SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

Content Area	Fifth Grade Mathematics
Recommended Days of Instruction	**Fourth Nine Weeks**

Standards/Indicators Addressed:

Standard 5-6: The student will demonstrate through the mathematical processes an understanding of investigation design, the effect of data-collection methods on a data set, the interpretation and application of the measures of central tendency, and the application of basic concepts of probability.

5-6.1* Design a mathematical investigation to address a question. (B6)
5-6.2* Analyze how data-collection methods affect the nature of the data set. (B4)
5-6.3* Apply procedures to calculate the measures of central tendency (mean, median, and mode). (C3)
5-6.4* Interpret the meaning and application of the measures of central tendency. (B2)
5-6.5* Represent the probability of a single-stage event in words and fractions. (B2)
5-6.6* Conclude why the sum of the probabilities of the outcomes of an experiment must equal 1. (B2)

* These indicators are covered in the following 3 Modules for this Nine Weeks Period.
Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.

Module 4-1 Collection and Representation			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 4-1 Lesson A 5-6.1 Design a mathematical investigation to address a question. (B6) 5-6.2 Analyze how data-collection methods affect the nature of the data set. (B4)	**STANDARD SUPPORT DOCUMENT** http://www.ed.sc.gov/agancy/standard-and-learning/academic standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment	See Instructional Planning Guide Module 4-1 Introductory Lesson A	See Instructional Planning Guide Module 4-1 Lesson A Assessing the Lesson

	Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 5-6		

Module 4-2 Data Analysis			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 4-2 Lesson A 5-6.3 Apply procedures to calculate the measures of central tendency (mean, median, mode). (C3)	**STANDARD SUPPORT DOCUMENT** http://www.ed.sc.gov/agancy/standard-and-learning/academic standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment	See Instructional Planning Guide Module 4-2 Introductory Lesson A	See Instructional Planning Guide Module 4-2 Lesson A Assessing the Lesson

	Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 5-6		
Module 4-2 Lesson B 5-6.4 Interpret the meaning and application of the measures of central tendency. (B2)		See Instructional Planning Guide Module 4-2 Introductory Lesson B	See Instructional Planning Guide Module 4-2 Lesson B Assessing the Lesson

Module 4-3 Probability			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 4-3 Lesson A 5-6.5 Represent the probability of a single-stage event in words and fractions. (B2)	**STANDARD SUPPORT DOCUMENT** http://www.ed.sc.gov/agancy/standard-and-learning/academic standards/math/index.html NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series 3-5 Teaching Student-Centered Mathematics Grades 3-5 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle Blackline Masters for Van de Walle Series www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) NCTM, Mathematics Assessment	See Instructional Planning Guide Module 4-3 Introductory Lesson A See Instructional Planning Guide Module 4-3, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 4-3 Lesson A Assessing the Lesson

	Sampler: Grades 3-5 ETA Cuisenaire, Hands-On Standards: Grades 5-6		
Module 4-3 Lesson B 5-6.6 Conclude why the sum of the probabilities of the outcomes of an experiment must equal 1. (B2)		See Instructional Planning Guide Module 4-3 Introductory Lesson B	See Instructional Planning Guide Module 4- 3 Lesson B Assessing the Lesson

MODULE

4-1

Collection and Representation

This module addresses the following indicators:

5-6.1 Design a mathematical investigation to address a question. (B6)
5-6.2 Analyze how data-collection methods affect the nature of the data set. (B4)

This module contains 1 lesson. This lesson is **INTRODUCTORY ONLY.** Lessons in S^3 begin to build the conceptual foundation students need. **ADDITONAL LESSONS will be required** to fully develop the concepts

I. *Planning the Module*

- **Continuum of Knowledge (Same for 5-6.1 and 5-6.2)**

In third grade, students compared the benefits of using tables, bar graphs and dot plots as representations of a given data set (3-6.5). In fourth grade, students compared how data collection methods impact survey results (4-6.1).
In fifth grade, students design a mathematical investigation to address a question and analyze how data collection methods affect the nature of the data set (5-6.1).

- **Key Concepts/Key Terms**

- *Survey
- *Sample Size
- *Investigation
- *Data
- *Data Set
- Data collection methods

**** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.***

II. Teaching the lessons

1. Teaching Lesson A-Mathematical Investigation

Using real world, meaningful data will engage students in the investigation process. There is a natural connection to the scientific method.

The nature of the data refers to the makeup of the data. For example, if the question is "do you think that The Simpsons is an offensive show?" How does your data collection method as it relates to the target population affect the data? Asking students between 12 and 15 years old will yield different data than asking adults between 50 – 60 years old.

For this indicator (5-6.1), it is **essential** for students to:

- Analyze a question to determine relevant and irrelevant components

- Generate a hypothesis or prediction
- Determine whether data will be collected through observation, survey, or experiment.
- Determine the best way to organize the data
- Determine the target population
- Determine when to collect data
- Determine how the methods used affect the data sets.
- Compare data sets collected in different ways
- Determine materials needed, if any

For this indicator, it is **not essential** for students to:

- Write the question

For this indicator (5-6.2), it is **essential** for students to:

- Understand how the interpretation of the question impacts the data
- Understand how when their asked the question impacts the data
- Understand how their method of measurement (if needed) impacts the data
- Understand how the method of recording impacts the data
- Understand how the selection of a sample population
- Compare data sets collect in different ways

For this indicator, it is **not essential** for students to:

