SOUTH CAROLINA SUPPORT SYSTEM INSTRUCTIONAL GUIDE

Content Area	**Sixth Grade Math**

Second Nine Weeks

Standards/Indicators Addressed:

Standard 6-3: The student will demonstrate through the mathematical processes an understanding of writing, interpreting, and using mathematical expressions, equations, and inequalities.

6-3.1* Analyze numeric and algebraic patterns and pattern relationships. (B4)
6-3.2* Apply order of operations to simplify whole-number expressions. (C3)
6-3.5* Use inverse operations to solve one-step equations that have whole-number solutions and variables with whole-number coefficients. (C3)

Standard 6-4: The student will demonstrate through the mathematical processes an understanding of shape, location, and movement within a coordinate system; similarity, complementary, and supplementary angles; and the relationship between line and rotational symmetry.

6-4.1* Represent with ordered pairs of integers the location of points in a coordinate grid. (C2)
6-4.2* Apply strategies and procedures to find the coordinates of the missing vertex of a square, rectangle, or right triangle when given the coordinates of the polygon's other vertices. (C3)
6-4.3* Generalize the relationship between line symmetry and rotational symmetry for two-dimensional shapes. (B2)
6-4.4* Construct two-dimensional shapes with line or rotational symmetry. (B6)
6-4.5* Identify the transformation(s) used to move a polygon from one location to another in the coordinate plane. (A1)
6-4.6* Explain how transformations affect the location of the original polygon in the coordinate plane. (B2)

* These indicators are covered in the following 4 Modules for this Nine Weeks Period.
Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.

Module 2-1 Patterns, Relationships, and Functions			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-1 Lesson A: **6-3.1** Analyze numeric and algebraic patterns and pattern relationships. (B4)	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 2-1 Introductory Lesson A See Module 2-1, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 2-1 Lesson A Assessment
Module 2-1 Lesson B: **6-3.1** Analyze numeric and algebraic patterns and pattern relationships. (B4)		See Instructional Planning Guide Module 2-1, Introductory Lesson B See Instructional Planning Guide Module 2-1, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 2-1 Lesson B Assessment

Module 2-2 Solve Mathematical Situations			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-2 Lesson A: **6-3.2** Apply order of operations to simplify whole-number expressions. (C3)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and	See Instructional Planning Guide Module 2-2 Introductory Lesson A See Instructional Planning Guide Module 2-2, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 2-2 Lesson A Assessment

Module 2-2 Lesson B: **6-3.5** Use inverse operations to solve one-step equations that have whole-number solutions and variables with whole-number coefficients. (C3)	Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 2-2 Introductory Lesson B See Instructional Planning Guide Module 2-2, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 2-2 Lesson B Assessment

Module 2-3 Geometry: Planes and Transformational Geometry			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-3 Lesson A: **6-4.1** Represent with ordered pairs of integers the location of points in a coordinate grid. (C2)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 2-4 Introductory Lesson A See Instructional Planning Guide Module 2-4, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 2-4 Lesson A Assessment
Module 2-3 Lesson B: **6-4.2** Apply strategies and procedures to find the coordinates of the missing vertex of a square, rectangle, or right Triangle when given the coordinates of the polygon's other vertices. (C3)		See Instructional Planning Guide Module 2-4, Introductory Lesson B See Instructional Planning Guide Module 2-4, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 2-4 Lesson B Assessment
Module 2-3 Lesson C **6-4.3** Generalize the relationship between line symmetry and rotational symmetry for two-dimensional shapes. (B2)		See Instructional Planning Guide Module 2-4, Introductory Lesson C See Instructional Planning Guide Module 2-4, Lesson C Additional Instructional Strategies	See Instructional Planning Guide Module 2-4 Lesson C Assessment

Module 2-3 continued			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-3 Lesson D **6-4.4** Construct two-dimensional shapes with line or rotational symmetry. (B6)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 2-4, Introductory Lesson D See Instructional Planning Guide Module 2-4, Lesson D Additional Instructional Strategies	See Instructional Planning Guide Module 2-4 Lesson D Assessment
Module 2-3 Lesson E **6-4.5** Identify the transformation(s) used to move a polygon from one location to another in the coordinate plane. (A1)		See Instructional Planning Guide Module 2-4, Introductory Lesson E See Instructional Planning Guide Module 2-4, Lesson E Additional Instructional Strategies	See Instructional Planning Guide Module 2-4 Lesson E Assessment
Module 2-3 Lesson F **6-4.6** Explain how transformations affect the location of the original		See Instructional Planning Guide Module 2-4, Introductory Lesson F See Instructional Planning Guide	See Instructional Planning Guide Module 2-4 Lesson F Assessment

polygon in the coordinate plane. (B2)		Module 2-4, Lesson F Additional Instructional Strategies	

MODULE

2-1

This module addresses the following indicators:

6-3.1 Analyze numeric and algebraic patterns and pattern relationships. (B4)

This module contains 1 lesson. These Lessons are **INTRODUCTORY ONLY.** Lessons in S3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

- **Continuum of Knowledge**

  6-3.1 Analyze numeric and algebraic patterns and pattern relationships

  - The study of patterns is extensive throughout elementary school. Students begin the process of transitioning from the concrete to the abstract and symbolic in the 5th grade, and learn to represent patterns in words, symbols, and algebraic expressions/equations for the first time.
  - The focus in the 6th grade should be on patterns that relate to linear functions which have a constant rate of change. This will lay the foundation for the study of slope in the 7th and 8th grades.

- **Key Concepts/Key Terms**
  ***** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

  Algebraic patterns*
  Expressions*
  Equations*
  Inequalities*
  Linear functions
  Rate of change
  Function
  Rule*
  Functional Relationship

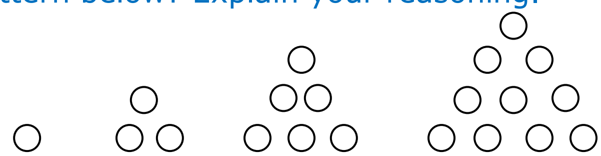
II. Teaching the Lesson(s)

Teaching Lesson A: Simulate a real-world pattern and discover its mathematical rule

Teacher Notes: Teachers should encourage 6th grade students to explain their observations of a pattern in their own words. This verbalization will enable students to begin to write a mathematical rule for a pattern. Students should also be encouraged to represent patterns using tables, graphs, and equations. Discussions should be used to determine which representation makes it easier to describe the pattern, extend the pattern, and/or make predictions with the pattern.

Students will need an in-depth experience discussing real world patterns and patterns which provide concrete examples before they can begin to represent them symbolically. Teachers will need to model many examples that involve moving from the concrete to the symbolic. For example:

- This pattern may be represented using concrete models. Which of the following numeric patterns best represents the geometric pattern below? Explain your reasoning.



A) 1, 3, 9, 12 B) 1, 2, 4, 8

C) 1, 3, 6, 9 D) 1, 3, 6, 10

- Students should explain their observations of a pattern in their own words. This verbalization will enable students to begin to write a mathematical rule for a pattern later.

For this indicator, it is **essential** for students to**:**

- Solve arithmetic (adding/subtracting) and geometric (multiplying by common ratio) sequences
- Represent patterns using tables, graphs, and equations.
- Write mathematical rules for patterns from numeric and pictorial patterns
- Determine which representation makes it easier to describe, extend, and or make predictions using the patterns

a. Indicators with Taxonomy

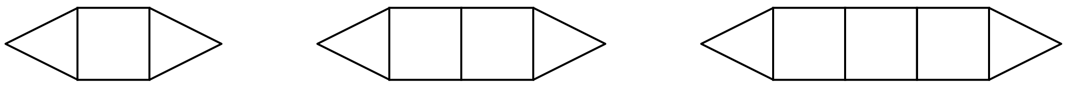
6-3.1 Analyze numeric and algebraic patterns and pattern relationships. (B4)

Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson A: Simulate a real-world pattern and discover its mathematical rule**: (**Adapted from NCTM Navigating through Algebra "Watch Them Grow")

Materials Needed: Pattern blocks

1. Give the following prompt to students: Mrs. Fox has decided to decorate the hall way with a picture of a bookworm. For every book that a student reads she will add another piece to the worm. The students are excited to see how long they can make the worm before the end of the year. This is what the worm looks like:



2. Give the students pattern blocks and ask them to determine the number of blocks needed to create each of the worms for 1-3 books. Have the students find the number of squares and triangles used to create each worm and have them organize the information in some way.

