# SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

| **Content Area** | Fifth Grade Mathematics |
|---|---|
| **Recommended Days of Instruction** | **First Nine Weeks** |

**Standards/Indicators Addressed:**

**Standard 5-2**: The student will demonstrate through the mathematical processes an understanding of the place value system; the division of whole numbers; the addition and subtraction of decimals; the relationships among whole numbers, fractions, and decimals; and accurate, efficient, and generalizable methods of adding and subtracting fractions.

5-2.1* Analyze the magnitude of a digit on the basis of its place value, using whole numbers and decimal numbers through thousandths. (B4)
5-2.2* Apply an algorithm to divide whole numbers fluently. (C3)
5-2.3* Understand the relationship among the divisor, dividend, and quotient. (B2)
5-2.4* Compare whole numbers, decimals, and fractions by using the symbols <, >, and =. (B2)
5-2.5* Apply an algorithm to add and subtract decimals through thousandths. (C3)
5-2.6* Classify numbers as prime, composite, or neither. (B2)
5-2.7* Generate strategies to find the greatest common factor and the least common multiple of two whole numbers. (B6)
5-2.8* Generate strategies to add and subtract fractions with like and unlike denominators. (B6)
5-2.9* Apply divisibility rules for 3, 6, and 9. (C3)

* These indicators are covered in the following 4 Modules for this Nine Weeks Period.
Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.

| **Module 1-1 Number Structure and Relationships – Whole Numbers, Fractions, and Decimals** | | | |
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| **Indicator** | **Recommended Resources** | **Suggested Instructional Strategies** | **Assessment Guidelines** |

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| **Module 1-1 Lesson A**<br><br>5-2.1 Analyze the magnitude of a digit on the basis of its place value, using whole numbers and decimal numbers through thousandths. (B4) | **STANDARD SUPPORT DOCUMENT**<br>http://www.ed.sc.gov/agancy/standard-and-learning/academic standards/math/index.html<br><br>NCTM's Online Illuminations<br>http://illuminations.nctm.org<br><br>NCTM's Navigations Series 3-5<br><br>Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle<br><br>Blackline Masters for Van de Walle Series | See Instructional Planning Guide Module 1-1 Introductory Lesson A<br><br>See Instructional Planning Guide Module 1-1, Lesson A Additional Instructional Strategies | See Instructional Planning Guide Module 1-1 Lesson A Assessing the Lesson |

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| **Module 1-1 Lesson B**<br><br>5-2.4 Compare whole numbers, decimals, and fractions by using the symbols <, >, and =. (B2) | www.ablongman.com/<br><br>vandewalleseries<br><br>NCTM's Principals and Standards for School Mathematics (PSSM)<br><br>NCTM, Mathematics Assessment Sampler: Grades 3-5<br><br>ETA Cuisenaire, Hands-On Standards: Grades 5-6 | See Instructional Planning Guide Module 1-1 Introductory Lesson B | See Instructional Planning Guide Module 1-1 Lesson B Assessing the Lesson |

| **Module 1-2 Division** | | | |
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| **Indicator** | **Recommended Resources** | **Suggested Instructional Strategies** | **Assessment Guidelines** |
| **Module 1-2 Lesson A**<br><br>5-2.2 Apply an algorithm to divide whole numbers fluently. (C3)<br><br>5-2.3 Understand the relationship among the divisor, dividend, and quotient. (B2) | **STANDARD SUPPORT DOCUMENT**<br>http://www.ed.sc.gov/agancy/standard-and-learning/academic standards/math/index.html<br><br>NCTM's Online Illuminations http://illuminations.nctm.org<br><br>NCTM's Navigations Series 3-5<br><br>Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle<br><br>Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries | See Instructional Planning Guide Module 1-2 Introductory Lesson A | See Instructional Planning Guide Module 1-2 Lesson A Assessing the Lesson |

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| **Module 1-2 Lesson B**<br><br>5-2.9 Apply divisibility rules for 3, 6, and 9. (C3) | NCTM's Principals and Standards for School Mathematics (PSSM)<br><br>NCTM, Mathematics Assessment Sampler: Grades 3-5<br><br>ETA Cuisenaire, Hands-On Standards: Grades 5-6 | See Instructional Planning Guide Module 1-2 Introductory Lesson B<br><br>See Instructional Planning Guide Module 1-2, Lesson B Additional Instructional Strategies | See Instructional Planning Guide Module 1- 2 Lesson B Assessing the Lesson |

| Module 1-3 Number Structure and Relationships - Whole Numbers | | | |
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| **Indicator** | **Recommended Resources** | **Suggested Instructional Strategies** | **Assessment Guidelines** |
| **Module 1-3 Lesson A**<br><br>5-2.7 Generate strategies to find the greatest common factor and the least common multiple of two whole numbers. (B6) | **STANDARD SUPPORT DOCUMENT**<br>http://www.ed.sc.gov/agency/standard-and-learning/academic standards/math/index.html<br><br>NCTM's Online Illuminations http://illuminations.nctm.org<br><br>NCTM's Navigations Series 3-5<br><br>Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle<br><br>Blackline Masters for Van de Walle Series | See Instructional Planning Guide Module 1-3 Introductory Lesson A<br><br>See Instructional Planning Guide Module 1-3, Lesson A Additional Instructional Strategies | See Instructional Planning Guide Module 1-3 Lesson A Assessing the Lesson |

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| **Module 1-3 Lesson B**<br><br>5-2.6 Classify numbers as prime, composite, or neither. (B2) | www.ablongman.com/vandewalleseries<br><br>NCTM's Principals and Standards for School Mathematics (PSSM)<br><br>NCTM, Mathematics Assessment Sampler: Grades 3-5<br><br>ETA Cuisenaire, Hands-On Standards: Grades 5-6 | See Instructional Planning Guide Module 1-3 Introductory Lesson B | See Instructional Planning Guide Module 1- 3 Lesson B Assessing the Lesson |

| Module 1-4 Operations – Addition and Subtraction | | | |
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| **Indicator** | **Recommended Resources** | **Suggested Instructional Strategies** | **Assessment Guidelines** |
| **Module 1-4 Lesson A**<br><br>5-2.5 Apply an algorithm to add and subtract decimals through thousandths. (C3) | **STANDARD SUPPORT DOCUMENT**<br>http://www.ed.sc.gov/agancy/standard-and-learning/academic standards/math/index.html<br><br>NCTM's Online Illuminations http://illuminations.nctm.org<br><br>NCTM's Navigations Series 3-5<br><br>Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle<br><br>Blackline Masters for Van de Walle Series | See Instructional Planning Guide Module 1-4 Introductory Lesson A | See Instructional Planning Guide Module 1-4 Lesson A Assessing the Lesson |

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| **Module 1-4 Lesson B**<br><br>5-2.8 Generate strategies to add and subtract fractions with like and unlike denominators. (B6) | www.ablongman.com/vandewalleseries<br><br>NCTM's Principals and Standards for School Mathematics (PSSM)<br><br>NCTM, Mathematics Assessment Sampler: Grades 3-5<br><br>ETA Cuisenaire, Hands-On Standards: Grades 5-6 | See Instructional Planning Guide Module 1-4 Introductory Lesson B<br><br>See Instructional Planning Guide Module 1-4, Lesson B Additional Instructional Strategies | See Instructional Planning Guide Module 1- 4 Lesson B Assessing the Lesson |

# MODULE 1-1

## Number Structure and Relationships – Whole Numbers, Fractions, and Decimals

**This module addresses the following indicators:**

5-2.1 Analyze the magnitude of a digit on the basis of its place value, using whole numbers and decimal numbers through thousandths. (B4)
5-2.4 Compare whole numbers, decimals, and fractions by using the symbols <, >, and =. (B2)

This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY.** Lessons in $S^3$ begin to build the conceptual foundation students need. **ADDITONAL LESSONS will be required** to fully develop the concepts.