- Conduct an experiment

a. Indicators with Taxonomy

5-6.1 Design a mathematical investigation to address a question. (B6)
Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual Knowledge

5-6.2 Analyze how data-collection methods affect the nature of the data set. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

Materials Needed

Handout at the end of the lesson to use in a survey collection

Lesson

<u>What Traits Do We Value Most in Friends?</u>

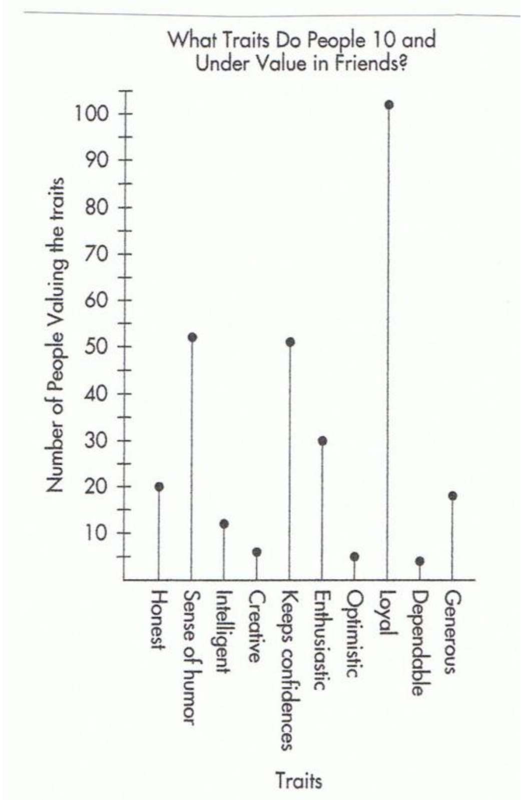
Conduct a survey of a certain number (you should decide on a good sample size based on the school's population size) of random students in grade 3, grade 4 and grade 5. For example, if there are 100 students in each grade, you may choose to survey ¼ of them or 25 students in each grade.

Use the chart at the end of the lesson when surveying. *<u>An example of the chart is below.</u>*

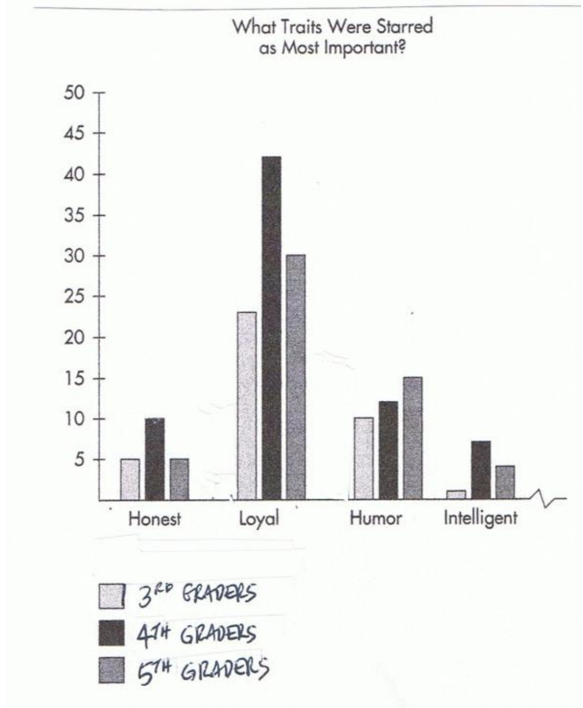
Grade Level ____	
Check the **3** traits below that you value MOST in your friends.	
___ Honest	___ Keeps confidences
___ Loyal	___ Enthusiastic
___ Sense of Humor	___ Optismistic
___ Intelligent	___ Dependable
___ Creative	___ Generous
Of the three traits you checked, put a STAR next to the one you feel is MOST IMPORTANT.	
Thank you for completing this survey.	

Students should tally the results by grade level. Tally all three choices for each survey response. ON A SEPARATE TALLY chart, tally the most important (starred) traits.

Create a bar graph of the tallied results. Create a separate bar graph for each grade level. The traits will be on the horizontal (x) axis. The number of students will be on the vertical (y) axis. For example, one of the graphs may look like this:


What Traits Do People 10 and Under Value in Friends?
Number of People Valuing the traits
100
90
80
70
60
50
40
30
20
10
Honest
Sense of humor
Intelligent
Creative
Keeps confidences
Enthusiastic
Optimistic
Loyal
Dependable
Generous
Traits

In order to compare the most important (starred) traits, create a triple bar graph. The most important traits will be listed across the horizontal (x) axis and the number of people choosing those traits will be the vertical (y) axis.
For example, it may look something like:



Hold a class discussion on the way data can be considered biased. Ask questions such as:

- What would have happened if you would have surveyed only 4th graders?
- If you had surveyed a much smaller sample size?
- If you had surveyed only boys or only girls?

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Data can be gathered through:
1) The U.S. Census Bureau at www.census.gov. This website contains statistical information by state, county or voting district.
2) The World Fact Book at www.odci.gov/cia/publications/factbook/index.html. This website provides demographic information for every nation in the world, including population, age distributions, death and birth rates, and information on the economy, government, transportation and geography. Maps are included as well.
3) Internet Movie Database at www.imdb.com. This website offers information about movies of all genres.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

(1) Miranda wants to take a survey of how many of the 36 people in her class have her favorite video game, *Leonardo's Speedway*. What would be a good sample size that she could use to avoid a biased result?