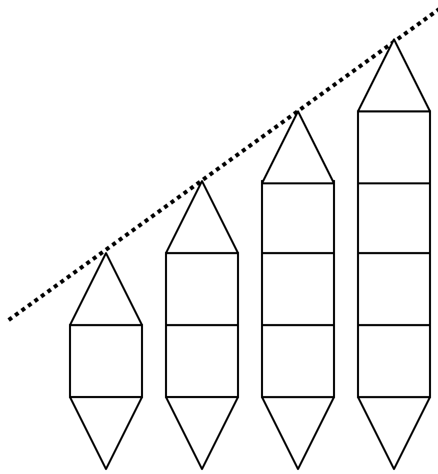
3. Discuss what the worm will look like when 4 books have been read. Discuss it orally and have the students give a written description as well.

4. Have the students predict the number of pieces that will be needed to when students have read 12 books. Have them explain their reasoning in writing and then discuss it as a class.

5. As a class, write a rule that gives the total number of pieces needed to build the worm for any number of books. Begin with verbal descriptions of the rule and then write the rule symbolically.

<u>Teacher Note</u>: When possible, it is important to make connections among the various major concepts. The following extension is a good link between the "Patterns" and "Data" concepts.

EXTENSION: Have the students plot the number of pieces needed for each number of books read on a graph. Ask the students to describe what they see and ask them why the graph formed a straight line. Make sure to discuss the rate of change being constant = causing a straight line on the graph. If you turn the worms on end and put them side by side you will be able to see the line that forms.



c. Misconceptions/Common Errors

Geometric patterns are sequences that involve multiplying by a common ratio not based on shapes.

d. Additional Instructional Strategies

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual Manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictoral representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- National Library of Virtual Manipulatives http://nlvm.usu.edu/
- Patterns In Context http://www.learner.org/courses/learningmath/algebra/session2/index.html
- Pattern Functions and Algebra http://almarolh.googlepages.com/patternsfunctionsandalgebra
- Investigating Growing Patterns http://www.mathwire.com/archives/algebra.html

f. Assessing the Lesson

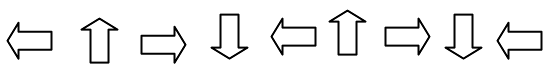
Provide students with multiple opportunities to explore patterns concretely, moving towards pictorial, and then abstract. Give students a new pattern using pattern blocks, unifix cubes, straws, toothpicks or another manipulative, and have them continue the pattern. Have them record in their notebooks or by some other means what the pattern looks like (drawing a picture) and explaining the pattern in words.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

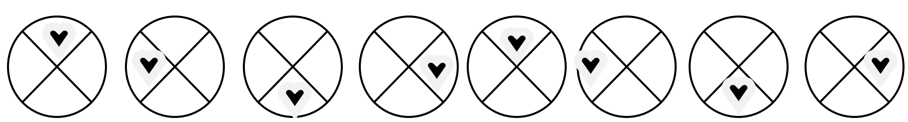
1. If this pattern continues, what will the 11^{th} shape look like? Draw it and describe the pattern.



2. Fill in the missing numbers for each of the patterns below.

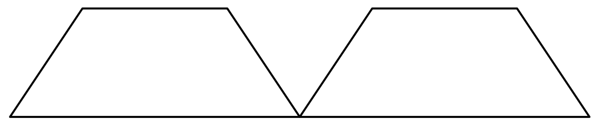
   a. 2, 5, 11, 23, ___, 95, 191
   b. 1, 2, 5, 10, 17, _____, 37
   c. 25, 20, _____, _____, 19, 14, 16, 11, 13, 8

3. A rule was used to make the pattern of figures shown below.



   a. Draw the next 5 figures in the pattern. Describe the rule used to make the pattern.
   b. Draw the 100^{th} figure in the pattern. Explain how you know what the figure should look like.

4. Look at the following trapezoids and complete the table.



# of trapezoids	# of sides
1	4
2	8
3	12
4	
5	
6	
7	
8	
9	
10	

a. If you had 50 trapezoids, how many sides would you have? Explain how you know.
b. Write an equation that would help you find the number of sides for any number of trapezoids.

MODULE

2-2

This module addresses the following indicators:

6-3.2 Apply order of operations to simplify whole-number expressions. (C3)

6-3.5 Use inverse operations to solve one-step equations that have whole-number solutions and variables with whole-number coefficients. (C3)

This module contains 2 lessons. These Lessons are **INTRODUCTORY ONLY.** Lessons in S3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

  6-3.2 Apply order of operations to simplify whole-number expression.
  - In 4th and 5th grades, students used variables to write mathematical expressions in symbolic form and to write an open sentence representing a given mathematical relationship. Sixth grade is the first time students are introduced to using order of operations to evaluate a numerical expression (6-3.2).
  - In 7th grade, students will use inverse operations to solve two-step equations and two-step inequalities. (7-3.4)

  6-3.5 Use inverse operations to solve one-step equations that have whole-number solutions and variables with whole-number coefficients. (C3)
  - In fourth grade, students apply procedures to find the value of an unknown letter or symbol in whole number equations (4- 3.5).
  - In 7th grade, students will use inverse operations to solve two-step equations and two-step inequalities. (7-3.4) They will utilize order operations as a tool to solve equations and inequalities (particularly two-step).

- **Key Concepts/Key Terms**
  * These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

  Order of operations *
  Exponents *
  All grouping symbols { [( )] }
  Inverse operation
  Coefficient
  Evaluate
  Solve
  Solution
  Additive inverse
  Multiplicative inverse

II. Teaching the Lesson(s)

In 4th and 5th grades, students did not evaluate expressions. Therefore, "Evaluating an expression", "Solve the expression" and "Find the solution to the expression" are written differently, but communicate the same meaning. These will be new and important phrases for students to understand.

NOTE: Evaluation/assessment at this grade level is ONLY to be done with WHOLE-NUMBER expressions.

Order of operations is sometimes a difficult concept for middle school students to understand. This may be a result of the way it is often introduced. It is *recommended* that the introductory lesson(s) provide opportunity(ies) for students to discover and understand why an agreement for the order of operations is necessary and important.

Many students are simply introduced to the concept of order of operations with the phrase "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally", often referred to as PEMDAS. While this is a helpful mnemonic device, it easily leads to common misconceptions.

Many students interpret PEMDAS *literally*, as: (a) M before D means multiplication is always done before division, and (b) A before S means addition is always done before subtraction. Students that are taught this mnemonic device up front seldom have any prior experiences to connect to this new concept. They don't always learn correctly or come to fully understand that the operations of multiplication and division (or addition and subtraction) are performed in the order that they are written, from left to right. Therefore, it is vital that teachers provide students with opportunites to discover why an agreement for the order of operations is necessary; before introducing the mnemonic device.

It is suggested that order of operations be introduced using a table (see below). Students tend to accept new learning more readily when it is presented in a way that makes sense to them. By using a table, students are able to visually prioritize the operations, and infer that the higher in the table an operation is located, the earlier in the process it is simplified? (Expressed? Solved?).

Placing the 'order' of the operations in a table, allows students to create a step-by-step progression for how the operations must occur. Additionally, the table format enables students to see that all grouping symbols must be done first – not just the "P", or parentheses, as it implies in the mnemonic.

After students have mastered an understanding of this format, the PEMDAS mnemonic may then be introduced if desired.

Order of Operations

Level 1	{ [( )] } All Grouping Symbols
Level 2	Exponents
Level 3	Multiplication and Division Proceeding from Left to Right
Level 4	Addition and Subtraction Proceeding from Left to Right

Teaching Lesson A: Order of Operations using Calculators, Problem Solving and Table Format

For this indicator, it is **essential** for students to**:**

- Solve problems that involve all operations with whole numbers
- Work with whole numbers expressions only
- Understand the reasoning behind order of operations

For this indicator, it is **not essential** for students to:

- Include negatives, fractions or decimals.

a. Indicators with Taxonomy

6-3.2 Apply order of operations to simplify whole-number expressions. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson A: Order of Operations using Calculators, Problem Solving and Table Format

Materials Needed: Calculators (basic and scientific)

To introduce order of operations, have students answer a relatively easy problem such as 4 + 2(3). Answers will likely be equally divided among 10 and 18. Then, give one student a scientific calculator and give another student a basic four function calculator. At this time, both answers are confirmed. Next, present a problem solving situation involving the problem: Kim has 4 dollars. She receives 3 dollars each from two of her friends. How much money does she have now? Most students will see that she would now have 10 dollars. Now, return to the original problem. Then introduce the table format for order of operations as detailed earlier.