## *I. Planning the Module*

- **Continuum of Knowledge:**

5-2.1
Fourth grade is the first time students are introduced to the concept of decimals. Students compare decimals through hundredths by suing the terms is less than, is greater than and is equal to and the symbols < ,> or = (4-2.7).

In fifth grade, students compare whole number, decimals, and fractions by using the symbols <, > and = (5-2.4) and apply an algorithm to add and subtract decimals through thousandths (5-2.5).

In sixth grade, students generate strategies to multiply and divide fractions and decimals (6-2.5).

5-2.4
Fourth grade is the first time students are introduced to the concept of decimals. Students compare decimals through hundredths by using the terms is less than, is greater than and is equal to and the symbols < ,> or = (4-2.7).

In fifth grade, students compare whole number, decimals, and fractions by using the symbols <, > and = (5-2.4) and apply an algorithm to add and subtract decimals through thousandths (5-2.5).

In sixth grade, students generate strategies to multiply and divide fractions and decimals (6-2.5).

- **Key Concepts/ Key Terms**

These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use and conversation for students.

Place value
* Tenths
* Hundredths
* Thousandths
Decimal
*Magnitude
*Fraction (mixed, improper, proper)
Whole Number
*<, > and = ( less than, greater than, equal to)
Numerator
Denominator

## *II. Teaching the Lessons*

### *1. Teaching Lesson A*: The Role of the Decimal Point

Students have analyzed the magnitude of a digit based on its place value since kindergarten. Therefore, this concept is not new. The only change and where emphasis should be placed is on the decimal portion. Decimals were introduced for the first time in fourth grade and students analyzed digits through hundredths. Fourth grade students also generated strategies to add and subtract decimals through hundredths.

In fourth grade students had opportunities to analyze decimal numbers as a part of a whole using concrete and pictorial models. The focus was on conceptual understanding of decimals through hundredths.

Students have had experiences with concrete models of whole numbers, fractions, and decimals. They have previously used symbols to compare whole numbers to other whole numbers.

In fifth grade, students should have an understanding through thousandths. Fifth grade students should also examine the relationship between the place value structure of whole numbers and the place value structure of decimals through thousandths.

Students in fifth grade should build on prior concrete experiences and learn to move fluently and confidently among and between the representations of whole numbers, fractions and decimals using symbols for comparison. Students should be able to decompose whole numbers and extend this notion to decimal numbers. (Principles and Standards for School Mathematics, p. 150) Again, however, student work with decimals should be limited to thousandths and there is no limit on the magnitude of fractions. Sound educational practice dictates that fractions should be of reasonable size and that emphasis is on understanding the relative magnitude NOT on applying some memorized pneumonic device when making comparisons.

Fifth grade builds on the conceptual understanding of decimals and extends decimal place value through thousandths.

Fifth grade is the first time students have compared whole numbers to decimals and fractions and decimals and fractions to each other. Classroom experiences should also include mixed numerals and improper fractions. Because indicator 5-2.1 limits decimal experiences through thousandths, decimals should not exceed thousandths place.

**5-2.1**
For this indicator, it is **essential** for students to:

- Understand place value through thousandths.
- Understand the relationship between the place value structure of whole numbers and the place value structure of decimals through thousandths.
- The role of the decimal point is to designate the unit position.

For this indicator, it is **not essential** for students to:
None noted

**5-2.4**
For this indicator, it is **essential** for students to:

- Compare fraction to whole numbers, decimals to fractions, whole number to decimal, etc…
- Compare whole numbers and fractions to decimals that do not exceed the thousandths place (as per Indicator 5-2.1)
- Understand mixed numerals and improper fractions
- Use a number line to make comparisons
- Compare fraction to fraction with compatible denominators i.e. 2/5 and 1/10

For this indicator, it is **not essential** for students to:
None noted

### *a. Indicators with Taxonomy*

5-2.1 Analyze the magnitude of a digit on the basis of its place value, using whole numbers and decimal numbers through thousandths. → (B4)

*Cognitive Process Dimension: Analyze*
*Knowledge Dimension: Conceptual Knowledge*

### *b. Introductory Lesson*

***Materials Needed:***

- Base Ten Blocks
- Overhead Base Ten Blocks
- Decimal model sheet

- Write place values *hundreds* through *thousandths* across the top of the board, including the decimal point

**Possible Literature Connections**
*Zero the Hero* by Jane Lester

The zero doesn't know where it fits in so other numbers jump into to help it find its place.

*A Grain of Rice* by Helena Clare Pittman

A clever, cheerful, hard working farmer's son wins the hand of a Chinese princess by outwitting her father the Emperor, who treasures his daughter more than all the rice in China.

**Lesson A**

Adapted from: Teaching Student Centered Mathematics: Grades 3-5, Van de Walle, John A. & Lovin, LouAnn H., Pearson Learning, 2006, pages 183-185.

Before introducing decimals to students, it is advisable to review some ideas of whole number place value (the 10 to 1 relationship between the value of any two adjacent positions. See Module 1-1, Grade 4).

Having established the progression to larger pieces, focus on the idea that each piece to the right gets smaller by 1/10. The critical question is "Is there ever a smallest piece?" Couldn't any piece be divided into 10 smaller pieces? There is no smallest strip or smallest square. The goal of this discussion is to help students see that a 10 to 1 relationship can extend indefinitely in two directions.

Important Points to Remember:
✓ Any unit can be chosen as the ones unit.
✓ The role of the decimal point is to designate the unit position.

Allow students to work in pairs. Distribute base ten blocks to each pair.

Inform students that the thousands cube represents "one". Ask students pairs to discuss how they would describe the hundreds flat, tens strip and cube in relationship to the whole. Allow groups to share. Because of their work with fractions, they may report in fraction form. After groups have shared, inform students there is another form of fractions called *decimals* and demonstrate how to write the decimal notation, lining the numbers up with the place value name written across the top of the board. Do not stress the fraction decimal relationship at this time.

Ask student pairs to use their hundreds flat, tens strip and cubes to form another decimal. Allow student groups to share, demonstrate on the overhead and write on the board, again lining up with the place values previously listed on the board. Keep a horizontal list of examples on the board.

Next, tell students to look closely at the place value chart and list of examples on the board. Ask students to share what they notice about the place value chart – this may include their knowledge of whole number place value as well. The object here is for students to see the relationship between the whole number and decimal place values. Even though not required, ask students to predict what they think the next decimal place value will be based on their knowledge of whole number place value. Again, the purpose is for them to recognize the pattern and relationships.

Next, make the connection to picture form. Give student groups grid paper. Ask them to build a decimal with the base ten blocks and challenge their partner to label and represent the decimal on the decimal models paper. Have students save their work for use in the lesson below that deals with comparison of decimals.

### *c. Misconceptions/Common Errors*

No typical student misconceptions noted at this time.

### *d. Additional Instructional Strategies/Differentiation*

A review of the 10 to 1 relationship between the values of any two adjacent positions in whole number place value may be needed.

For additional instruction strategies, refer to NCTM, Navigating through Number and Operations: Grades 3-5. 2006

Adapted from Anderson 5:
With a partner, write numerals zero through nine and a decimal point on index cards. Teacher calls out a number. With partners, the students will build their number by placing their cards in proper order. Have partners explain as you move to the right of the decimal in a number. (It gets smaller by 1/10).

### *e. Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives.

Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

There is no specific technology recommended for this lesson at this time.

### *f. Assessing the Lesson*

*Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.*

Use an exit ticket to formatively assess this lesson. One possible question might be: What are your "take aways" from today's lesson?

## *2. Teaching Lesson B:* Compare Whole Numbers, Decimals, and Fractions

### *a. Indicators with Taxonomy*

5-2.4 Compare whole numbers, decimals, and fractions by using the symbols <, >, and =. (B2)

*Cognitive Process Dimension: Understand*
*Knowledge Dimension: Conceptual Knowledge, etc.*

### *b. Introductory Lesson*

### *Materials Needed*

- Index cards with whole numbers, fractions and decimals written on them (teacher made or student made)
- Number lines

**Lesson B**

The number line is useful for comparing whole numbers, decimals, and fractions. Students should be able to draw and mark a number line with whole numbers, with decimals, and with fractions and with a combination of all three types. They should be able to translate between forms and use whichever form is more convenient or more appropriate in the context of problems. Once students are comfortable with placing a combination of all three types on a number line, then have students compare using the symbols of <, >, and =.