(2) At Gloria's school, the principal wants to select a new school mascot. There are 400 students at her school, and Gloria has surveyed all 70 of the 4th grade students. The principal refuses to name the mascot according to Gloria's survey results. Why is the principal correct?

III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The objective of this indicator (5-6.1) is to <u>design</u> which is in the "create conceptual" knowledge cell of the Revised Taxonomy. To design is to devise a procedure for accomplishing some task. The learning progression to **design** requires students to <u>analyze</u> the question and determine relevant and irrelevant data in order to outline their next steps. They <u>generate</u> a hypothesis or prediction based on the question. Students <u>determine</u> a target population and when to collect data. They <u>recognize</u> the limitations of various forms of representation (5-1.8) and use that understanding to <u>select</u> the most appropriate methods of collecting and organizing the data. They <u>explain</u> and <u>justify</u> their design using correct, clear and complete oral and written mathematical language (5-1.5).

The objective of this indicator (5-6.2) is to <u>analyze</u> which is the "analyze conceptual" knowledge cell of the Revised Taxonomy. To analyze is to break down material into its parts and determine how the parts relate to one another and to an overall. The learning progression to **analyze** requires students to <u>understand</u> the intent of the question that the data answers. They <u>evaluate</u> the data based on the sample population, when the data was collected, how it was measured and how it was recorded. They use this information to <u>construct</u> arguments (5-1.2) about the impact on the nature of the data. They <u>explain</u> and <u>justify</u> their answers using correct, clear and complete oral and written mathematical language (5-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Padma's school only has enough money to buy footballs or basketballs, but not both; so Padma takes a survey to find out which is the preferred sport. Padma stands on the blacktop next to the basketball courts with a clipboard and starts asking students whether they prefer basketball or football. She finds that almost everyone walking by prefers basketball and concludes that the school should not spend any more money for footballs. Principal Milner, however, says her survey has not produced reliable results. Why do you think this is?

2. Performance Assessment on one of the following topics where students will:
   1) design an investigation by selected a sample size
   2) conduct the survey – may be in school or outside of school
   3) compile results
   4) effectively demonstrate results through written or graphical representation
   5) respond to the following prompt: "Do you think there was any bias in your survey after looking at your survey results? What is your evidence?

Topics may include, but are not limited to:

- Which bubble gum is best?
- Which pizza is best?
- Which comic strips are most popular?
- What do we think about school issues?
- What is the typical student in our class like?
- What type of car is most popular?

Grade Level ____	
Check the **3** traits below that you value MOST in your friends.	
___ Honest	___ Keeps confidences
___ Loyal	___ Enthusiastic
___ Sense of Humor	___ Optismistic
___ Intelligent	___ Dependable
___ Creative	___ Generous
Of the three traits you checked, put a STAR next to the one you feel is MOST IMPORTANT.	
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___ Sense of Humor	___ Optismistic
___ Intelligent	___ Dependable
___ Creative	___ Generous
Of the three traits you checked, put a STAR next to the one you feel is MOST IMPORTANT.	
Thank you for completing this survey.	

MODULE

4-2

Data Analysis

This module addresses the following indicators:

5-6.3 Apply procedures to calculate the measures of central tendency (mean, median, and mode). (C3)
5-6.4 Interpret the meaning and application of the measures of central tendency. (B2)

This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY.** Lessons in S^3 begin to build the conceptual foundation students need. **ADDITONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

There are no related indicators prior to fifth grade.

In fifth grade, students apply procedures to calculate the measure of central tendency (mean, median, and mode) (5-6.3). They also interpret the meaning and application of the measures of central tendency (5-6.4). In sixth grade, students analyze which measure of central tendency (mean, median, or mode) is the most appropriate for a given purpose (6-6.3). In seventh grade, students apply procedures to calculate the interquartile range (7-6.3) and interpret the interquartile range for data (7-6.4). In eighth grade, students interpret graphic and tabular data representations by using range and the measures of central tendency (8-6.8).

- **Key Concepts/Key Terms**

  - *Mean
  - *Median
  - *Mode
  - *Measure of central tendency
  - *Averages

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lessons

1. Teaching Lesson A-Measures of Central Tendency

Although the focus of the indicator is to apply procedures, students should explore these concepts from a conceptual standpoint as well. For example, mean may be modeled using the leveling concept. Students should use paper squares to "build" a situation. For this activity, each square will represent one dollar ($1). They should create a bar graph with 30 squares and with 5 or 6 bars (so the answer comes out even) – each bar should have less than 10 squares in each and each bar should have at least one square. This bar graph should represent items purchased in a store and their cost. For example, one bar may represent a video game on sale for $7. This bar would be 7 squares high. Another bar may represent a toy car

costing $5. This bar would be 5 squares high. Students should keep track of the amount in each individual bar – they will need this data later.) The next task is to manipulate the heights of the bars to end up with equal bar heights. Then, explain the heights or sizes of the leveled bars is the mean of the data – the amount each item would cost if all items cost the same amount but the total of all the items remained the same. If they have five bars, the height should be six. If they have six bars, the height should be five.
To explore problems where the mean is not a whole number, have students build graphs where the numbers of bars is 4. The key is to allow students to discover how to split the divide squares into fractional parts to create equal bars.

