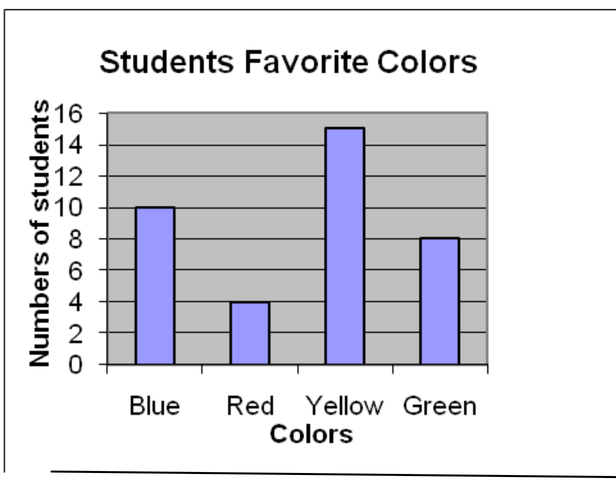
Although it is not essential for students to create examples of each type of data, having students create examples supports conceptual understanding and supports retention of facts.

Examples of data.

Tally Chart

STUDENTS FAVORITE COLORS

Blue	~~||||~~ ~~||||~~
Red	| | | |
Yellow	~~||||~~ ~~||||~~ ~~||||~~
Green	~~||||~~ | | |



For this indicator (4-6.4), it is **essential** for students to:

- Understand the vocabulary terms categorical data and numerical data.
- Distinguish between the two types of data
- Match the types to the appropriate graph

For this indicator, it is **not essential** for students to:

- Formulate questions that address numerical and categorical data.
- Construct categorical and numerical data.

For this indicator (4-6.5), it is **essential** for students to:

- Understand the characteristics of numerical data
- Understand the characteristics of categorical data
- Match terms with graphical and verbal representations (the favorite pizza topping for teachers)
- Explore many different examples/models

For this indicator, it is **not essential** for students to:

- Construct and organize categorical and numerical data.

a. Indicators with Taxonomy

4-6.4 Distinguish between categorical and numerical data. (A4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Factual Knowledge

4-6.5 Match categorical and numerical data to appropriate graphs. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

<u>Materials Needed</u>

Handout of graphical representations

Lesson-Part A

The following lesson IS NOT a discovery lesson.. it begins with giving students definitions and examples. Part B is where students will match categorical and numerical data to graphs.

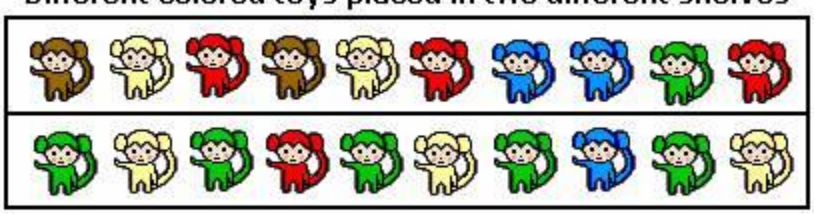
Definition of Categorical Data

Categorical data are values that are words and that represent possible responses with respect to a given category. Frequency counts can be made of the values for a given category

Example of Categorical Data

- The following diagram shows the different colored toys placed in two different shelves.

Different colored toys placed in two different shelves



These toys can be sorted according to their color in different categories as:

Categorical data

Color	Number of toys
Brown	2
Yellow	5
Red	4
Blue	3
Green	6

Example on Categorical Data

The table shows the details of the hair color and eye color of a number of students in a class. Find the total number of students with black hair, ignoring the eye color.

Hair color → / Eye color ↓	Blonde	Black	Red	Brown
Green	2	2	0	1
Black	0	1	0	2
Brown	2	1	4	3
Blue	2	1	1	0

Definition of Numerical Data

Numerical Data can be defined as a collection of data that is measured using numbers. It can involve data such as counts, measurements and ratings.

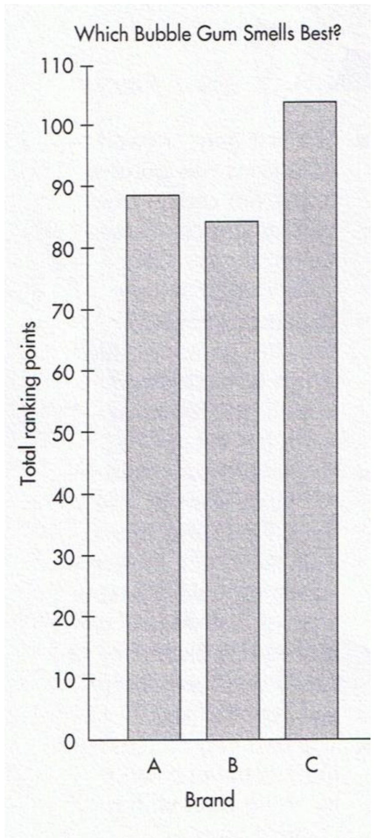
Examples of Numerical Data

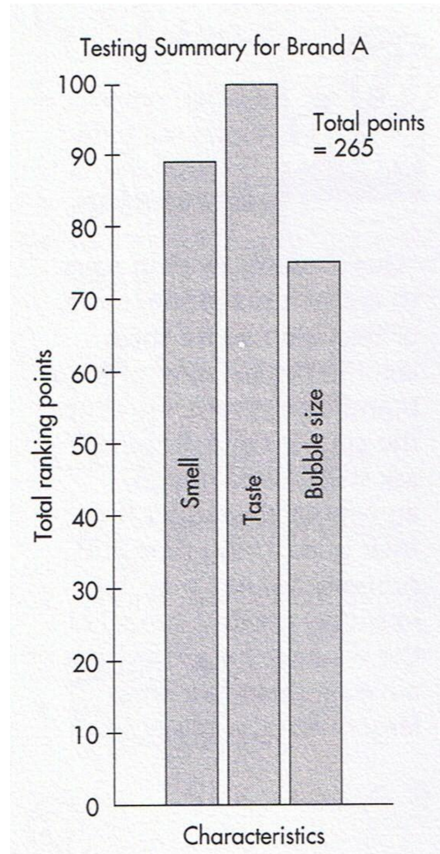
- The data shown below are Mark's scores on five Math tests conducted in 10 weeks. 45, 23, 67, 82, 71. The data helps us compare his scores and learn his progress.