### *c. Misconceptions/Common Errors*

Results of National Assessment of Educational Progress (NAEP) examinations reveal that students have difficulties with the fraction-decimal relationship. Division of the numerator by the denominator may be a means for converting fractions to decimals, but it does not contribute to the understanding of the resulting equivalence. Therefore, other methods for comparing whole numbers, decimals, and fractions are encouraged.

When comparing fractions, students may not have an understanding of the role of the numerator and denominator and will only focus on the denominator when comparing fractions. For example, with 3/8 and ¾ , student may incorrectly state that 3/8 is larger because 8 is larger than 4.

### *d. Additional Instructional Strategies/Differentiation*

Students use number cards to generate a number using 5 (this can vary based on ability of students) of the cards and then call on two students to come to the front of the room to make a true statement using the two numbers. (ex. 5.4356 is greater than 3.5353) You could vary this lesson by the assigned number of (cards) in the digits and also by use of the decimal point card.

Adapted from Anderson 5:
Write common fractions, decimal and whole numbers on cards. Distribute cards to student groups of 3 or 4. Have students put their cards in order from greatest to least and least to greatest. Regroup students in pairs. Have students compare their numbers using the symbols <, > or =. Share with whole group to check.

### *e. Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

There is no specific technology recommended for this lesson at this time.

### f. *Assessing the Lesson*

*Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.*

Using the index cards with the fractions and decimals written on them, have students record 5 number sentences using <,>, and = on a sheet of paper and submit as an exit ticket.

## *III. Assessing the Module*

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

5-2.1
The objective of this indicator is to analyze which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To analyze means to determine relevant features and relationships. The learning progression to **analyze** requires students to understand place value of whole numbers and decimals and be able to locate the correct place value. Students represent the place values using concrete and/or pictorial models and generalize the connections (5-1.6) between the two place value systems. They use these connections to generate statements (5-1.4) about the magnitude of numbers. Students explain and justify their answers (5-1.3) and use correct, complete and clearly written and oral language to communicate their ideas (5-1.5).

5-2.4
The objective of this indicator is to compare which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning therefore, students should not just learn procedural strategies for comparing but they should build number sense around these types of numbers. The learning progression to **compare** requires students to recognize the place value of digits through the thousandths and understand the magnitude of fractions, decimals and whole numbers. Students use their conceptual understanding to compare without dependent on a traditional algorithm and use concrete models to support understanding where appropriate. Students recognize mathematical symbols <, >, and = and their meanings. As students analyze (5-1.1) place value patterns, fractional relationships, they construct arguments and explain and justify their answer to classmates and their teacher (5-1.3). Students should use correct, complete and clearly written and oral mathematical language to communicate their reasoning (5-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

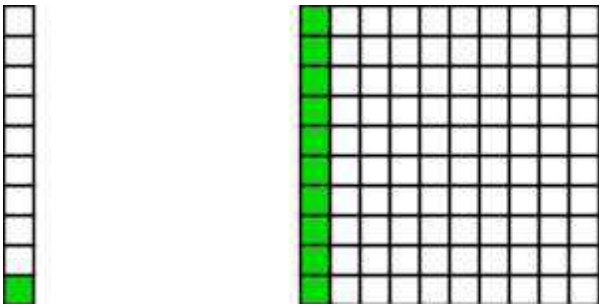
1. Use >, <, or = to compare the pairs of decimals and then explain each answer.

   16.2 _____ 16.20 Explain:

   16.02 _____ 16.20 Explain:

2. Which decimal is closer to 1, 0.99 or 0.999? Explain your reasoning.

3. The models below represent the same decimal value. How is this possible?



4. Bob is making a snack tray. He bought $\frac{3}{4}$ pound of cheese and 0.875 pound of meat. Write a number sentence that compares the cheese and meat.

5. Which of the following statements is true?

   A $0.5 < \frac{5}{8}$ B $0.15 > \frac{3}{10}$ C $\frac{4}{5} < 0.8$ D $\frac{1}{3} < 0.25$

**6.** What is the magnitude of the underlined digit?

325.01<u>2</u>

A. two
B. two tenths
C. two hundredths
D. **two thousandths**

7. Expand the following number to show the magnitude of each digit?

10,569.783

Possible answers

***10,000**
**500**
**60**
**9**
**0.7**
**0.08**
**0.003**

**or**

***10,000+500+60+9+0.7+0.8+0.003**

# MODULE

# 1-2

**This module addresses the following indicators:**

5-2.2 Apply an algorithm to divide whole numbers fluently. (C3)
5-2.3 Understand the relationship among the divisor, dividend, and quotient. (B2)
5-2.9 Apply divisibility rules for 3, 6, and 9. (C3)

This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY.** Lessons in $S^3$ begin to build the conceptual foundation students need. **ADDITONAL LESSONS will be required** to fully develop the concepts.

## *I. Planning the Module*

- ***Continuum of Knowledge***

5-2.2
In second grade focused on interpreting models of sharing equally (division) as repeated subtraction (2-2.6). In third grade, students compared the inverse relationship between multiplication and division (3-2.8).Fourth grade is the first time students are formally introduced to the concept of division. In fourth grade, students generate strategies to divide whole numbers by single-digit divisors (4-2.5) and they apply divisibility rules for 2, 5, and 10 (4-2.2).
In fifth grade, students apply an algorithm to divide whole numbers fluently (5-2.2) and understand the relationship among the divisor, dividend and quotient (5-2.3).
In sixth grade, students generate strategies to multiply and divide fractions and decimals (6-2.5).

5-2.3
In second grade focused on interpreting models of sharing equally (division) as repeated subtraction (2-2.6). In third grade, students compared the inverse relationship between multiplication and division (3-2.8).Fourth grade is the first time students are formally introduced to the concept of division. In fourth grade, students generate strategies to divide whole numbers by single-digit divisors (4-2.5) and they apply divisibility rules for 2, 5, and 10 (4-2.2).
In fifth grade, students apply an algorithm to divide whole numbers fluently (5-2.2) and understand the relationship among the divisor, dividend and quotient (5-2.3).
In sixth grade, students generate strategies to multiply and divide fractions and decimals (6-2.5).

5-2.9
Fourth grade is the first year students are formally introduced to the concept of division. Students apply divisibility rules for 2, 5 and 10 and generate strategies to divide whole numbers by single-digit divisors.
In fifth grade, students apply an algorithm to divide whole numbers fluently (5-2.2) and understand the relationship among the divisor, dividend and quotient (5-2.3)

- **Key Concepts/ Key Terms**

These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use and conversation for students.

*Algorithm
*Divisor
*Dividend
*Quotient
Relationship
*Divisibility
*Remainder
*Evenly Divisible
*Division
Even
Odd

## *I. Teaching the Lessons*

### 1. *Teaching Lesson A: Interactive Division*

Fourth grade students generated strategies to divide whole numbers by a single digit divisor with no remainders. That means their learning experiences involved strictly concrete and pictorial models for division – an emphasis on understanding division.

In fourth grade, students were introduced to the concept of divisibility rules for 2, 5, and 10.

In fifth grade student work should link those previous concrete and pictorial experiences from fourth grade to the symbolic. While fifth grade students should become fluent with division, sound educational practice dictates that the magnitude of the divisor and dividend should be reasonable. Division should not be a laborious task to be dreaded by students. In the contrary, students should see and understand division as a means to problem solving. Fifth grade learning experiences should involve quotients both with and without remainders. If students have a conceptual understanding of division, they have an understanding of the relationship among the divisor, dividend, quotient and remainder.

