SOUTH CAROLINA SUPPORT SYSTEM INSTRUCTIONAL GUIDE

Content Area	**Sixth Grade Math**

First Nine Weeks

Standards/Indicators Addressed:

Standard 6-2: The student will demonstrate through the mathematical processes an understanding of the concepts of whole-number percentages, integers, and ratio and rate; the addition and subtraction of fractions; accurate, efficient, and generalizable methods of multiplying and dividing fractions and decimals; and the use of exponential notation to represent whole numbers.

6-2.1* Understand whole-number percentages through 100. (B2)
6-2.2* Understand integers. (B2)
6-2.3* Compare rational numbers and whole-number percentages through 100 by using the symbols ≤, ≥, <, >, and =. (B2)
6-2.4* Apply an algorithm to add and subtract fractions. (C3) *Connects to 3.2, 3.3, 3.4, and 3.5*
6-2.5* Generate strategies to multiply and divide fractions and decimals. (B6)
6-2.6* Understand the relationship between ratio/rate and multiplication/division. (B2) (Introduced – to be revisited in 4th nine weeks)
6-2.7* Apply strategies and procedures to determine values of powers of 10, up to 10^6. (C3)
6-2.8* Represent the prime factorization of numbers by using exponents. (C2)
6-2.9* Represent whole numbers in exponential form. (C2)

Standard: 6-3: The student will demonstrate through the mathematical processes an understanding of writing, interpreting, and using mathematical expressions, equations, and inequalities.

6-3.3* Represent algebraic relationships with variables in expressions, simple equations, and simple inequalities. (B2) **(Introduced – to be revisited in 2nd and 3rd nine weeks** *Connects to 2.4 and 2.7***)**
6-3.4* Use the commutative, associative, and distributive properties to show that two expressions are equivalent. (C3) **(Introduced – to be revisited in 2nd and 3rd nine weeks** *Connects to 2.4 and 2.7***)**

Standard 6-5: The student will demonstrate through the mathematical processes an understanding of surface area; the perimeter and area of irregular shapes; the relationships among the circumference, diameter, and radius of a circle; the use of proportions to determine unit rates; and the use of scale to determine distance.

6-5.6* Use proportions to determine unit rates. (C3) **(Introduced – to be revisited in 4th nine weeks)** *Connects to 2.6*
6-5.7* Use a scale to determine distance. (C3) **(Introduced – to be revisited in 4th nine weeks)** *Connects to 2.6*

* These indicators are covered in the following 5 Modules for this Nine Weeks Period.

<table>
<tr><td colspan="4">Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.</td></tr>
<tr><th colspan="4">Module 1-1 Number Structure and Relationships – Rational Numbers</th></tr>
<tr><th>Indicator</th><th>Recommended Resources</th><th>Suggested Instructional Strategies</th><th>Assessment Guidelines</th></tr>
<tr><td>Module 1-1 Lesson A:

6-2.1 Understand whole-number percentages through 100. (B2)</td><td rowspan="3">NCTM's Online Illuminations
http://illuminations.nctm.org

NCTM's Navigations Series

SC Mathematics Support Document

<u>Teaching Student-Centered Mathematics Grades 5-8</u> and <u>Teaching Elementary and Middle School Mathematics Developmentally 6th Edition</u>, John Van de Walle

NCTM’s <u>Principals and Standards for School Mathematics</u> (PSSM)

Textbook Correlations – See Appendix A</td><td>See Instructional Planning Guide Module 1-1 <u>Introductory Lesson A</u>

See Module 1-1, Lesson A <u>Additional Instructional Strategies</u></td><td>See Instructional Planning Guide Module 1-1 <u>Lesson A Assessment</u></td></tr>
<tr><td>Module 1-1 Lesson B:

6-2.2 Understand integers. (B2)</td><td>See Instructional Planning Guide Module 1-1, <u>Introductory Lesson B</u>

See Instructional Planning Guide Module 1-1, Lesson B <u>Additional Instructional Strategies</u></td><td>See Instructional Planning Guide Module 1-1 <u>Lesson B Assessment</u></td></tr>
<tr><td>Module 1-1 Lesson C:

6-2.3 Compare rational numbers and whole-number percentages through 100 by using the symbols ≤, ≥, <, >, and =. (B2)</td><td>See Instructional Planning Guide Module 1-1 <u>Introductory Lesson C</u>

See Instructional Planning Guide Module 1-1, Lesson C <u>Additional Instructional Strategies</u></td><td>See Instructional Planning Guide Module 1-1 <u>Lesson C Assessment</u></td></tr>
</table>

Module 1-1 Lesson D **6-2.9** Represent whole numbers in exponential form. (C2)		See Instructional Planning Guide Module 1-1 Introductory Lesson D See Instructional Planning Guide Module 1-1, Lesson D Additional Instructional Strategies	See Instructional Planning Guide Module 1-1 Lesson D Assessment
Module 1-2 Operations – Addition and Subtraction			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 1-2 **Lesson A and B:** **6-2.4** Apply an algorithm to add and subtract fractions. (C3) *Connects to 3.2, 3.3, 3.4, and 3.5*	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 1-2 Introductory Lesson A and B See Instructional Planning Guide Module 1-2, Lesson A and B Additional Instructional Strategies	See Instructional Planning Guide Module 1-2 Lesson A and B Assessment

Module 1-4 Representations, Properties, and Proportional Reasoning **Introduced – to be revisited in 2nd and 3rd nine weeks** *Connects to 2.4 and 2.7*			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 1-4 Lesson A: **6-3.3** Represent algebraic relationships with variables in expressions, simple equations, and simple inequalities. (B2)	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics	See Instructional Planning Guide Module 1-3 Introductory Lesson A See Instructional Planning Guide Module 1-3, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 1-3 Lesson A Assessment

Module 1-4 Lesson B: **6-3.4** Use the commutative, associative, and distributive properties to show that two expressions are equivalent. (C3)	Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 1-3, Introductory Lesson B See Instructional Planning Guide Module 1-3, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 1-3 Lesson B Assessment
Module 1-3 Operations – Multiplication and Division			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 1-3 Lesson A: **6-2.5** Generate strategies to multiply and divide fractions and decimals. (B6)	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and	See Instructional Planning Guide Module 1-4, Introductory Lesson C See Instructional Planning Guide Module 1-4, Lesson C Additional Instructional Strategies	See Instructional Planning Guide Module 1-4 Lesson C Assessment

Module 1-3 Lesson B: **6-2.6** Understand the relationship between ratio/rate and multiplication/division. (B2)	Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 1-4, Introductory Lesson B See Instructional Planning Guide Module 1-4, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 1-4 Lesson B Assessment
Module 1-3 Lesson C: **6-2.7** Apply strategies and procedures to determine values of powers of 10, up to 10^6. (C3)		See Instructional Planning Guide Module 1-4, Introductory Lesson C See Instructional Planning Guide Module 1-4, Lesson C Additional Instructional Strategies	See Instructional Planning Guide Module 1-4 Lesson C Assessment
Module 1-3 Lesson D **6-2.8** Represent the prime factorization of numbers by using exponents. (C2)		See Instructional Planning Guide Module 1-4, Introductory Lesson D See Instructional Planning Guide Module 1-4, Lesson D Additional Instructional Strategies	See Instructional Planning Guide Module 1-4 Lesson D Assessment

Module 1-5 Proportional Reasoning

Introduced – to be revisited in 4th nine weeks. *Connects to 2.6*

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines
Module 1-5 Lesson A **6-5.6** Use proportions to	NCTM's Online Illuminations http://illuminations.nctm.org	See Instructional Planning Guide Module 1-5 Introductory Lesson A	See Instructional Planning Guide Module 1-5

determine unit rates. (C3)	NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 1-5, Lesson A Additional Instructional Strategies	Lesson A Assessment
Module 1-5 Lesson B **6-5.7** Use a scale to determine distance. (C3)		See Instructional Planning Guide Module 1-5, Introductory Lesson B See Instructional Planning Guide Module 1-5, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 1-5 Lesson B Assessment

MODULE

1-1

dule addresses the following indicators:

6-2.1 Understand whole-number percentages through 100. (B2)
6-2.2 Understand integers. (B2)
6-2.3 Compare rational numbers and whole-number percentages through 100 by using the symbols ≤, ≥, <, >, and =. (B2)
6-2.9 Represent whole numbers in exponential form. (C2)

This module contains 5 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

1. Continuum of Knowledge

6-2.1 Understand whole-number percentages through 100. (B2)

- The sixth grade is the first time students are introduced to the concept of percents.
- In sixth grade, students understand whole number percentages through 100(6-2.1).
- In seventh grade, students understand fractional percentages and percentages greater than one hundred (7-2.1).

6-2.2 Understand integers. (B2)

- The sixth grade is the first time students are introduced to the concept of integers.
- In the sixth grade, students understand integers (6-2.2).
- In the seventh grade, students will generate strategies to add, subtract, multiply, and divide integers (7-2.8).

6-2.3 Compare rational numbers and whole number percentages through 100 by using the symbols ≤, ≥, <, >, and =. (B2)

- In the fourth grade, students compare decimals through hundredths by using the terms *is less than*, *is greater than*, and *is equal to* and the symbols <, >, and =(4-2.7).
- In the fifth grade, students compare whole numbers, decimals, and fractions by using <, >, and = (5-2.4).
- In the sixth grade, students will compare rational numbers and whole number percentages through 100 by using the symbols ≤, ≥, <, >, and = (6-2.3). This is the first time students compare rational numbers and percentages.
- In the seventh grade, students will compare rational numbers, percentages, and square roots of perfect squares by using the symbols ≤, ≥, <, >, and = (7-2.3).

6-2.9 Represent whole numbers in exponential form. (C2)

- In fifth grade, students classified numbers as prime, composite, or neither (5-2.6) and generated strategies to find the greatest common factor and the least common multiple of two whole numbers (5-2.7).
- In the sixth grade, represent the prime factorization of numbers by using exponents

2. Key Concepts/Key Terms

*These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and in use in conversation with students.

- * denominator
- * percent (per hundred)
- * fraction
- * decimal
- * integer
- * whole number
- * positive
- * negative
- * ratio
- * exponential form
- * base
- * exponent
- rational numbers

Symbols:
$\leq$, $\geq$, <, >, =, and %

II. Teaching the Lesson(s)

1. Teaching Lesson A –Working with Whole Number Percents

Sixth grade is the first time students are introduced to percents. Students will begin their study of whole number percentages through 100. Percent literally means per hundred. Percentages can be represented as a fraction with a denominator of 100, as a decimal, or as a whole number followed by the % symbol. Students will extend their learning from fifth grade to compare whole-number percentages through 100 as well as rational numbers by using the symbols ≤, ≥, <, >, and =. Student have previously used <, >, and =. Students will need to understand the differences between <, > and ≤, ≥. By providing experiences for students, they will see the symbols are similar but different. Students should discuss the symbolic comparison as well as the relative magnitude of the numbers being compared.

For this indicator, it is **essential** for students to:

- Understand that percentage mean out of 100 or part of a whole divided into 100 parts
- Connect the concept of percentages to fractions and decimals
- Connect percentages with familiar fractional equivalents
- Convert between fractions/decimals/percents

For this indicator, it is **not essential** for students to:

- Work with percentages with decimals for example 33.3 %

a. Indicators with Taxonomy

6-2.1 Understand whole-number percentages through 100. *(B2)*

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson A

Materials needed:

- Percent Beads (Made by stringing 50 red and 50 green plastic ¼ inch beads together in a circle. Use dental floss for string. Alternate the beads in groups of 5-5 red, 5 green, etc.)-one per student (NOTE: If strung properly, the circle o beads should fit around the outside of a standard coffee filter.)
- Coffee filters-one per student
- Straight Edge/Ruler- one per student
- Hundreds Disk- Attached

Allow students to string the beads. This is a good way to introduce the concept of percent. Tell students that percent is French for one hundred. One hundred percent means all of something. Therefore, the 100 beds on their string are 100% of their place value beads. Write on the board, "100 beads = 100% of the Place Value Beads". Ask students what percent would 5 beads be, 25 beads be, etc. Continue to quiz and write on the board. Since this is the students' first introduction to percent, they will need practice making the symbol. (It would be a good idea to have students find several pictures of the percent symbol to bring to class the next day. When they share, they should also have information to share about in what context the symbol was used-sale paper, stock market, etc.

Next, have the students fold the coffee filter in half and then in half again- making four equal sections. Have them trace the crease marks with a straight edge and put a dot at the intersection of the two lines. Pose questions such as what fractional part does one of the sections represent, two of the sections, three of the sections?

Have students place their Percent Beads around the outside of the coffee filter. Line the beads up so that a colored section begins on the crease lines. Begin the discussion by asking the following questions:

- What fractional part does one of the sections represent? (one-fourth). How do you know?
- What fractional part of the beads goes around the edge of one section? (25/100 or ¼) How do you know? (The groups of 5 colored beads makes it easier to count.) Write " ¼ = 25/100=" on the board.
- What percent of all the beads do 25 beads represent? (25%). How do you know? Continue writing " ¼ = 25/100 = 25% = .25

Select a new number of beads and continue the discussion.

Ask the students to shade a hundreds disk and exchange with a partner. The partner is to write all they know about fractions, decimals and percents with regard to the shaded part. Students exchange back and check each other's work. Teacher moves around the room posing individual questions to check for understanding questions to check for understanding. A hundreds disk is attached.

NOTE: In order to help students think of fraction/decimal /percent relationships, experiences should also include square models_ the commercial product "Decimal Squares" is an excellent manipulative.

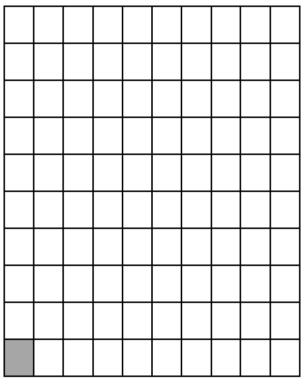
c. Student Misconceptions/Common Errors

- When there is only one number behind the decimal, students commonly forget that the tenths place is also referred to 10-hundredths. Thus, you may see them represent .5 as 5% instead of 50%.
- Also, many times they leave the decimal when writing the percent. Ex: .45 may be written as .45%

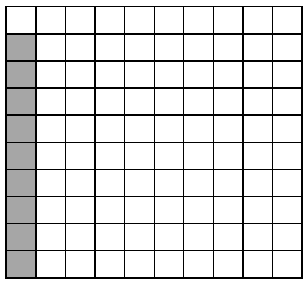
d. Additional Instructional Strategies –

- Starting with a review benchmark fractions will give students a review of fractions and decimals in a familiar setting. This will help them as them as they transfer their understanding to other less familiar percents, decimals and fractions.

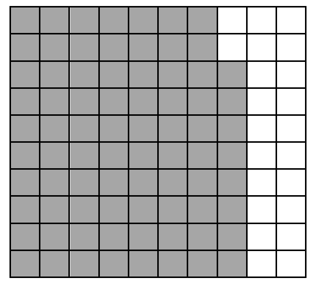
- **Decimal squares** Decimal squares may be used to help build understand of percentages. Students are connecting the graphical representation of percentages to the symbolic (number only) form



1%



9%



78%

- Connect the concept of percent to money by guiding students to see that if a penny is 1% of a dollar, then it is less than 1% of any amount greater than a dollar extend their learning by using coins to help the students see that any amount greater than a coin is more than 100% of that coin.

- **"Grid and Percent It" lesson from NCTM**
  http://illuminations.nctm.org/LessonDetail.aspx?id=L249

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/category_g_3_t_1.html)

- Percentages (http://www.mathsisfun.com/percentage-menu.html)
- AAA Math (http://www.aaamath.com/pct.htm)

f. Assessing the Lesson

Formative –See questions listed within the lesson.

2. Teaching Lesson B – Understanding Integers

Students' knowledge of number is extended to numbers less than zero which are best represented by real world situations. Students should identify situations where numbers less than zero are used. At this stage, student exposure to numbers less than zero is limited to the set of integers. Students are **not** expected to perform operations on integers at this stage of their mathematical development. Students should be able to describe numbers less than zero using real world models to aid in their development of understanding integers. Students will generate strategies to compute integers in seventh grade and apply algorithms for computation in eighth grade. Because the computation of integers is abstract, students need to develop an understanding of integers in context so that they can easily connect that concrete understanding to the abstract process of computation. Students need to see that we use integers in math every day.

The concept of integers should be introduced to children using familiar models. Students may find it helpful to represent integers on a number line, with two color counters, or with everyday tools such as a thermometer. Because of the increased use of the metric system, sixth grade students may have already been informally introduced to the concept of negative numbers as they relate to temperature. Situations that can be visualized and make sense are best for introducing the concept of integers. Other models or situations that may be used to introduce this concept are distance, altitude, balances of money (quiz shows), and sports events. Horizontal number lines are useful when comparing the magnitude of numbers. Numbers to the right of zero are positive; numbers to the left of zero are negative. Vertical number lines can be used by asking students to visualize a thermometer. Students can see that numbers above zero are positive as numbers below zero are negative.

After students have experienced integers, they can begin to study rational numbers. For students to compare rational numbers, they will first need to understand what a rational number is - any number that can be written as a fraction, a ratio of two integers $\frac{a}{b}$, where b is never zero.