Adapted from National Council of Teachers of Mathematics, *Teaching Mathematics in the Middle School*, "A New Approach to an Old Order". Volume 8, Number 4, 2002.

c. Misconceptions/Common Errors – Students commonly make an error in solving for order of operations (because of the mnemonic device) that multiplication must be completed prior to division and the same with addition and subtraction.

d. Additional Instructional Strategies

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual Manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictoral representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- Interactive Math Lesson for Order of Operations http://www.mathgoodies.com/lessons/vol7/order_operations.html
- Operation Order Game http://www.funbrain.com/algebra/index.html
- Order of Operations step by step (includes first glance, in depth, examples, and workout) http://www.math.com/school/subject2/lessons/S2U1L2GL.html
- Mystery Picture Game using order of operations (click next to start) http://www.dositey.com/math/m/mystery2SMA.htm
- Order of Operations Basketball (make sure to choose order of operations from the left side of the screen) http://www.scienceacademy.com/BI/

f. Assessing the Lesson

Formative - Have students write in their notebook or on an exit slip describing the order of operations. Also, give them a couple problems to solve and explain how they solved each one.

Teaching Lesson B: Experimenting with Inverse Operations: *Adapted from Van de Walle's Teaching Student-Centered Mathematics Volume 3*

Teacher Notes: Although students work with informal equations in the early grades, sixth grade is the first year they are introduced to both inverse operations and their use in solving one-step equations. As stated earlier, there is often a common misunderstanding in regards to the concept of equivalence in the 6th grade. Prior to 6th grade, students have simply written an answer after the equal sign and often have the misconception that an equal sign means "provide an answer after performing an operation" rather than seeing the equal sign as an indicator of equality. It is imperative that the 6th grade student understand that the equal sign does not mean perform an operation, but that there is a relationship of equivalence between the two terms on either side of that equal sign. Therefore, it is highly recommended that students be given the opportunity to review and discuss the concept of equality to identify and address any misconceptions *prior* to the introduction of solving one-step equations.

Because this is the first experience students have had with the additive inverse (the sum of a number and its opposite is zero), they will need sufficient opportunities to develop an understanding of and demonstrate the usefulness of this property. The connection must be made between the inverse operation (additive identity) and its use to isolate the variable. Keep in mind that 6th grade is the first time that the concept of solving equations is introduced. Therefore it is extremely important that students be given the opportunity to build on prior knowledge, use manipulatives to model the equations and the process of solving one-step equations, and then gradually progress to solving numerically by using inverse operations.

Although this is the introductory year that students solve one-step equations using inverse operations, it is expected that students reach fluency with this concept before leaving the 6th grade

For this indicator, it is **essential** for students to**:**

- Add, subtract, multiply, and divide whole numbers
- Understand the concept of a variable and how to solve for it
- Understand additive inverse (the sum of a number and its opposite is 0)
- Use manipulatives to model equations and the process of solving one-step equation
- Solving equations using inverse operations
- Check their solutions

For this indicator, it is **not essential** for students to:

- Include negatives, fractions, or decimals

a. Indicators with Taxonomy

6-3.5 Use inverse operations to solve one-step equations that have whole-number solutions and variables with whole-number coefficients.

Cognitive Dimension: Apply
Knowledge Dimension: Procedural

b. Introductory Lesson

Materials (optional): actual balance scale with different weights or shapes

To solve an equation means to find value (s) for the variable that make the equation true. Van de Walle recommends using pan balance to develop skills in solving equations with one variable.

"The balance makes it reasonably clear to students that if you add or subtract a value from one side, you must add or subtract a like value from the other side to keep the scales balanced" (p. 280).

Action: Show a balance with variable expressions in each side. Use only one variable. Make the tasks such that a solution by trial and error is not reasonable.
For example, the solution to, $3X + 2 = 11 - X$, is not a whole number. (Use whole numbers only!)
Suggest that adjustments be made to the quantities in each pan as long as the balance is maintained. Begin with simple equations such as, $12 + n = 27$, in order to help students develop skills and explain their rationale. Students should also be challenged to devise a method of proving that their solutions are correct. (Solutions can be tested by substitution in the original equation.)

Other simple examples: $21 - n = 13$; $n + 323 = 412$;
$512 \div x = 2$; $25 \times n = 725$

You may wish to attempt an example such as: $x - 17 = 31 - x$

Input/Output tables or function tables can also be useful when working on inverse operations. If concrete materials are needed, use an actual balance scale with different weights or shapes.

c. Misconceptions/Common Errors

Students think that an equal sign means to provide an answer rather than seeing it as an indicator of equality.

Students often solve an equation and do not understand why they are doing it. A common question that suggests this lack of understanding if often, "Is my answer right?" Students who ask this lack a conceptual understanding of the concept of equivalency and the purpose of the procedure of solving.

d. Additional Instructional Strategies

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual Manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictoral representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- National Library of Virtual Manipulatives (Pan balance) http://nlvm.usu.edu/en/nav/category_g_3_t_2.html
- Everything Balances Out in the End (Illuminations lesson) http://illuminations.nctm.org/LessonDetail.aspx?id=U170
- Extending to Symbols (Illuminations lesson) http://illuminations.nctm.org/LessonDetail.aspx?id=L755
- Solve One-Step Equations http://www.learningwave.com/lwonline/algebra_section1/onestep.html
- One Step Equation Basketball Game http://www.math-play.com/One-Step-Equation-Game.html
- Modeling One-step equations (Interactive) http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=109
- Battleship game solving one-step equations (Watch for negative numbers here.) http://www.quia.com/ba/36544.html

f. Assessing the Lesson

Formative
On an exit ticket or notebook entry, have students explain how using the inverse operation helps solve equations. Ask them to relate the pan balance to this process.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

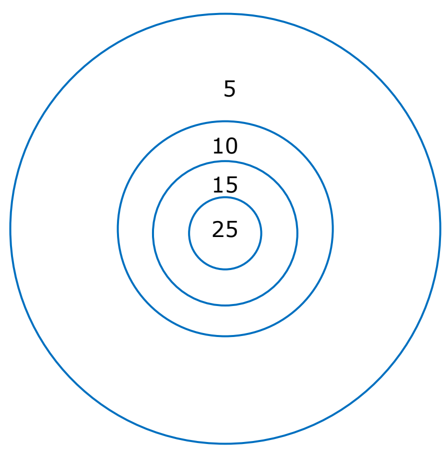
6-3.2

The objective of this indicator is apply, which is in the "apply procedural" of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus is to gain computational fluency with order of operations, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **apply** requires student to be fluently in all whole number operations. Given an expression, students explore various ways to simplify the expression. Students explain and justify their process of simplifying to their classmates and their teacher. They use inductive reasoning to generalize connections among strategies with an emphasis on the need for a common process to simplify. Students analyze the order of operations and gain of understanding of the structure and purpose of each level. They use this understanding to generate and solve more complex problems (6-1.1).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Jay shot 4 arrows at the target. His total score was 45. Which of these scores is **not** a possible result of Jay's 4 shots? How do you know?

   a. 25 + (2 x 5) + 10
   b. 15 + (3 x 10)
   c. (2 x 15) + 10 + 5
   d. 25 + 5 + (2 x 10)



2. Matt and Tom had to solve the following expression prior to leaving class. Matt said the answer was 21 and Tom said the answer was 31. Which student solved the expression correctly? How do you know?

$$5 + 6 \times 3 - 2$$

3. Evaluate the following expression:

$$((2 \times 4) + 3) \times 6 \div 2$$

4. Solve each equation.

1. $x + 5 = 7$
2. $a - 15 = 5$
3. $8 = \frac{x}{5}$
4. $6x = 24$

2-3

Geometry: Planes and Transformational Geometry

This module addresses the following indicators:
6-4.1 Represent with ordered pairs of integers the location of points in a coordinate grid. (C2)
6-4.2 Apply strategies and procedures to find the coordinates of the missing vertex of a square, rectangle, or right triangle when given the coordinates of the polygon's other vertices. (C3)
6-4.3 Generalize the relationship between line symmetry and rotational symmetry for two-dimensional shapes. (B2)
6-4.4 Construct two-dimensional shapes with line or rotational symmetry. (B6)
6-4.5 Identify the transformation(s) used to move a polygon from one location to another in the coordinate plane. (A1)
6-4.6 Explain how transformations affect the location of the original polygon in the coordinate plane. (B2)

This module contains four lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

6-4.1 Represent with ordered pairs of integers the location of points in a coordinate grid. (C2)

- In the fourth grade, students represented with ordered pairs of whole numbers the location of points in the first quadrant of a coordinate grid (4-4.7).

6-4.2 Apply strategies and procedures to find the coordinates of the missing vertex of a square, rectangle, or right triangle when given the coordinates of the polygon's other vertices. (C3)

- In fourth grade, students analyzed the quadrilaterals squares, rectangles, trapezoids, rhombuses and parallelograms according to their properties (4-4.1). In fifth grade, students applied the relationship of quadrilaterals to make logical arguments about their properties (5-4.1).