By applying an algorithm to divide whole numbers fluently, students should be able to explain what each number in a division algorithm means and understand the relationship among the divisor, dividend, quotient and remainder. For example, how the quotient becomes larger when the divisor is changed to a smaller digit or how the quotient becomes smaller when the divisor is changed to a larger digit. Students should also understand that if they are unable to efficiently find the answer to a problem such as 39 ÷ 3, they can decompose the dividend 39 to 30 + 9 then divide each easily by 3 so that 30 ÷ 3 = 10 and 9 ÷ 3 = 3 so that the quotients of 10 and 3 can be added to get 13. Again, the emphasis is on understanding and dividing fluently – not on pages of symbolic manipulation.

After fifth grade students are comfortable with applying an algorithm to divide whole numbers, the divisibility rules for 3, 6, and 9 should be introduced as a way of quickly determining by what numbers a whole number may be evenly divided.

***Teacher Note*:** Multiplication is not mentioned in the fifth grade standards; however, students should maintain multiplication fluency as part of the division algorithm.

**5-2.2**

For this indicator, it is **essential** for students to:

- Recall basic division facts
- Estimate and determine the reasonableness of the quotient
- Connect experiences with concrete models to symbolic representations (numbers only)
- Gain computational fluency

For this indicator, it is **not essential** for students to:

None noted

**5-2.3**

For this indicator, it is **essential** for students to:

- Recall and understand the meaning of dividend, divisor and quotient
- Understand the relationship between dividend and the numerator of a fraction and the divisor and denominator of a fraction

For this indicator, it is **not essential** for students to:

- Divide fractions

***a. Indicators with Taxonomy***

5-2.2 Apply an algorithm to divide whole numbers fluently. (C3)
*Cognitive Process Dimension: Apply*
*Knowledge Dimension: Procedural Knowledge*

5-2.3 Understand the relationship among the divisor, dividend, and quotient. (B2)
*Cognitive Process Dimension: Understand*
*Knowledge Dimension: Conceptual Knowledge*

***b. Introductory Lesson***

***Materials Needed***
None

**Possible Literature Connections**

*A Remainder of One*, by Elinor J. Pinczes

*When the queen of her bugs demands that her army march in even lines, Private Joe divides the marchers into more and more lines so that he will not be left out of the parade. Great book for whole group lesson on division with remainders of one. Children can use manipulatives and follow right along!*

*Divide and Ride* by Stuart J. Murphy

*Scream down the Dare-Devil Coaster and whirl around in the Twin Spin cars! Join in the carnival fun as 11 friends divide up to fit on the 2-to-a-seat roller coaster and the 4-to-a-cup teacups ride. Making new friends and practicing predivision skills have never been so exciting!*

***Lesson***

With the exception of "demonstrate fluency", students in fourth grade had experiences similar to those required at 5th grade. Therefore, in 5th grade, the goal is for students to have increased experiences and to refine their explanations with regard to their division strategies. Students may begin with a simple division game to review concepts. Tell the class to stand up. Ask them to look around and, without talking to become three groups, four groups etc. Review vocabulary by reminding students that the entire class represents the dividend, the number of groups is the divisor, and the number of students in each group is the quotient. The remaining students that have no groups are the remainder (leftover). This division game can be used throughout the year to practice and illustrate division concepts. Ask the students to explain what they notice about the size of the number of groups compared to the number of students in each group. For example, the greater the number of groups (divisor) the smaller the number in each group (quotient). Again, this is a review of 4th grade concepts and therefore, the emphasis is on student explanations.

The best type of problems come from the children's own experiences. Therefore, making up a problem similar to the following but which has features related to your school, would be the best approach:

Mrs. Richards' class feeds the rabbits 16 small pieces of carrot each day. If there are 128 small pieces of carrot in the refrigerator, how many days of food are on hand for the rabbits? Explain your thinking.

Since the emphasis is on clear, refined explanations, allow students time to share openly with the class and to write explanations in their math journals. As students share in class, listen for opportunities to question and praise clear thinking and to question and clarify misconceptions.

Use the same problem and change the divisor, ask the students to explain what they think will happen to the quotient and to justify their responses. Then, use the original problem and change the dividend. Again, ask the students to explain what they think will happen to the quotient and to justify their responses.

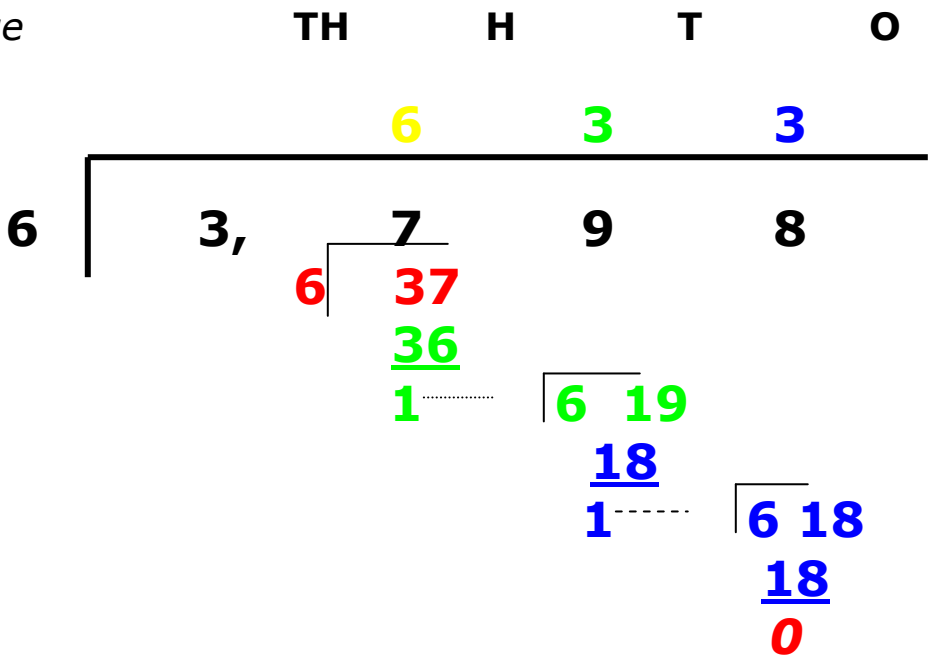
NOTE: It is more important to spend time thoroughly estimating, justifying estimations, working, and explaining work for one or two

good problems than to work a page of skill and drill practice problems. Completing a series of division problems does not mean the students understand what is happening. These standards require understanding and an ability to explain thinking. That will transfer to the students being able to create and solve their own division problems.

**NOTE: The following is <u>one</u> example of an explanation of the division process. Since students began division of two numbers by one number in 4th grade, they should have an algorithm they are comfortable explaining. The following is provided just for teacher information.**

NOTE: Indicator 5-2.2: <u>Apply an algorithm</u> to divide whole numbers fluently. This does not mean students must use the traditional long division algorithm.

*Place value*




*I am trading 3 thousands and 7 hundreds for 37 hundreds so they may be divisible by 6.*

*I am trading 1 hundred and 9 tens for 19 tens so they may be divisible by 6.*

*I am trading in 1 ten and 8 ones for 18 ones so they may be divisible by 6.*

*Students should know that the quotient will have no remainders because the dividend is an even number, and when the digits, 3,798 are summed, they add up to 27, a number divisible by 3.*

### *c. Misconceptions/Common Errors*

Although students have seen these words before, they may have difficulty relating these terms to the numerator and denominator in a fraction.

### *d. Additional Instructional Strategies/Differentiation*

**Index Card Division**:

Number cards 1-9 (4 of each)
Score sheet

1 or 2 players

Skill: division of 2 digit by 1 digit numbers

Object of game: to reach 100 in the fewest division problems

Directions:

1. Prepare a score sheet
2. Player One Player Two
3. Quotient / Score Quotient / Score
4. Turn over 3 cards and lay them down in a row, from left to right.
5. Use the 3 cards to generate a division problem. The two cards on the left form a 2-digit number. This is the dividend. The number on the card at the right is the divisor.
6. Divide the 2-digit number by the 1-digit number and record the results. This result is you quotient. Remainders are ignored. Paper may or may not be used.
7. Add you quotient to you previous score and record your new score. If this is the first round, your previous score was 0.
8. Players continue play until one player's score is 100.
9. This game could vary based on the number of cards per student. You could also change it to the first to reach 500 (or any other number)

### *e. Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

There is no specific technology recommended for this lesson at this time.