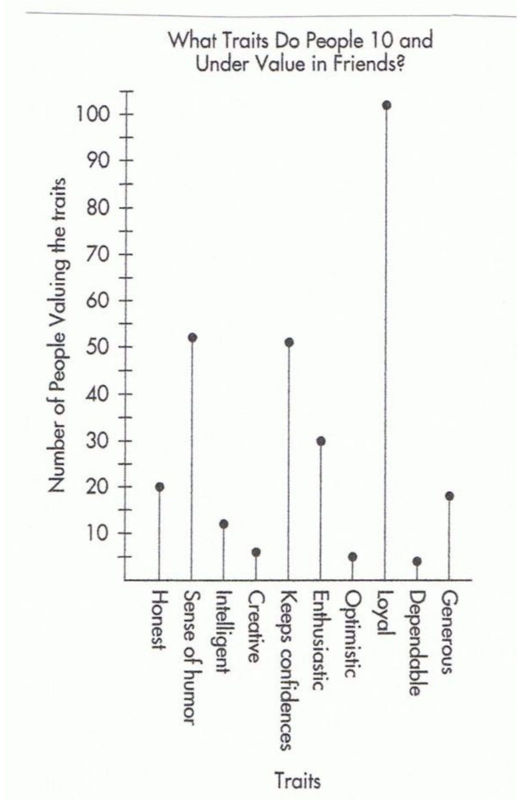
- Number of baskets scored by a basketball player are: 8, 15, 11, 13, 9, 12, and 10. The range can be found when using numerical data. Now, find the range of the data.
  Solution:
  Step 1: *Highest score = 15*
  Step 2: *lowest score = 8*
  Step 3: Range = 15 - 8 = 7. [Range = highest score - lowest score.]

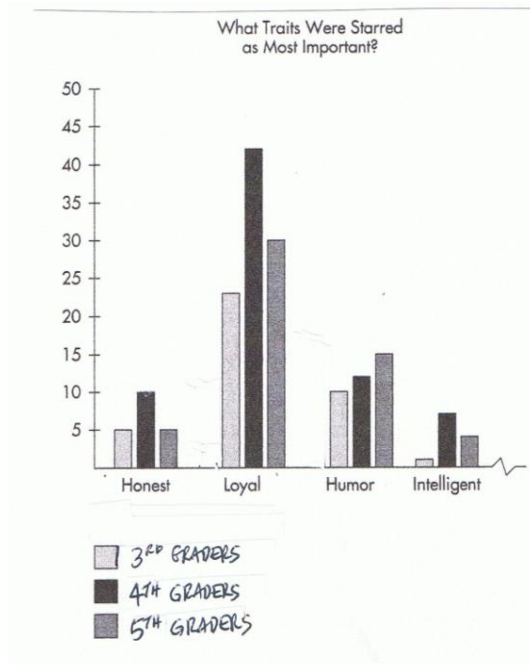
Lesson - Part B

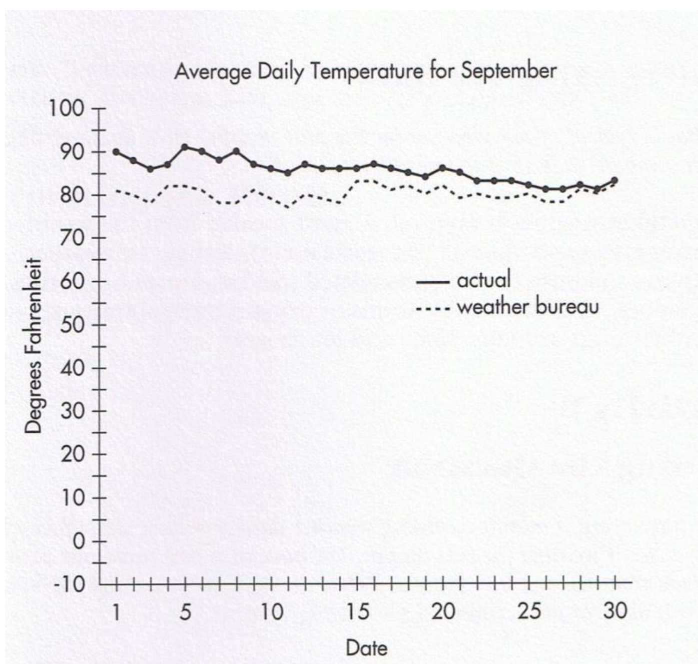
Look at the following graphs and determine if the information is categorical data or numerical data. Explain how you know.






What Traits Do People 10 and Under Value in Friends?
Number of People Valuing the traits
100
90
80
70
60
50
40
30
20
10
Honest
Sense of humor
Intelligent
Creative
Keeps confidences
Enthusiastic
Optimistic
Loyal
Dependable
Generous
Traits


What Traits Were Starred as Most Important?
50
45
40
35
30
25
20
15
10
5
Honest
Loyal
Humor
Intelligent
3RD GRADERS
4TH GRADERS
5TH GRADERS


Average Daily Temperature for September
Degrees Fahrenheit
100
90
80
70
60
50
40
30
20
10
0
-10
actual
weather bureau
1
5
10
15
20
25
30
Date

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Numerical and Categorical Data
http://illuminations.nctm.org/LessonDetail.aspx?id=U116

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Call these types of data out one by one and have students hold up their responses on index cards… one side of the index card should say "categorical" and the other should say "numerical."

Are the following types of data sets categorical or numerical?

- Number of children in families
- Pulse rates of top athletes
- Months in which people have birthdays (Jan, Feb, Mar, and so on)
- Time in minutes that students spend watching television a day
- Favorite pizza toppings of teachers

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

4-6.1
The objective of this indicator is compare, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Understand

requires students to construct meaning and conceptual knowledge is not bound by specific examples; therefore, the focus is on building an understanding of the impact of methods as opposed to mastering a particular method. The learning progression to **compare** requires students to explore a variety of data collection methods. Students use correct, complete, and clearly written and oral mathematical language to pose questions and communicate ideas (4-1.5) about these methods. They generate descriptions and mathematical statements about relationships between and among various methods and *explain* and *justify* how data-collection methods impact survey on the basis of mathematical properties, structures, and relationships. (4–1.3)

4-6.3
The objective of this indicator is to organize which is in the "analyze conceptual " knowledge cell of the Revised Taxonomy. Conceptual knowledge is not bound by specific examples. Therefore, the student's conceptual knowledge of organizing data in tables, line graphs and bar graphs should be explored using a variety of examples. The learning progression to **organize** requires students to understand the characteristics of tables, bar graphs, and line graphs. Students analyze data and determine how the data should be categorized or sorted. Students should recognize the limitations of each type of representation (3-1.8) and use that understanding to select the most appropriate method for organizing the data. They explain and justify answers on the basis of mathematical properties, structures, and relationships (3-3.3), and use correct, complete, and clearly written and oral mathematical language to pose questions, communicate ideas, and extend problem situations (3-1.5) with their classmates and teacher.

4-6.4
The objective of this indicator is distinguish which is in the "analyze factual" knowledge cell of the Revised Taxonomy. Analyze factual means to break down factual knowledge and determine relevant and irrelevant characteristics. The learning progression to **distinguish** requires students to analyze information (4-1.1) to determine how the data has been organized. Based on their observations, students generalize the connection between the organization of the data and terms categorical or numerical data (4-1.6). They explore a variety of examples and generate conjectures about the data. Students explain and justify their reasoning on the basis of mathematical properties, structures, and relationships (4-1.3) using correct, complete, and clearly written and oral mathematical language to communicate their ideas (4-1.5) to their classmates and teacher.