6-4.3 Generalize the relationship between line symmetry and rotational symmetry for two-dimensional shapes. (B2)

- In fifth grade, students analyzed shapes to determine line symmetry and/or rotational symmetry (5-4.6).

6-4.4 Construct two-dimensional shapes with line or rotational symmetry. (B6)

- In fifth grade, students analyzed shapes to determine line symmetry and/or rotational symmetry (5-4.6).

6-4.5 Identify the transformation(s) used to move a polygon from one location to another in the coordinate plane. (A1)

- In fifth grade students predicted the results of multiple transformations on a geometric shape when combinations of translation, reflection and rotation are used (5-4.5).
- In seventh grade, students create tessellations using transformations (6-4.9).

6-4.6 Explain how transformations affect the location of the original polygon in the coordinate plane. (B2)

- In fifth grade students predicted the results of multiple transformations on a geometric shape when combinations of translation, reflection and rotation are used (5-4.5).

- In seventh grade, students create tessellations using transformations (6-4.9).

- **Key Concepts/Key Terms**
  Coordinate plane/ coordinate grid
  *Quadrant (I, II, III, IV)
  *Ordered pair
  *Coordinates
  *Origin
  *X-axis
  *Y-axis
  *Translation
  *Rotation
  *Reflection
  *Transformation
  Angle of rotation
  *Vertex (vertices)
  Rotational symmetry
  *Line of symmetry
  Line Symmetry
  Center of rotation
  Clockwise/counterclockwise
  *Integers
  Polygons
  *X-coordinate
  *Y- coordinate

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lesson(s)

1. Teaching Lesson A "Points on the Cartesian Plane"

6-4.1 For this indicator, it is **essential** for students to:

- Understanding the meaning and concept of integers.
- Understand that a coordinate grid/plane is made up of one horizontal line and one vertical line with the number lines intersecting where both are zero.
- Label the terms important to the coordinate plane; origin, x-and y-axis, and Quadrants counterclockwise.
- Plot points in a coordinate grid.
- Write the coordinates of the ordered pairs as x coordinate first, then y coordinate second.
- Write the coordinate given the graph

For this indicator, it is **not essential** for students to:

- Find the coordinates of a missing vertex of a square, rectangle, or right triangle when given the coordinates of the polygon's other vertices.

a. Indicators with Taxonomy

6-4.1 Represent with ordered pairs of integers the location of points in a coordinate grid. (C2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson – "Points on the Cartesian Plane"

Materials Needed:
Sentence strips (or other materials to make number lines)
Class set of coordinate grids

Introductory Lesson: "Points on the Cartesian Plane"

Review the 1st quadrant that has been used since 4th grade.

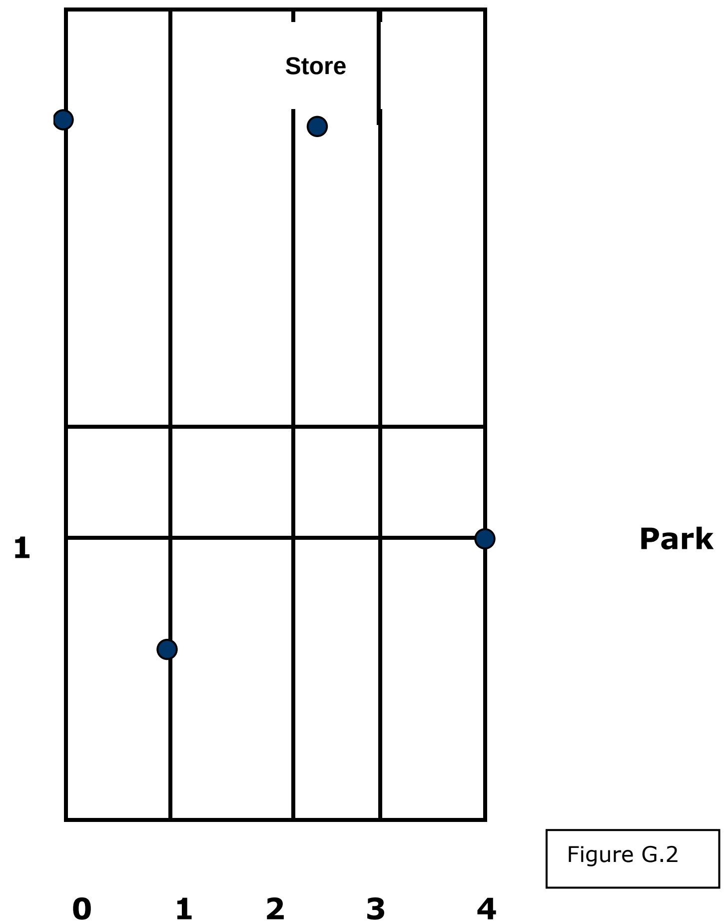


Figure G.2

Use a transparency of the above Figure G. 2 to quickly review ordered pairs by having students name and label an ordered pair for each of the dots/locations on the grid. Students should be able to use ordered pairs of numbers to represent the sites.

Point out to students that this coordinate grid is only the first quadrant of a Cartesian Plane (See Figure G.3). In coordinate geometry, this represents the first quadrant only (See *Figure G.3* below), and the numbers are always given in a certain way: first the horizontal distance or number (*X* axis), followed by the vertical distance or number (*Y* axis). A strategy for remembering this is that "you must go into the building (along the x-axis) before you can ride the elevator up (along the y-axis)". For example in *Figure G.2*, the school would be designated (1,0), the library would be (0,2), the store would be (2,2) and the park would be (4,1). As students' ideas about the number system expand to include negative numbers, they will be able to locate points in all four quadrants of the Cartesian Plane (See *Figure G.3*).

Cartesian Plane

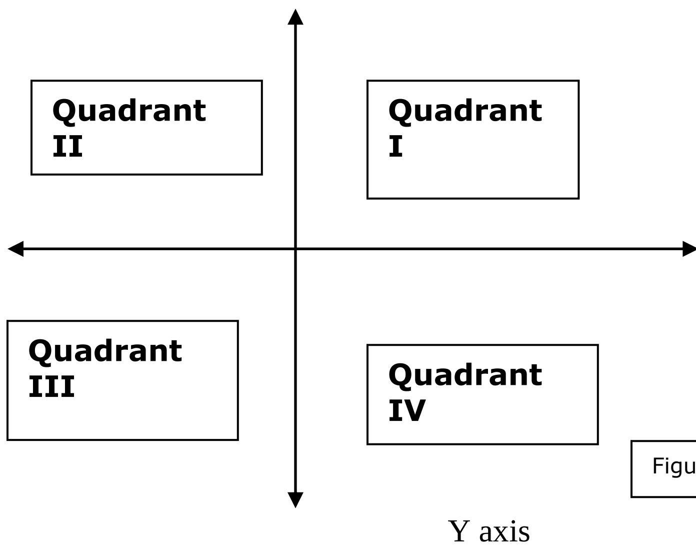


Figure G.3

Following instructions from the support document, have students build a coordinate grid by creating two number lines (-10 to 10) and placing one horizontally and one vertically with the number lines intersecting where both are zero. This allows students to see that the coordinate plane is made up of something they are already familiar with, a number line. Students should then label the terms important to the coordinate plane; *origin, x- and y- axis, Quadrants I, II, III,* and *IV*. Quadrants are labeled counterclockwise with Quadrant I being in the upper right section of the coordinate plane. Students can now be asked to go to (or point to) the location of a specific ordered pair and class discussions can begin to take place in regards to the signs common to numbers in each quadrant, etc.

To practice naming coordinates or points in all four quadrants, play "Advanced Hidden Treasure" from 5th grade Everyday Mathematics. Students will need games boards that have coordinate grids showing all four quadrants and a pencil, red pen, or crayon. (They can make their own by drawing two playing grids on graph paper or provide copies for them.) The object of the game is for each player to "hide" a point on a grid. Players try to find each other's hidden point.

Directions:

1. Each player writes on his or her own pair of playing grids. Players sit so they cannot see what the other is writing.
2. Each player secretly marks a point on Grid 1. These are the "hidden" points.
3. Player 1 guesses the location of Player 2's hidden point by naming an ordered pair. To name (4, -7), say "4 comma negative 7."
4. If Player 2's hidden point is at that location, player one wins.
5. If the hidden point is not at that location, Player 2 marks the guess in pencil on Grid 1. Player 2 counts the least number of "square sides" needed to travel from the hidden point to the guessed point and tells it to Player 1. Repeat steps 3-5 with Player 2 guessing and Player 1 answering.

You may want to play against the class or have two volunteers come up and play in front of the class to model the game. Make an overhead transparency of two grids (one labeled Grid 1: Hide your point here and the other labeled Grid 2: Guess other player's point here.) Play the game on the overhead so students can see how the grids are completed and how a player's guesses are answered.