For this indicator, it is **essential** for students to:

- Have a strong number sense with respect to whole numbers, fractions, and decimals.

- Recall the definition of integer
- Understand the relationship between integers and other types of numbers
- Identify real world situations that involve integers

For this indicator, it is **not essential** for students to:

- Perform operations of integers

a. Indicator with Taxonomy:

6-2.2 Understand integers. (*B2)*
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson B

Materials Needed:

- thermometers

- Present a problem situation: "The lowest temperature ever recorded in Florida was minus 2 degrees. The lowest temperature in Georgia was minus 17 degrees. Which temperature was the lower, or colder, of the two?"
- Display a thermometer that shows values less than zero. Turn it horizontally to show how it is a number line with positive and negative temperatures.
- Display a number line with positive and negative numerals.
- Ask: "When you use a whole number line, do the numbers' values increase or decrease as you move to the right?" (increase) "What happens when you move to the left on the line?" (The values of numbers decrease.)
- Refer to the horizontally placed thermometer. Ask: "What happens as you move to the left on this number line?" (The values of the numbers decrease; the temperatures get colder and colder.)
- Ask: "What can you conclude about the lowest recorded temperatures in Georgia and Florida?" (Georgia's temperature was colder than Florida's.)
- Have children use the number line to answer these questions:
  Which is more or warmer?
  Minus 4 or minus 8?
  Minus 14 or minus 3?
  Minus 17 or minus 19?
  Which is less or colder?
  Minus 4 or minus 7?
  Minus 16 or minus 12?
  Minus 12 or minus 1?
- Have children use weather reports in newspapers to locate and compare the highest and the lowest temperatures for their state, province, or nation.

- Look in encyclopedias or almanacs for record low temperatures and put them on display a number line or thermometer.
- Check the Weather Channel website for record highs and lows for various locations.

c. Misconceptions/Common Errors

- Students often misunderstand that negative integers decrease in magnitude as they move away from zero. For example, it is difficult for them to understand that -17 is less than -10.

d. Additional Instructional Strategies

Checks and Bills

Extension lesson: small groups
Objective: students recognize situations in which they have negative values of money.
Materials: play money in $1, $5, $10, and $20 denominations; cards with statements such as: You receive a rebate of $2.00 from Wrangler J." "You receive $5.00 for doing yard work." You pay $5.00 for an afternoon at Waterworld." "You buy a baseball for $6.00."
NOTE: One simple way to generate scenarios is to ask each student in your class to create 3 "gain money" and 3 "lose money" situations. Type these up and cut them out for use in the game. That way, you don't have to create everything yourself.

- Two to four persons take turns in clockwise order. One player also serves as banker. Cards are shuffled and put face down.
- Each player begins with $50 in bills. A bank containing $100 is set up for each group.
- The first player draws the top card and tells whether it is a positive card (money received) or a negative card (money is paid) and how much will be gained or lost. For example, "I have a negative card that will cost me $4.00." Players receive money from the bank for positive cards and give money to the bank for negative cards.
- Play rotates until one player is out of money, until the allotted time has elapsed, or one player has $100.

Playing Football

Materials:

- poster board marked like a football field
- a miniature football marker

- Present this situation: "The Lawrencetown football team fielded a punt on their own 23-yard line. A successful pass play gained 24 yards. On first down, a running play resulted in a loss of 6 yards. The next play gained 12 yards, but the fourth play lost 2 yards. Where is the ball?" Then ask: "What is the next play you would call if you were the coach?" [Students can model each play on the poster board football field.]
- Have students create their own game situations. Tell them that each group must prepare a written explanation of its game situation and its reason for the play it called. (Other game situations students may consider are the time left in the game and the game's score.)
- Have students present their game situation.
- Have each group prepare a four-play scenario that includes gains and losses to exchange with another group. Discuss responses to these scenarios.

Further Strategies

- Representing integers on a number line including rational numbers(fractions and decimals) helps student gain a understanding of how integers relate to other numbers.
- Use two color counters to represent integers
- Use real world examples such as temperature(reading a thermometer), checkbook(deposits/withdrawals), distance, altitude(above/below sea level), and sports events(football-gains and loss of yards)

A5

- Read-Aloud Strategy: The teacher will read <u>Less Than Zero</u> by Stuart J. Murphy to introduce the unit on integers. Throughout the book, point out the concepts that will be taught in the unit.

- Students can write about situations that can be represented by 0, positive, and negative numbers. Instruct students to complete a *K-W-L Chart* to show what they know about integers. Record responses from various students and have the class discuss the results.

- Brainstorm situations that use integers. For example:

  Sports scores: If your favorite football team gained 6 yards during the big game, it could be represented by +6. If your team loses 6 yards, it could be represented by -6.

  Weight gain and loss: Kellie gained 7 pounds during summer vacation (+7), but lost the 7 pounds after she returned to school (-7).
  Profit and loss: A gift shop lost $8,300 (-8,300) the first year of business but recovered the amount (+8,300) the second year.

e. Technology

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- Introduction to Integers (http://www.mathleague.com/help/integers/integers.htm)
- Learn Integers at your own pace (http://www.mathgoodies.com/lessons/vol5/intro_integers.html)
- Integer Bars or Cuisenaire Rods (http://www.arcytech.org/java/integers/)
- National Library of Virtual Manipulatives (http://nlvm.usu.edu/)

f. Assessing the Lesson

i. Formative – Refer to questions throughout the lesson. Use responses to guide instruction.

3. Teaching Lesson C – Comparing Fractions, Decimals, and Percents

For this indicator, it is **essential** for students to:

- Understand the meaning of rational numbers
- Understand the difference between ≤ and ≥
- Translate numbers to same form before comparing numbers, where appropriate
- Translate between the fraction and percents

For this indicator, it is **not essential** for students to:

- Work with repeating decimals; use numbers with terminating decimals

a. Indicators with Taxonomy

6-2.3 Compare rational numbers and whole-number percentages through 100 by using the symbols ≤, ≥, <, >, and =.
(B2)

Cognitive Process Dimension: Understand

Knowledge Dimension: Conceptual Knowledge

Teacher note: Essentially, students must compare fractions, decimals, and percents.

b. Introductory Lesson C –Comparing Fractions, Decimals, and Percents

Materials:

- Large index cards labeled with the integers -10 through 10.

1. Have the students make a human number line using the index cards.
2. Call out inequalities and have the students step forward if it applies to their number. For instance: P is a number that is greater than 4, or D is a number that is less than or equal to 0.
3. Show the students how to symbolize the inequalities that you are calling out. For instance P>4 and D≤ 0. As you show the symbolization, have the students with numbers that fit the inequality step forward.
4. Have students begin to symbolize contextual situations using inequalities. For instance, Dameon is younger than his sister Ellen. D< E. However, Dameon is older than 3 years less than Ellen's age. D> E-3

   ****Discussion about ≤ and ≥ will need to take place prior to this lesson as this is the first time students have worked with these mathematical symbols for inequalities.**

c. Misconceptions/Common Errors

Students often have a misconception the equal sign means "equal to". In most cases, students begin to see the left side of an equation as the problem and the right side as the answer. However, students need to be provided with experiences that the equal sign means "the same as". (i.e. 5 x 8 = 10 x 4; five times eight is the same as ten times 4) Understanding this concept will help students as they begin to compare numbers. For more information, refer to Van de Walle's <u>Elementary and Middle School Mathematics Teaching Developmentally</u>, pps. 260-262.

d. Additional Instructional Strategies

e. Technology

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- National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/category_g_3_t_1.html)
- Games (http://education.jlab.org/sminequality/index.html)
- Basic Rules for Solving Inequalities (http://www.sosmath.com/algebra/inequalities/ineq01/ineq01.html)
- www.decimalsquares.com

f. Assessing the Lesson

Formative – Use questions throughout the introductory lesson to check for understanding. Include clarification questions on the meaning and use of ≥ and ≤, as well as the meaning of the equal sign. Have students describe a solution set for inequalities such as, b – 5 < 3; 7 + f >7; x ≤ 10; s > -4; 42/6 ≥ r. Probe for explanations; use number lines to help as needed. It may also be helpful to have students name numbers that are NOT a solution to the inequalities and explain why to better check for understanding.

4. Teaching Lesson D – Exponents

a. Indicators with Taxonomy

6-2.9 Represent whole numbers in exponential form. (C2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Procedural Knowledge

Exponential form: Students should experience this shorthand method for writing numbers expressed as repeated multiplication such as $64 = 4 \times 4 \times 4 = 4^3$. In this way students should be able to convert whole numbers into their exponential form. Students should be able to convert whole numbers into their exponential form.

For this indicator, it is **essential** for students to:

- Understand exponential form
- Translate from whole number to exponential form
- Translate from exponential form to whole number

For this indicator, it is **not essential** for students to:
None noted

b. Introductory Lesson D – Exponents

Materials

- sheet of paper per pair of students
- calculator (for those who need to "prove" the multiplication)
- handout for recording work – 1 for each student (optional; students can create the table as they work if you choose. If you do provide the handout, leave the table blank and have students fill it in.)
- transparency for you to use

Have students work in pairs so they can discuss the math they are doing.

Begin by having students set up their tables. They should start with just the first two rows, leaving room for 2 more rows later.

Number of folds	1	2	3	4	5	6	7	8
Number of sections	*2*	*4*	*8*	*16*	*21*	*64*	*128*	*256*
(This will be a LONG column. See next page.)								

Tell students to fold a piece of paper in half and then unfold it. They can see that there are two sections. This is the first entry in their table.
Ask them to predict how many sections would be formed if the paper could be folded in half eight times. If they need to, they can adjust their prediction as they work. If they do make adjustments, they should record why.

After they've recorded the number of sections formed by one fold, they should continue to fold the paper in half as many times as possible, recording the number of sections each time. *NOTE: It's likely they'll only be able to fold the paper five times or so. By then, they should be able to identify the pattern in the table.*

Debrief the folding investigation with the following questions:

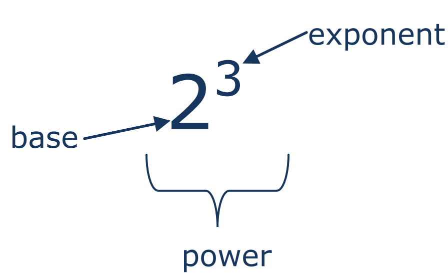
- Each time you made a new fold, how did the number of sections change? How did this help you complete the table?
- How did you do with your prediction for eight folds? What adjustments (if any) did you make to your prediction? At what points did you make adjustments and why?

Return students' attention to the table where they recorded their work. Tell students they can use the pattern they found to write the number of sections a little differently. Have students label the third row "Written as an expression." Get them started by doing the first couple of columns together. Ask them what they notice about the multiplication expressions they're writing. Ask them to describe the expressions. *(The text is turned "sideways" to make the string of factors fit in one line. You may want to have students do this in their own tables. It makes it easier to see the connection that the number of folds is the same as the number of times 2 is used as a factor.)* If students need to use a calculator to "prove" that they number of sections really is the product of the expression, let them do that. They need not use the exponent key – they may simply use the "x" key to multiply.

Tell students that mathematicians have a short hand way to write strings of the same factor. It's called exponential form and it has two parts: the base and the exponent. Have students label the fourth row "Exponential form – powers of 2." Show them how to write the first three columns, then ask them to complete the row. (The completed table is on the next page.)

Number of folds *[exponent (add this later – see notes)]*	1	2	3	4	5	6	7	8
Number of sections *(standard form)*	*2*	*4*	*8*	*16*	*32*	*64*	*128*	*256*
Written as a multiplication expression *Possible descriptions:* *The expression tells how many times the number of sections, which is the product, uses the factor of 2.* *The number of folds tells how many times the factor of 2 is used.*	2	2 • 2	2 • 2 • 2	2 • 2 • 2 • 2	2 • 2 • 2 • 2 • 2	2 • 2 • 2 • 2 • 2 • 2	2 • 2 • 2 • 2 • 2 • 2 • 2	2 • 2 • 2 • 2 • 2 • 2 • 2 • 2
Exponential form- powers of 2	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8

Label the parts of the expression as shown and have students put it in their notes below their tables:



Be sure to tell students how to read the expression.

Ask them to first discuss with their partners, and then write definitions for base, exponent, and power in their own words based on the work they recorded in their tables. Have pairs of students form quads and have them share and refine those definitions. Circulate as students work and ask questions to guide them.

Finally, provide "official" definitions and ask students to compare theirs to those. Ask them to make any final refinements.

Call students' attention to their tables. Ask them what they notice about the number of folds and the exponents on the powers of 2. They should notice that the number of folds denotes the exponent. They can add that to the first row of the table. It's also important for them to notice that 2^1 is an exponential expression, but that we usually just "leave off" the exponent of 1 because it's understood. This will help later on when they encounter variables with coefficients of 1.

Write the expression $2^4 = 16$ and read it for them so that they have an example of the correct way to read the expression. Ask them to write it in their notes: "Two to the fourth power is/equals 16."

c. Misconceptions/Common Errors –

- The most common error students make when solving for exponents is multiplying the exponent by the base number or even attempting to add the two numbers. Another common error to watch for would be students who write the exponent the same size and position as the base number.

- Students may believe that there is only one exponential form for every number. Explore numbers like 64 that have multiple exponential forms such as 2^6, 4^3, and 8^2.

d. Additional Instructional Strategies

- Have students create a table similar to the one they completed for the folding investigation. Give pairs of students a number between 3 and 9 to work on. They will also need a calculator so that it doesn't become tedious. As pairs complete their tables, have them find another pair who worked with the same number so they can check each other.

Base: 3 (each pair gets a # 3 – 9)

exponent	1	2	3	4	5	6	7	8
power (exponential form)	3^1	3^2						
written as a multiplication expression	3	$3 \bullet 3$						
standard form	3	9						

- Show students how to use the exponent key on a scientific calculator.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- National Library of Virtual Manipulatives (http://nlvm.usu.edu/)
- Math Goodies: Exponents (http://www.mathgoodies.com/lessons/vol3/exponents.html)

f. Assessing the Lesson

- Exit slip: Have students write about the concept of exponential notation learned throughout the lesson, providing examples.
- Notebook entry: Explain why 2^5 and 5^2 do not give the same result.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

Assessment Guidelines

6-2.1 Understand whole-number percentages through 100. (B2)

The objective of this indicator is to understand which is in the "understand conceptual" knowledge cell of the Revised Bloom's Taxonomy. To understand is to construct meaning. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge should include a variety of examples. The learning progression to **understand** requires students to recall the meaning of fractions and decimals. Students use the definition of percent to generate examples of percentages by generalizing connections (6-1.7) to real world situations where percentages are needed. They analyze these situations and explore how the percentages can be represented as fraction and decimals. As students analyze these situations, they use inductive and/or deductive reasoning to formulate mathematical arguments (6-1.3) about the relationship between fractions, decimals and percents. Students understand equivalent symbolic expressions as distinct symbolic forms (percent, fraction, decimal) that represent the same relationship (6-1.4).

6-2.2 Understand integers. (B2)

The objective of this indicator is to understand, which is in the "conceptual knowledge" of the Revised Taxonomy. Conceptual knowledge is not bound by specific examples and shows the interrelationship of among integers, whole numbers, fractions and decimals (rational numbers). The learning progression to understand requires students to recall the characteristics of whole numbers, fractions, and decimals. Students generate examples by generalizing connections (6-1.7) of real world situations where positive and negative numbers are needed. Then students should use correct and clearly written or spoken words (6-1.6) to create a definition of integers. In order to understand integers, students also examine non-examples of integers. They generalize connections (6-1.7) by representing integers (whole numbers), fractions, and decimals on the number line. Using this understanding, students evaluate their definition of integers by posing questions to prove or disprove their conjecture (6-1.2).

6-2.3 Compare rational numbers and whole-number percentages through 100 by using the symbols ≤, ≥, <, >, and =. (B2)

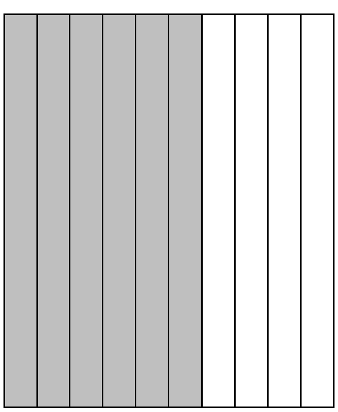
The objective of this indicator is to compare which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning therefore, students should not just learn procedural strategies for comparing but they should build number sense around these types of numbers. The learning progression to **compare** requires students to recognize and understand rational numbers and whole number percentages through 100. Students understand the magnitude of rational number and whole numbers. Students use their conceptual understanding to compare without dependent on a traditional algorithm and use concrete models to support understanding where appropriate. Students recognize mathematical symbols $<$, $>$, $\geq$, $\leq$ and = and their meanings. As students analyze (5-1.1) the relationships to compare percentages and rational numbers, they construct arguments and explain and justify their answer to classmates and their teacher (5-1.3). Students should use correct, complete and clearly written and oral mathematical language to communicate their reasoning (5-1.5).