### *f. Assessing the Lesson*

*Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.*

Using an exit ticket, have students solve a problem that you've written based on their experiences. They should solve the problem using an algorithm of their choice and explain their thinking. Also, have them explain the relationship between the divisor, dividend and quotient in the problem.

## 2. *Teaching Lesson B: Strategies for Division Rules*

**5-2.9**
For this indicator, it is **essential** for students to:
- Be able to divide whole numbers with fluently in order to apply the divisibility rules
- Understand the divisibility rule for 3 Understand the divisibility rule for 6
- Understand the divisibility rule for 9

For this indicator, it is **not essential** for students to:
None Noted

### *a. Indicators with Taxonomy*

5-2.9 Apply divisibility rules for 3, 6, and 9. (C3)

*Cognitive Process Dimension: Apply*
*Knowledge Dimension: Procedural Knowledge*

### *b. Introductory Lesson*

***Materials Needed***
An index card with the following set of division problems – one card per student pair:

1 card:

1,434 ÷ 3 =

4,773 ÷ 3 =

765 ÷ 3 =

765 ÷ 3 =

1,235 ÷ 3 =

**Possible Literacy Connections**

*One Hundred Hungry Ants* by Elinor J. Pinczes
The ants divide themselves into groups of tens.

*Divide and Ride* by Stuart Murphy
A family goes to park and figures out how 11 of them get on the same ride.

*A Remainder of One* by Elinor J. Pinczes
Numerical division to practical problem.

Tell students that mathematicians have found ways to tell by looking at a number whether or not it will come out even when it is divided by certain digits. Challenge the students to be mathematicians and see if they can "figure" out the rule.

Then give each student pair an index card with the above division problems written on it. Ask the students, "By looking at the problems on the card, can you tell me which digit are we looking for a rule for first?" (3) "So, your challenge is to see if you can tell how you know which numbers are evenly divisible by three without doing the division."

Allow the student pairs time to work. Afterwards, allow students to share their strategies.

Repeat this process by making up division problems for 6 and for 9. Make up problems for the 9 first because the rule is similar to 3. Students will determine the strategy quicker and gain more confidence. Next follow with the 6. Do not tell the students the rule will

be different. Allow them time to figure this out. Be certain to include dividends that are only divisible by 2 and only divisible by 3.

NOTE: The following divisibility rules are provided for teacher information and should be discovered by students – not memorized:

- If the sum of the digits in the dividend is divisible by three, then the dividend is divisible by three. (1,434 = 1 + 4 + 3 + 4 = 12, then 1,434 is divisible by 3)
- If the dividend is divisible by both 2 and 3, then it is divisible by 6. ( 798 = 7+9+8= 24, then 897 is divisible by both 2 and 3.
- If the sum of the digits is divisible by 9 then the number is divisible by 9. ( 2,781 = 2 + 7 + 8 + 1 = 18, then 2,781 is divisible by 9.

### *c. Misconceptions/Common Errors*

No typical student misconceptions noted at this time.

### *d. Additional Instructional Strategies/Differentiation*

To practice the rules the following could be used:

*Materials Needed:*

Index cards with division problems that are divisible by 3, 6 OR 9.

*Divisibility War* is a game used to practice and review divisibility rules.

- The game is played with two to six players.
- Cards are shuffled and dealt face down to all players. Any extra cards will be set aside.
- Players take turns turning over cards and then stating whether the number on the top card is divisible by 3, 6, 0R 9.
- Players may challenge a statement of divisibility.
- The player with the most correct answers wins the round.

### *e. Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

There is no specific technology recommended for this lesson at this time.

### *f. Assessing the Lesson*

*Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.*

For an exit ticket, have students write a number that is divisible by 3, 6, and 9.

## *III. Assessing the Module*

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

**5-2.6**
The objective of this indicator is to classify which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning and conceptual knowledge is not bound by specific examples; therefore, students should gain a conceptual understanding of prime and composite number in order to classify any number. The learning progression to **classify** requires students to understand the characteristics of prime and composite numbers. Students analyze a variety of examples and compare the characteristics of each type of number. As students classify number, they explain and justify their answers (5-1.3) using correct, clear and complete oral and written mathematical language (5-1.5).

**5-2.7**
At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.
The objective of this indicator is to generate which is in the "create conceptual" knowledge cell of the Revised Taxonomy. The create means to put ideas together into a new structure; therefore, students use prior knowledge to generate new strategies. The learning progression to **generate** requires students to recall basic multiplication facts and understand how to write a number as the product of its factors. Students explore the concepts of LCM and GCF in context (story problems) and analyze information (5-1.1) from these experiences. They generate mathematical statements (5-1.4) about the relationships they observe then explain and justify their strategies (5-1.3) to their classmates and their teachers. Students recognize the limitations of various strategies and representations (5-1.8) and use correct, complete and clearly written and oral language to communicate their ideas (5-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Name a number between 300 and 400 that is divisible by 3.

2. Three friends, Kara, Cody, and Quinn worked on Mr. Cornell's fence. He paid them a total of $75. Will the 3 friends be able to evenly divide the money?

3. Name one 3-digit number that is divisible by 6.

4. Which of these numbers is divisible by 9?
   A 1111
   B 19
   C 334
   D 490
   E 495

5. Is 1,043 divisible by 9? How do you know?

6. Molly has one hundred forty-nine chocolate hearts that she wants to give to her five best friends, but she wants to make sure that each friend gets the same amount. If there is any candy left over, Molly will give it to her family. How many hearts will each friend receive? How many will her family receive?

7. What is 208 divided by 12?

8. KK Doughnuts was hired to cater the grand opening of a Burger Place. If a total of 300 doughnuts were ordered, and each box holds 12 doughnuts, how many boxes would they need to hold all the doughnuts?

9. Tommy had a problem. He just couldn't figure out how to solve the last equation in his math homework without using a calculator. Help him out, and explain to Tommy how you would solve 300÷15.

10. Justin and Chris are throwing a party and decide to grill hamburgers. Including themselves, there will be 35 people at the party. They want to make sure that there is plenty of food for everyone, so they buy 105 hamburger patties. How many hamburgers will each person get?

# MODULE

# 1–3

## Number Structure and Relationships - Whole Numbers

**This module addresses the following indicators:**
5-2.7 Generate strategies to find the greatest common factor and the least

common multiple of two whole numbers. (B6)

5-2.6 Classify numbers as prime, composite, or neither. (B2)

This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY.** Lessons in $S^3$ begin to build the conceptual foundation students need. **ADDITONAL LESSONS will be required** to fully develop the concepts.

## I. Planning the Module

- **Continuum of Knowledge:**

5-2.7
In fifth grade, students generate strategies to find the greatest common factor and the least common multiple of two whole numbers.
In sixth grade, students apply an algorithm to add and subtract fractions (6-2.4).

5-2.6
Students have not worked with prime or composite numbers prior to fifth grade.

- **Key Concepts/ Key Terms**

These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use and conversation for students.
*Factors
*Multiples
*Greatest Common Factor GCF
*Least Common Multiple LCM
Product
*Factor Tree
*Prime Number
*Composite Number

## II. Teaching the Lessons

### *1. Teaching Lesson A: Strategies for Greatest Common Factors and Least Common Multiples*

Students should be familiar with the terms *factor* and *multiple* since the concept of multiplication was introduced in third grade. In addition, fourth grade students were expected to explain the effect on the product when one of the factors were changed. As a result, fifth grade students should build on that knowledge when generating strategies to find the greatest common factor and the least common multiple of two whole numbers.