4-6.5
The objective of this indicator is *match,* which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To match detect

similarities and differences in order to construct meaning; therefore, student's conceptual knowledge of matching categorical and numerical data should be explored using a variety of examples. The learning progression to **match** requires students to recall the meaning of the terms categorical and numerical data. Students analyze representations (4-1.1) to determine key characteristics and generalize the connections between those characteristics and the term numerical and categorical. Student *construct* arguments (4-1.2) about these connections and explain and justify their answers (4-1.3) using correct, complete, and clearly written and oral mathematical language to pose questions, communicate ideas, and extend problem situations (4.1.5) with their classmates and teacher.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Padma's school only has enough money to buy footballs or basketballs, but not both; so Padma takes a survey to find out which is the preferred sport. Padma stands on the blacktop next to the basketball courts with a clipboard and starts asking students whether they prefer basketball or football. She finds that almost everyone walking by prefers basketball and concludes that the school should not spend any more money for footballs. Principal Milner, however, says her survey has not produced reliable results. Why do you think this is?

2. What is your classmates' favorite cookie? To find out, conduct a survey of 12 people in your class to serve as a sample. Now make a bar graph to show your results. Use an interval greater than 1 for the y-axis.

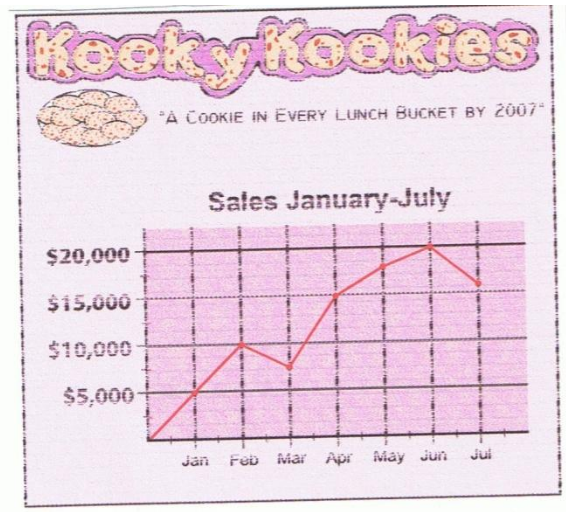
3. In Jenny's school, there are ninety 4th grade students and three 4th grade teachers. Jenny thinks the 4th grade students spend too much time doing homework, and she wants to prepare a report for the student council meeting. She needs to survey the 4th grade students to find out how much homework they are doing, but she doesn't want to spend the time to survey all 90 students. Help her decide how many students and from which classrooms she needs to survey to come up with reasonable results.

4. Will's brother, Sam, is running for school president on the platform that he will improve the athletic department. His opponent, Che, has promised more vending machines. Will wants to take a poll to see where Sam stands in the

rankings, but he's not sure where he should gather his sample data. Which of the groups below should Will survey so that he doesn't get skewed results?

a. Students on the football team.
b. Students in the Crazy For Candy club.
c. Students in the schoolyard at recess.

5.



Which table represents the graph most accurately?

Month	Profit ($)
January	5,000
February	10,000
March	7,500
April	15,000
May	18,000
June	20,000
July	16,000

Month	Profit ($)
January	5,000
February	10,000
March	7,000
April	15,000
May	20,000
June	20,000
July	15,000

6. Are the following types of data sets categorical or numerical?

- Ratings such as how much people value or like movies
- Favorite color t-shirt
- Kinds of pets people have

7. What type of data (categorical or numerical) is usually represented in a bar graph?
What type of data (categorical or numerical) is usually represented in a line graph?

MODULE

4-4

Data Analysis

This module addresses the following indicators:

4-6.2 Interpret data in tables, line graphs, bar graphs, and double bar graphs whose scale increments are greater than or equal to 1. (B2)

* This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**
  In first grade, students use survey questions to collect data (1-6.1). In second grade, students created survey questions to collect data (2-6.1). In third grade, students organized data in tables, bar graphs, and dot plots and compared the benefits of each form of representation (3-6.2).
  In fourth grade, interpret data in tables, line graphs, bar graphs, and double bar graphs whose scale increments are greater than or equal to 1 (4-6.2). Students also compare how data-collection methods impact survey results (4-6.1).

  In fifth grade, students interpret the meaning and application of the measures of central tendency. (5-6.4)

- **Key Concepts/Key Terms**

  - *Numerical Data
  - *Categorical Data
  - Increments
  - Scale
  - *Table
  - *Bar Graph
  - *Double Bar Graph
  - *Line Graph
  - Trends/Change
  - Increasing
  - Decreasing
  - Varying

  ** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lesson(s)

1. Teaching Lesson A-Interpreting Data

Graphs convey factual information and provide opportunity to make inferences that are not observed directly. (van de walle, page 331)
The interpreting of graphs is as much about the facts as it is the inferences made.

<u>Tables</u> – Way of organizing data that usually leads to some other type of graphical representation
<u>Bar Graphs</u> – way of organizing categorical data – data that has no numeric ordering
<u>Double Bar Graphs</u> – way of organizing categorical data about two different data sets
<u>Line Graphs</u> – way of organizing continuous data – data that show change over time for example. Line graphs are used when there is a numeric value associated with equally spaced points along a continuous number scale.

(Adapted From Teaching Student Centered Mathematics Grades 3-5, Van de Walle, John A. and Lovin, LouAnn, 2006, pages 331-334.)

The teacher may use the following steps to help student interpret graphs: (1) students clarify, paraphrase what the graph tells them, represent and translate data from graphical form to verbal form, (2) students write sentences about what the graph reports using key words such increasing, decreasing, or varying; (3) then answer the question posed to them in the problem.
An additional focus is on students using appropriate mathematical terminology in written or oral form.

For this indicator, it is <u>**essential**</u> for students to:
- Use the mathematical terms increasing, decreasing, or varying to show change over time to interpret data.
- Imply more than simply reading data in a graph.
- Clarify, paraphrase, represent, and translate data from graphical to verbal form.

For this indicator, it is <u>**not essential**</u> for students to:
- Construct double bar graphs, line graphs, bar graph only interpret data.

a. Indicators with Taxonomy

Indicator → 4-6.2 Interpret data in tables, line graphs, bar graphs, and double bar graphs whose scale increments are greater than or equal to 1. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

Materials Needed

Handout of tables and graphs for Part A
Handout of situations in Part B

Lesson- <u>Part A</u>

Given the tables and graphs below, determine which are representative of the same data.