For this indicator, it is **essential** for students to:

- Add whole numbers fluently
- Understanding the meaning of mean, median and mode
- Understand that each measure of central tendency is considered an average
- Compare the measures of central tendency

For this indicator, it is **not essential** for students to:

- Interpret the meaning of each measure of central tendency in real world problems.

a. Indicators with Taxonomy

Indicator → 5-6.3 Apply procedures to calculate the measures of central tendency (mean, median, and mode). (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson

Teacher Note

Think about mean as a number "that represents what all the data items would be if they were leveled out… The concept of the mean is more in keeping with the notion of a measure of the "center" of the data or a measure of central tendency". (Teaching Student Centered Mathematics, Grades 3-5, Van de Walle, pg 325).

Materials Needed

- 1" paper squares OR small post it notes – 30 per pair, trio or quad group

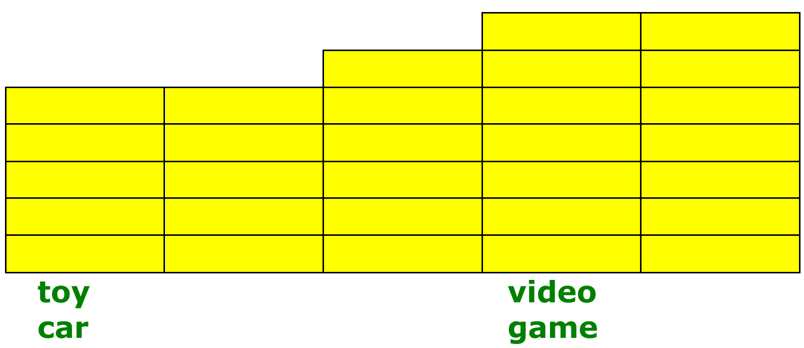
- Squares that can be manipulated on a whiteboard or cloned on a SmartBoard for use. (Students will need these for explanation of their groups findings at the end).

Lesson – Finding Mean (only)

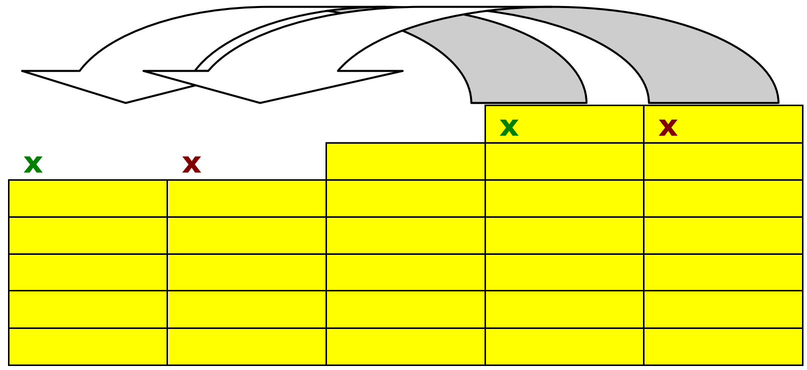
Opening/Teacher Notes: "Suppose that the average number of family members for the students in your class is 5. One way to interpret this is to think about distributing the entire collection of [family members] to each of the students so that each would have a 'family' of the same size. To say that you have an average of 93 for the four tests in your class is like spreading out the total of all your points evenly across the four tests".

<u>"On Level Ground"</u> → Students should use paper squares to "build" a situation. For this activity, each square will represent one dollar ($1). They should create a bar graph with 30 squares and with 5 or 6 bars – each bar should have less than 10 squares in each and each bar should have at least one square. This bar graph should represent items purchased in a store and their cost. For example, one bar may represent a video game on sale for $7. This bar would be 7 squares high. Another bar may represent a toy car costing $5. This bar would be 5 squares high.

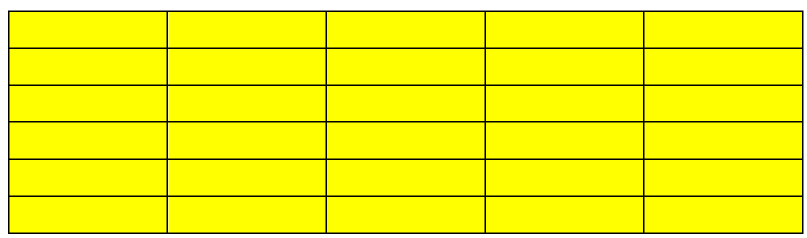
<u>An example of what student's may create is:</u>



(Students should keep track of the amount in each individual bar – they will need this data later.) The task is next to manipulate the heights of the bars to end up with equal bar heights. Then, explain the heights or sizes of the leveled bars is the mean of the data – the amount each item would cost if all items cost the same amount but the total of all the items remained the same.



When the two top squares are moved to columns 1 and 2, then the result is….

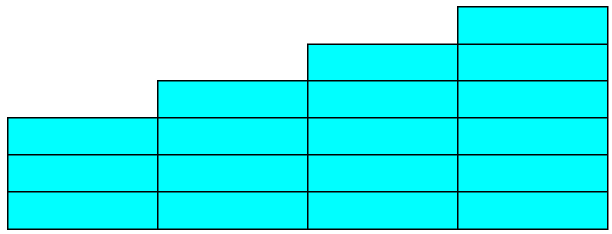


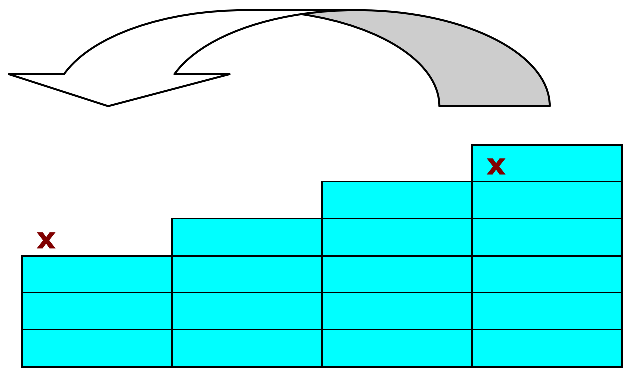
This levels out the bar heights. The mean is 6 squares or $6 since there are 6 squares in each column.