As the verb "Generate" implies in Indicator 5-2.7, students should be given opportunities to generate and share strategies as they develop a conceptual understanding of the greatest common factor and the least common multiple of two whole numbers. Experiences involving least common multiples and greatest common factors provide opportunities for students to work with rational numbers in a variety of problem solving situations. This will later help fifth grade students to begin generating strategies to add and subtract fractions with like and unlike denominators (indicator 5-2.8) as well as simplifying fractions. The emphasis is on student understanding, not memorizing a process, thus the need for students to generate their own strategies.

Continuing to use models and pictorial representations in fifth grade will help students connect to the symbolic representation of the concept of applying algorithms for simplifying fractions and adding and subtracting fractions with unlike denominators in later grades.

Fifth grade is the first year students classify whole numbers as prime, composite, or neither. In order to do so, many opportunities must be provided for students to conceptually understand these classifications. Experiences such as constructing arrays for whole numbers and categorizing the arrays into 2 groups of arrays with "Factors of 1 and Itself" (The number 11 only has 2 factors, 1 and 11) and arrays with "More than 1 Factor and Itself" (The number 10 has 4 factors of 1, 2, 5, 10) will enhance students' conceptual understanding. The number 1 is neither prime nor composite because it has only one factor - itself.

Initially using concrete or pictorial representations of multiplication arrays will enable students to concretely see and begin to classify numbers as prime, composite, or neither as the chart below indicates.

| | **Composite** | **Prime** |
|---|---|---|
| | More than 1 Factor and Itself | 1 Factor and Itself |
| **Numbers** | | |
| | **Factors** | **Factors** |
| **2** | | 1,2 |
| **3** | | 1,3 |
| **4** | 1,2,4 | |
| **5** | | 1,5 |
| **6** | 1,2,3,6 | |
| **7** | | 1,7 |
| **8** | 1,2,4,8 | |
| **9** | 1,3,9 | |
| **10** | 1,2,5,10 | |

**5-2.7**
For this indicator, it is **essential** for students to:
- Recall and understand basic multiplication facts
- Understand the meaning of multiples
- Explore GCF and LCM in context in order to support understanding

For this indicator, it is **not essential** for students to:
None noted

### *a. Indicators with Taxonomy*

5-2.7 Generate strategies to find the greatest common factor and the least common multiple of two whole numbers. (B6)

*Cognitive Process Dimension: Create*
*Knowledge Dimension: Conceptual Knowledge*

### *b. Introductory Lesson*

***Materials Needed***
- Unifix cubes

One way that students could generate their own strategies to find LCM and GCF with concrete materials is:

*LCM:*
Hot dogs are sold 10 to a package, while buns are usually sold 8 to a package. What is the least number of buns and hot dogs one should buy in order to have one hot dog for each bun with none left over?

Give children snap cubes and let them figure it out. However, one solution is make rows of 8 green cubes and 10 red cubes. Keep lining up the rows until they come out even. (you would have five green links over four red links; that would be 40 cubes in a row. Thus the LCM is 40)

*GCF*
Two small bands are to be combined and march in a parade. A 8 - member band will march behind an 12 - member band. The combined bands must have the same number of columns. What is the greatest number of columns in which they can march and have more than one person in each column?

Again use cubes and allow the student to figure out. (One strategy is to use a link of 8 green cubes and 12 red cubes - then think of all the ways 8 can be divided and test to see if 12 can be divided the same way. Once the student has determined all the ways each can be divided, then select the one that is the greatest and common to both.

Of course, each of the above explanations are dependent on students understanding the terms "factor" and "multiple" - which should have occurred in previous grades.

### *c. Misconceptions/Common Errors*

Students typically confuse the concepts of greatest common factor and multiples. Therefore, when engaging in classroom discussion, require students to use those terms in their explanations. Also, as a teacher pose student questions that help students make the connection between "factor" and the "parts of a multiplication problem". Student experiences should help them see that multiplies are derived from multiplying or using repeated addition.

### *d. Additional Instructional Strategies/Differentiation*

*LCM Flash Cards*

Make flash cards with pair of numbers. Most should be less than 16. For each card, students try to give the Least Common Multiple. Be sure to include pairs that are prime; pairs in which one is a multiple of the other and pairs that have a common divisor.

*Tiling a Bathroom*

*Materials Needed*
- Tiles
- 1 inch grid paper (Tiles should fit in the grids)

NOTE: Because this standard should not be taught in isolation, the following lesson involves finding equivalent models for fractions and creates the need for determining the least common multiple of two whole numbers (denominators).

Tell students that you have been retiling your bathroom floor. Last night you tiled $\frac{1}{3}$ of the floor and the night before you tiled $\frac{1}{4}$ of the floor. You were wondering how much you tiled in both nights.

Give student pairs a supply of tiles and a sheet of grid paper. Challenge students to determine a way to show how much of your bathroom floor you tiled in both nights. Allow student groups to share their responses. (Note: Students had experience in $4^{th}$ grade determining equivalent fractions. So, the emphasis here is for them to use their knowledge of equivalent fractions to combine fractions.) Student responses will vary.

As students share their responses, post their responses on the board horizontally and in ascending order according to the denominator. For example, the list might include: $\frac{7}{12}$, $\frac{14}{24}$, $\frac{21}{36}$, etc.

After students have all shared responses and all correct responses are posted, call students' attention to the list of correct responses, and write the numerals "3" and "4" before the list (those were the denominators from your tiling scenario). Ask student pairs to talk about the relationship they see between the denominators of your tiling fractions and the denominators of their responses. Allow groups to share.

At another place on the board write the numerals "3" and "4" vertically. After each, list 4-5 multiples. Again ask students to compare this list with their previous fraction responses to your tiling problem. Ask students to share their observations.

Share with students that as they just found out, it is necessary to have common denominators before adding fractions. One way to find common denominators is to begin by finding the least common multiple of the numerals in the denominators of the fractions to be added. Emphasize that statement by calling their attention to the

information you have listed on the board. Students will need a variety of opportunities to experience finding common multiples. This introductory lesson is only a starter. However, again, that concept should not be taught in isolation.

### *e. Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- http://www.aaamath.com/g72b-grt-com-fac.html for additional practice
- http://www.onlinemathlearning.com/fractions-math-games.html for additional practice.

### *f. Assessing the Lesson*

*Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.*

While students are working on contextual solving problems involving finding GCF and LCM, use the green, yellow, and red cup system of formative assessment.

## 2. *Teaching Lesson B:* Classifying Numbers as Prime, Composite, or Neither

**5-2.6**

For this indicator, it is **essential** for students to:

- Describe the characteristics of a composite number
- Describe the characteristics of a prime number
- Compare the characteristics of prime and composite numbers
- Determine if a number is prime, composite or neither
- Give example of prime and composite numbers

For this indicator, it is **not essential** for students to:
None noted

### *a. Indicators with Taxonomy*

5-2.6 Classify numbers as prime, composite, or neither. (B2)

*Cognitive Process Dimension: Understand*
*Knowledge Dimension: Conceptual Knowledge*

### *b. Introductory Lesson*

**Materials Needed:**
- Hundreds chart
- Chart markers

Give each student a copy of the hundreds chart. Have them start by putting a square around the 1. Next, they should circle the number 2 and cross out the multiples of 2. Then have the students circle the 3 and proceed to cross out the multiples of 3. The 4 is already crossed out, so have them move on to the 5. Have the students circle the 5 and cross out all multiples of 5. Continue on using this method until all the numbers are either circled or crossed out. As you are reading the instructions and guiding the students through this activity, you can closely monitor them. Ask the students to determine what the circled numbers have in common and how the circled numbers are different from the crossed out numbers. When it is determined that all the circled numbers have itself and one as factors, the terms "prime" and "composite" can be introduced.

The students can explain the characteristics of prime numbers and composite numbers through their writing. In their notebooks, have the students define prime and composite numbers and provide examples to further illustrate their understanding of this concept. The students should also explain in elementary terms why the number "2" is the only even prime and why 1 is neither prime nor composite. Have the students explain by writing why all odd numbers are not prime. They could illustrate this concept by drawing a Venn diagram. Using 2 circles that do not intersect, have the students label one circle for prime numbers and the other for composite numbers. The students should use the numbers 1-100 for this activity. They should put any number they found as neither prime nor composite outside the Venn.