In this game, each student has a coordinate grid. Players secretly put their initials on five intersections of their own grid. Players (keeping their grids hidden) take turns trying to "hit" the other player's targets by naming points using coordinates. The other player indicates whether the shot was a "hit" or "miss" (just as in Battleship). When a player scores a "hit", he or she receives another turn. Each player keeps track of where he or she has taken shots, recording an X for a hit and an O for a miss. Then game ends when one player has hit all of the other player's initials (targets).

Close the lesson by discussing strategies used to plot and find points on the grids. Ask students how someone could determine in which quadrant a point would be located using the ordered pair.

c. Misconceptions/Common Errors

- Students get confused with quadrants and that they are labeled counterclockwise.
- Another common error to watch for is simply following the ordered pairs when using a combination of negative and positive numbers.
- Students may think that all quadrants have the same coordinates.
- It is important for students to start at the origin because this helps them with direction.
- Students often confuse the ordered pairs (x,y), incorrectly graphing it as (y,x) instead.

d. Additional Instructional Strategies/Differentiation

- A review of plotting points in the first quadrant will be helpful. Then extend the horizontal axis and discuss how the other quadrants are a result of this extension. Then do the same thing for the y axis.
- When finding the coordinates of a point in a coordinate grid a strategy that would assist students in finding and writing the coordinate pair correctly would be to allow students to use a ruler. The students would line up the point with the x –axis to determine the x coordinate and then line up the point with the y-axis to determine the y coordinate. The student would then write the coordinates as an ordered pair with the x coordinate first, then the y coordinate second.

e. Technology-

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and the virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- "Finding Your Way Around" (Illuminations lesson) Objectives are to practice graphing ordered pairs, practice naming ordered pairs, and use the coordinate plane to create geometric figures. http://illuminations.nctm.org/LessonDetail.aspx?id=L280
- National Library of Virtual Manipulatives Geoboard Coordinate Grid http://nlvm.usu.edu/en/nav/frames_asid_303_g_3_t_3.html
- Coordinate Plane (First Glance, In-depth, Examples, Workout) http://www.math.com/school/subject2/lessons/S2U4L1GL.html
- Coordinate Plane Game http://www.math-play.com/Coordinate%20Plane%20Game/Coordinate%20Plane%20Game.html
- Interactive Coordinate Plane http://www.mathopenref.com/coordplane.html
- Coordinate Game http://www.oswego.org/ocsd-web/games/BillyBug2/bug2.html
- Interactive tool with SC Mathematics standards; click on the indicator you are addressing. http://www.shodor.org/interactivate/standards/organization/19/

f. Assessing the Lesson

Throughout the class activity and game look for the following in students…question them: do they label the axis appropriately? Can they locate the x- and y- coordinates? Do they plot points in more than one quadrant? Can they name points correctly? Can they name the quadrants correctly?

2. Teaching Lesson B "Finding missing coordinates"

6-4.2 For this indicator, it is **essential** for students to:

- Understand integers.
- Have an understanding of the signs common to numbers in each quadrant.
- Use ordered pairs containing integers.
- Use polygons that have been orientated horizontally.
- Locate points in a coordinate plane.
- Plot points in a coordinate plane.
- Find the coordinates of a missing vertex when a polygon has vertices in more than one quadrant.
- Find the coordinates of a missing vertex then the other coordinates are giving in word form (list) or on a graph
- Understand the characteristics of the square, rectangle and the right triangle.

For this indicator, it is **not essential** for students to:

- Find coordinates that contain fractional points.
- Predict the result of transformations.

a. Indicators with Taxonomy

6-4.2 Apply strategies and procedures to find the coordinates of the missing vertex of a square, rectangle, or right triangle when given the coordinates of the polygon's other vertices. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson - "Finding missing coordinates"

Materials Needed:
Class set of coordinate grids
Overhead coordinate grid
Class set of transparencies

Wet or dry erase markers

Introductory Lesson: "Finding Missing Coordinates"

Make a class set of blank coordinate grids and paper clip a blank transparency to each one. Each student will also need a wet or dry erase marker to use to write on the transparency. You will also need to make a transparency of a coordinate graph for the overhead projector. Introduce the coordinate graph by discussing the vertices of different polygons (square, rectangle, or right triangle) on the coordinate graph. Review of the properties of these polygons may be necessary. (Rectangles have four 90 degree angles, squares have four 90 degree angles and four equal sides, and right triangles have three sides and one 90 degree angle.) Focusing on one polygon at a time, give students the coordinates for all the vertices except one and ask the students to identify the missing vertex. You can extend the activity by asking students to plot the vertices of a congruent polygon that you have already plotted. Ask students to describe how they can prove that the new shape is congruent. (Use simple shapes like rectangles and squares.) Have students plot the points for similar polygons. You can also have students apply transformations to figures, plot the new vertices, and describe how they changed.

NOTE: Use whole number integers and keep polygons horizontal. Also, provide example situations where polygons cross quadrants.

Close the lesson by discussing the procedures used for finding the coordinates for the unknown point.

c. Misconceptions/Common Errors

- Students may think that coordinate points in different quadrants have the same integer value.

d. Additional Instructional Strategies/Differentiation

- Using graphing paper or coordinate grid white boards, have students draw any square, rectangle, and right triangle. Then have the students identify the coordinates of each.
- Give student groups 3 of the 4 points to make a variety of squares and rectangles. Have the students plot the points on graph paper or coordinate grid white boards. Have students locate the fourth point by drawing a line from to point. Do the same for a right triangle except only give the students 2 of the 3 points.
- Online lesson plan "Quadrilaterals in the Coordinate Plane" where students plot part of the polygon and find the missing vertices to

finish. (may need to copy and paste link) http://mdk12.org/instruction/curriculum/pdfs/clg2activity008.pdf

- Online lesson: Use Patterns to Find Missing Points (only uses 1st quadrant…could be adapted for others as well) http://www.sw-georgia.resa.k12.ga.us/patterns.pdf

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and the virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- National Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/category_g_3_t_3.html
- Interactive tool with SC Mathematics standards; click on the indicator you are addressing. http://www.shodor.org/interactivate/standards/organization/19/

f. Assessing the Lesson

As students work to find the missing coordinates of polygons, ask them to explain their thinking and their procedure for doing so.

3. Teaching Lesson C "Line and rotational symmetry"

6-4.3 For this indicator, it is **essential** for students to:

- Identify shapes that have line symmetry.
- Identify shapes that have rotational symmetry.
- Identify counter-examples (shapes with no line of symmetry)
- Understand that all regular polygons have rotational symmetry.
- A shape that rotates onto itself before turning 360° has rotational symmetry.
- Identify shapes that have both types of symmetry

6-4.4 For this indicator, it is **essential** for students to:

- Understand line symmetry.
- Understand rotational symmetry.
- Understand the properties of regular polygons.
- Be able to explain the relationships they may find among two-dimensional shapes that have both line and rotational symmetry.

- Construct shapes with both types of symmetry

For this indicator, it is **not essential** for students to:

- Identify the type of transformation used to create the shape.

a. Indicators with Taxonomy

6-4.3 Generalize the relationship between line symmetry and rotational symmetry for two-dimensional shapes. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

6-4.4 Construct two-dimensional shapes with line or rotational symmetry. → *B6*

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson - "Line and rotational symmetry"

Materials Needed:
Die-cut letters or copies of printed letters
Coordinate graphs
Copies of several regular polygons

Introductory Lesson: "Line and Rotational Symmetry"

line symmetry: another name is "reflection symmetry"- ½ is the reflection of the other ½; the line of symmetry is the imaginary line where you could fold the image and have both halves match exactly

rotational symmetry: the shape or image can be rotated less than 360^0 and it still looks the same

Have students identify the capital letters of the alphabet that have line symmetry, rotational symmetry, or both***. Note: There are three letters that have rotational symmetry and not line symmetry. (N, S, and Z) There are twelve letters that have line symmetry and not rotational symmetry. (A, B, C, E, K, M, T, U, V, W, and Y) There are four letters that have both rotational and line symmetry. (H, I, O, and X) There are seven letters that do not have rotational symmetry nor line symmetry. (F, G, J, L, P, Q, and R)*** Help students make conjectures about symmetry by asking, "What is the relationship between the letters that have both rotational and line symmetry and the lines of symmetry the letter contains?"

Note: It turns out that any letter with two different lines of symmetry will also have rotational symmetry.