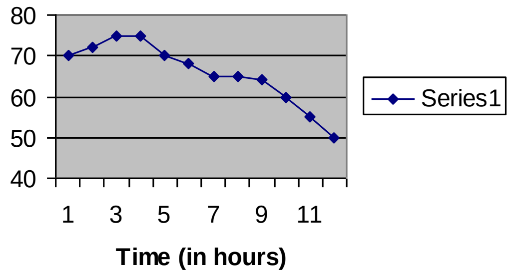
Then, write a description of what each representation "tells" you. Think about:

- Increments/Scales
- Connections
- Trends
- Data Types
-

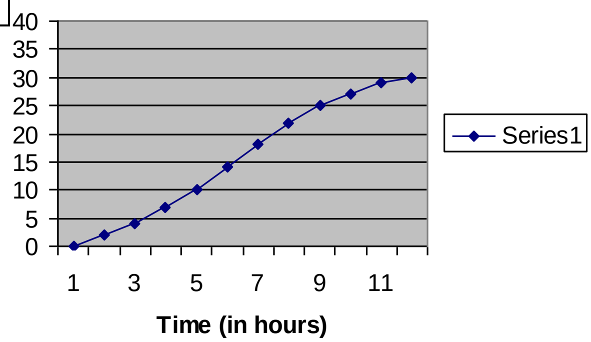
Which type of graph is most useful? When and why?

HANDOUT

1.



2.



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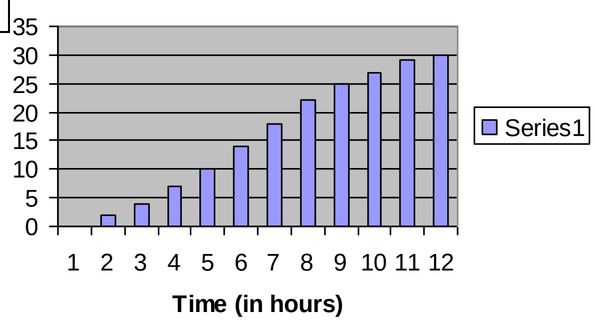
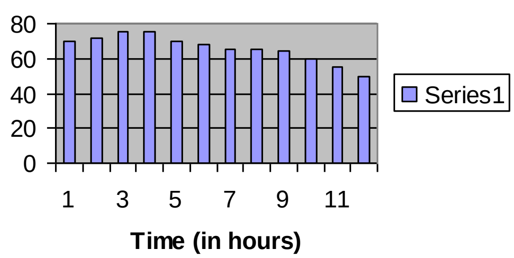


4.



5.

Time (in hours)	Gallons of Gas Used
1:00 PM	0
2:00 PM	2
3:00 PM	4
4:00 PM	7
5:00 PM	10
6:00 PM	14
7:00 PM	18
8:00 PM	22
9:00 PM	25
10:00 PM	27
11:00 PM	29
12:00 AM	30

6.

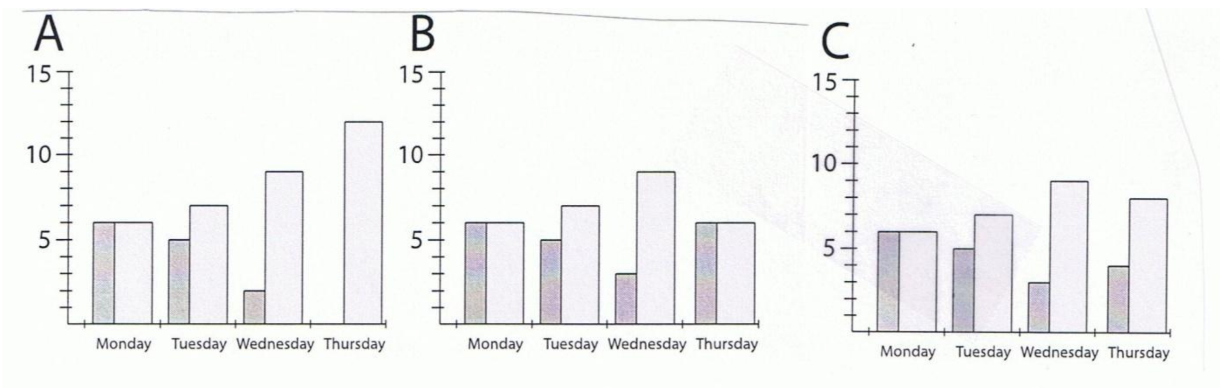
Time (in hours)	Degrees (in Fahrenheit)
1:00 PM	70
2:00 PM	72
3:00 PM	75
4:00 PM	75
5:00 PM	70
6:00 PM	68
7:00 PM	65
8:00 PM	65
9:00 PM	64
10:00 PM	60
11:00 PM	55
12:00 AM	50

Part B:

Anna and her brother, Frank, like to share a package of 12 strawberries every day after school. Anna notices that Frank has started to take more and more of his fair share of strawberries over the past few days.

- *Discuss what possible values or combinations of strawberries split between Anna and Frank might be. Which combinations will "fit" which day to reflect Frank getting more and more of his fair share.*

Which graph represents Frank's increasing rate of strawberry consumption? Anna = darker bars, Frank = lighter bars



Part C:

Which Graph is Best? →

Explain that data is called categorical data if it can be put into categories. Usually categorical data is shown in a bar graph. Explain that data is called numerical data if it uses numbers (like measuring or counting) and is measured over time. <u>Usually</u> numerical data is shown in a line graph.

Tell the students that you have some different data sets and you want some help deciding the best way to graph them and justification on the decision.

1.

Molly took a survey to find out what fruit the students in her class like. 20 students voted: 8 chose an apple, 4 chose a banana, 3 chose an orange, and 5 chose grapes.

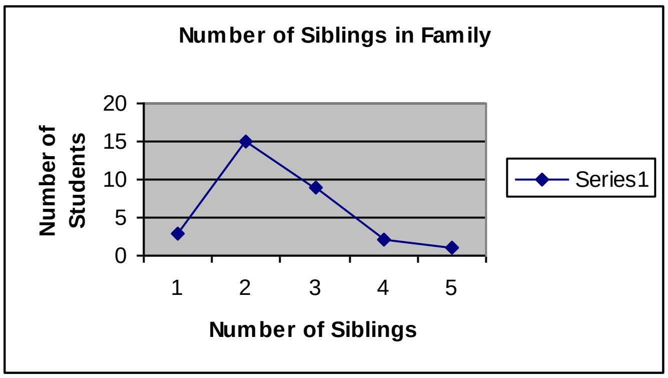
2.

Jamaal was growing a tulip in a cup for his mother. After a week, the bulb started growing. Then he measured the height of the plant each day. On day 8 the plant was 3 centimeters tall. On day 9, the plant was 4 cm tall. On day 10 it was still 4 cm tall. On day 11 it was 5 cm tall, on day 12 it was 7 cm tall, on day 13 it was 8 cm tall, and on day 14 it was 10 cm tall.