6-2.9 Represent whole numbers in exponential form. (C2)

The objective of this indicator is represent which is in the "understand procedural" knowledge of the Revised Taxonomy**.** To understand a procedural implies not only knowing the steps of the procedural but also understanding the purpose and value of using it. The learning progression to **represent** requires students to recall the structure of exponential form. Students explore a variety of problems. They analyze these situations and use inductive reasoning (6-1.3) to generalize a mathematical statements (6-1.5) about the relationship between exponential and whole number form. Students understand that the exponential form is an equivalent symbolic expression that represents the same the number but in a different form (6-1.4). Students engage in meaningful practice to support retention.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Write the fraction, decimal, and percent that correspond with the figure.

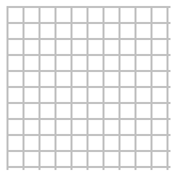


Fraction________

Decimal________

Percent ________

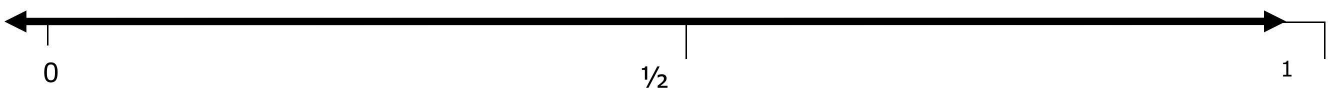
2. Shade 43%. Write the fraction and the decimal.



Fraction______

Decimal ______

3. Place 2/3, 0.55, and 75% on the number line.



4. The table below shows the temperature on four winter mornings in the Berkshire Mountains.

Winter Temperatures in Brookshire Mountains

Day	Temperature at 6:00 AM
Thursday	-9°C
Friday	-10°C
Saturday	-18°C
Sunday	-12°C

Which day had the warmest morning? How do you know? (*Thursday…explanation should relate to conceptual understanding of numbers less than 0.)*

(Adapted from *Mathematics Assessment Sampler*, NCTM)

5. Corina was investigating information about natural wonders of the world. She found that Mount Everest is the highest mountain in the world. It is 29,028 feet ABOVE sea level. She found that Mariana Trench in the Pacific Ocean is the lowest point on earth. It is 35,840 feet BELOW sea level. If Corina could throw a rock from the top of Mount Everest to the bottom of the Marianas Trench, how many feet would it fall? Draw a diagram and explain your answer.

(*64,868 feet. The diagram should show 29,028 feet above a representation of sea level as 0, and 35,840 further below the representation of sea level. This assessment requires an understanding of the relative position of positive and negative numbers through a common application. No rules or algorithms are necessary in order to solve this question*.)

(Adapted from *Mathematics Assessment Sampler*, NCTM)

6. On a number line, 0.6 is closest to which of the following: ¼, ½, or 1? Justify your answer. *(1/2. The justification should include the use of models, benchmark fractions, or equivalent forms to compare fractions or could explain that 0.5 is ½ and 0.6 is just a little larger.)*

7. Students in Mrs. Johnson's class were asked to tell why 4/5 is greater than 2/3. Whose reason is best? Explain your answer. Kelly said, "Because 4 is greater than 2." Keri said, "Because 5 is larger than 3." Kim said, "Because 4/5 is closer to 1 than 2/3." Kevin said, "Because 4 + 5 is more than 2 +3." *(Students should choose Kim's explanation.)*

8. Tell whether each number sentence is true or false. If the sentence is false, change it to make it true. Justify your answers.

- $8 \geq 7 + 1$
- $9 \div 3 \leq 3$
- ¾ < 75%
- $27 + 3 \geq 5 \times 6$
- ⅜ + ¼ = 87.5%
- $3 \leq -1$
- $54/9 > 10$
- 45% = 45/100

9. Write each number in expanded form and solve.

a. 6^2
b. 4^3
c. 10^6
d. 9^1

10. Write the following numbers in exponential and standard notation.

1. 12 x 12 x 12
2. 5 x 5 x 5 x 5
3. 3 x 3 x 3 x 3 x 3 x 3

11. A student evaluated 4^3 to be 12. What was the student's error? What would you do to help this student to understand?

12. $3^2 + 4^3$ = ________

MODULE

1-2

This module addresses the following indicators:

6-2.4 Apply an algorithm to add and subtract fractions. (C3)

This module contains 1 lesson. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

Indicator 6-2.4
Apply an algorithm to add and subtract fractions.

In the fourth grade, students apply strategies and procedures to find equivalent forms of fractions (4-2.8) and represent improper fractions, mixed numbers, and decimals (4-2.11). In the fifth grade, students generated strategies to add and subtract fractions with like and unlike denominators (5-2.8)

In the sixth grade, this is the first time students are required to perform addition and subtraction of fractions symbolically (6-2.4). Students also generate strategies to multiply and divide fractions and decimals (6-2.5). In the seventh grade, students will apply an algorithm to multiply and divide fractions and decimals (7-2.9).

- **Key Concepts/terms**

*These are vocabulary terms that are reasonable for students to know and be able t use. Terms without the * are additional terms for teacher awareness, knowledge and in use in conversation with students.

*equivalent fraction
*algorithm
*numerator
*denominator
*common denominator
*least common denominator

II. Teaching the Lesson

1. Teaching Lesson A – Add and Subtract Fractions

In fourth grade, students had experience writing equivalent fractions and representing improper fractions and mixed numbers. Students were given sufficient experiences with concrete and pictorial models to fully grasp these concepts which are extremely important prerequisites to adding and subtracting fractions. In fifth grade, students generated strategies to add and subtract fractions. As a result of sharing those generated strategies, students developed an understanding of the concepts. In sixth grade the emphasis is on applying an algorithm. As a result, by the end of sixth grade students should exhibit fluency when solving a wide range of addition and subtraction problems involving fractions.

Sixth grade is the first time students are required to perform addition and subtraction of fractions symbolically (numbers only). As a result, children need experiences that will enable them to make the link between those concrete and pictorial models used in fifth grade and the new symbolic operations. Students often have difficulty with the algorithm to add and subtract fractions and will forget that a common denominator is a necessity. Students should work with fractions in all forms including mixed fractions. Students should also be given opportunities to apply addition and subtraction of fractions in context – not merely perform the operation for the sake of adding and subtracting fractions. That means a problem situation should be introduced and students should explore possible solution strategies. Students should then share their strategies with the whole class as the teacher facilitates. The students' understanding of computation increases when they develop their own methods and discuss those methods with others. Finally, when the traditional algorithm is shown, some students will begin to use it while others will revert back to an invented algorithm until they are comfortable enough to move on.

Sixth grade students should build on their prior experiences with fraction models, benchmark fractions, and fraction equivalencies when they begin to estimate sums and differences of fractions. Emphasis should be placed on students explaining the method they used to estimate. Estimation should be used as a tool to check the reasonableness of answers to contextual problems.

For this indicator, it is **essential** for students to:

- Work with fractions in all forms including mixed numbers, proper and improper fractions.
- Subtracting with regrouping
- Add and subtract fractions in word problems
- Use estimation strategies to determine the reasonableness of their answers.

For this indicator, it is **not essential** for students to:

- Perform operations involving more than four fractions with compatible denominators.

- Perform operations involving more than three fractions with non-compatible denominators.

a. Indicators with Taxonomy

6-2.4 Apply an algorithm to add and subtract fractions. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson

Materials Needed:

- Fraction pattern blocks (yellow hexagon, red trapezoid, blue rhombus, green equilateral triangle, brown right trapezoid, purple right triangle)
- Overhead fraction pattern blocks
- Isometric dot paper 1 cm
- Isometric dot paper 2 cm
  (NOTE: The pattern blocks fit on the 2 cm paper perfectly)
- Transparencies of both isometric dot papers
- Colored pencils
- Teacher sheets w/ examples to get you started (at end of module after the assessment items)

Arrange desks in groups of 4 so that students may work together and communicate ideas.

Begin by reviewing some fractions vocabulary: whole, numerator, and denominator. Then name the yellow hexagon as one whole and tell the students to find and record the relationship of each of the other pieces to the whole. They should record their findings on the isometric grid paper by drawing the shapes. Lead a class discussion that focuses on the relationship of part to whole and the function of the numerator and denominator (as opposed to only their position). Students should also realize that 1 hexagon = 1 hexagon (i.e., the whole is itself a fraction equal to $\frac{1}{1}$).

Then ask the students to find the relationship of each piece to the other pieces. Remind them that the hexagon is still the whole. They should record their findings and be prepared to share them.

This process should be repeated using a different whole (possibly 1 hexagon and 1 red trapezoid, or 2 hexagons). The goal is to help students work flexibly with fractions and understand that the fraction always depends on the whole.

Help students generate pairs of equivalent fractions. Work on the overhead while they work at their desks with their own pattern blocks. CHANGE THE WHOLE ON THEM so they will think about what they're doing. After generating several examples, ask students if they can find a relationship between the pairs of fractions.

Please see **Teacher sheet—equivalent fractions** for s short list of examples. It is absolutely necessary that you "play" with the blocks ahead of time so that you can generate examples that are right for your students.

The relationship that should be established through their working with the blocks and your modeling with them is that equal fractions have a common factor in both the numerator and denominator—and that factor, which is equal to one whole ($\frac{1}{1}$) can be "seen" in the pieces. This lays the groundwork for renaming fractions when the denominators are unlike.

Name the hexagon as one whole and tell the students to model how they would add several different pairs of fractions with common denominators. The students should use the blocks at their desks to "prove" their answers. They should also simplify their answers by replacing blocks in the sum so that the fewest number of identical blocks is used. When they are comfortable with 1 hexagon as the whole, change the name of the whole and have them model further problems. Please see **Teacher Sheet—addition examples 1**.   The list of examples given is by no means exhaustive. Again, you need to "play" to come up with further examples of your own.

Use similar examples for subtracting fractions with common denominators. Again, rename the whole so that students continue to develop the idea that a fraction is dependent upon the whole being used.

Require students to record responses to the following questions:

- Describe what happens when fractions with the same denominator are added.
- Describe what happens when they are subtracted.
- Is this what you (students) expected?
- Explain making trades to simplify your answers.
- Describe what happens when the whole is something other than 1 hexagon.

Move to adding and subtracting fractions with different denominators. Students should work together and use the pattern blocks to solve problems like the ones on **Teacher sheet—addition examples 2.** As with like fractions, you need to generate your own, **much longer** list of examples for both addition and subtraction. Students need a lot of experience at this level in order to understand the algorithm later. Don't focus on the algorithm; focus instead on manipulating the blocks to rename the pieces so that students get to the place where they have like fractions and can concentrate on the effects of adding and subtracting.

Students need to record what they're doing so that they are able to reflect on their thinking later. Provide isometric dot paper and colored pencils for them. Either kind of isometric dot paper will work well, but the 2cm isometric dot paper is the same size as the pattern blocks, and some students will likely want to physically place the blocks on the paper.

c. Misconceptions/Common Errors

- Teachers commonly tell students that in order to add or subtract fractions, you must first get a common denominator. While this is true for the traditional algorithm, it may not be true for other strategies. Therefore, a correct statement may be, "In order to use the *standard algorithm* for adding and subtracting fractions, you must first find a common denominator" because the algorithm is designed to only work with common denominators.

- For students, a common error is adding both numerators and denominators. Use models to show ½ + 1/3 ≠ 2/5. Ask students what is wrong…allow them to find the error.

- Students may struggle with finding a least common denominator. The last time students encountered this concept was in fifth grade (5-2.7).

d. Additional Instructional Strategies –

- In order to move students to adding or subtracting with unlike denominators, consider beginning with a problem where only one fraction needs to be changed (i.e. 5/8 + 2/4). Let students use any method to find the solution. The key question to ask is, "How can we change this problem into one that is just like the easy ones where the parts are the same?" Have students use models or drawings to explain why both problems (original and converted) will have the same answer.

- Next, try some examples where both fractions must be changed (i.e. 2/3 + ¼). Continue providing examples and practice using fractions where both must be changed. Have models on-hand to use if needed (i.e. fraction strips, fraction bars, counters, fraction circles, etc.).

- The parameters in the non-essentials keep the task from becoming tedious and unmanageable for students.

- Teachers commonly tell students that in order to add or subtract fractions, you must first get a common denominator. While this is true for the traditional algorithm, it may not be true for other strategies. Therefore, a correct statement may be, "In order to use the *standard algorithm* for adding and subtracting fractions, you must first find a common denominator" because the algorithm is designed to only work with common denominators.

- Students need experiences that will enable them to make the link between concrete and pictorial models used in the fifth grade and the new symbolic operations. Students can work in pairs to examine how the model and algorithm are related. One student creates the models, the other student applies the algorithm then they discuss how the processes are similar. Having student create pictorial models is an essential step. Since students will not have access to concrete models on state assessment, it is beneficial for students to be able to draw representations of problem in order to access their understanding of the procedure.

- In sixth grade the emphasis is on applying an algorithm. As a result, by the end of sixth grade students should exhibit fluency when solving a wide range of addition and subtraction problems involving fractions.

- Encourage students to use estimation strategies and the benchmark fractions(0, ½, and 1) to determine the reasonableness of their answers. For example, given 3/5 + 1/3, the student's conceptual understanding of the relationship between these fractions and the benchmark fractions would lead them to conclude that 3/5 is more than ½ because the numerator is greater than 2.5 and 1/3 is less than ½ because the numerator is less than 1.5; therefore, their sum must be less than one.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html)

- Soccer Shootout (game) (http://www.funbrain.com/fractop/index.html)

f. Assessing the <u>Lesson</u>

Formative –
Require students to discuss and record responses to the following questions:

- Compare adding and subtracting fractions with like denominators to fractions with unlike denominators.
  - Tell what was the same and what was different.
  - Why do you think fractions need to have the same denominator before you can add or subtract them?
- Describe the strategy(ies) you or your group used to add and subtract these fractions.
- Write your strategy(ies) so that someone who wasn't sure what to do to add or subtract unlike fractions would to be able to follow your directions.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

6-2.4 Apply an algorithm to add and subtract fractions. (C3)

The objective of this indicator is apply, which is in the "apply procedural" of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus is to gain computational fluency with addition and subtraction of fractions, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **apply** requires students to understand fractional forms such as mixed numbers, proper fractions, and improper fractions. Students should apply their conceptual knowledge of fractions to transfer their understanding of concrete and/or pictorial representations to symbolic representations (numbers only) by generalizing connections among a variety of representational forms and real world situations (6-1.7). Students use these procedures in context as opposed to only rote computational exercises and use correct and clearly written or spoken words to communicate about these significant mathematical tasks (6-1.6). Students engage in repeated practice using pictorial models, if needed, to support learning. Lastly, students should evaluate the reasonableness of their answers using appropriate estimation strategies.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Susie works at a pizza shop. She cuts 2 pizzas into eight equal sections each. Customers ate 7/8 of each pizza. Susie says that 7/8 + 7/8 = 14/16, so 14/16 of all the pizza was eaten. Is Susie's addition correct? Explain. (This is tricky to explain because Susie's conclusion that 14/16 of all pizza was eaten is correct. However, we know that 7/8 + 7/8 = 14/8, not 14/16. So, where does the contradiction come from? Susie did not say what her unit "whole" was in her sentence. If we say that 7/8 of one pizza + 7/8 of one pizza = 14/16 of one pizza, that is NOT correct. Susie used two different "wholes" in this sentence. She can

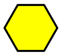
correctly say that 7/8 of one pizza + 7/8 of one pizza = 14/16 of all the pizza. But, she needs to say what she is taking a fraction of.)

2. A local magazine sells advertising space. It charges advertisers according to the fraction of a page their ad will fill. For page 20 in the magazine, advertisers have purchased 1/8 of the page and 1/16 of the page. What fraction of the page will be used for ads? What fraction of the page will remain for other uses? Explain your reasoning. (1/8 + 1/16 = 3/16 of the page will be used for ads. 1 – 3/16 = 13/16 of the page will remain. Students should be able to explain how the algorithm they chose to solve this problem worked. It doesn't mean that they must use the traditional algorithm.)

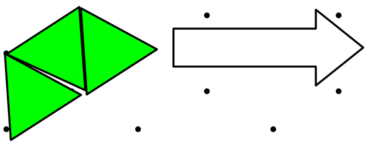
3. Josh purchased a skateboard that was on sale at 25% off. What fraction of the original price did he pay?

5. Mario worked 5 ¾ hours on his math project Saturday. Sunday, he worked for another 2 ⅛ hours. Estimate the number of hours that he has worked on his project.

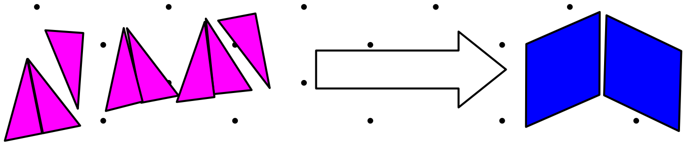
Teacher sheet—addition examples 1

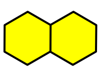
 is 1

$\frac{1}{6}+\frac{2}{6}=\frac{3}{6}$ → $\frac{3}{6}=\frac{1}{2}$ because there is a common factor of $\frac{3}{3}$.

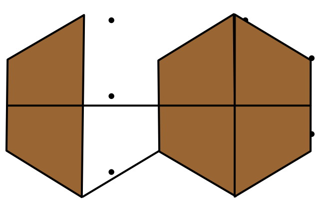


$\frac{3}{12}+\frac{5}{12}=\frac{8}{12}$ → $\frac{8}{12}=\frac{2}{3}$ because there is a common factor of $\frac{4}{4}$.