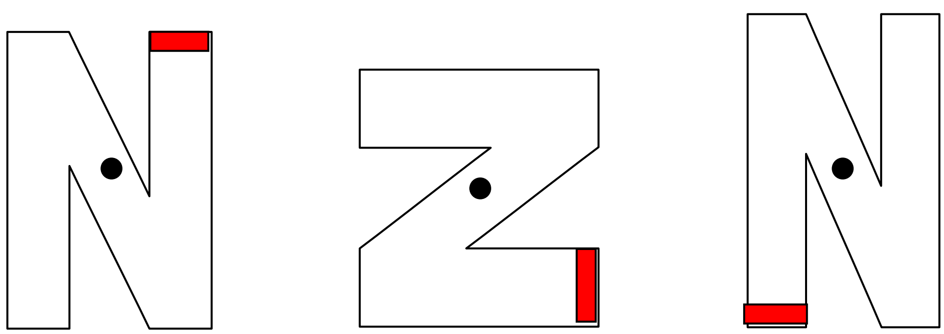
Have students identify all the lines of symmetry that regular polygons contain. Students can prove the lines of symmetry by folding the figure and making two congruent parts. Ask probing questions that will lead the students to explore symmetry and make connections to the following: The number of sides in a regular polygon and the lines of symmetry are the same. Draw regular polygons with dotted lines showing lines of symmetry.

Using geoboards, have students construct two-dimensional shapes that have line symmetry, rotational symmetry, or both. Also, have students fold a piece of paper in ½ and cut out a shape. This shape will at least have one line of symmetry. Have them identify any others in their shape and determine whether or not their shape has rotational symmetry as well. In closing, ask students what relationships they may find among two-dimensional shapes that have both line and rotational symmetry.

Examples:

LETTERS

"N" has rotational symmetry because it locks in before rotating 360^0

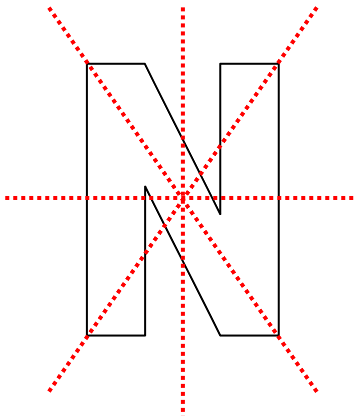


Starting position

Rotate clockwise 90^0

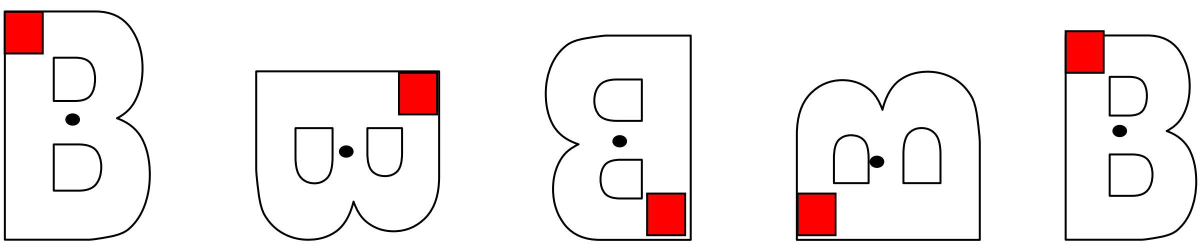
Rotate clockwise another 90^0 for a total rotation of 180^0 and the shape "locks in place."

"N" does not have line symmetry. It can't be "cut" or "folded" so that one half is a reflection of the other.



Having die cut letters will make it easier to "prove" symmetry or lack of symmetry. It isn't always easy to see it in a drawing.

"B" does NOT have rotational symmetry. It never locks in place until it rotates a complete 360^0 turn.



Starting position

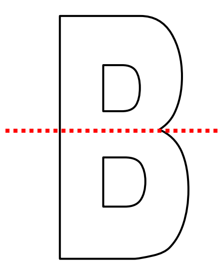
rotate clockwise 90^0

rotate clockwise another $90^0 \rightarrow 180^0$

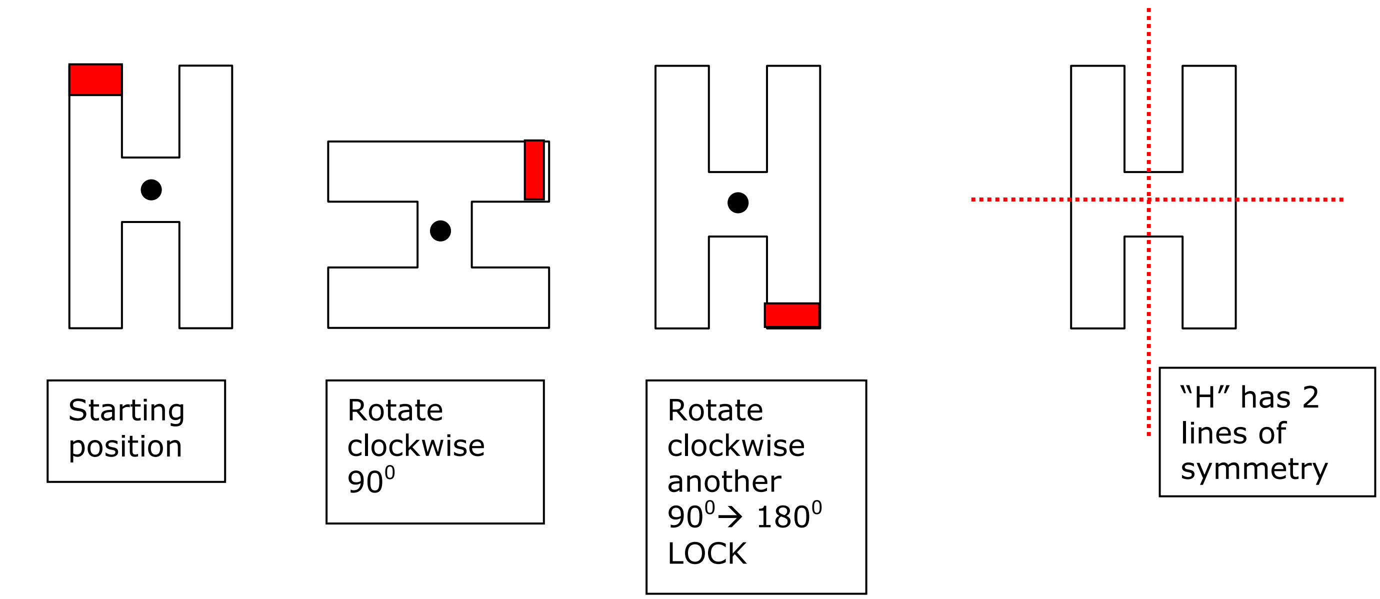
rotate clockwise another $90^0 \rightarrow 270^0$

rotate clockwise another $90^0 \rightarrow 360^0$

"B has line symmetry because it can be "cut" or "folded" so that one half is a reflection of the other. It has only one line of symmetry.

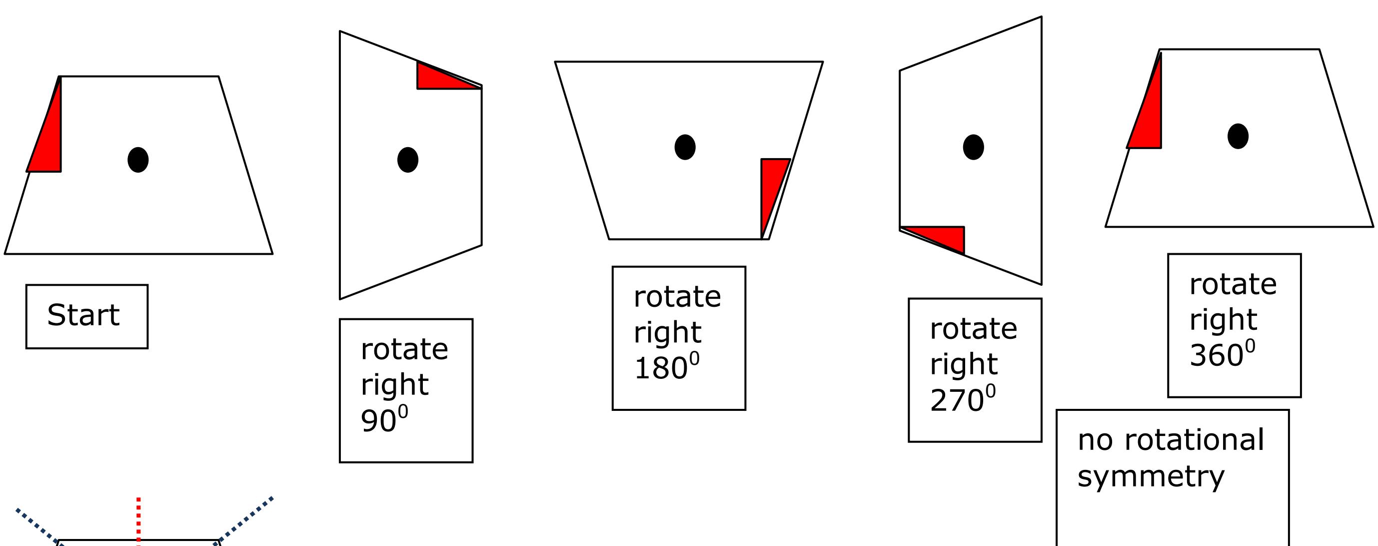


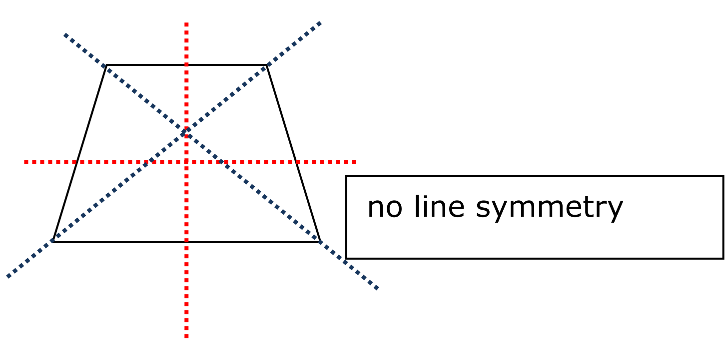
"H" has both rotational and line symmetry.