Students should notice that with 30 squares or $30, situations with 5 bars had a mean of 6 squares or $6 and those with 6 bars had a mean of 5 squares or $5.

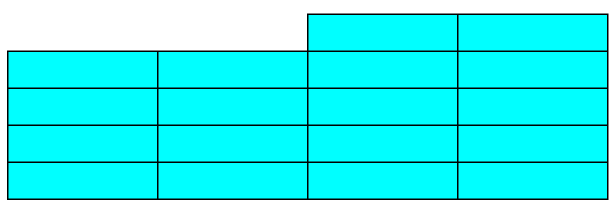
For the next part, students should work in their pairs, trios or quads to answer the following: What happens when the mean is not a whole number, how is that represented with bars and leveling? Allow students to use up to the 30 blocks or up to $30. They should be able to use 2, 4, or 6 bars in their situations of items and cost. Have them build their bar graph of sorts and make note of how many bars they used and what the height of each one is. Then, they should manipulate the data (the squares) to try to level out the bars. *The trick here is to let students determine that they may have to cut a square or more squares to equal out the bar heights.* Once all groups have found their means, allow share out of the situation each group created, from the set up to how they "calculated" mean by manipulating the squares. It may be helpful to have some squares on the SmartBoard or the WhiteBoard for them to manipulate.

For example:

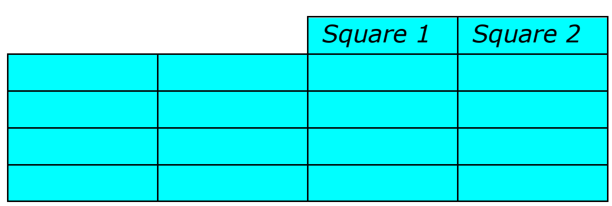




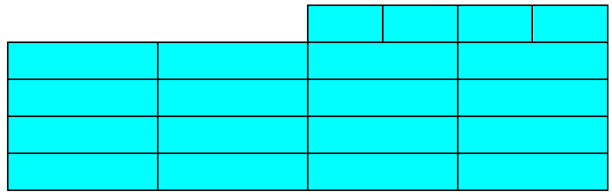
Becomes…



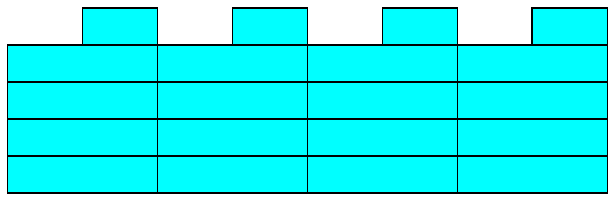
Now, you have two squares to split over 4 columns….



They can be cut in halves and distributed….



When distributed, you have ½ on each column…



Giving you a mean of 4 ½ since there are 4 ½ squares in each column.

Note: This can be done with connecting cubes, but they <u>cannot</u> be broken apart to find the mean of anything with other than whole number results.

Lesson – Finding Median and Mode

Have students write out the costs of the items from their first situation created above. ***(They should have recorded that data as directed in the above lesson.)*** Have them write each amount on a separate post it note. Each post it represents one item and the cost of the item is written on the post it. They should arrange the post it notes as a group in order from least to greatest. Instruct them that finding the median is finding the exact middle. For student groups who originally had 5 bars it would be the middle number – leaving 2 numbers on the left and 2 on the right of it. For the groups with 6 bars originally, the middle is actually between 2 numbers, between the 3^{rd} and 4^{th} number in sequence. To find the number between these two, the two numbers must be "averaged" – or the mean of just those two numbers should be found. This could be done for each group with their second situations data from the previous "mean' activity.

Have students give you the mean of their first situation. Record those in a bar graph of your own as the teacher. Keep in mind, the first situation yielded a mean of 5 or 6. The mode of the data should be easy to see/recognize/find.

c. Misconceptions/Common Errors

Students have the misconception that only the mean is the average. All measures of central tendency are considered averages.

When the formula is introduced without conceptual understanding, there are many errors that can occur. Students get mean, median and mode mixed up or cannot remember when each is used or when each is more effective.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Lesson on mean, median, and mode from Illuminations from NCTM.
http://illuminations.nctm.org/LessonDetail.aspx?id=L297

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Several student bookbags were weighed as part of a class project. Seven of the bookbags weighed the following:
7 pounds, 9 pounds, 9 pounds, 12 pounds, 13 pounds
Find the mean of the weights.
Find the median of the weights.
Find the mode of the weights.
Record your solutions and an explanation of your solutions in your mathematics notebook or journal. Please leave your journal/notebook with the teacher.

2. *Teaching Lesson B*

Having students interpret measures of central tendency in context build a deeper conceptual understanding of these concepts.

For this indicator, it is <u>essential</u> for students to:

- Understanding the meaning of mean, median and mode
- Understand that each measure of central tendency is considered an average
- Compare the measures of central tendency
- Understand which measure is most appropriate for a given situation
- Interpret the meaning of each measure of central tendency in real world problems.