### *c. Misconceptions/Common Errors*

No typical student misconceptions noted at this time.

### *d. Additional Instructional Strategies/Differentiation*

While additional learning opportunities are needed, no suggestions are included at this time.

### *e. Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- http://www.aaamath.com/fra63a-primecomp.html
- http://illuminations.nctm.org/mathlets/factor/index.html

(Note: Remember to check the sites prior to student use as they do change periodically.)

### *f. Assessing the Lesson*

*Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.*

Use an exit ticket and ask a question similar to the following:
Elias is concerned because he is not prepared for the math quiz he's taking in a couple of minutes. He can't remember what prime and composite numbers are. Explain to him what you know about prime and composite numbers.

## *III. Assessing the Module*

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

**5-2.6**
The objective of this indicator is to generate which is in the "create conceptual" knowledge cell of the Revised Taxonomy. The create means to put ideas together into a new structure; therefore, students use prior knowledge to generate new strategies. The learning progression to **generate** requires students to recall basic multiplication facts and

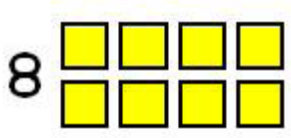
understand how to write a number as the product of its factors. Students explore the concepts of LCM and GCF in context (story problems) and analyze information (5-1.1) from these experiences. They generate mathematical statements (5-1.4) about the relationships they observe then explain and justify their strategies (5-1.3) to their classmates and their teachers. Students recognize the limitations of various strategies and representations (5-1.8) and use correct, complete and clearly written and oral language to communicate their ideas (5-1.5).

**5-2.7**

The objective of this indicator is to generate which is in the "create conceptual" knowledge cell of the Revised Taxonomy. The create means to put ideas together into a new structure; therefore, students use prior knowledge to generate new strategies. The learning progression to **generate** requires students to recall and understand the meaning of numerator and denominator. Students also should understand how to find equivalent fractions. Using concrete and/or pictorial models, students apply their understanding of fractional relationships to solve problems in context. As students analyze information (5-1.1) from these experiences, they generate conjectures and mathematical statements (5-1.4) about the relationships they observe then explain and justify their strategies (5-1.3) to their classmates and their teacher. Students should recognize the limitations of various strategies and representations (5-1.8) and develop strategies for determining the reasonableness of their answers. Students should use correct, complete and clearly written and oral language to communicate their ideas (5-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Look at the array. Is 8 a prime or composite number? Why or why not?




2. Harriet's favorite type of number is a prime number. In order to enter her clubhouse, you have to name a prime number. Which of the following numbers will enable you to enter Harriet's clubhouse?

A 2
B 4
C 6
D 8
E 9

3. Is 26 a prime or composite number? Explain how you know.

4. Use a drawing to show how you find the Greatest Common Factor for 4 and 8.

5. Show how you might use a hundreds chart to find the Least Common Multiple of 12 and 18.

**Hundreds Chart**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

# MODULE

# 1-4

# Operations – Addition and Subtraction

**This module addresses the following indicators:**

5-2.5 Apply an algorithm to add and subtract decimals through thousandths. (C3)

5-2.8 Generate strategies to add and subtract fractions with like and unlike denominators. (B6)

This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY.** Lessons in $S^3$ begin to build the conceptual foundation students need. **ADDITONAL LESSONS will be required** to fully develop the concepts.

# I. Planning the Module

- **Continuum of Knowledge:**

**5-2.5**

Fourth grade is the first time students are introduced to the concept of decimals. Students compare decimals through hundredths by suing the terms is less than, is greater than and is equal to and the symbols < ,> or = (4-2.7) and generated strategies to add and subtract decimals through hundredths.

In fifth grade, students compare whole number, decimals, and fractions by using the symbols <, > and = (5-2.4) and apply an algorithm to add and subtract decimals through thousandths (5-2.5).

In sixth grade, students generate strategies to multiply and divide fractions and decimals (6-2.5).

**5-2.8**

In fourth grade, students apply strategies and procedures to find equivalent forms of fractions (4-2.8).

In fifth grade, students generate strategies to add and subtract fractions with like and unlike denominators (5-2.8). Students generate strategies to find the greatest common factor and the least common multiple of two whole numbers (5-2.7).

In sixth grade, students apply an algorithm to add and subtract fractions (6-2.4).

- **Key Concepts/ Key Terms**

These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use and conversation for students.

Sum
Difference
*Tenths
*Hundredths
*Thousandths
*Algorithm
*Like denominator
*Unlike denominator
Addend

## II. Teaching the Lessons

### 1. Teaching Lesson A: Computation Strategies for Adding and Subtracting Decimals

Fourth grade students were introduced to the concept of decimals for the first time. Besides creating concrete and pictorial models to gain an understanding of decimals through hundredths, they generated strategies to add and subtract decimals through hundredths.

Fifth grade students should be able to fluently add and subtract decimals through thousandths. Fifth grade should place an emphasis on symbolic manipulation (numbers only when adding and subtracting through thousandths). Classroom experiences should be provided that enable students to make the connection between the concrete/pictorial experiences of fourth grade and the symbolic operations of fifth grade.

Fifth grade is the first year students begin to develop computational strategies for adding and subtracting fractions with like and unlike denominators. In fourth grade students generated equivalent fractions which laid the foundation for their work this year. As the verb "Generate" implies in indicator 5-2.5, students should generate and share their own strategies for adding and subtracting fractions. That means that all addition and subtraction work with fractions during fifth grade should be with concrete and pictorial models. Also, problems should be posed in context – not adding and subtracting for the sake of doing so – but having a reason, a problem to solve that requires addition or subtraction of fractions. Ample time should be provided for students to share their strategies and learn from each other. Such sharing and discussing leads students to discover an efficient strategy that they understand – not just memorize a strategy that they soon forget.

For this indicator, it is **essential** for students to:

- Understand place value through thousandths.
- Understand the relationship between the place value structure of whole numbers and the place value structure of decimals through thousandths.
- The role of the decimal point is to designate the unit position.

For this indicator, it is **not essential** for students to:
None noted

#### *a. Indicators with Taxonomy*

5-2.4 Apply an algorithm to add and subtract decimals through thousandths. (C3)
*Cognitive Process Dimension: Apply*
*Knowledge Dimension: Procedural Knowledge*

### *b. Introductory Lesson*

***Materials Needed***
(Adapted from: Teaching Student Centered Mathematics: Grades 3-5, Van de Walle, John A. & Lovin, LouAnn H., Pearson Learning, 2006, page 198.)

Give students a sum involving different numbers of decimal places. For example, 73.46 + 6.2 + 0.582. The first task is to make an estimate and explain how the estimate was made. The second task is to compute the exact answer and explain how that was done (no calculators). In the third and final task, students devise a method for adding and subtracting decimal numbers that they can use with any two or more numbers.

When students have completed these three tasks, have students share their strategies for computation and test them on a new computation that you provide.

The same task should be repeated for subtraction.

### *c. Misconceptions/Common Errors*

No typical student misconceptions noted at this time.

### *d. Additional Instructional Strategies/Differentiation*

While additional learning opportunities are needed, no suggestions are included at this time.

### *e. Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources: Additional Practice

- http://www.funbrain.com/football/
- http://www.math-play.com/Decimal-Game.html (Christmas theme)

### *f. Assessing the Lesson*

*Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.*

(Adapted from: Teaching Student Centered Mathematics: Grades 3-5, Van de Walle, John A. & Lovin, LouAnn H., Pearson Learning, 2006, page 198.)

Have students complete an exit ticket using a problem such as the following:

Ken bought a model plane for$4.99, glue for $1.29, and paint for $2.19. There was no sales tax. How much change should he have received from $10.00?