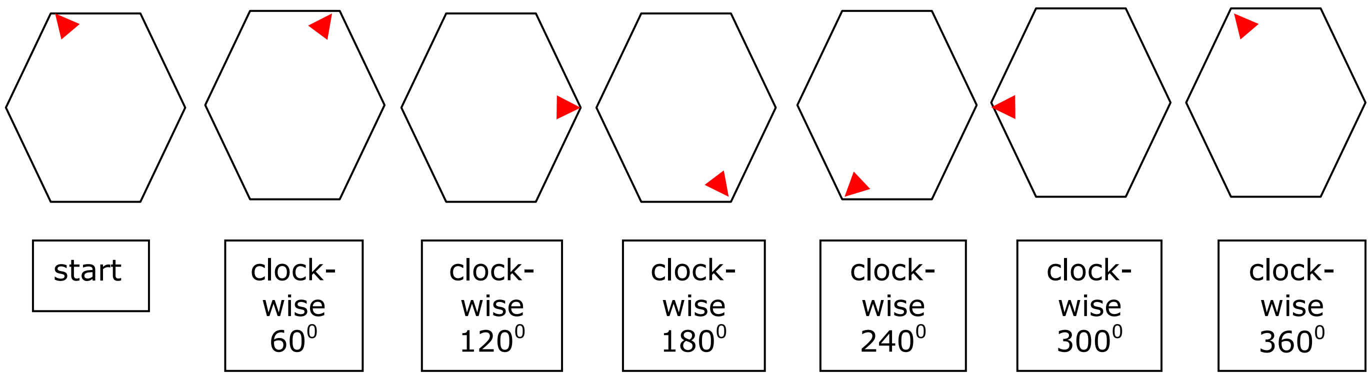
SHAPES

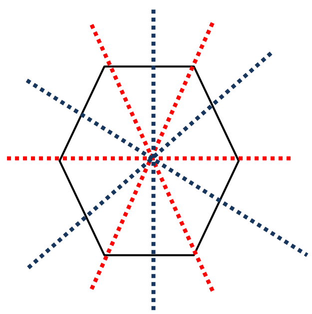
This trapezoid has neither rotational nor line symmetry.





This regular hexagon has multiple rotational symmetry and multiple lines of symmetry.





3 lines of symmetry through the vertices

3 lines of symmetry through the sides

c. Misconceptions/Common Errors

- "Students often create figures with rotational symmetry, but often have difficulty describing the regularity they see. They should be using language about turns and angles to describe these figures." *(Principles and Standards for School Mathematics, 167-168)*

- Students sometimes use the "eyeball" method to determine symmetry. Although this is a valid strategy for making predictions, they need experiences with materials they can touch, fold, and rotate to check for line symmetry and rotational symmetry.

d. Additional Instructional Strategies/Differentiation

- A strategy to review line symmetry is to use mirrors. When you place a mirror on a picture or design so that the mirror is perpendicular to the table, you see a shape with symmetry when you look in the mirror. Help students see that if a shape can be folded on a line so that the two halves match, then it is said to have line symmetry. Notice that the fold line is actually a line of reflection—the portion of the shape on one side of the line is reflected onto the other side. That is the connection between line symmetry and transformations. Another strategy can be done with Geoboards. First, stretch a band down the center or from corner to corner. Make a design on one side of the line and its mirror image on the other. Check with a mirror.

- Another strategy for reviewing symmetry is to have students identify all the lines of symmetry that regular polygons contain. Students can prove the lines of symmetry by folding the figure and making two congruent parts.

- Using Geoboards have students construct two-dimensional shapes that have line symmetry, rotational symmetry, or both. Also, have students fold a piece of paper in ½ and cut out a shape. This shape will at least have one line of symmetry. Have them identify any others in their shape and determine whether or not their shape has rotational symmetry as well. Ask students what relationships they may find among two-dimensional shapes that have both line and rotational symmetry.

- Online lesson for both rotational and line symmetry
http://74.125.45.132/search?q=cache:OAp54BGFGy0J:academic.evergreen.edu/projects/wallpainting/New%2520curriculum/4-7%2520All/4-7%2520Math/4-7%2520Math%2520Lesson%2520Rotational.doc+line+and+rotational+symmetry&hl=en&ct=clnk&cd=21&gl=us

  - Use the clock to reinforce the concept of a rotation. Tell students to start at 12 and rotate clock wise (move the clock hand) to 3 for 90 degrees, to 6 for 180 degrees, to 9 for 270 degrees, and back to 12 for 360 degrees. Have students practice rotations with pattern blocks. Make sure they identify a point of rotation.

  - Have students fold a plain piece of paper "hotdog" style and write their name in block letters(concave polygons) making sure the letters touch the fold. Also have students connect the letters. Let students trace their names with a black marker and turn paper over and trace again. This creates a reflection (line symmetry). Allow students to color and decorate their name reflections. Use MIRA to show reflections. Resource: Elementary and Middle School Mathematics (6th Edition) by John A. Van DeWalle, Activity 21.17, p. 432 (Pattern Block Mirror Symmetry)

  - Have students cut snowflakes and determine lines and points of symmetry.

  - Introduce the idea of symmetry using flags that represent countries from around the world. Talk about the patterns in the arrangement of colors within each individual flag, identify geometric shapes, and compare their sizes. Have students create their own flags. Each student's flag must contain at least 2 types of symmetry. On the back of the flag, the students must write a description of the types symmetry used and what the colors and shapes represent for that particular student. Students should also write about what they learned from this assignment.

e. Technology-

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- National Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/category_g_3_t_3.html
- "Finding Lines of Symmetry" (Illuminations lesson with virtual geoboard) http://illuminations.nctm.org/LessonDetail.aspx?ID=L556
- Webquest exploring both line and rotational symmetry http://www.adrianbruce.com/Symmetry/
- Two-Dimension Rotational Symmetry video http://www.mathexpression.com/rotational-symmetry.html#GoToVideo (practice questions: http://www.mathexpression.com/2d-rotational-symmetry.html)
- Symmetry and patterns in rugs http://mathforum.org/geometry/rugs/symmetry/
- Interactive tool with SC Mathematics standards; click on the indicator you are addressing. http://www.shodor.org/interactivate/standards/organization/19/

f. Assessing the Lesson

Use probing questions throughout the lesson to check for understanding of line and rotational symmetry. Students should be able to verbally describe both and the relationship of the two.

4. Teaching Lesson D "Transformations of polygons in a plane"

6-4.5 Identify the transformation(s) used to move a polygon from one location to another in the coordinate plane. (A1)

For this indicator, it is **essential** for students to:

- Recall the meaning of reflection, rotation and translation
- Visualize what a reflection, rotation and translation look like
- Understand that the polygon should remain the same size and shape after the transformation

For this indicator, it is **not essential** for students to:

- Rename the vertices of the polygon.
- Rename the coordinates based on the ordered pair without a given picture.

6-4.6 Explain how transformations affect the location of the original polygon in the coordinate plane. (B2)

For this indicator, it is **essential** for students to:

- To name the new coordinates that result from a translation. Examples: know that a movement to the right or left affects the x-coordinate and a movement up or down affects the y-coordinate. Right adds to the x-coordinate, left subtracts from the x-coordinate. Up adds to the y-coordinate and down subtracts from the y-coordinate.
- To name the new coordinates that result from a reflection. A reflection over the x-axis results in the y-coordinates becoming the opposite while the x-coordinate remains the same. A reflection over the y-axis results in the x-coordinate becoming the opposite while the y-coordinate remains the same.
- To identify the amount of degrees (90^0, 180^0, 270^0, and 360^0) a polygon rotated around a central point. Students should know that the angle is formed by the line segments that are connected to the point of rotation.
- Name the new coordinates for a reflection and a translation of the vertices of a polygon when given the ordered pairs or from a picture.
- Use appropriate terminology when explaining the effects.