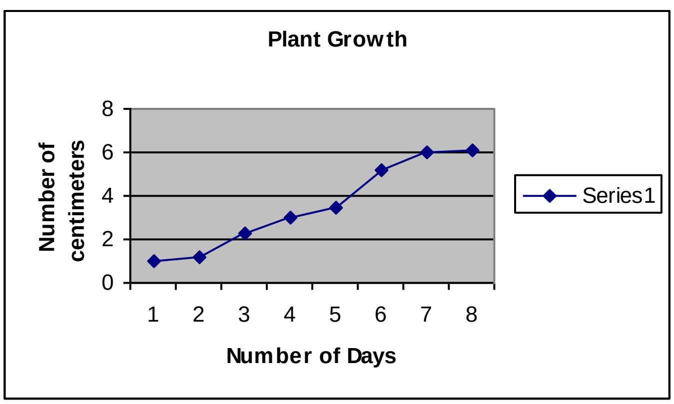
c. ***Misconceptions/Common Errors***

(these misconceptions were stated in module 2-5, but are important here as well)

Students sometimes tend to graph discrete data using a line graph which is for continuous data. Remember in a line graph, EVERY point on the line should have a value. Don't use a line graph when a bar graph may be a better choice. **Consider the graph below (it is an INCORRECT use of a line graph**. For example, is there a value at 1½ siblings? NO, but it looks like there is on the graph. (Adapted From Teaching Student Centered Mathematics Grades 3-5, Van de Walle, John A. and Lovin, LouAnn, page 334.)



The following is a CORRECT use of a line graph. Notice how it grows every day and in between whole number of days, as plants do – they continually grow.



d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

These two links will allow students to have more experiences with bar graphs.

http://nlvm.usu.edu/en/nav/frames_asid_323_g_2_t_1.html?from=category_g_2_t_1.html

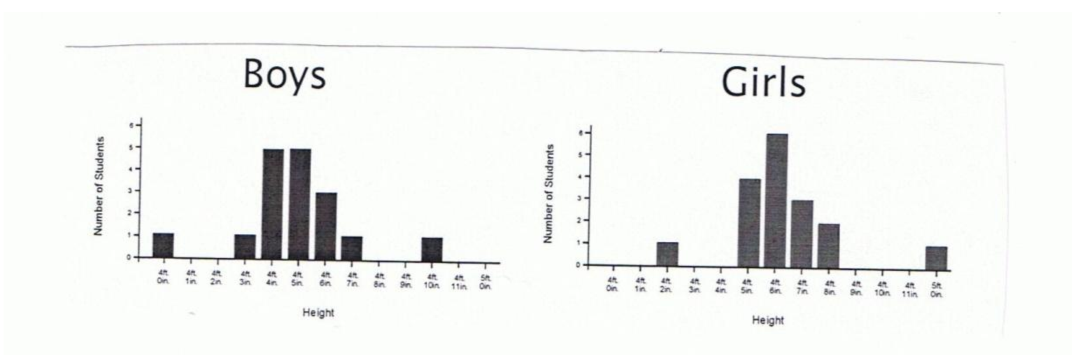
http://illuminations.nctm.org/ActivityDetail.aspx?ID=63
The following link is a lesson on analyzing data in tables and graphs.

http://illuminations.nctm.org/LessonDetail.aspx?id=U82

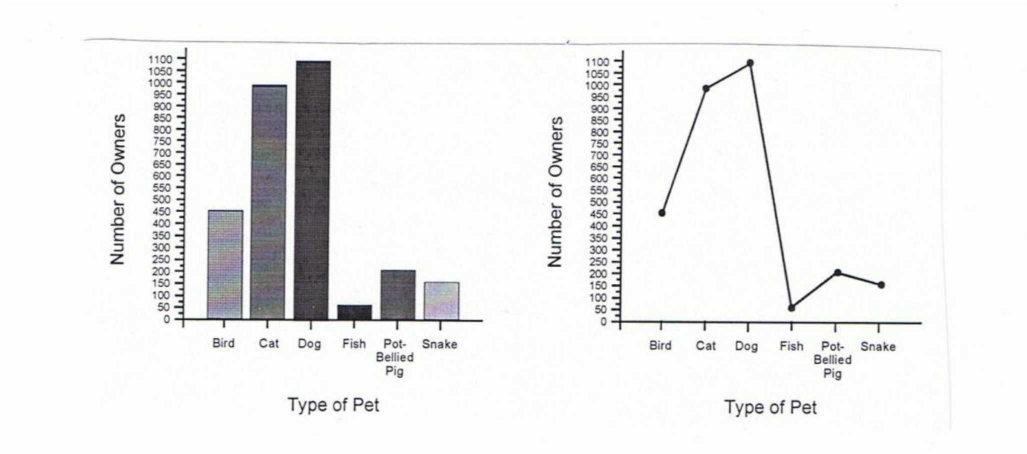
f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Give students the following graphs and the following prompts. Have them respond to the graphs.



What type of graph is this?
What information does it show?
What trends do you see?
Is the data increasing, decreasing, varying or staying the same?



Which type of data is it?
Which type of graph would be most appropriate and why?

III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The objective of this indicator is to <u>interpret,</u> which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge of interpreting data in tables, line graphs, bar graphs and double bar graph should be explored using a variety of examples. The learning progression to **interpret** requires students to <u>understand</u> the characteristics of each type of representation and <u>recognize</u> the limitations of each (4-1.8). Students <u>analyze</u> the representation (4-3.1) to detect changes in the data. Students <u>generate</u> descriptions and mathematical statements based on their observations and <u>explain</u> and <u>justify</u> their interpretation (4-3.3) using correct, complete, and clearly written and oral mathematical language (4-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

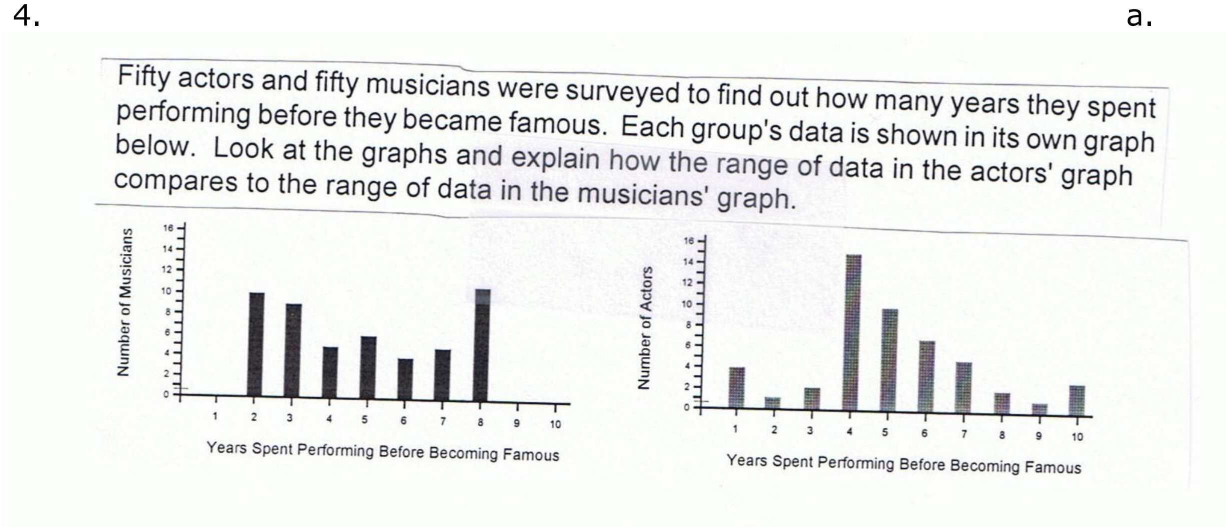
For 1-3, Decide if the data would be best shown in a table, bar graph or line graph. Then they should decide if it is categorical data or numerical data.