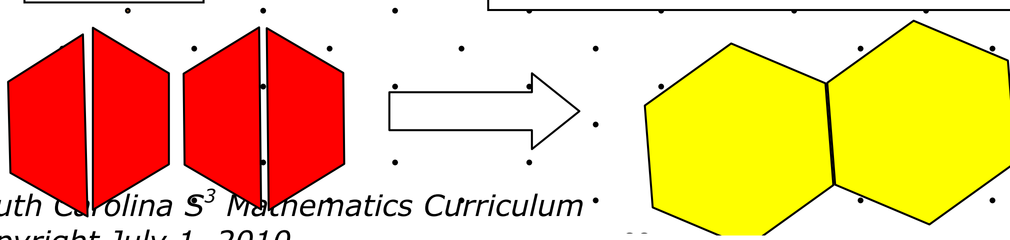


 is 1.

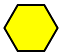
$\frac{3}{8}+\frac{4}{8}=\frac{7}{8}$ → No common factor, so it stays $\frac{7}{8}$.



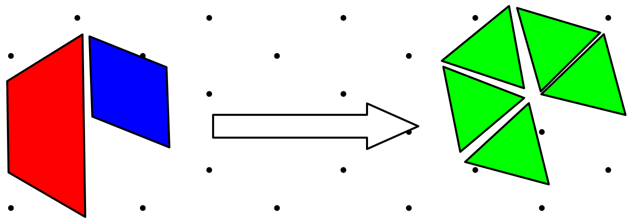
$\frac{1}{4}+\frac{3}{4}=\frac{4}{4}$ → $\frac{4}{4}=\frac{1}{1}$ or 1 whole because of the common factor $\frac{4}{4}$.



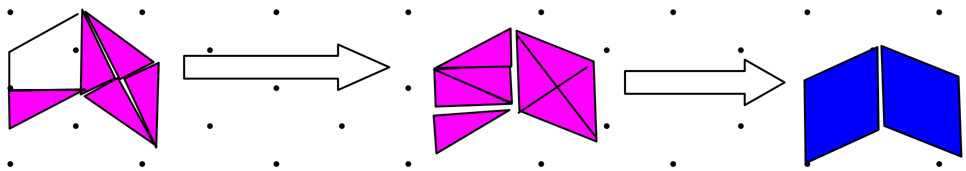
Teacher sheet—addition examples 2

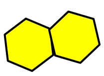
 is 1.

$$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$$

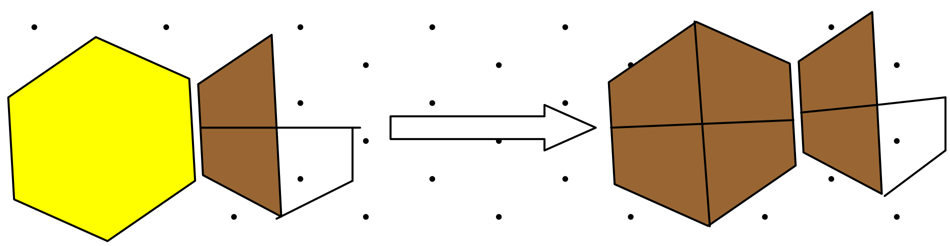


$$\frac{1}{4}+\frac{5}{12}=\frac{8}{12}=\frac{2}{3}$$

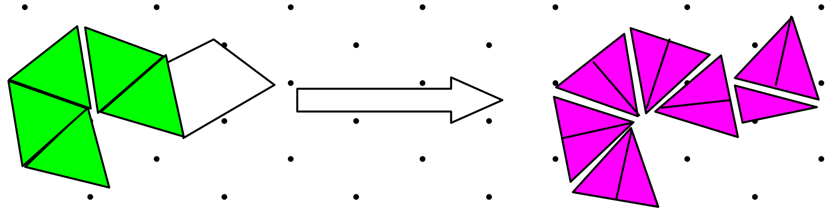


 is 1

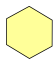
$$\frac{1}{2}+\frac{3}{8}=\frac{7}{8}$$



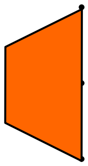
$$\frac{5}{12}+\frac{1}{8}=\frac{13}{24}$$



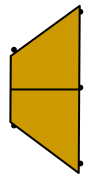
Teacher sheet—equivalent factions

 is 1

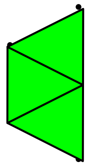
“See” the $\frac{2}{2}$? 2 right trapezoids to build one red.



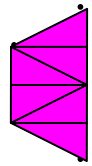
$\frac{1}{2} = \frac{2}{4}, \frac{3}{6}, \frac{6}{12}$



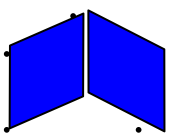
$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$



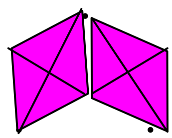
$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$



$\frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$

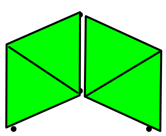


$\frac{2}{3} = \frac{4}{6}, \frac{8}{12}$



$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$

4 right triangles build one rhombus, so multiply by $\frac{4}{4}$.



$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$

MODULE

1-3

This module addresses the following indicators:

6-2.5 Generate strategies to multiply and divide fractions and decimals. (B6)
6-2.6 Understand the relationship between ratio/rate and multiplication/division. (B2)
6-2.7 Apply strategies and procedures to determine values of powers of 10, up to 10^6. (C3)
6-2.8 Represent the prime factorization of numbers by using exponents. (C2)

This module contains 7 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

6-2.5 Generate strategies to multiply and divide fractions and decimals. (B6)

The sixth grade is the first time students are introduced to the concept of multiplying and dividing fractions and decimals with the emphasis on generating strategies (6-2.5).

In seventh grade, students will apply an algorithm to multiply and divide fractions and decimals (7-2.9).

6-2.6 Understand the relationship between ratio/rate and multiplication/division. (B2)

The sixth grade is the first time students are introduced to the concept of ratio and rate (6-2.6).

In seventh grade, students will apply ratios, rates, and proportions to discounts, taxes, tips, interest, unit costs, and similar shapes (7-2.5)

6-2.7 Apply strategies and procedures to determine values of powers of 10, up to 10^6. (C3)

In third grade, students use basic number combinations to compute related multiplication problems that involve multiples of 10(3-2.10).

Sixth grade is the first time students will be introduced to the concept of applying strategies and procedures to determine values of powers of 10, up to 10^6 (6-2.7).

In seventh grade, students will translate between standard form and exponential form (7-2.6) and translate between standard form and scientific notation (7-2.7).

6-2.8 Represent the prime factorization of numbers by using exponents. (C2)

In fifth grade, students classified numbers as prime, composite, or neither (5-2.6) and generated strategies to find the greatest common factor and the least common multiple of two whole numbers (5-2.7).

Key concepts/terms

*These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and in use in conversation with students.

- * Denominator
- * numerator
- * Quotient
- * Product
- * ratio
- * rate
- * multiplication
- * division
- * Base
- * Exponent
- * Powers
- * Squared
- * Cubed
- * exponential notation
- * Factor
- * Multiples
- * Product
- * Prime
- * Composite
- * Prime factorization

II. Teaching the Lessons

Sixth grade is the first time students have been introduced to the concept of multiplying and dividing fractions and decimals with the emphasis on generating strategies. In seventh grade students will apply an algorithm for multiplication and division of fractions and decimals.

Students in the sixth grade should **not** multiply or divide fractions and decimals symbolically. It is essential that students develop an understanding of the concepts of multiplication and division of fractions and decimals by sharing their generated strategies. Students are encouraged to explore and discover various methods. In 7th grade, students will apply algorithms to multiply and divide fractions or decimals. In order to do so, **students need opportunities to investigate contextual problems without first being shown an algorithm. That means multiple problem situations should be introduced and students should**

explore possible solution strategies. Students should then share their strategies with the whole class as the teacher facilitates. The students' understanding of computation increases when they develop their own methods and discuss those methods with others. Finally in seventh grade, when the expectation is to apply an algorithm, some students will begin to use the traditional algorithms while others will revert back to an invented algorithm until they are comfortable enough to move on.

For this indicator, it is **essential** for students to:

- Understand the meaning and concept of fractions and decimals
- Explore and discover various methods
- Develop an understanding of the concepts of multiplication and division of fractions and decimals by sharing their generated strategies.
- Understand that multiplication does not always result in a larger answer and division does not result in a smaller answer

For this indicator, it is **not essential** for students to:

- Multiply or divide fractions and decimals symbolically.

1. Teaching Lesson A, Part 1 – Multiplying a Fraction and a Whole number

a. Indicators with Taxonomy

6-2.5 Generate strategies to multiply and divide fractions and decimals. (B6)

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson A, Part 1– Multiplying a Fraction and a Whole Number

Materials Needed:

- paper and pencil
- some concrete manipulative to represent the cookies (color tiles, counters, etc.)

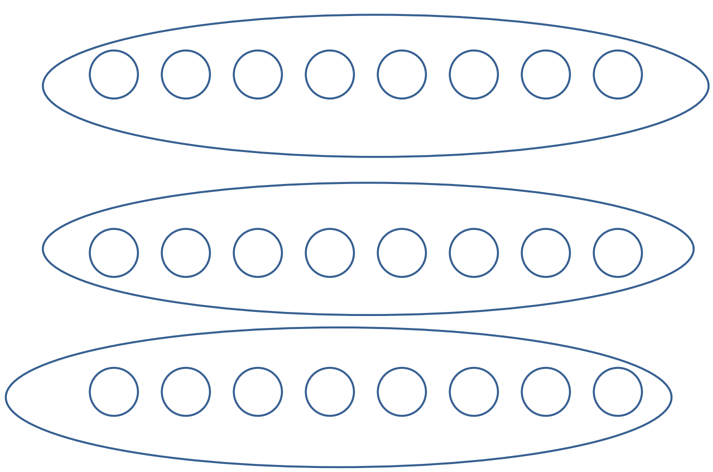
Students should work in pairs so they can discuss the strategies they are using to solve the problem.
NOTE: One problem situation is described below. You will need to create/find others.

1. Give the students the following prompt:

   Susan baked 24 cookies for a bake sale. At the end of the bake sale, she had one-third of her cookies left. How many cookies were left?

2. Allow the students time to solve the problem in pairs while you circulate and interact with them, asking questions and probing thinking. As pairs finish up, have them form quads to compare strategies and answers. Both pairs and quads should be recording their strategies.
3. Have quads share how they solved the problem and show their work to the class. Make sure to discuss the different strategies and methods used to solve the problem.
4. As you debrief the discussion, point out to students that what they did was find that 1/3 of 24 is 8, and that it should be written as a multiplication sentence: 1/3 • 24= 8. Also ask them about the relationship of the product to the factors. They are accustomed to multiplying whole numbers and getting a greater product. In this case, the product is less than the whole amount with which they started. Does that make sense? Why?
5. Have pairs of students create and solve a similar story problem: multiplying a fraction by a whole number. Again, have pairs join to make quads and discuss their work. One way to have students share is to give them a large piece of paper and have them write on it their story problem and the work they did to solve it. Post the charts around the room and have students do a gallery walk to examine their classmates' work.

This example is for you as the teacher. It's one way students might choose to solve the problem.



24 cookies to start

1/3 means 3 groups (denominator tells how many groups)

1/3 → 1 of the 3 groups

8 cookies

Introductory Lesson A, Part 2– Multiplying two Fractions

Materials Needed:

- pencil and paper
- something students might use as a model for a cake

Students should work in pairs so they can discuss the strategies they are using to solve the problem.
NOTE: One problem situation is described below. You will need to create/find others.

1. Remind students of the work they did when multiplying a fraction by a whole number. Things to include: finding a fraction of another number means you are multiplying and the product was smaller than the whole.

2. Have students predict what would happen if you multiply two fractions.

3. Give the students the following prompt:

Darius' mother baked a rectangular birthday cake for his birthday. He took his cake to school to share with his classmates who ate $\frac{1}{2}$ of the cake. At home that night, Darius and his family ate $\frac{2}{3}$ of the remaining cake. How much of the whole cake did Darius and family eat?

4. Allow the students time to solve the problem in pairs while you circulate and interact with them, asking questions and probing thinking. As pairs finish up, have them form quads to compare strategies and answers. Both pairs and quads should be recording their strategies.

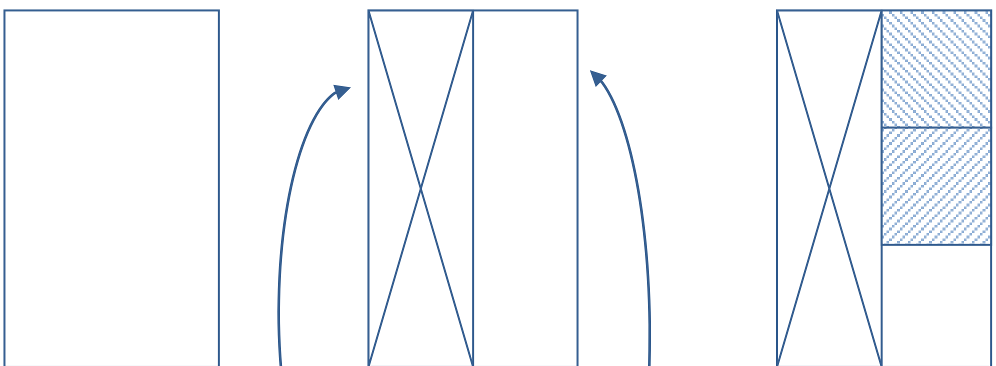
5. Have quads share how they solved the problem and show their work to the class. Make sure to discuss the different strategies and methods used to solve the problem.

6. As you debrief the discussion, point out to students that what they did was find that 2/3 of ½ is 2/6 or 1/3, and that it should be written as a multiplication sentence: 2/3 • ½ = 2/6 or 1/3. Also ask them about the relationship of the product to the factors. Is it greater than or less than what they started with? Why? Does that make sense? Why?

7. Have pairs of students create and solve a similar story problem: multiplying a fraction by a fraction. Again, have pairs join to make quads and discuss their work. One way to have students share is to give them a large piece of paper

and have them write on it their story problem and the work they did to solve it. Post the charts around the room and have students do a gallery walk to examine their classmates' work.

This example is for you, the teacher. It's one way students may use to solve the problem.



The whole cake.

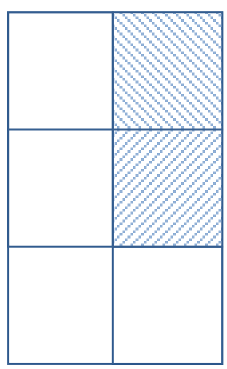
The class ate 1/2.

This ½ is gone.

This ½ is left.

The family ate 2/3 of the ½ that was left.

What part of the whole cake is that?



The family ate 2/6 of the whole cake. That's the same as 1/3.

c. Misconceptions/Common Errors

Multiplication does not always result in a larger product and division does not always result in a smaller quotient. Multiplication and division of fractions may result in a smaller number.

d. Additional Instructional Strategies

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- Pizza Fraction Multiplication (http://math.rice.edu/~lanius/fractions/frac5.html)
- National Library of Virtual Manipulatives (http://nlvm.usu.edu/)

f. Assessing the Lesson

The assessment is embedded in the lesson in the sharing and debriefing of student strategies used to solve the initial story problem and the story problems created by the students.

2. Teaching Lesson B - Multiply Decimals

a. Indicators with Taxonomy

6-2.5 Generate strategies to multiply and divide fractions and decimals. (B6)

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson B – Multiply decimals

Materials Needed:

- Base ten blocks
- Centimeter Grid Paper
- any materials students might use to model the story problem
- Calculators

Students may need a quick place value review before beginning. It's important they understand that a number such as 3.5 means 3 wholes and 5/10 of a whole.

Students should work in pairs so they can discuss the strategies they are using to solve the problem.
NOTE: One problem situation is described below. You will need to create/find others.

1. Make materials available to students. Give them the following story problem:

    Max runs 1.2 km every week. How many kilometers does Max run in a typical month?

2. Allow the students time to solve the problem in pairs while you circulate and interact with them, asking questions and probing thinking. As pairs finish up, have them form quads to compare strategies and answers. Both pairs and quads should be recording their strategies.

3. Have quads share how they solved the problem and show their work to the class. Make sure to discuss the different strategies and methods used to solve the problem.

4. If students' strategies center on repeated addition, ask them if they remember the shortcut for adding the same number over and over again. Have them check their strategies and challenge them to model the problem as a multiplication problem. As you debrief the discussion, be sure to write addition and multiplication sentences that describe the models students are sharing.

5. Have pairs of students create and solve a similar story problem. Again, have pairs join to make quads and discuss their work. One way to have students share is to give them a large piece of paper and have them write on it their story problem and the work they did to solve it. Post the charts around the room and have students do a gallery walk to examine their classmates' work.