For this indicator, it is <u>not essential</u> for students to:
None noted
Student Misconceptions/Errors

- Students have the misconception that only the mean is the average. All measures of central tendency are considered averages.
- When the formula is introduced without conceptual understanding, there are many errors that can occur. Students get mean, median and mode mixed up or cannot remember when each is used or when each is more effective.

a. *Indicators with Taxonomy*

Indicator → 5-6.4 Interpret the meaning and application of the measures of central tendency. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. *Introductory Lesson*

Materials Needed

Lesson

Give students 4 sets of data, including the mean, median, mode and range of each set. The task is for students to discuss what each measure of center tells you about the given data set. What do they each mean about the data? Suggestion: Start with data set A, make conjectures and then move to B and so on. This activity is an introduction to student thinking and discussion.

Data Set →	*A: 1,3,3,4,5, 5,9,9,10,11*	*B: 10,12,17,24,25, 32,34,34,40,47, 54,68,71,79,80, 85,86,87,96,99*	*C: 5,9,10,11,40*	*D: 0,1,2,3,3, 3,4,4,5,5*
Mean	*6*	*54*	*15*	*3*
Median	*5*	*50.5*	*10*	*3*
Mode	*3,5,9*	*34*	*None*	*3*
Range	*10*	*89*	*35*	*5*

Questions that would prove useful are a lot of "why's"

- *Why is the mean and median so close on A, but not so close in C?*
- *Why are the mean, median and mode all the same in D?*
- *Why is the median represented by a decimal in B?*
- *What affects the data set the most in set C?*
- *Does the range of a data set with few entries, such as in set C, affect the median or the mean the most? Possible response: the range is greater because 40 is so far from 11, that is an outlier. Because of that outlier, the mean is affected and higher. The median is not affected because it is a data point and not added in to a sum….*

c. Misconceptions/Common Errors

Students have the misconception that only the mean is the average. All measures of central tendency are considered averages.

When the formula is introduced without conceptual understanding, there are many errors that can occur. Students get mean, median and mode mixed up or cannot remember when each is used or when each is more effective.

Students forget to put the data in order before finding the median or think there are two medians, one when you put them in order and one when you don't put them in order.

d. Additional Instructional Strategies/Differentiation

These activities are adapted from A5 and are for both indicators.

1. Measure each student's height in inches. Working in trios, have the students represent the data in graph form. Ask: What can you tell about the data by looking at your graph? Discuss the different graphs made by each group. Ask: Which graph is easier for you to read and

understand the data? Why? Have them predict the mean, median and mode. Then find the mean, median, and mode (centimeters can also be used).

2. Discuss how a final grade is found for report cards. Students can use their own grades for a subject or make up some to find the mean. Then they can find the median and mode. Discuss what each measure of center tells about the grades.
3. Sports statistics can be used as an example of mean. Baseball or basketball cards can be used to find mean, median, and mode.
4. Survey the class about the number of books they read last month. Ask: What might affect how many books you read in a month? Ask the students to represent the data any way they want. Have them predict which measure of central tendency will give them the most accurate average for the class? Why do they think that? Find the mean, mode, and median of the data collected. Ask: What would happen to our data if everyone read 2 more books that month? Half the class read 4 more books that month? Why?

e. Technology

There are no specific recommendations for technology for this lesson at this time.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Consider the data set: 20, 32, 35, 37, 40, 43, 49
What number could be added to the data set to greatly affect the mean? Why would the number affect the mean?
Would the median change a lot or a little? Why is that the case?
Teacher should take the responses up on an exit card (i.e. index card, ½ sheet of paper, etc.), review them and return the following day (unmarked!) for student discussion.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The objective of this indicator (5-6.3) is to <u>apply</u> which is in the "apply conceptual" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is to apply, the learning progression should integrate strategies to build conceptual and procedural knowledge.

The learning progression to **apply** requires students to recall and understand the meaning of mean, median and mode. Students explore concrete and pictorial models and generalize connections (5-1.6) between these models and the concepts of mean, median and mode. They generate mathematical descriptions (5-1.4) for each average and recognize the limitations of each (5-1.8). Students engage in meaningful practice to gain computational fluency and explain and justify their answers (5-1.3) using correct, clear and complete mathematical language (5-1.5).

The objective of this indicator (5-6.4) is to interpret, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To interpret involves changing from one form of representation to another (e.g. from graphic or tabular to verbal). The learning progression to **interpret** requires students to recall and understand the meaning of mean, median and mode. Students recognize and understand the limitations (5-1.8) of each measure and use their understanding analyze data in graphical, tabular and numeric form. They use inductive and deductive reasoning to reach a conclusion. Students explain and justify (5-1.3) their answers using correct, clear and complete oral and written mathematical language (5-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Given the number of team homeruns from the last 5 seasons, what is the mean number of homeruns hit every season? *Answer: 36*

Season	Team homeruns
1	41
2	28
3	28
4	34
5	49

If the next two seasons team homeruns were 58 and 52, would the mean go up or down?

Answer: up because both numbers are higher than the mean of 36.

2. An elementary school is selling candy bars for a fund raiser. The table below shows the number of candy bars sold in one week. Something spilled on the total for Thursday and it cannot be read. Before the spill, we had calculated or found the median to be 18. And, we remember that Thursday's count was not 18.

Day	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Candy	18	18	15		23	31	31

What might Thursday's count have been? Explain your answer and your thinking.

Answer: any number 17 or less.