1. TJ and Fran wanted to know what type of ice cream was the favorite in their class, so they conducted a survey. They found out that 6 students like vanilla the best, 3 students like strawberry, 5 like chocolate, 4 like cookies and cream, and 1 likes butter pecan.

2. Mrs. Kim's class measured the temperature outside on the playground each hour during the school day. At 8:00 the temperature was 45 º. At 9:00 it was 49º. At 10:00 it was 53º, at 11:00 it was 60º, at noon it was 64º, at 1:00 it was 68º, and at 2:00 it was 70º.

3. Jamie found out that 4 students in her class have birthdays in January, none in February, 2 students have birthdays in March, 4 in April, 1 in May, 3 in June, 5 in July, none in August, 2 in September, 1 in October, 1 in November, and 3 students have birthdays in December.

4. a.



b. What type of graph could be used to display the information below more efficiently? Explain your reasoning.

MODULE

4-5

Probability

This module addresses the following indicators:

4-6.6 Predict on the basis of data whether events are *likely, unlikely, certain, impossible*, or *equally likely* to occur. (B2)
4-6.7 Analyze possible outcomes for a simple event. (B4)

* This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be** required to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

4-6.6
Predict on the basis of data whether events are likely, unlikely, certain, impossible, or equally likely to occur.

In first grade, students have used the terms *likely* and *unlikely* to discuss whether or not events will occur (1-6.4). In the second grade, students predicted on the basis of data whether events are *more likely* or *less likely* to occur (2-6.4). In third grade, students progressed to understanding whether event are likely, unlikely, certain, or impossible when the probability of an even is 0 or 1 (3-6.7) and used the terms *likely, unlikely, certain,* or *impossible* when making predictions (3-6.6)
In fourth grade, students use those terms and now include the term *equally likely* when making predication based on data. (4-6.6)
In fifth grade, students represent the probability of a single stage event in words and fractions. (5-6.5)

4-6.7
Analyze possible outcomes for a simple event.

In third grade, this mathematical concept is not formally addressed.

In fourth grade, is the first time students have been formally introduced to the concept of possible outcomes. (4-6.7)

In fifth grade, students conclude why the sum of the probabilities of the outcomes of an experiment must equal 1. (5-6.6)

- **Key Concepts/Key Terms**

    - Outcome*
    - Likely*
    - Unlikely*
    - Certain*
    - Impossible*
    - Equally likely*
    - Simple events*

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lesson(s)

1. Teaching Lesson A

For this indicator, it is **essential** for students to:

- Understand that probability is the chance that a given event will occur
- Understand that an event is something that might happen
- Understand that an outcome is a possible result of an experiment
- Understand that an event that is likely has a good chance of happening
- Understand that an event that is unlikely does not have a good chance of happening
- Understand that an event that is impossible will never happen
- Understand that an event that is certain will always happen
- Develop a conceptual understanding of these mathematical terms likely, unlikely, equally likely, certain, and impossible.
- Use the language of probability to predict whether events are likely, unlikely, certain, impossible, or equally likely to occur.

For this indicator, it is **not essential** for students to:

- Use fractions to predict the probability of events.

a. Indicators with Taxonomy

4-6.6 Predict on the basis of data whether events are *likely, unlikely, certain, impossible*, or *equally likely* to occur. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory lesson

Adapted from: Van de Walle, John A. & Lovin, LouAnn H.,2006. Teaching Student Centered Mathematics: Grades 3-5

Before students attempt to assign numeric probabilities to events, it is important that they have the basic idea that some events are certain to happen, some are impossible, and others have different chances of occurring that fall between these extremes.

During a discussion of probability, it is a good idea to discuss the extremes of chance situations—impossible and certain. Between these extremes are events that are possible but not certain. Have students suggest events that fall into these three categories: impossible, certain, and in between impossible and certain. To develop the idea of probability being on a continuum, students need to see that some possible events are more likely or less likely than others.

The use of devices that can be analyzed (spinners, number cubes, coins to toss, colored disks drawn from a bag, etc.) can help students make predictions about the likelihood of an event. The following activity is a game of chance with unequal outcomes. However, students will not readily be able to predict which result is most likely, so it provides a good opportunity for discussion.

Suggested Literature Connections

These books can be used to introduce or reinforce the concept of probability.

- *It's Probably Penny by Loreen Leedy*
  Lisa's class is learning about probability. For part of her homework, she has to think of an event that will happen, one that might happen, and one that can't happen.
- *A Very Improbable Story: A Math Adventure by Edward Einhorn*
  Waking up one morning and finding a talking cat on his head, Ethan is informed that the cat will not leave until he has won a game of probability. Challenge students to think of events that are likely, equally, likely unlikely, certain, or impossible.
- *Probably Pistachio, by Stuart J. Murphy*
  This book will help students learn about predicting the likelihood of any given event

- Use blank cubes to make 2 Number Cubes per pair of students with the sides 1, 1, 2, 3, 3, 3
- Add Then Tally recording sheet (sample below)

Add Then Tally

2										
3										
4										
5										
6										

Pairs of students take turns rolling the two cubes and record the sum of the two numbers. Students should record the results on the tally sheets. They continue to roll the cubes until one of the rows is full. They can repeat the game on a new tally sheet as long as time permits.

Discussion after the students play the game is important. Ask questions such as:

- Which numbers "won" the most often?
- Which number "lost" the most often?
- If you were to play the game again, which number would you pick to win and why?

All outcomes are possible. A sum of 4 is most likely. Sums of 2 or 3 are the least likely. However, since few if any students will analyze the possible outcomes, their predictions for future games will tell you a lot about their probabilistic reasoning. Students who observe that 4 comes up a lot and therefore, is the best choice to win have abandoned earlier subjective ideas about luck or of chance having a memory.

c. Misconceptions/Common Errors

An understanding that chance has no memory can only come with experience and with discussion with peers. Some adults still do not believe this. For example, many select lottery numbers because they have not come up recently.

d. Additional Instructional Strategies/Differentiation

Adapted from: Van de Walle, John A. & Lovin, LouAnn H.,2006. Teaching Student Centered Mathematics: Grades 3-5

Example Lesson 1

Materials Needed
Spinner (all white)
Spinner (all blue)
Spinner (half blue, half white)
Spinner (less than ¼ blue)
Spinner (more than ¾ blue)

To begin refining the concept that was started in the previous activity that some events are more or less likely to occur than others, introduce the idea of a continuum of likelihood between impossible and certain. Draw a long line segment on the board. Label the left "Impossible" and the right "Certain." Write "Chances of Spinning Blue" above the line. Call this the "probability line" or the "chance line."