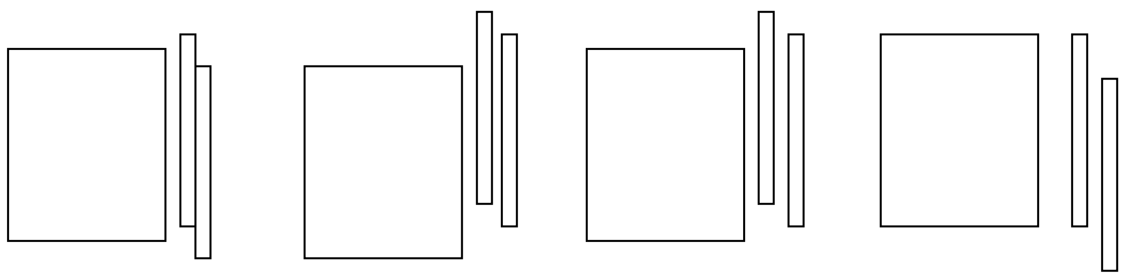
6. As you debrief the series of story problems created by both yourself and your students, ask students to describe the relationship of the factors to the product as

well as the placement of the decimal in the factors and the product. Also, that whole numbers are decimal numbers as well.

These examples are for you, the teacher. They represent ways students may solve the problem.

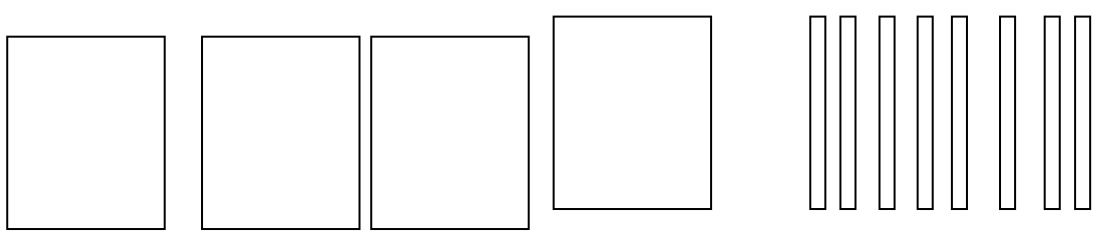
If base ten blocks are used, the flat can be one whole and the rod can be one tenth.

It takes 4 groups to model the 1.2 km Max runs each week.



1.2 + 1.2 + 1.2 + 1.2 = 4.8

A student might write an addition sentence to describe his model.

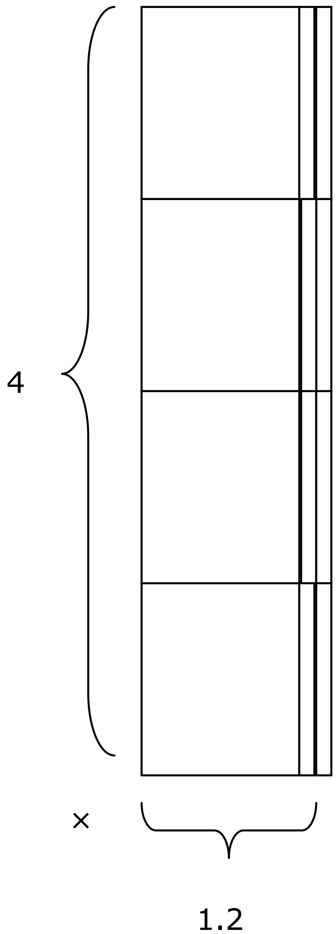


4 + .8 = 4.8

A student might pull the 4 flats together and the 8 rods together and write a sentence describing that.

One possible model of multiplication is an array.

1.2 • 4 → 4 rows of 1.2



c. Misconceptions/Common Errors

Multiplication does not always result in a larger product and division does not always result in a smaller quotient. Multiplication and division of fractions may result in a smaller number.

d. Additional Instructional Strategies

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- National Library of Virtual Manipulatives (http://nlvm.usu.edu/)
- Football Game (http://www.funbrain.com/cgi-bin/fb.cgi?A1=start3&A2=Medium&ALG=No&INSTRUCTS=1) Have manipulatives or graph paper available for students to generate a strategy to solve the problems.
- Decimal Squares Interactive Games (http://www.decimalsquares.com/dsGames/)
- Explore decimal multiplication by building rectangles using base 10 blocks (http://argyll.epsb.ca/jreed/math7/strand1/1201.htm)

f. Assessing the Lesson

The assessment is embedded in the lesson in the sharing and debriefing of student strategies used to solve the initial story problem and the story problems created by the students.

3. Teaching Lesson C, Part 1- Dividing a Whole Number by a Fraction

a. Indicators with Taxonomy

6-2.5 Generate strategies to multiply and divide fractions and decimals. (B6)

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson C, Part 1 – Dividing a Whole Number by a Fraction

Materials Needed:

- any manipulative that may be used to model fractions, including pattern blocks or fraction strips
- iso-dot paper would be helpful if students choose to use pattern blocks
- graph or grid paper
- colored pencils (optional)

Students should work in pairs so they can discuss the strategies they are using to solve the problem.
NOTE: One problem situation is described below. You will need to create/find others.

1. Discuss with the class: What is division of whole numbers? What does 6 divided by 2 mean? What does 12 divided by 2 mean? What does 12 divided by 6 mean? The intent is to encourage student understanding that division involves breaking a whole (the dividend) into groups specified by the divisor.

2. Have students make a story problem for one of the above examples and sketch the solution. Ask volunteers to read their story problems and explain their solutions.

3. Give students the following story problem to solve.

> *Duncan has four pounds of candy and decides to use it all to make 1/3 pound bags to give away. How many bags can he make with his four pounds of candy?*

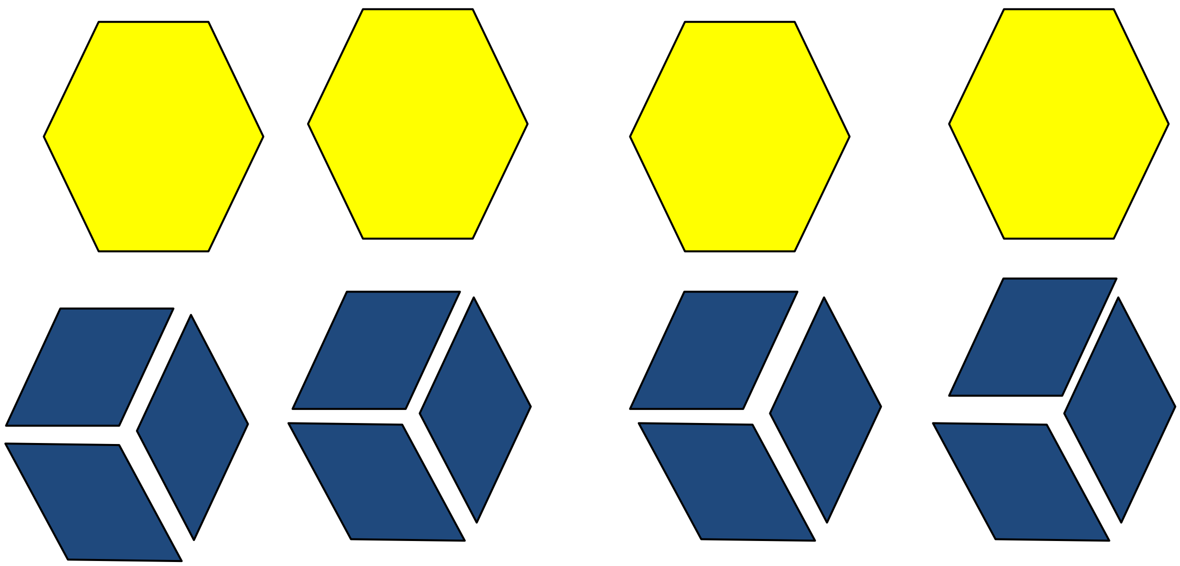
Allow the students time to solve the problem in pairs while you circulate and interact with them, asking questions and probing thinking. As pairs finish up, have them form quads to compare strategies and answers. Both pairs and quads should be recording their strategies.

4. Have quads share how they solved the problem and show their work to the class. Make sure to discuss the different strategies and methods used to solve the problem. As you debrief the discussion, be sure to write number sentences that describe the models and strategies students are sharing.

5. Have pairs of students create and solve a similar story problem. Again, have pairs join to make quads and discuss their work. One way to have students share is to give them a large piece of paper and have them write on it their story problem and the work they did to solve it. Post the charts around the room and have students do a gallery walk to examine their classmates' work.

These examples are for you, the teacher. They represent ways students may solve the problem.

Using pattern blocks – the hexagon represents 1 pound of candy.



There are 4 pounds of candy. The candy has to be divided into 1/3 pound bags. That's each pound split into 3 bags. So, that's 12 bags of candy.

Drawing a circular or rectangular model.

Pretty much the same as above, but using 4 circles or rectangles to represent the pounds of candy and dividing each pound into thirds.

b. Introductory Lesson C, Part 2 – Dividing a Fraction by a Fraction

1. Give students the following story problem to solve.

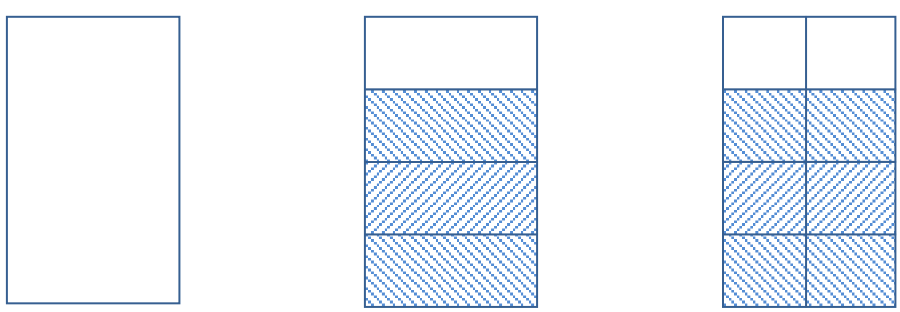
    Jose wants to make some baggies of popcorn to have for snacks. He has ¾ of a box of popcorn, and he's decided to put 1/8 of the whole box in each baggie. How many baggies will he fill?

Allow the students time to solve the problem in pairs while you circulate and interact with them, asking questions and probing thinking. As pairs finish up, have them form quads to compare strategies and answers. Both pairs and quads should be recording their strategies.

2. Have quads share how they solved the problem and show their work to the class. Make sure to discuss the different strategies and methods used to solve the problem. As you debrief the discussion, be sure to write number sentences that describe the models and strategies students are sharing.

3. Have pairs of students create and solve a similar story problem. Again, have pairs join to make quads and discuss their work. One way to have students share is to give them a large piece of paper and have them write on it their story problem and the work they did to solve it. Post the charts around the room and have students do a gallery walk to examine their classmates' work.

This example is for you, the teacher. It's one way students might solve the problem.



6 baggies can be filled from the ¾ box of popcorn

box of popcorn

¾ full

each baggie uses 1/8 of the can

c. Misconceptions/Common Errors

Multiplication does not always result in a larger product and division does not always result in a smaller quotient. Multiplication and division of fractions may result in a smaller number.

d. Additional Instructional Strategies

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- National Library of Virtual Manipulatives (http://nlvm.usu.edu/)
- Interactive lesson on dividing fractions…There are some good visuals here, but be cautious of algorithm. Use this as a teacher resource rather than letting students loose on the site. (http://www.homeschoolmath.net/teaching/f/dividing_fractions_2.php)

f. Assessing the Lesson

The assessment is embedded in the lesson in the sharing and debriefing of student strategies used to solve the initial story problem and the story problems created by the students.

4. Teaching Lesson D – Dividing Decimals

a. Indicators with Taxonomy

6-2.5 Generate strategies to multiply and divide fractions and decimals. (B6)

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

- Base ten blocks
- Centimeter Grid Paper
- any materials students might use to model the story problem
- Calculators

Students may need a quick place value review before beginning. It's important they understand that a number such as 3.5 means 3 wholes and 5/10 of a whole.

Students should work in pairs so they can discuss the strategies they are using to solve the problem.
NOTE: One problem situation is described below. You will need to create/find others.

1. Give students the following story problem to solve.

> *Andrea has 4.6 pieces of plywood. She wants to use it for two sets of bookshelves that are the same size. How much wood does she have for each set of shelves?*

Allow the students time to solve the problem in pairs while you circulate and interact with them, asking questions and probing thinking. As pairs finish up, have them form quads to compare strategies and answers. Both pairs and quads should be recording their strategies.

2. Have quads share how they solved the problem and show their work to the class. Make sure to discuss the different strategies and methods used to solve the problem. As you debrief the discussion, be sure to write number sentences that describe the models and strategies students are sharing.

3. Have pairs of students create and solve a similar story problem. Again, have pairs join to make quads and discuss their work. One way to have students share is to give them a large piece of paper and have them write on it their story problem and the work they did to solve it. Post the charts around the room and have students do a gallery walk to examine their classmates' work.

c. *Misconceptions/Common Errors*

Teachers should be alert to the misconception that all division results in "a smaller number". Division with fractions and decimals <u>may</u> result in a smaller quotient.

d. *Additional Instructional Strategies*

While additional learning opportunities are needed, no suggestions are included at this time.

e. *Technology*

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- National Library of Virtual Manipulatives (http://nlvm.usu.edu/)
- Decimal division Football (http://www.funbrain.com/cgi-bin/fb.cgi?A1=start4&A2=Hard&ALG=No&INSTRUCTS=1) Have students use any strategy to solve the problem prior to typing their answer in the box provided.

f. Assessing the Lesson

The assessment is embedded in the lesson in the sharing and debriefing of student strategies used to solve the initial story problem and the story problems created by the students.

2. Teaching Lesson E – Ratios and Rates

a. Indicators with Taxonomy

6-2.6 Understand the relationship between ratio/rate and multiplication/division. (B2)

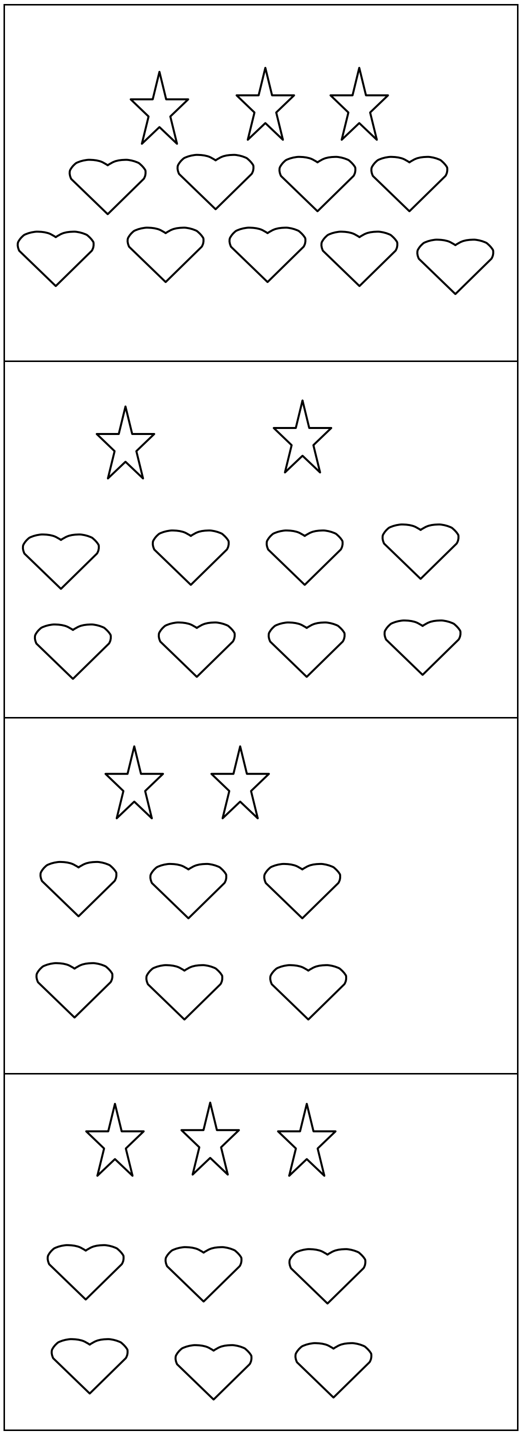
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual

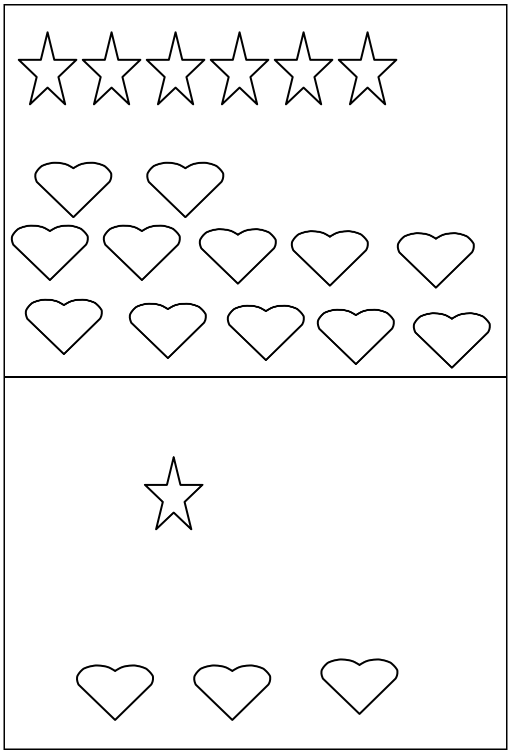
b. Introductory Lesson – The following lesson introduces students to the idea of ratios and fractions. To further cover this indicator, use lessons from the additional instructional strategies section.