3.
a. Each Saturday morning Jasmine has a jewelry stand at the mall. 8, 8, 6, 8, 7, 5, 6, 8, 9, and 5 are the sizes of rings she sold Saturday morning. Find the mean, median, and mode.

b. Which of these measures might be best to consider when she is reordering rings for next Saturday? Explain your answer.

4. If the mean of a set of data is 50 and the median is 52, which measure would be most affected by an outlier of 200 being added to the data set?

5. If the mode of a set of data is 30, the mean is 15 and the median is 9.5, what could you say about the mode and the range of numbers?

MODULE

4-3

Probability

This module addresses the following indicators:

5-6.5 Represent the probability of a single-stage event in words and fractions. (B2)
5-6.6 Conclude why the sum of the probabilities of the outcomes of an experiment must equal 1. (B2)

This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY.** Lessons in S^3 begin to build the conceptual foundation students need. **ADDITONAL LESSONS will be required** to fully develop the concepts.

I. Planning the module

- **Continuum of Knowledge**

In third grade, students predicted on the basis of data whether events are likely, unlikely, certain or impossible to occur (3-6.6). In fourth grade, students predicted on the basis of data whether events are likely, unlikely, certain, impossible or equally likely to occur (4-6.6). They also analyzed the possible outcome for a simple event (4-6.7).

In fifth grade, students represent the probability of a single-stage even in words and fractions (5-6.5).

In sixth grade, students use theoretical probability to determine the sample space and probability for one- and two-stage events such as tree diagrams, models, lists, charts, and pictures (6-6.4). It should be noted that sixth grade is the first time students have been introduced to two-stage events.

In seventh grade, students apply procedures to calculate (7-6.5) and interpret (7-6.6) the probability for mutually exclusive simple and compound events. In eighth grade, they apply procedures to calculate (8-6.5) and interpret (8-6.6) probability for two dependent events.

- **Key Concepts/Key Terms**

  - *Probability
  - *One stage event
  - *Likely, unlikely, certain, impossible, or equally likely
  - *Outcome(s)
  - *Sample Space
  - *Sum
  - *Experiment

  ** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the lesson(s)

1. Teaching Lesson A

For the purpose of this indicator, students should explore experiments that require only a single device such as a number cube, a spinner, or drawing a number cube from a bag. These are referred to as single-stage events because there is only one activity to determine an outcome.

In indicator 5-2.8, students generate strategies for adding and subtracting fractions with like and unlike denominators; therefore, students may explore this indicator through the use of concrete or pictorial models as opposed to a formal algorithm for adding fractions. Through a variety of experiences and the use of hands-on models, students should conclude that the sum of the probability is one.

For this indicator (5-6.5), it is **essential** for students to:

- Understand the meaning of probability
- Connect probability with the terms likely, unlikely, certain, impossible or equally likely
- Write a fraction
- Explain the meaning of a probability

For this indicator, it is **not essential** for students to:

- Perform operations with fractions

a. Indicators with Taxonomy

5-6.5 Represent the probability of a single-stage event in words and fractions. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

Suggested Literature Connection:

Do You Wanna Bet?

(Adapted from: Teaching Student Centered Mathematics: Grades 3-5, Van de Walle, John A. & Lovin, LouAnn H., Pearson Learning, 2006, page 344-345.)

In order for students to determine a numeric probability, it is important that they can identify all possible outcomes of an experiment and consider their relative chances of occurring.

The sample space for an experiment is the set of all possible outcomes for that experiment. For example, if a bag contains 2 red, 3 yellow, and 5 blue tiles, the sample space consists of all 10 tiles. The event of drawing a yellow tile has 3 elements in the sample space and the event of drawing a blue tile has 5 elements in the sample space.

For the purpose of this indicator, students should explore experiments that require only a single device such as a number cube, a spinner, or drawing a number cube from a bag. These are referred to as single-stage events because there is only one activity to determine an outcome.

Materials Needed

One bag per pair of students with 10 different colored tiles (such as 6 red, 2 green, 1 yellow, and 2 blue)
10 counters per pair

Create a Game

The task is to separate the possible outcomes into two lists, one for each of the two players. For example, player A might be assigned the red tiles while player B is assigned the green, yellow, and blue tiles. The assignments should be recorded. The players take turns drawing a tile from the bag and then replacing it. When players draw a tile of their color, they win one counter. If it is not their color, they opponent wins a counter. Start with ten counters. Players take turns drawing and replacing tiles until all ten counters have been won.

Repeat the activity with different tile combinations so that you can observe how students divide the events and on what basis. This observation is a form of formative assessment. Try situations such as 2 red, 3 blue, 7 yellow, where there is no possible way to create a fair game. Will this bother students? What do they believe about their chances of winning? For a 2-3-5 bag of tiles, will students separate the three colors to create a fair game? Allow students to replay the game with the same tiles but with a change of how the colors are divided.

Create a Game can also be played with different spinners.

Once students have had hands-on experiences with single-stage events, introduce represent the probability of a single-stage event in words and fractions using the Create a Game configuration. For example, the probability of choosing red in the example given is 6 out of 10, $\frac{6}{10}$ or $\frac{3}{5}$. Repeat this process for other examples.

Note: Even though another activity is given for indicator 5-6.6, this lesson could be used to introduce the sum of probabilities always equals 1.

c. Misconceptions/Common Errors

It may be difficult for students to identify all possible outcomes of an experiment and consider their relative chances of occurring.

d. Additional Instructional Strategies/Differentiation

For additional instruction strategies, refer to Navigating through Data Analysis and Probability: Grades 3-5, NCTM, 2006.