Chances of Spinning Blue

Impossible●---●Certain

Next, show students a spinner that is all white. Ask "What is the chance of spinning blue with this spinner?" Indicate the left end of the probability line as showing this chance. Repeat with an all-blue spinner, indicating the right end, labeled Certain. Next show a spinner that is half blue and half white. Ask "What is the chance of spinning blue with this spinner?" The discussion should develop a consensus that it is equally likely that blue will come up as not blue. Place a mark exactly at the center of the line to indicate this chance. (See line segment below.) You may want to note that this represents a chance of ½ although the position of the mark is sufficient.

Impossible●-------------------------|----------------------------●Certain

Repeat the preceding discussion with a spinner that is less than ¼ blue and with one that is nearly all blue. Ask students where they would put a mark on the line to indicate the chance of spinning blue for each of these. These marks should be close to the ends of the line segment. (See line segment below.)

Impossible●----|---------------------|------------------------|----●Certain

To review these ideas, show the spinners one at a time and ask which marks represent the chance of getting blue for the spinner.

Example Lesson 2: Design a Bag

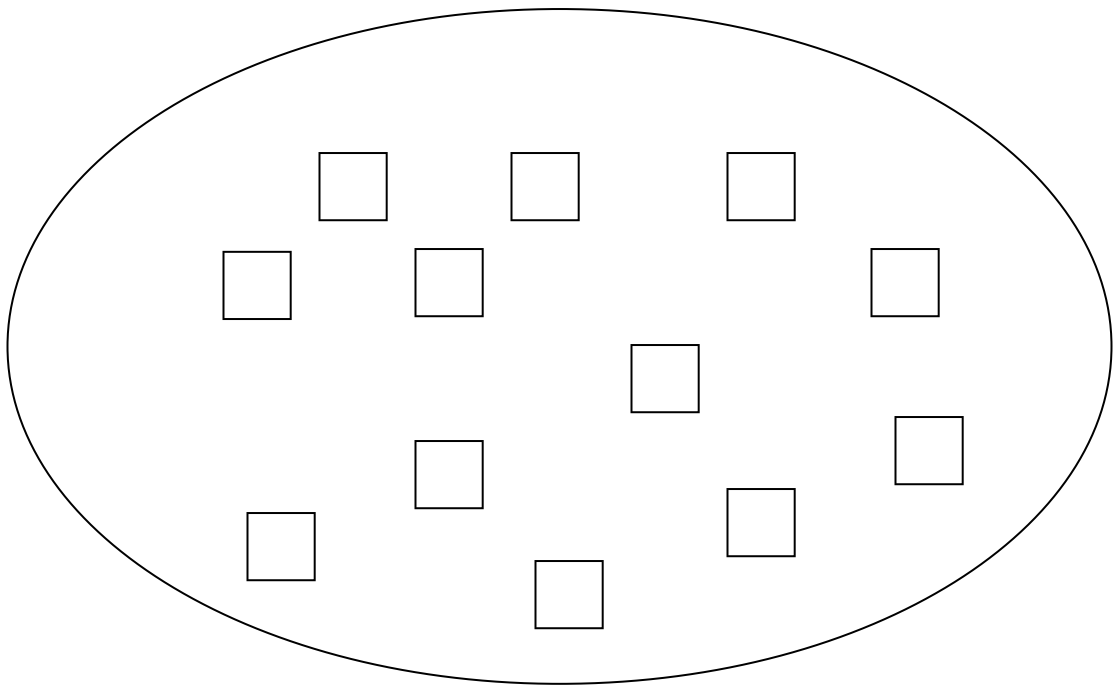
Materials Needed:
bag activity sheet
colored pencils/crayons
colored tiles/cubes
bags

Students will design a bag that they think will create chances for various positions on the probability line. Students should use a bag of colored tiles or cubes before transferring their information onto the sheet. Even if drawing tiles from a bag seems new to your students, do not provide additional hints to help them with their reasoning.

Provide pairs of students with a copy of a worksheet similar to the sample below, colored tiles, and a bag. The sheet can be sketched by hand. Be sure there are 12 squares drawn on the bag. On the board, mark a place on the probability line to represent unlikely. At this time, do NOT use fraction language with the students. Students should mark this position on their worksheet probability line. Students should color the square indicated by "Color" at the top of the page. Explain that they are going to decide what color tiles should be put in the bags of 12 total tiles so that the chance of drawing this designated color is about the same as the chance indicated on the line. Talk about how many would be colored if the mark was very close to the middle of the line. Show how the real bags will be filled based on the design on the page. Emphasize that the tiles will be shaken up so that which particular squares on the bag design are colored makes no difference. (This activity can also be repeated with likely, certain, impossible, equally likely.) Another variation of this activity might be to use spinners instead of tiles and bags.

Names____

Color: ☐



Impossible●---●Certain

Testing Bag Designs

After students have finished, have them discuss their designs in "Design a Bag." Some students may think the colors of the other tiles make a difference. Discuss this point. Choose the bag design that the most students agree on for unlikely (or which ever you choose). Have students conduct the experiment based on the Bag Design that was agreed upon by most. Have them tally the times they get the designated color and times they don't. This should be repeated at least 10 times replacing each tile after it is drawn and recorded.

Discuss how individual experiments turned out. Make a large bar graph or tally graph of the data for all of the groups together. Discuss the advantages of repeating an experiment many times.

Additional activities are available in Teaching Student Centered Mathematics: Grades 3-5, Van de Walle, John A. & Lovin, LouAnn H., 2006. and Navigating through Data Analysis and Probability in Grades 3-5, NCTM, 2006.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

This is a suggestion for a resource:

- http://www.nsa.gov/academia/_files/collected_learning/elementary/patterns/games_galore.pdf
- Probability-Bright Balloons http://www.harcourtschool.com/activity/bright_balloons/

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Use the discussions during the activities to formally assess students. Also, the green, yellow and red cups would be useful during these activities.

2. Teaching Lesson B: Bruce's Game

- An example of a simple event activity is using games spinners with certain portions shaded and considering possible outcomes the spinner will land on.
- Analysis also means knowing how the possible outcomes relate to each other. For example, three 5th graders and two 6th graders names are put in a hat because they are running for class president. What are the chances that a 5th grader will be chosen? Answers should include simple ratios like: 1 out of 5; 1 out of 3, and 3 out of 5.
- Providing students with real-world concrete materials to analyze the possible outcomes for a simple event will be helpful. Using

pictorial representations such as a tree diagram is an excellent way to help students visualize the possible outcomes of a simple event.

For this indicator, it is ***essential*** *for students to:*

- *Understand the meaning of simple event*
- *Understand the meaning of outcomes*
- *Understand how the possible outcomes relate to each other.*
- *Explore probability through experiments that have possible outcomes*

For this indicator, it is ***not essential*** *for students to:*

- *Use fractions to analyze the possible outcomes for a simple event.*

a. Indicators with Taxonomy

4-6.7 Analyze possible outcomes for a simple event. (B4)

Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

Suggested Literature Connection:

- The Sundae Scoop- Stuart J. Murphy
  Lauren, James and Emily help out and are amazed by how many different kinds of sundaes you can make with just two ice-cream flavors, two sauces and two types of toppings. But when supplies run low, the number of combinations changes. Determining how many different combinations can be made from given sets of items is an important first step in understanding probability.