Materials Needed:

- Ratio Cards (See below) - One set per pair of students - already cut out
- Chart paper

Students take turns selecting a card and finding another card on which the ratio of the two types of objects is the same. The students then determine a method to record the ratio depicted by the cards and prepare to share their reasoning with the class. Students can tape their pairs of cards to the chart paper and write an explanation of why they think the pair belongs together. This task moves students to a numeric approach rather than a visual one and introduces the notion of ratios as rates. This activity introduces the concept of unit rate.





c. ***Misconceptions/Common Errors –*** It is important to relate ratios to equivalent fractions. It is also important for students to understand that all fractions are ratios but not all ratios are fractions. The difference is that fractions always represent part-to-whole relationships. On the other hand, ratios can represent part-to-whole OR part-to-part relationships. For example, if there are 20 students in the class and 14 are males, the ratio of males to the class is 14/20 (relationship of part to whole, thus a fraction or a ratio). The ratio of males to females is 14/6 (relationship of part to part and thus not a fraction). In the cases just cited, the ratio compared two measures of the same type of thing. However, a ratio can also be a rate (as in a unit rate) or a comparison of the measures of two different things or quantities – the measuring unit is different for each value (miles per gallon, for example).

d. ***Additional Instructional Strategies***

Additional practice and lessons for ratios can be found on the following web sites:

A. Go to Cynthia Lanius' web site on ratios - http://math.rice.edu/~lanius/proportions/

- The students should be able to complete the first six practice exercises. The last three exercises deal with solving proportions, which isn't covered in the standards until 7th grade.

B. Go to AAAmath.com web site on ratios - http://www.aaamath.com/rat62a-ratios.html

- This site gives a review of ratios and there are practice activities and games for students to use. If you use this site, make manipulatives available and connect it to the use of the ratio cards in the introductory lesson.

e. Technology

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- See links for additional lessons within the introductory lesson.
- Algebra lab: Rates and Ratios (http://www.algebralab.org/studyaids/studyaid.aspx?file=Algebra1_2-8.xml)
- Ratios http://www.math.com/school/subject1/lessons/S1U2L1GL.html

f. Assessing the Lesson

Exit question: (ask for explanations also)

Jeannine has a bag with 3 videocassettes, 4 marbles, 7 books, and 1 orange.

1) What is the ratio of books to marbles?
Expressed as a fraction, with the numerator equal to the first quantity and the denominator equal to the second, the answer would be 7/4.
Two other ways of writing the ratio are 7 to 4, and 7:4.

2) What is the ratio of videocassettes to the total number of items in the bag?
There are 3 videocassettes, and 3 + 4 + 7 + 1 = 15 items total.
The answer can be expressed as 3/15, 3 to 15, or 3:15

3. Teaching Lesson F

a. Indicators with Taxonomy

6-2.7 Apply strategies and procedures to determine values of powers of 10, up to 10^6. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson

Materials Needed:

- Overhead calculator
- Scientific calculators
- Colored pencils
- Data packet, "I've Got the Power!" [4 pages—suggest running front and back]
- Transparencies of "I've Got the Power!"

Arrange desks in groups of four so that students may work together and communicate ideas. Begin by BRIEFLY reviewing what factors, multiples, and products are and how they are related to each other.

Direct students to complete the "Product" column of the table on p. 1 of the data packet, "I've Got the Power!" (In third grade students were introduced to the concept of multiplying by 10 and in fourth grade they worked with multiplying by multiples of 10.).

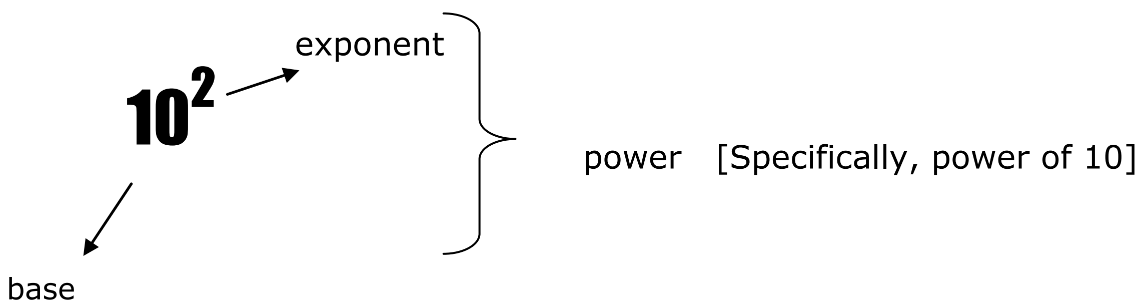
Ask students to discuss the "Think about it…Write about it!!" prompt on the data sheet. If students seem "stuck", tell them they may use the colored pencils to identify possible patterns. Have each group record its thoughts on a sheet of butcher or chart paper and post them at the front of the room.

Discoveries:

- The number of zeroes in each set of factors is equal to the number of zeroes in the product.
- The decimal in the product is "moving" to the right.
- Each product is the first term in the next number sentence in the factors column.
- In each row, beginning with the third, the first term could be stretched into some number of 10s→THIS IS HUGE!!

The goal is to help them see the combinations of 10 in the factors. Focus their attention on the 4th row in the factors column. Ask the students if they know another way to represent "100." They should be able to expand 100 into 10 x 10. Repeat this process with each row. There is room for them to write the strings of 10s in the factors column. This will lead to being able to understand the term "power of 10" as well as the definitions of *base, exponent*, and *power*.

Go the section of the data sheet labeled "Vocabulary." It is VERY likely that students have been exposed to the terms **base** and **exponent** and that they may be able to generate their own definitions. Invite definitions from the class. Be sure the definitions the class decides on include the "job description" and not only the position. Then tell them to record this model and the definitions for base and exponent on their papers. From the model, they should be able to generate a definition for **power**. It would be valuable to post the "different" definitions where students can see and compare them, as well as the "official" definitions.



Go to the section labeled "Calculating powers."

Teach the children the key sequence for raising a base to a power. You need to know what that sequence is for whatever model calculator you are using with them. Tell them to record that sequence in the space provided. The only base you will use for this exercise is the number 10. Use proper vocabulary as you teach the sequence.

EXAMPLE: On the TI-34, there is a caret (^) key. To raise a base to a power, you must enter the base, press the caret (**^**) key, and enter the exponent. Students should record that sequence, possibly as written below:

$\boxed{10} \Rightarrow \boxed{\wedge} \Rightarrow \boxed{3} \Rightarrow \boxed{=} \Rightarrow \boxed{1{,}000}$

Direct students to complete the "Calculating Powers" table as well as the "Think about it…Write about it!!" prompts.

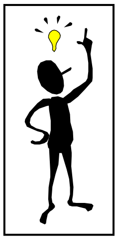
Discoveries:

- The exponent in each row is the same as the number of zeroes in the product for that row.
- The product columns in the two tables are the same.
- If you compare the tables row by row, the number of 10s in the "10 string" in the first table is the same as the exponent in the corresponding row in the second table.

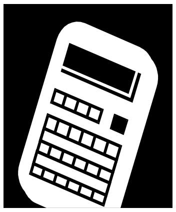
The goal is to help them make the connections between the exponent and the "10 strings." They should be able to generate the "rule" for powers of 10, i.e., the exponent tells you how many zeroes there are after the 1. Tell the children to record their thoughts and any further proof in the section, "Powers of 10 Rule!" Leave time for the students to share their "proof" with their classmates.

I've Got the Power!

Factors	*Product*
1 x 1	
1 x 10	
10 x 10	
100 x 10	
1000 x 10	
10,000 x 10	
100,000 x 10	



Think about it…Write about it!! **Describe any patterns you see happening in the table.**



Calculating powers→key sequence.

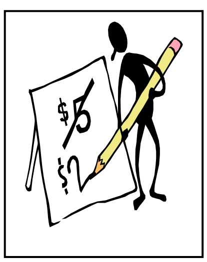
Write the key sequence for calculating powers here.

Vocabulary.

Calculating powers.

Use a calculator and the key sequence you recorded earlier to complete the table.

Base	Exponent	Product
10	0	
10	1	
10	2	
10	3	
10	4	
10	5	
10	6	



Think about it...Write about it!! Describe any patterns you see happening in the table.

Compare the two tables you have completed. Describe any patterns you see.

Powers of 10 Rule!

Can you write a rule for finding the product of ANY power of 10? Use your calculator to help "prove" your rule.

Record your work on this page and be ready to share!

c. Misconceptions/Common Errors

- Students need to understand that a power of 10 is a movement of a decimal not just a certain number of zeroes.
- The most common error students make when solving for exponents is multiplying the exponent by the base number or even attempting to add the two numbers.
- Another common error to watch for would be students who write the exponent the same size and position as the base number.

d. Additional Instructional Strategies

- Exponents Powers of 10 Online Lesson
  http://www.studyzone.org/mtestprep/math8/e/exponentten6l.cfm

e. Technology

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- Celebrate Powers of 10 Day: Interactive website that includes activities and Powers of 10 video. The video shows more than 10^6 and delves into negative powers. However, it is a great way for students to relate powers of 10 to real world situations. In order to view the video, you will be asked to register using your email address. http://www.powersof10.com/
- Secret Worlds: The Universe Within – video starting at the Milky Way Galaxy working towards the earth. By using the manual option, you can decide where to start and stop. This will start at 10^{23} and decrease to negative exponents. Another great relation to the real world. There are student activities, teacher resources, and tutorials on this site as well. http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/
- Powers of 10 Lesson http://www.themathpage.com/arith/powers-of-10.htm
- Powerful Facts about Powers of 10
  http://www.calacademy.org/exhibits/powers_of_ten/#facts

f. Assessing the <u>Lesson</u>

- Notebook entry: Explain the relationship of adding zeros, the placement of the decimal, and Powers of 10. Use examples if you need to.

- How far back would you travel in time if you went back approximately 10^3 minutes? (About 17 hours)
- How far back would you travel in time if you went back approximately 10^6 minutes? (About 2 years)

4. Teaching Lesson D

Indicators with Taxonomy

6-2.8 Represent the prime factorization of numbers by using exponents. (C2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Procedural Knowledge

a. Introductory Lesson G – Prime factorization

Students should work in pairs so they can talk about the math they are doing.

1. Begin by accessing students' prior learning. Ask them to talk to their partners about the following two math words: prime number, composite number, and factor. When students have had about a minute and a half to talk, ask for volunteers to share what they know. Record on a chart or on the board what students know about the two words. The definitions need to be clear.

2. Have students consider the number 16.
   - Is it prime or composite? [composite] How do you know? [too many factors]
   - Ask students to write a multiplication sentence whose product is 16 that uses only prime numbers.

Give students time to complete this task. Then bring them back together and have a volunteer give the solution.

3. The solution is $2 \bullet 2 \bullet 2 \bullet 2 = 16$.
   - How might this expression be written in a more concise form? [Students have worked with exponential form at this point, so they should be able to come up with 2^4.] Ask them questions like *How many 2s are in the expression? Is there a shorter way to do this?* to jog their memories.

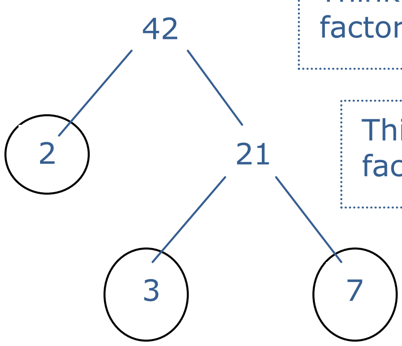
4. Repeat this exercise with the numbers 15, 20, and 28. As students work, circulate and interact with them, asking questions and probing their thinking. When students are finished, bring them back together and have them share their solutions.

$15 \rightarrow 3 \bullet 5 \qquad 20 \rightarrow 2 \bullet 2 \bullet 5 \rightarrow 2^2 \bullet 5 \qquad 28 \rightarrow 2 \bullet 2 \bullet 7 \rightarrow 2^2 \bullet 7$

5. Tell students they have found the ***prime factorization*** for each of the numbers 16, 20 and 28. Based on their findings, ask them to develop a definition for the vocabulary term prime factorization. From there, develop a class definition.

It is likely that students will need a procedure for finding the prime factorization of more challenging numbers. The following examples are provided for your consideration.

Factor Tree

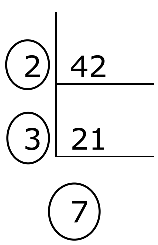


Think of a prime factor of 42 – 2. Divide by 2 to get the other factor – 21.

Think of a prime factor of 21 – 3. Divide by 3 to get the other factor – 7.

Both 3 and 7 are prime, so the tree is finished. "Collect" the prime factors and write the multiplication sentence: 2 • 3 • 7.

Factor Ladder



Think of a prime factor of 42 – 2. Divide by 2 to get the other factor – 21.

Think of a prime factor of 21 – 3. Divide by 3 to get the other factor – 7.

"Collect" the prime factors and write the multiplication sentence: 2 • 3 • 7.

Problems using prime factorization.

Present this problem to the class:

The band is packing boxes of mixed fruit to sale as a fundraiser. They want to include apples and oranges in each box. Apples are shipped 16 in a box, and oranges are shipped 20 in a box. Name three possible ways the fruit could be boxed so that there are an equal number of apples and oranges. What are the least numbers of boxes they should buy in order to have an equal number of apples and oranges? (LCM)

1. Use the prime factorization chains and trees to find the LCM of 16 and 20. Find the product of the list of prime numbers that contains the prime factorization of both numbers.
    a. $16 = 2 \times 2 \times 2 \times 2$ or 2^4
    b. $20 = 2 \times 2 \times 5$ or $2^2 \times 5$
    c. LCM $= 2 \times 2 \times 2 \times 2 \times 5 = 80$ OR $2^4 \times 5 = 80$
2. To determine the number of boxes to buy, divide the LCM by the number of pieces of fruit in each box.
    a. $80 \div 16 = 5$ boxes of apples
    b. $80 \div 20 = 4$ boxes of oranges

Present this problem to the class:

Stephanie is making flower arrangements for the tables at the sixth grade honor roll banquet. She wants the same number of daisies and carnations in each arrangement. She has 16 daisies and 20 carnations. What is the greatest number of arrangements she can make without any flowers left over? How many daisies and carnations will be in each arrangement?

The greatest common factor (GCF) is the greatest factor that two or more numbers have in common. Finding the GCF will help you solve this problem.

1. Use the prime factorization chains and trees to find the GCF of 16 and 20. Find the product of the list of prime factors the numbers have in common.
    a. $16 = \underline{2 \times 2} \times 2 \times 2$
    b. $20 = \underline{2 \times 2} \times 5$
    c. GCF $= 2 \times 2 = 4$
2. Stephanie can make 4 arrangements.
3. To determine the number of each flower in the arrangements, divide the total number of flowers by the GCF.
    a. $16 \div 4 = 4$ daisies
    b. $20 \div 4 = 5$ carnations

b. Misconceptions/Common Errors –

No typical student misconceptions noted at this time.

c. Additional Instructional Strategies

- If you need to review prime and composite, use the following: http://www.aaamath.com/fra63a-primecomp.html

d. Technology

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- Prime Factorization interactive lesson that includes prime numbers chart, information about prime and composite numbers, prime factorization tool, and divisibility rules. http://www.mathsisfun.com/prime-factorization.html
- BrainPop movie on Prime Factorization (you will need to register to use this site) http://www.brainpop.com/math/numbersandoperations/primefactorization/preview.weml
- Prime Factors…includes: What is it?, Tips and Tricks, When students ask (why do we need to know this), and lesson ideas (introducing the concept and developing the concept). http://www.eduplace.com/math/mathsteps/5/b/index.html

- Factor Tree practice from National Library of Virtual Manipulatives http://nlvm.usu.edu/en/NAV/frames_asid_202_g_3_t_1.html
- Prime Factorization of the first 1000 integers http://www.sosmath.com/tables/factor/factor.html
- Prime Factorization Game: Match column one (standard notation) with its equivalent prime factorization in column two. http://www.quia.com/cm/26221.html
- Prime Factorization Calculator http://www.mathwarehouse.com/arithmetic/numbers/prime-number/prime-factorization-calculator.php
- Online Video Lesson: http://primefactorization.org/

e. Assessing the Lesson

1.Rosey claims the prime factorization of 30 is 2 x 3 x 5. Lon claims there is another one: 1 x 2 x 1 x 3 x 1 x 5. Who is correct? Why?

2. Find the prime factorization of 36 and 45. What is the GCF and LCM of 36 and 45?
3. Find the prime factorization of 30 and 75. What is the GCF and LCM of 30 and 75?
4. How does finding the prime factorization of two numbers help you find the GCF and LCM?