**2. *Teaching Lesson B:*** Strategies to Add and Subtract Fractions with Like and Unlike Denominators

For this indicator, it is **essential** for students to:
- Use concrete or pictorial models to represent fractions and operations with fractions
- Estimate to determine the reasonableness of their answer
- Understand the meaning of numerator (tell the number of parts) and denominator (the type of part)
- Understand the concept of equivalent fractions

For this indicator, it is **not essential** for students to:
- Gain computational fluency

### *a. Indicators with Taxonomy*

5-2.8 Generate strategies to add and subtract fractions with like and unlike denominators. (B6)

*Cognitive Process Dimension: Create*
*Knowledge Dimension: Conceptual Knowledge*

### b. Introductory Lesson

(Adapted from: Teaching Student Centered Mathematics: Grades 3-5, Van de Walle, John A. & Lovin, LouAnn H., Pearson Learning, 2006, pages 160-164.)

**Materials Needed**
Various manipulatives, drawing paper, etc.

**Possible Literature Connections:**

Use Hershey's Milk Chocolate Fractions Bar Book by Jerry Pallota or Skittles Riddles Math by Barbara Barbieri McGrath and Roger Glass and create addition and subtraction problems using Skittles® pieces or Hershey® bars as manipulatives.

The following guidelines should be kept in mind when developing computational strategies with fractions:

- Begin with simple contextual tasks.
- Connect the meaning of fraction computation with whole-number computation.
- Let estimation and informal methods play a big role in the development of strategies.
- Explore each of the operations using models.

Expect students to use a variety of methods and that the methods will vary widely with the fractions encountered in the problems.

Pose problems such as the following to students:

- Paul and his brother were eating the same kind of candy bar. Paul had ¾ of his candy bar. His brother still had ½ of his candy bar. How much candy did they two boys have together?
- John and Jill ordered two identical pizzas. John ate $\frac{5}{6}$ of a pizza and Jill ate $\frac{7}{8}$ of a pizza. How much pizza did they eat together?

***It is imperative that students share their strategies with the class!***

### *c. Misconceptions/Common Errors*

No typical student misconceptions noted at this time.

### *d. Additional Instructional Strategies/Differentiation*

**Adapted from Anderson Five Curriculum:**

1. Use fraction pieces to show adding and subtracting like or unlike fractions. Give students three wholes, six halves, nine thirds, 12 fourths, 18 sixths, 24 eighths, 36 twelfths. Have them place two fourths on one whole bar and three fourths on the 2nd whole bar. Have students recount for a total number since all are alike. The answer will be $\frac{5}{4}$. Do not change to a mixed

number at this time. Have students write the following sentence on their paper. Three fourths + two fourths = five fourths of a whole bar. Put other exercises on the board for them to build with fraction bars. You may use improper fractions at this time. Have students create their own sentences at this time. This same activity may be used for subtraction of fractions.

2. Use fraction bars to find a common part size needed for recounting. You will not be focusing on LCD at this time. Show $\frac{3}{4}$ on the first whole bar and $\frac{2}{6}$ on the second whole bar. Ask, "Can the fourths be traded for sixths? Can the sixths be traded for fourths? If not, is there another part size for which both fourths and sixths might be traded? Give students time to explore different sizes. They should find fourths can go to twelfths (3 to 1) and sixths to twelfths (2 to 1). Now that all parts are the same the students can proceed as in exercise 1 above. Have students draw a representation of their problem.
3. Have students create fractions problems by rolling dice, both adding and subtracting. Record the problem on index cards. Teams can use fractions pieces to work and record answers on the back. Have teams exchange cards and check answers from other teams.
4. Use Cuisenaire® rods to work addition problems. The orange bar = one whole, etc.

For additional instructional strategies, see NCTM, Navigating through Number and Operations in Grades 3-5.

### *e. Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

There is no specific technology recommended for this lesson at this time.

### *f. Assessing the Lesson*

*Formative Assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.*

Since this is a generating strategies indicator, any assessment, ra ther formative or summative, should involve concrete and pictorial models only. It is suggested that you use the green, yellow, and red cup system as students are working on the problems. Much insight can be gained into student thinking as students explain their thinking to their classmates.

## *III. Assessing the Module*

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

**5-2.5**

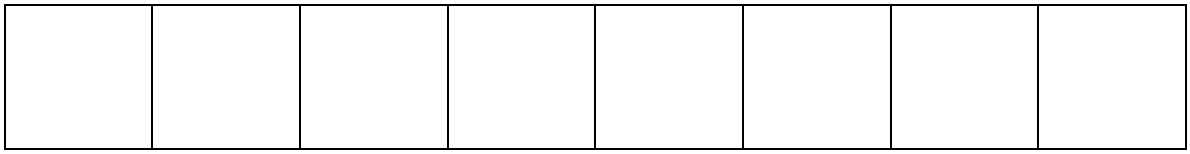
The objective of this indicator is to apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. To apply means to carry out a procedure in familiar and unfamiliar situations; therefore, students should be able to add decimals in a variety of place value combinations through the thousandths. The learning progression to **apply** requires students to recall place value structure for whole numbers and decimals. Students connect experiences with concrete and pictorial models from fourth grade to symbolic procedures. Students use these models to generalize connections (5-1.6) between their models, their generated strategies and the symbolic procedure. Students use estimation strategies to determine the reasonableness of their answers and explore these procedures in context to further deepen both procedural and conceptual knowledge. As students exchange mathematical ideas with their classmates/teachers and explain and justify their answers (5-1.3), they are supporting conceptual understanding and building computational fluency.

**5-2.8**

The objective of this indicator is to generate which is in the "create conceptual" knowledge cell of the Revised Taxonomy. The create means to put ideas together into a new structure; therefore, students use prior knowledge to generate new strategies. The learning progression to **generate** requires students to recall and understand the meaning of numerator and denominator. Students also should understand how to find equivalent fractions. Using concrete and/or pictorial models, students apply their understanding of fractional relationships to solve problems in context. As students analyze information (5-1.1) from these experiences, they generate conjectures and mathematical statements (5-1.4) about the relationships they observe then explain and justify their strategies (5-1.3) to their classmates and their teacher. Students should recognize the limitations of various strategies and representations (5-1.8) and develop strategies for determining the reasonableness of their answers. Students should use correct, complete and clearly written and oral language to communicate their ideas (5-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Use this model to illustrate your answer.



Susan ate $\frac{3}{4}$ of a candy bar. Kim ate $\frac{1}{8}$ of the same candy bar. Using the model, show how much they ate all together.

*Item #2 Adapted from: ETA Cuisenaire, 2006. Hands-On Standards: Grades 5-6*

2. Deon grows carrots in $\frac{1}{6}$ of his garden. He grows potatoes in another $\frac{1}{4}$ of the garden. The rest of his garden is planted in flowers. What fraction of Deon's garden is used to grow vegetables? Use a pictorial model to show your how you got your answer.

*Item #3 Adapted from: ETA Cuisenaire, 2006. Hands-On Standards: Grades 5-6*

3. Esmeralda rides $\frac{1}{4}$ mile from her house to her friend's house. Together they ride $\frac{3}{8}$ mile to school. How far does Esmeralda ride to school that day? Use a pictorial model to show how you got your answer.

*Item #4 Adapted from: ETA Cuisenaire, 2006. Hands-On Standards: Grades 5-6*

4. Jordan has painted $\frac{7}{10}$ of a fence. Mark has painted $\frac{1}{2}$ of another fence the same size. How much more has Jordan painted than Mark? Use a drawing to show you got your answer.

A $\frac{5}{8}$ B $\frac{1}{2}$ C $\frac{1}{5}$ D $\frac{1}{10}$

*Item #5 Adapted from: ETA Cuisenaire, 2006. Hands-On Standards: Grades 3-4*

5. Rachael and Jake take turns feeding the class hamster. One week Jake gives the pet 0.7 cups of food and the next week Rachael gives him 0.8 cups of food. How much food did the hamster get during those two weeks?

A 1.3 cups
B 1.5 cups
C 1.7 cups
D 1.9 cups

6. Erik had $2.78 in his piggy bank, then he put in $19.36 that was left over from his birthday money. How much money is in Erik's piggy bank now?