Lesson: Bruce's Games
Adapted from CSMP
http://ceure.buffalostate.edu/~csmp/CSMPProgram/Intermediate%20Disk/IG_III/IG-III%20Lesson%20Plans/IGIII-PStrand.pdf

Group size: whole class, small groups of 2 or 3, whole class

Materials: dice (2 each a different color) for each small group
Handout-Bruce's Games
Transparency

Engagement: Ask the students what kind of games they like to play and what makes them fun to play? Lead the discussion to the idea that games are more fun if they are fair. Tell them that you are going to look at a game that "Bruce" made up and determine if it is fair.

Exploration: Use this story for exploration. You might want to substitute students' names for people in the story.
Bruce likes to make up games. One day when he was playing with Linda and Dave, he made up a game using 2 dice. He said that they would take turns rolling the 2 dice and finding the sum. If the sum was 2, 3, 4, or 5, Dave would win. If the sum was 6, 7, or 8, Bruce would win and if the sum was 9, 10, 11, or 12, Linda would win. ("Does this sound fair to you? Why or why not?" Have the students rate each person's chance of winning from 1 to 10, 1 being never and 10 being all the time.) At first, Dave and Linda thought this would be a great game; but after playing for awhile, they weren't so sure anymore.
Play this game as a whole class with someone at the board recording who is winning each time. After about 30 rolls of the dice, ask the class if they think the game is fair. Who is winning? Why do they think he/she is winning so much?

Divide the class into groups of 2 or 3. Give each group 2 dice of different colors and the handout. Have each group label each side by the color of their die. Ask them how many spaces are in the grid? (36) Tell them that they are going to roll the dice and record the sum in the grid. If they roll a green 5 and a white 3, then they go across the grid to green 5 and up to white 3 and write 8 on the grid at the place where that ordered pair meets. Have them predict how many times they think they will have to roll the dice before they fill in the chart? Then let them start rolling the dice and filling in the chart. When they have been working for awhile, ask them if they need to change their prediction on how many times they will have to roll the dice? Why or why not? When they have filled in most of the chart, ask them to fill in the rest without rolling the dice. What patterns did they see? What are they noticing?

Explanation: Have the students circle all the different ways that Dave could win the game, then Bruce, and then Linda. Teacher does this on the overhead or interactive white board to demonstrate.
Ask the students "What are you noticing about the chances of each person's chances of winning?" "Is this a fair game? Why or why not?"

Extension: Ask the students to come up with a game similar to Bruce's except everybody has an equal chance of winning? (Find out the probability of each sum and then combine the sums together so that each person has the same probability.)

Bruce's Games

Use this chart to determine the probability of each person's chance of winning the game. Label the right side with one color of the die and the bottom with the other color of the die. Roll both dice and place the sum of the dice in the appropriate spot on the graph.

Before you start rolling the dice, predict how many times you will have to roll the dice to completely fill in the chart. ______________

When you finish the chart, answer these questions?

How many times did you have to roll the dice to fill in the chart? _________

What patterns do you see?

What are you noticing about the chances of each person's chances of winning?

Is this a fair game? Why or why not?

Sums						
6						
5						
4						
3						
2						
1						
	1	2	3	4	5	6

c. Misconceptions/Common Errors

It is important to remember that the intent of this indicator (4-6.7 Analyze possible outcomes for a simple event.)
Is that students should analyze possible outcomes which implies more than just knowing that the possible outcomes for flipping a coin are *heads* or *tails*. Analysis also means knowing how the possible outcomes relate to each other.

Student may still struggle with these concepts because they believe in luck and chance in their daily lives.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

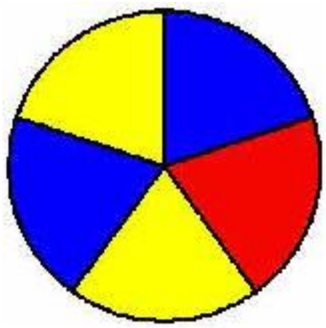
e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

One possible formative assessment strategy for this lesson is the use of an exit ticket. The spinner or a similar spinner could be shown on the Promethean board so that students could respond to the question.



What are the chances that the spinner will land on red? Explain your reasoning.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

4-6.6
Predict on the basis of data whether events are likely, unlikely, certain, impossible, or equally likely to occur.

The objective of this indicator is to predict, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To predict is to draw conclusions from presented information. Conceptual knowledge is not bound by specific examples; therefore, students use a variety of examples to make predictions. The learning progression to **predict** requires students to recall the meaning of likely, unlikely, certain, equally likely and impossible. Students explore teacher generated examples and construct arguments (3-1.2) about whether the event is likely, unlikely, certain, equally likely or impossible. They explain and justify their arguments (3-1.3) with their classmates and their teacher. Students generalize connections among examples of each event and deepen their conceptual understanding by generating examples of events that are likely, unlikely, certain, equally likely or impossible. Students use correct, complete, and clearly written and oral mathematical language to explain and justify their examples (4-1.5).

4-6.7
Analyze possible outcomes for a simple event.

The objective of this indicator is to analyze, which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. Analyze conceptual break down data into its parts and determine how the parts relate to one another and to the overall structure (single event); therefore, the student's conceptual knowledge of analyzing outcomes should be explored using a variety of examples. The learning progression to **analyze** requires students to understand the characteristics of simple events. Students generate conjectures about the outcomes to a given event. They then simulate real world experiments using concrete and/or pictorials models and generate mathematical statements about relationships they observe (4-1.4). They explain and justify their observations (4-1.3) to their classmates and their teacher and use correct, complete, and clearly written and oral mathematical language to pose questions, communicate ideas, and extend problem situations (4-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Israel rolls 2 number cubes. Each cube has faces numbered 1 to 6.

Which event is **most likely** to happen?
A. He rolls a sum of 12.
B. He rolls a sum of 7.
C. He rolls a sum of 1.

2. Royce has a bag with 8 red marbles, 4 blue marbles, 5 green marbles, and 9 yellow marbles all the same size. If he pulls out 1 marble without looking, which color is he most likely to choose?
   A red
   B blue
   C green
   D yellow

The following item #3 is adapted from NCTM, 2005. Mathematics Assessment Sampler: Grades 3-5, page185

3. On the swim team are 3 fourth grade students and 2 fifth grade students. Everyone's name is put in a hat, and a captain is chosen by picking one name. What are the chances that the captain is a fifth grader? Is it likely? Explain your answer.

4. Are the following outcomes equally likely?
   a. rolling 1 die and getting an even number/rolling 1 die and getting an odd number
   b. forecast says 80% chance of rain and no rain
   c. forecast says 0% chance of rain and no rain