5. Use prime factorization to help determine: "What is my number?"
   Clue 1: My number is a multiple of 2 and 7.
   Clue 2: My number is less than 100 but larger than 50.
   Clue 3: My number is the product of three different primes.
   (*Since the number is a multiple of 2 and 7, it must be a multiple of 14. The multiples of 14 between 50 and 100 are 56 = 2^3 x 7, 70 = 2 x 5 x 7, 84 = 2^2 x 3 x 7 and 98 = 2 x 7^2 . Of these numbers, only 70 has three different primes in its prime factorization. The number is 70.)*

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

6-2.5 Generate strategies to multiply and divide fractions and decimals. (B6)

The objective of this indicator is to generate which is in the "conceptual knowledge" of the Revised Taxonomy. To create is to put together elements to form a new, coherent whole or to make an original product. Conceptual knowledge is not bound by specific examples. The learning progression to **generate** requires students to recall concepts of multiplying, dividing, and relate parts to a whole. Students explore problem situations (story problems) and explore various strategies to solve those problems by applying their conceptual knowledge of fractions. Students translate their understanding of concrete and/or pictorial representations by generalizing connections between their models and real world situations (6-1.7). Students should use these procedures in context as opposed to only rote computational exercises and use correct and clearly written or spoken words to communicate about these significant mathematical tasks (6-1.6). Students formulate questions to prove or disprove their methods (6-1.2) and generate mathematical statement (6-1.6) about these operations. They should evaluate the reasonableness of their answers using appropriate estimation strategies.

6-2.6 Understand the relationship between ratio/rate and multiplication/division. (B2)

The objective of this indicator is to understand, which is in the "understand conceptual" of the Revised Taxonomy. To understand is to construct meaning about the interrelationship among multiplication/division and ratio/rate. The learning progression to **understand** requires students to understand and represent ratio and rate using appropriate forms. Students generalize connection among rate and ratio and real world problems. As students analyze these problems, their use inductive and deductive reasoning to generalize mathematical statements (6-1.5) summarizing how multiplication and division are used to solve problems involving ratios and rates.

6-2.7 Apply strategies and procedures to determine values of powers of 10, up to 10^6. (C3)

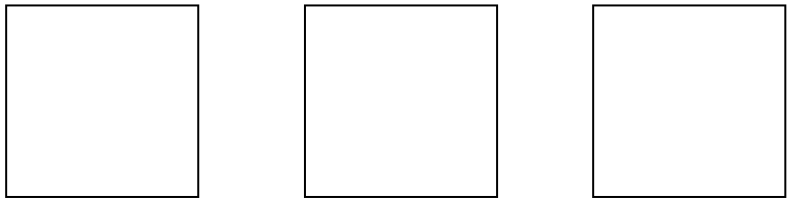
The objective of this indicator is apply, which is in the "apply procedural" of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus is to gain computational fluency in computing powers of 10, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **apply** requires students to understand the structure of exponent form (base and exponent). Students explore powers of 10 by analyze the relationship between the exponent form, expanded form and numerical value. Students generalize mathematical statements (6-1.5) about these relationship based on inductive and deductive reasoning (6-1.3). They understand that each is an equivalent symbolic expression that conveys the same meaning but in different forms. Students then develop strategies that can be used to compute powers of 10 fluently.

6-2.8 Represent the prime factorization of numbers by using exponents. (C2)

The objective of this indicator is represent which is in the "understand procedural" knowledge of the Revised Taxonomy**.** To understand a procedural implies not only knowing the steps of the procedural but also understanding the purpose and value of using it. The learning progression to **represent** requires students to recall the concept of prime and composite numbers by making connections to prior knowledge. Students explore problems situation where using the process of prime factorization is using. They analyze these situations and use inductive reasoning (6-1.3) to generalize a mathematical statement (6-1.5) about prime factorization. Students understand that the prime factorization is an equivalent symbolic expression that represents the same the number but in a different form (6-1.4). Students then rehearse strategies to find the prime factorization of a number and explain and justify their answers to their classmates and teacher.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Janice walks at an average speed of 0.6 miles per hour. At that speed, about how many miles can she walk in 3 hours? Shade the grids to model the problem given below and write the solution in the blank.

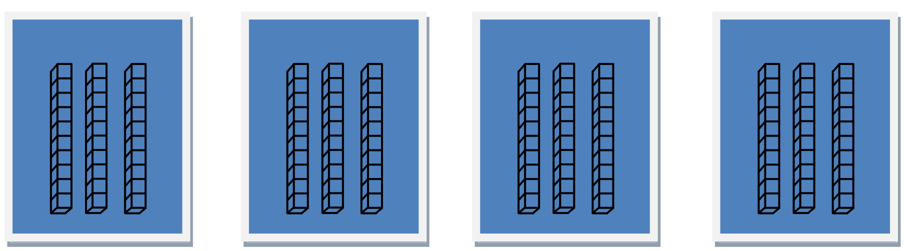


Answer________

2. Sam has 5 pounds of peanut brittle. He wants to pack it in ¾ pound packages. How many full packages can he make? Will he have any left over? How much? Use any strategy or materials you need in order to help you solve this problem. Explain your thinking.

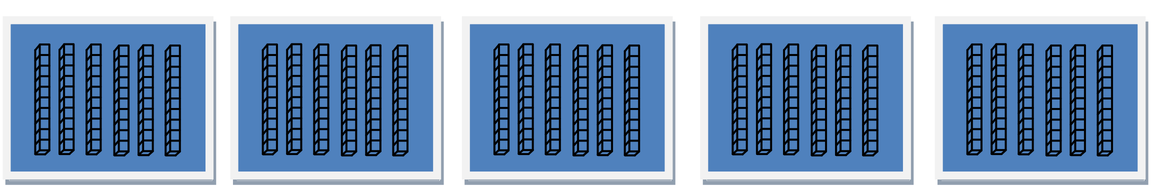
3. Sherri incorrectly solved the following problem: 9.1 ÷ 7. Her solution was 13. What is wrong with her quotient? Explain a strategy you would use to show Sherri how to correctly solve this problem. Show your strategy and solution.

4. Jalisa used base ten blocks to model the solution to a division problem. Identify the quotient that she modeled.



   A. 12 ÷ 3 = 4
   B. 120 ÷30 = 4
   C. 1.2 ÷0.3 = 4
   D. 12 ÷0.3 = 40

5. Identify the quotient that is modeled by the base ten blocks.



   A. 30 ÷ 5 = 6
   B. 3 ÷ 0.6 = 5
   C. 3 ÷ 0.5 = 6

D. $3 \div 5 = 0.6$

6. Melissa planted 10 flowers in 30 minutes. At this rate, how long would it take her to plant 50 flowers? Show how you found the solution.

7. Rudy was filling his fish tank. He was adding 1 quart every 4 minutes. How many quarts did he add in 28 minutes? Explain how you found your solution.

8. Complete the table.

Words	Exponential Notation	Base	Exponent	Repeated Factors	Standard Notation
Ten to the 1st power	10^1	10	1	10	10
Ten to the 2nd power		10		10 x 10	
Ten to the 3rd power					
		10	4		10,000
					100,000
	10^6				

9. Find the prime factorization of 36.

10. Find the prime factorization of 180.

11. Which shows the prime factorization of 30?

A) 2 x 15

B) 3 x 10

C) 2 x 3 x 5

D) 5 x 6

12. Find all the numbers less than 100 that have only 2s and 5s in their prime factorization. What do you notice about these numbers? *Answer:(10, 20, 40, 50, 80; they're all multiples of 10.)*

MODULE

1-4

This module addresses the following indicators:

6-3.3 **Represent algebraic relationships with variables in expressions, simple equations, and simple inequalities. (B2)**

6-3.4 **Use the commutative, associative, and distributive properties to show that two expressions are equivalent. (C3)**

This module contains 2 lessons. These Lessons are **INTRODUCTORY ONLY.** Lessons in S3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

  6-3.3 Represent algebraic relationships with variables in expressions, simple equations, and simple inequalities. (B2)

  - In fourth grade, students translated among, letter, symbols and words to represent quantities in simple mathematical expression or equations (4-3.4). In fifth grade, students represented numeric, algebraic and geometric pattern in words, symbols, algebraic expression and algebraic equations (5-3.1)

  - In seventh grade, students represent proportional relationships with graphs, tables, and equations (7-3.6) and represent algebraic relationships with equations and inequalities (8-3.2).

  6-3.4 Use the commutative, associative, and distributive properties to show that two expressions are equivalent.

  - In fifth grade, students will identify applications of commutative, associative, and distributive properties with whole numbers (5-3.4).

  - In eighth grade, students use commutative, associative, and distributive properties to examine the equivalence of a variety of algebraic expressions (8-3.3).

Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

- Equations*
- Inequality*
- Expression*
- Variable*
- Commutative Property*
- Associative Property*
- Distributive Property*
- Equivalence*

II. Teaching the Lesson(s)

Teaching Lesson A: Experiment with functional relationships using two variables

Teacher notes: In 4th and 5th grades, students used variables to write a mathematical expression in symbolic form. This knowledge should be reinforced in the 6th grade and refined to understand that variables are more than letters or symbols that represent a number. The focus in 6th grade is for students to see the "use" of variables for a specific unknown in representing algebraic relationships. This focus is then extended into writing simple one-step equations and inequalities that model a mathematical situation. This standard is the foundational step for algebraic concepts taught in 7th and 8th grades. Seventh grade will reinforce these concepts and focus on the study of proportional relationships with graphs, tables, and equations.

Please note that a more in depth understanding of the concept of inequality is crucial in the 6th grade. Students have been using the inequality symbols >, ≥, <, and ≤ since the 2nd grade in grade appropriate applications. It is imperative at this level that students' think of an inequality as much more than "the alligator eats the biggest piece". Sixth grade students must be encouraged to view inequalities as a way to describe and represent a relationship between/among quantities.

For this indicator, it is **essential** for students to**:**

- Write an equation or inequality from a picture
- Write an equation or inequality from a word problem
- Understand inequality symbols
- Understand the concept of equivalency
- Understand that algebraic relationships can be in the form of words, tables or graphs

For this indicator, it is **not essential** for students to:

- Solve or graph equations or inequalities

a. Indicators with Taxonomy

6-3.3 Represent algebraic relationships with variables in expressions, simple equations, and simple inequalities. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

Provide students with the following prompt:

Your school has decided to hire a DJ for the "end of school" dance. The DJ from the local radio station charges a $100 deposit in addition to his $25 per hour fee.

A. Have the students discuss how you would find the cost of the DJ for various numbers of hours.
B. Ask the students to give a general rule for finding the cost of the DJ. For instance: $100 + ($25 times the number of hours worked equals the total cost).
C. Have the students convert the rule into algebraic symbolization such as:

C = Cost   h = number of hours worked

C = 25h + 100

2. Allow the students to work in groups. Give the student groups the following figures and prompt:

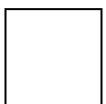


Figure 1

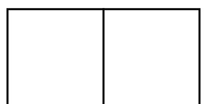


Figure 2

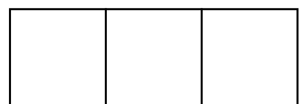


Figure 3

Make a table of values to find the rule which describes the pattern for finding the perimeter of the figure.

Figure Number	Number of Squares	Perimeter
1	1	4 units
2	2	6 units
3	3	8 units
4	4	
5		
6		
…		
10		

3. Have the students use the data table to predict the perimeter of a figure with 10 squares and describe how they could find the answer without drawing all ten squares. Have the students write a simple equation for finding the perimeter of a figure with ten squares. *For instance* $P = 10 \times 2 + 2$

4. Have the students write a simple equation for finding the perimeter of a figure with n squares. *For instance* $P = n \times 2 + 2$ *or* $P = 2n + 2$

c. Misconceptions/Common Errors

- Many students have a common misconception that different variables represent different numbers. For instances, some students think that the following equations will have different solutions for the variable: $5a + 10 = 45$ or $5y + 10 = 45$.

- Students also misunderstand the concept of equivalence. They must establish that the equal sign plays different roles based on the situation. In this instance, it does not mean "do something". It means, that there is a relationship of equivalence on either side of the equal sign.

d. Additional Instructional Strategies

EXTENSIONS:
The task list below provides ideas for experimental situations.
The task created by each situation is to try and develop a functional relationship between two variables by conducting an experiment. Data should be entered in a table and analyzed. The goal is to determine an equation (function) that can be used to make predictions.

- How long would it take for 100 students standing in a row to complete a wave like the ones done at football games? Experiment with different numbers of students from 5 to 25. Can the relationship predict how many students it would take for a given wave time?
- How far will a Matchbox car roll off of a ramp, based on the height the ramp is raised?
- How is the flight time of a paper airplane affected by the number of paper clips attached to the nose of the plane?
- What is the relationship between the number of dominoes in arrow and the time required for them to fall over? (Use multiples of 100 dominoes)
- Make wadded newspaper balls using different numbers of sheets of newspaper. Rubber bands help hold the paper in a ball. What is the relationship between the number of sheets and the distance the ball can be thrown?
- If colored water is dropped on a paper towel, what is the relationship between the number of drops and the diameter of the spot? Is the relationship different for different brands of towels?
- How much weight can a toothpick bridge hold? Lay toothpicks in a bunch to span a 2-inch gap between two boards. From the toothpicks, hang a

bag or other container into which weights can be added until the toothpicks break. Begin with only one toothpick.

e. Technology

Virtual Manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictoral representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- National Library of Virtual Manipulatives
  http://nlvm.usu.edu/en/nav/category_g_3_t_2.html

- Illuminations Lesson: Extending to Symbols: Students learn about the notion of equivalence in concrete and numerical settings. As students begin to use symbolic representations, they use variables as place holders or unknowns. This investigation illustrates the continued transition from the concrete balance view of equivalence to a more abstract view.
  http://illuminations.nctm.org/LessonDetail.aspx?id=L755

- Introduction to Algebra (variables, expressions, equations, etc.)
  http://www.mathleague.com/help/algebra/algebra.htm

- Math Focal Points Grade 6: Provides a list of online games and activities with a description and links to each one.
  http://msteacher.org/epubs/math/math14/expressions.aspx

- Writing Algebraic Equations
  http://www.mathgoodies.com/lessons/vol7/equations.html

- Practice with Solving Inequalities http://www.regentsprep.org/

- Introduction to Equations video
  http://www.classbrain.com/artteensb/publish/introduction_to_equations_56k.shtml

f. Assessing the Lesson

(Use the following problems as either notebook entries or a quick handout for assessing how students are progressing with expressions and equations.)

- What expression would you write for the following problem: Customers at Blockbuster Video receive a sale coupon that lets them buy any DVD for $3 off the regular price? (*r - $3)*
- Complete the table. If the sale price is $15, then the regular price is ______________.

Regular Price (r)	Sale Price (r – 3)
$10	
$11	
$12	
$16	
$18	
$20	

- Micah left for school with 4 boxes of pencils. Each box had 6 pencils. At school, he gave away 4 pencils from one box. Which equation below can be used to find the total number of pencils that were left?

a) 4 x 6 – 4 = p
b) 4 x 6 + 2 = p
c) 3 x 6 + 4 = p
d) 3 x 6 – 2 = p

- What value for n makes n/3 + 1 = 7 a true sentence?

Teaching Lesson B: It's the Property that Matters

Teacher Notes:
Students should demonstrate a clear understanding of the concepts of equivalence by using the commutative, associative, and distributive properties. These properties should be used in situations that involve all operations with whole numbers, addition and subtraction of fractions and decimals, and powers of 10 through 10^6.

For this indicator, it is **essential** for students to**:**

- Gain a conceptual understanding each rule (using examples and non-examples)
- Verbalize each rule using appropriate terminology
- Perform whole number computations

For this indicator, it is **not essential** for students to:
- Use properties in situations that involve multiplication/division of fractions and decimals
- Create a formal rule for each property using variables.
  For example, a + b = b + a

a. Indicators with Taxonomy

6-3.4 Use the commutative, associative, and distributive properties to show that two expressions are equivalent. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson

Materials Needed:

- Cuisenaire Rods (if available)
- Centimeter Cubes (connecting cubes are best)
- Centimeter grid paper
- Colored pencils
- Calculator (optional)

Give the following situation to students:

A variety of writing instruments are in one package. There are 2 glittery pencils and 5 striped pencils. There are also 4 pens in the package.

Part 1: Commutative Properties

1. Find 2 ways to represent (as an equation) the number of pencils in the package. Answer: 2 + 5 and 5 + 2

   Discuss the results when the expressions are evaluated. Tell students they have just demonstrated the Commutative Property of Addition.

   Ask: "What other operations are commutative?" Allow students time to work with subtraction, multiplication and division. They may choose to use a calculator to expedite their discoveries. Students may also build using the manipuatives to show their discoveries.

Teacher Note: For example, students could experiment with the following expressions:

a. 15 – 10 and 10 – 15
b. 4 x 6 and 6 x 4
c. 20 ÷ 5 and 5 ÷ 20
d.

Any examples should work….

Ask, "What other operation(s) are commutative?"

Answer: multiplication

Have students generate 3 number sentences to show the Commutative Property of Addition.

Have students generate 3 number sentences to show the Commutative Property of Multiplication.

Part 2: Associative Properties

2. Refer back to the writing instrument package:

   A variety of writing instruments are in one package. There are 2 glittery pencils and 5 striped pencils. There are also 4 pens in the package.

Two students used two different methods to represent the number of writing instruments in the package. The two expressions are:

$$(2 + 5) + 4 \quad \text{and} \quad 2 + (5 + 4)$$

Ask students to compare the models using the grid paper or other manipulatives provided.

Once compared, tell students they are using or showing the Associative Property of Addition.

Ask, "What other operations can be associative?" Allow students time to work with subtraction, multiplication and division. They may choose to use a calculator to expedite their discoveries. Students may also build using the manipuatives to show their discoveries.