Submitted from A5

Roll On

Materials Needed:

2 numbered cubes or dice
paper
pencil

Directions:

1. If you rolled the dice and added the numbers, what do you predict would be the most likely sum?
2. Create a tally sheet with the sums from 1 to 12.
3. Roll your dice 36 times and record a tally next to the sum each time it occurs.

A. What sum of sums occurred the most?
B. What sum of sums occurred the least?
C. Was you prediction correct? Why or why not?

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- www.shodor.org/interactivate/activities/**adjustablespinner**/
- http://www.mathwire.com/data/dicetoss1.html

f. Assessing the <u>Lesson</u>

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Formative Assessment Note: In "Create a Game" watch for students who omit an outcome from the two lists. Allow them to play the game anyway. When the omitted outcome occurs, listen to how they handle the situation. You want students to realize all outcomes are possible even though some may have small chances.

Something else to look for in "Create a Game" is the manner in which the outcomes are sorted between the two players. Do they seem to understand the chances of each and try to make the game fair or do luck and favorite colors influence decisions?

2. Teaching Lesson B: Sums of Probabilities

In indicator 5-2.8, students generate strategies for adding and subtracting fractions with like and unlike denominators; therefore, students may explore this indicator through the use of concrete or pictorial models as opposed to a formal algorithm for adding fractions.
Through a variety of experiences and the use of hands-on models, students should conclude that the sum of the probability is one.

*For this indicator (5-6.6), it is **<u>essential</u>** for students to:*
- *Understand the meaning of probability*
- *Understand how to write a probability as a fraction*
- *Understand how to represent adding of fractions using concrete and/or pictorial models*

*For this indicator, it is **<u>not essential</u>** for students to:*
- *Add fractions*

a. Indicators with Taxonomy

5-6.6 Conclude why the sum of the probabilities of the outcomes of an experiment must equal 1. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

Materials Needed:

- Index cards
- Opaque bag

Lesson: Sums of Probabilities

Write each letter in the word, Clemson, on a card. Place the cards in a bag. One card is chosen at random. What is the probability of choosing the letter T?

Probability of T = $\frac{0}{7}$ → $\frac{\text{zero favorable outcomes}}{\text{seven total possible outcomes}}$

Since there are no T's in the bag, choosing a T is impossible. The probability of choosing the letter T is 0.

What is the probability of choosing a letter in the word Clemson?

Probability of a letter in the word Clemson =

$\frac{7}{7}$ → $\frac{\text{7 favorable outcomes}}{\text{7 possible outcomes}}$

Since all of the letters in the word are in the bag, the event is certain. The probability of choosing a letter in Clemson is 1.

- If an event is impossible, the probability of it occurring is 0.
- If an event is certain, the probability of it occurring is 1.

Because all of the letters in the word Clemson are in the bag, you can find the probability of choosing a letter in Clemson by finding the sum of the probabilities of all the letters.

$P(C)=\frac{1}{7}$ $P(E)=\frac{1}{7}$ $P(L)=\frac{1}{7}$ $P(M)=\frac{1}{7}$ $P(N)=\frac{1}{7}$

$P(O)=\frac{1}{7}$ $P(S)=\frac{1}{7}$

$$\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=\frac{7}{7} or 1$$

Why does the sum of the probabilities equal to 1? Have students create a similar situation as the one given and problem statements involving probability based on information from their situation.

Repeat the above procedure allowing students more independence.

c. Misconceptions/Common Errors

After a couple of simple examples students can reach the conclusion that the sum of the probabilities is one. They don't have to "add" the probabilities of every event

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

The same technology listed for Lesson A of this module may be used for this indicator by taking it a step further and having students analyze why the sum of the probabilities must equal to one.

f. Assessing the Lesson

Formative Assessment is embedded within the lesson through questioning and observation, however, other formative assessment strategies should be employed.

Using an exit ticket, have students explain what they know about the sum of the probabilities of the outcomes of an experiment.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The objective of this indicator (5-6.5) is to represent which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To represent is to translate from one form (probability) to another (numerical and words). The learning progression to **represent** requires students to explore single stage events. Students recall and understand how to find possible outcomes. They use appropriate methods to record these outcomes. Students analyze this information (5-1.1) and generate descriptions (5-1.4) about part/whole relationships they observe. They translate these observations into a probability statement and then to numerical form (fraction) and explain and justify their answers using correct, clear and complete oral and written mathematical language (5-1.5).

The objective of this indicator (5-6.6) is to conclude which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To conclude is to draw a logical conclusion from presented information. The learning progression to **conclude** requires students to recall and understand the meaning of probability. Students explore a variety of single stage events and calculate the probability of individual outcomes. Students analyze this information (5-1.1) and look for relationships among the probabilities. They generate mathematical statements (5-1.4) about these relationships and use concrete models to prove or disprove these statements. They conclude that the sum of the probability of the outcomes of an experiment is 1 and explain and justify their answers (5-1.3) using correct, clear and complete oral and written mathematical language (5-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. If you tossed a penny once in the air, what is the probability of it landing face down (getting tails)? Write your answer in words and fraction form.

2. Why must all the probabilities of each outcome together equal 1?

3. There are 3 blue marbles, 4 green marbles, and 2 yellow marbles in a bag. If you took a marble out of the bag without looking, what is the probability you would get a green marble?

A $\frac{1}{3}$ B $\frac{4}{5}$ C $\frac{4}{9}$ D $\frac{5}{4}$