Teacher Note: For example, students could experiment with the following expressions:

a. (30 – 5) – 6 and 30 – (5 – 6)
b. (4 x 6) x 7 and 4 x (6 x 7)
c. (60 ÷ 5) ÷ 2 and 60 ÷ (5 ÷ 2)

Ask, "What other operation(s) are associative?"
Answer: multiplication

Have students generate 3 number sentences to show the Associative Property of Addition.

Have students generate 3 number sentences to show the Associative Property of Multiplication.

Part 3: Distributive Properties

3. Refer back to the writing instrument package:

A variety of writing instruments are in one package. There are 2 glittery pencils and 5 striped pencils. There are also 4 pens in the package.

What would the following expression represent:
(2 + 5 + 4) + (2 + 5 + 4) + (2 + 5 + 4)?

Answer: 3 packages of writing instruments.

Ask: "Could the same expression be represented by: 3 (2 + 5 + 4)" ? Discuss the 3 groups of (2 + 5 + 4) or the 3 groups of total writing instruments.

Have students evaluate the two expressions:

(2 + 5 + 4) + (2 + 5 + 4) + (2 + 5 + 4) AND 3 (2 + 5 + 4)

Have students discuss what they notice.

Ask, "What would happen if you evaluated the following expression: 3(2) + 3(5) + 3(4)?"

Have students discuss what they notice.

Recap for students:

(2 + 5 + 4) + (2 + 5 + 4) + (2 + 5 + 4) = 3 (2 + 5 + 4)
= 3(2) + 3(5) + 3(4)?

Tell students they are demonstrating the Distributive Property.

Ask students to evaluate the following expressions to determine which pairs are equivalent.

a. 2 (4 + 2) = 2(4) + 2(2)
b. (4 + 16) (5) = 4(5) + 16(5)
c. 6 (3 + 6) = 6(3) + 6

c. Misconceptions/Common Errors

Students might have difficulty naming the property that they use. The most important point here is that they understand what the different properties allow them to do and not do. (Connect to NCTM Standards: Grade 5, p. 46)

Students definitely need to explore operations and properties. For example, just because addition is commutative does not mean subtraction is, etc.

d. Additional Instructional Strategies

- Elementary and Middle School Mathematics Teaching Developmentally: Sixth Edition, Van de Walle, pps. 157-158 (covers commutative and distributive properties)
- Everyday Mathematics Grade 6 Volume 2: "Area Models for the Distributive Property" Teacher's Edition pps. 700 – 703. Also, see the continuation lesson on pps. 704 – 709
- Navigations Series: Navigating through Algebra in Grades 3-5, "I Spy Patterns" p. 48. Objectives: Students will partition the given array into two different parts; translate visual patterns into numerical expressions; and explore how equivalent numerical expressions represent the commutative and associative properties of operations.
- http://www.coolmath.com/prealgebra/06-properties/index.html

e. Technology

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- Basic Number Properties: Associative, Distributive, and Commutative http://www.purplemath.com/modules/numbprop.htm

- Number Properties (includes video for distributive)
http://www.onlinemathlearning.com/number-properties.html
- Commutative Property Video
http://www.glencoe.com/sec/math/brainpops/00112039/00112039.html
- Interactive Math lesson on all properties
http://www.aaamath.com/pro74b-propertiesmult.html
- Associative Property Video
http://www.glencoe.com/sec/math/brainpops/00112040/00112040.html
- Distributive Property Video
http://www.glencoe.com/sec/math/brainpops/00112041/00112041.html
OR
http://www.classbrain.com/artteensb/publish/distributive_property.shtml

f. Assessing the Lesson

Formative Assessment:

1. Which property is used to show equivalence?

a. 5 + (8 + 1) = (5 + 8) + 1
b. 5 + 8 = 8 + 5
c. 8 x 4 = 4 x 8
d. 7(3 + 5) = 7(3) + 7(5)
e. 3 x (3 x 2) = (3 x 3) x 2

2. Complete the following sentences to show equivalence using the stated property:

a. 6 (3 + 7) = ________________ (distributive property)
b. 5 + 14 = _________________ (commutative property of addition)
c. 3 + (6 + 4) = __________________ (associative property of addition)
d. 5 x (3 x 6) = __________________ (associative property of multiplication)
e. 8 x 5 = __________________ (commutative property of multiplication)

3. Respond to the following: Is the commutative property true for subtraction? On what evidence do you base your answer?

III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

6-3.3

The objective of this indicator is to represent, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand means to construct meaning; therefore, the students' focus is on building conceptual knowledge of the relationships between the forms. The learning progression to **represent** requires students to understand the concepts of equivalency and inequalities. Students analyze algebraic relationships (words, tables and graphs) to determine known and unknown values and the operations involved. They generate descriptions of the observed relationship and generalize the connection (6-1.7) between their description and structure of expression, equations or inequalities. Students explain and justify their ideas with their classmates and teachers using correct and clearly written or spoken words, variables and notation to communicate their ideas (6-1.6). Students then compare the relationships (words, tables and graphs) to their equation, inequality or expression to verify that each form conveys the same meaning.

6-3.4

The objective of this indicator is to use which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is to use which is a knowledge of specific steps and details, learning experiences should integrate both memorization and concept building strategies to support retention. The learning progression to **use** requires student to explore a variety of examples of these number properties using a various types of numbers. They analyze these examples and generalize connections (6-1.7) about what they observe using correct and clearly written or spoken language (6-1.6) to communicate their understanding. Students do not generalize these connections using formal rules involving variables. Students connect these statements to the terms commutative, associative and distributive. Students then develop meaningful and personal strategies that enable them to recall these relationships.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

(Item 1 adapted from NCTM Mathematics Assessment Sampler Grades 3-5)

1. The two number sentences shown below are true.

$$\square - \bigcirc = 6$$

$$\bigcirc + \bigcirc = 2$$

If both equations shown above are true, which of the following equations must also be true? (Circle your choice and explain why.) **Students should circle the first equation.*

$$\square \times \bigcirc = \square$$

$$\bigcirc \times 2 = \bigcirc$$

$$\square + \square = 12$$

$$\square + \bigcirc = \square$$

2. Which of these problems could be solved by using the open sentence:
   A – 5 = ?
   a. Janis is 5 years older than Seth. If A is Seth's age, how old is Janis?
   b. Todd is 5 years younger than Amelia. If A is Amelia's age in years, how old is Todd?
   c. Isaac is 5 times as old as Bert. If A is Bert's age in years, how old is Isaac?
   d. Nathan is one-fifth as old as Leslie. If A is Nathan's age, how old is Leslie?

3. Your school is having a fall festival. Admission into the festival is $2 and each game inside the festival costs $0.25. Write an <u>inequality</u> that represents the possible number of games that can be played having $10.

4. Which rule represents the table?

School Fees

# days in school	#students who paid their fees
1	6
2	10
3	14
4	18

A. 4x
B. X + 5
C. 2x + 4
D. 4x + 2

5. Which rule represents the table?

Math Club

# of people on a team	# of correct answers
0	1
2	7
4	13
6	19
8	25

A. X + 1
B. 2x + 1
C. 3x + 1
D. 4 x + 1

6. Use the commutative, associative, or distributive property to explain the reasons for the equivalence of each of the numerical expressions.

i. 10 + (5 + 5 + 5) = (10 + 5 + 5) + 5

ii. (5 + 12) + 6 = 5 + (12 + 6)

iii. (7 + 8) * 3 = (7*3) + (8*3)

iv. (12 + 20) + 5 = 5 + (12 + 20)

7. Use the distributive property to solve: A person wants to know the cost of 5 hot dogs at $1.97 each, plus 5 orders of fries at $0.53 each. *Students should notice that they can combine the cost of the hot dogs and fries and then multiply times 5. Thus, a(b+c) or 5 ($1.97 + $0.53).*

8. Which property (ies) might assist you in solving the following problem mentally? Explain why?

A person has four items in his or her cart. The prices of the items are $0.74, $0.32, $0.26, and $0.28. How much would these items cost? Students might use their knowledge of the commutative and associative properties for addition to rearrange and regroup the addends to make it easier to find the sum of $0.74 + $0.32 + $0.26 + $0.28. For example, students could add $0.74 and $0.26 to get $1.00, add $0.32 and $0.28 to get $.60, and then add $1.00 and $0.60 to get $1.60.)

MODULE

1-5

This module addresses the following indicators:

6-5.6 Use proportions to determine unit rates. (C3)

6-5.7 Use a scale to determine distance. (C3)

This module contains two lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

  6-5.6 Use proportions to determine unit rates.

  - There are no previous indicators that relate to this indicator.
  - In seventh grade, students use ratio and proportion to solve problems involving scale factors and rates (7-5.1).

  6-5.7 Use a scale to determine distance

  - There are no previous indicators that relate to this indicator.
  - In seventh grade, students use ratio and proportion to solve problems involving scale factors and rates (7-5.1).

- **Key Concepts/Key Terms**
  *Rate
  Proportional Reasoning
  *Proportion
  Representative Fraction
  Statement of Equivalency
  Bar Scale
  *Unit Rate
  *Distance
  *Ratio
  *Per

**These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students*

II. Teaching the Lessons

1. Teaching Lesson A

For this indicator, it is **essential** for students to:

- Understand concept of a ratio written as a fraction
- Understand unit rate as one unit
- Connecting the concept of equivalent ratios to equivalent fractions
- Connecting equivalent ratios to a proportion

- Interpret their answers. For example, $\frac{250 miles}{5 hour}$ means he drove 250 miles in 5 hours.
- Work with answers that are in whole number form

For this indicator, it is **not essential** for students to:

- Use the Cross-Products Property to solve proportions.
- Work with answers that are in decimal or fractional form

a. Indicators with Taxonomy

6-5.6 Use proportions to determine unit rates. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson

Materials Needed:

None

Literature Connections:

Introductory Lesson: "Rate: Using Proportions to Determine Unit Rates"

Pose the following to student groups/pairs:

I've decided to change my cell phone carrier so I started comparing prices. I was mainly concerned with the cost if I go over the limit on my minutes. One phone company charges $.70 for every 15 minutes over and another charges $1.00 for 20 minutes over. Which company has the cheaper rate? (.70 for 15 minutes equals $2.80 per hour; $1 for 20 minutes equals $3 per hour)

Allow groups/pairs to solve the problem any way they wish as long as they can explain their reasoning. Have groups/pairs share their solution strategies. Listen for opportunities to probe student thinking with regard to proportional reasoning. Listen for opportunities to probe student thinking with regard to unit rate.

Some students may have chosen to solve the problem by determining the cost per minute – unit rate.

If not suggested by students, ask, "What would be the cost per minute for each?" ($.70 for 15 minutes is approximately $.47 per minute and $1.00 for 20 minutes is $.28 per minute.) Allow time for students to determine and share strategies.

Some may have chosen to solve the problem by determining equivalent ratios (they were introduced to scale in the 6th grade).

If not suggested by students, ask, "Would it have been easier if I had told you the cost per hour for each – in other words, if I had said that one charges $2.80 per hour and the other charges $3 per hour?" (yes) "When we need to compare ratios, it is easier if they are expressed in terms of the same unit. In this case the unit would be an hour – 60 minutes. So the question then becomes if 15 minutes cost $.70, how much does 60 minutes cost?"

$.70 is to 15 minutes as how much (x) is to 60 min

$$\frac{\$0.70}{15\,\text{min}} = \frac{x}{60\,\text{min}}$$

Have students set up the proportion for 20 minutes at $1.00.

Explain equivalent ratio method for solving. Relate equivalent ratio method to equivalent fractions.

Ask, "Are both methods correct? Is one easier than the other? (It depends on personal preference and the numbers used.)

c. Misconceptions/Common Errors

Students may invert the units when setting up their ratio.

d. Additional Instructional Strategies

- Use real world examples, such as: miles per hour, beats per minute, miles per gallon, cost/lb., etc. Use grocery store ads to comparison shop (Who has the best deal?).

- When using proportions to determine unit rates, students should determine the rate for <u>one</u> unit. For example, if it takes George 2 hours to drive 230 miles, how far can he drive in 1 hour?

- Understanding Rational Numbers and Proportions (Illuminations lesson) Students use real-world models to develop an understanding of fractions, decimals, percents, unit rates, proportions, and problem solving. http://illuminations.nctm.org/LessonDetail.aspx?id=L284

- Teaching Ratios and Proportions http://www.homeschoolmath.net/teaching/proportions.php

e. Technology

Taking Its Toll (rates – distance) Illuminations lesson
http://illuminations.nctm.org/LessonDetail.aspx?id=L571

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Student understanding of this lesson is assessed through questioning and listening during the class dialogue.

2. Teaching Lesson B

For this indicator, it is **essential** for students to:

- Understand the meaning of ratio
- Understanding the meaning of proportion
- Set up a ratio
- Set up a proportion
- Read a scale on a map
- Understand the meaning of the scale
- Work with answers that are in whole number form
- Use an appropriate strategy to solve the proportion

For this indicator, it is **not essential** for students to:

- None noted

a. Indicators with Taxonomy

6-5.7 Use a scale to determine distance. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson

Materials Needed:

Three maps of the same state or region, each with a different scale

Introductory Lesson:

1. Show three different maps of the same state or region with different scales.
2. Tell the students the scale on each map (1 in = ? mi). Write it on the board or overhead for each map.
3. Identify the location of the same two towns on each map.
4. Have student volunteers use rulers to measure the distance between the two towns on each map. The students need to think about proper measurement and how accurate their measurements should be.
5. Use the scale on each map and the corresponding measured distance to determine the actual distance between the two towns. Figure out the actual distance based on the scale and measurement for each map.
6. The actual distance should be very similar for each of the maps. Discuss why this occurs when all three maps are different sizes. Students should be able to tell you that the distance between the two towns is a certain distance, so no matter which map you use, the distance should be the same. The scales are different to reflect this fact. The measured distances on the maps were all different and the scales were all different, but the distance between the two towns should be the same.
7. Discuss why we would use different scales. Discuss the fact that different scales can be used to represent the same thing (example – different sizes of maps).

c. Misconceptions/Common Errors

Students may invert the units when setting up their ratio.

d. Additional Instructional Strategies

- How Big is Barbie? – Students measure various dimensions of a male and a female doll's body and scale them proportionally to average human measurements. http://www.mathprojects.com/Downloads/Pre-Algebra/HowBigIsBarbie.pdf
- Junior Architects (Illuminations Lesson) This is a series of activities and lessons that build students towards a 3D scale drawing. Read online description before proceeding with your students. http://illuminations.nctm.org/LessonDetail.aspx?id=U172

- Marketing Careers: Working with Scale Drawings (Students calculate the dimensions of a toy car to create a scale drawing) http://www.learnnc.org/lp/pages/2625
- Scale Drawings Activity (Students will solve problems by using ratios and rates; find measures by using scale drawings, and list several ways in which scale drawings can be used) http://www.poahonline.org/lessons/mclarty/mclarty-scale%20drawing%20lesson.pdf

e. Technology

- Scaling Away (Illuminations Lesson) http://illuminations.nctm.org/LessonDetail.aspx?id=L584

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the <u>Lesson</u>

On an exit ticket , as a notebooking entry, or for homework: Provide students with a drawing of either a simple geometric figure or a simple shape (i.e. sailboat). Their task will be to create a new drawing that is either larger or smaller (let them choose) than the given one. Make sure to provide dimensions for the drawing. Students can set up ratios to determine the other dimensions by solving the proportion.

III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

6-5.6 Use proportions to determine unit rates. (C3)

The objective of this indicator is <u>use</u>, which is in the "apply procedural" cell of the Revised Bloom's Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus is to gain computational fluency with problems involving the use of proportions to solve problems with and rates, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **use** requires students recall the definition of ratio and proportion and how to use proportions (equivalent ratios) to solve simple problems involving unit rates. Students should be given a variety of situations that involve rates and be able to generalize connections among real-world situations (6-1.7). Then students use correct and clearly written or spoken words (6-1.6) to explain their reasoning.

6-5.7 Use a scale to determine distance. (C3)

The objective of this indicator is use, which is in the "apply procedural" cell of the Revised Bloom's Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus is to gain computational fluency with problems involving the use of proportions to solve problems with and rates, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **use** requires students recall the definition of ratio and proportion and how to use proportions (equivalent ratios) to solve simple problems involving unit rates. Students should be given a variety of situations that involve scale factors and rates and be able to generalize connections among real-world situations (6-1.7). Then students should use correct and clearly written or spoken words (6-1.6) to explain their reasoning.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. A 12-pack of cola costs $3 at Stellar Supermart.
   - Complete the rate table below to find the per-unit rates.

Dollars		3.00	1.00
Cans	1	12	

   - At this price, how much would 30 cans of cola cost? (*$7.50*)
   - How many cans could you buy for $2? (*8 cans*)

2. Write a proportion to find each unit rate.

   a) Malia drove 180 miles in 3 hours.
      Proportion______________ Unit Rate_____________

   b) Jeffrey grilled 54 hot dogs for 18 people
      Proportion_____________ Unit rate____________

   c) Darlene typed 200 words typed in 5 minutes
      Proportion__________________ Unit Rate______________

3. Find the distances from:
   a) Greenville to Spartanburg.___________
   b) Florence to Charleston._____________
   c) Greenville to Charleston_____________
   d) Sumter to Myrtle Beach _____________