For this indicator, it is **not essential** for students to:

- To name the new coordinates of a rotation from only ordered pairs

a. Indicators with Taxonomy

6-4.5 Identify the transformation(s) used to move a polygon from one location to another in the coordinate plane. (A1)

Cognitive Process Dimension: Remember
Knowledge Dimension: Factual Knowledge

6-4.6 Explain how transformations affect the location of the original polygon in the coordinate plane. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson - "Transformations of polygons in a plane"

Materials Needed:

- Die-cut letters or copies of printed letters
- Coordinate graphs
- Copies of several regular polygons
- Miras (A piece of Plexiglas that stands perpendicular to the paper and functions like a see through mirror allowing you to trace a reflection.)

- Patty Paper (Squares of paper with wax on one side that are normally put between uncooked hamburger patties.)

Introductory Lesson: "Transformations of Polygons in a Plane"

This activity is a continuation from the introductory lesson on symmetry. Using the die-cut letters or copies, have students move the letters from one place to another using reflection, translation, or rotation and describe the movement. You may need to review these as flip, slide, turn.

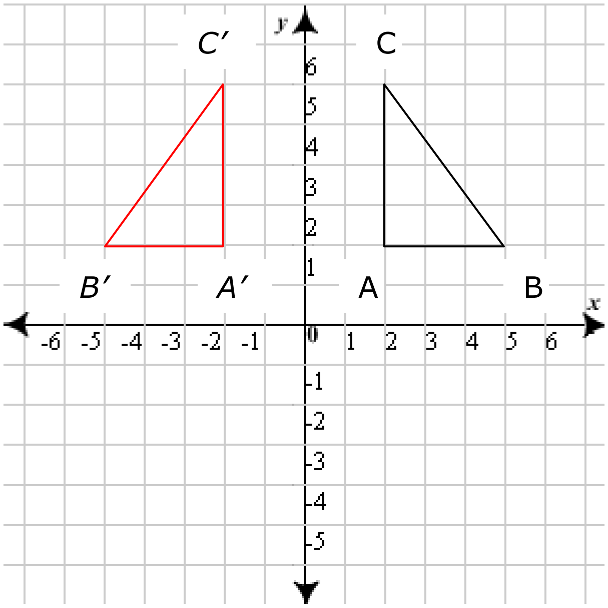
When students describe the rotation of a figure, they give the direction, the angle of rotation, and the center of rotation. It is important for 6th grade students to use correct terminology (translation, reflection, rotation) to describe the change made to the figure or polygon.

Miras can be used to draw reflections. Patty paper can be used to help students explore the change a figure makes when rotated. Students can trace the figure, rotate the paper, and compare the original figure with the rotated figure. When students trace the figure and rotate it, they will be able to see the original figure through the paper in order to describe the effects of the rotation. Patty paper can also be used to explore the effects of reflection and translation.

For transformations, have students place the polygons in the first quadrant and record the points of each vertex. Then, have students move the polygons from one place to another in the first quadrant using reflection, translation, or rotation and describe the new location of the polygon. Students need to explain how transformations affect the location of the original polygon.

Examples:

reflection



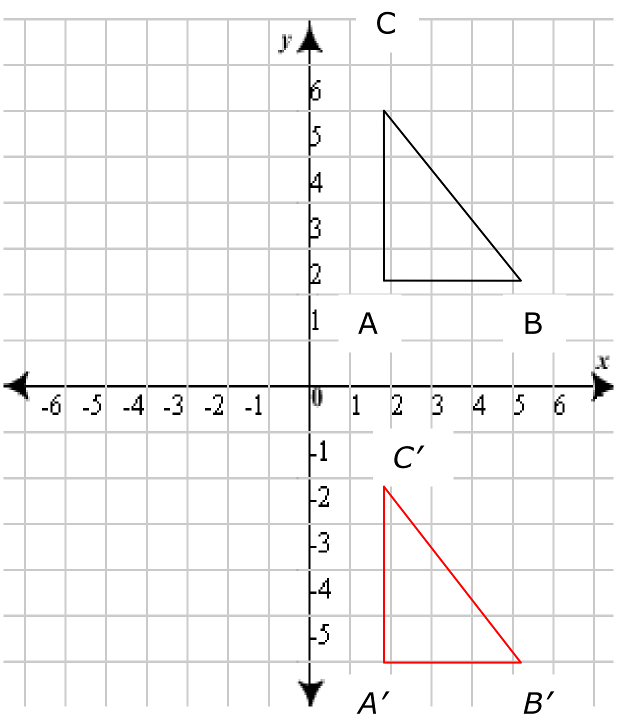
Δ ABC: A (2, 2), B (5, 2), C (2, 6)

If you reflect Δ ABC over the y-axis:

- The coordinates for *Δ A′B′C′* are *A′ (- 2, 2), B′ (- 5 , 2), C′ (- 2, 6)*
- ΔABC is located in Quadrant I and *Δ A′B′C′* is located in Quadrant II.
- The x-coordinates in *Δ A′B′C′* are the opposites of those in the Δ ABC.
- The y-coordinates are the same for both triangles.
- The size and shape of the triangle are NOT affected by the reflection.

NOTE: A shape can be reflected over any line, not just an axis. A shape may also be reflected over a point, such as the origin.

translation



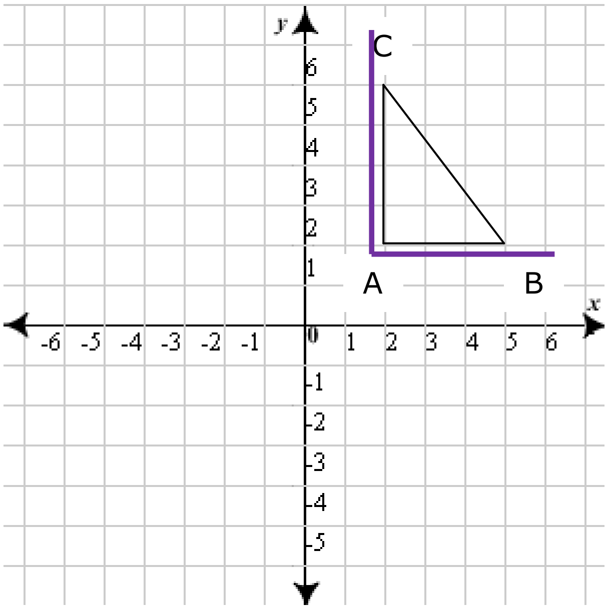
Δ ABC: A (2, 2), B (5, 2), C (2, 6)

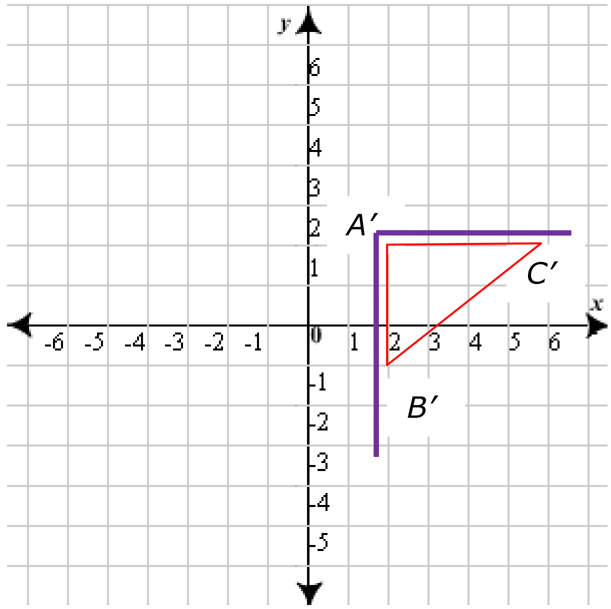
If you translate ΔABC down 8 units:

- The coordinates for *Δ A′B′C′* are *A′ (2, -6), B′ (5 , - 6), C′ (2, - 1)*
- Δ ABC is located in Quadrant I and *Δ A′B′C′* is located in Quadrant IV.
- The x-coordinates in Δ ABC are the same as the x-coordinates in *Δ A′B′C′.*
- The y-coordinates in *Δ A′B′C′* are 8 units below the y-coordinates in Δ ABC.
- The size and shape of the triangle are NOT affected by the reflection.

rotations

A figure may be rotated using a vertex as the point of rotation. In this example, vertex A is used.





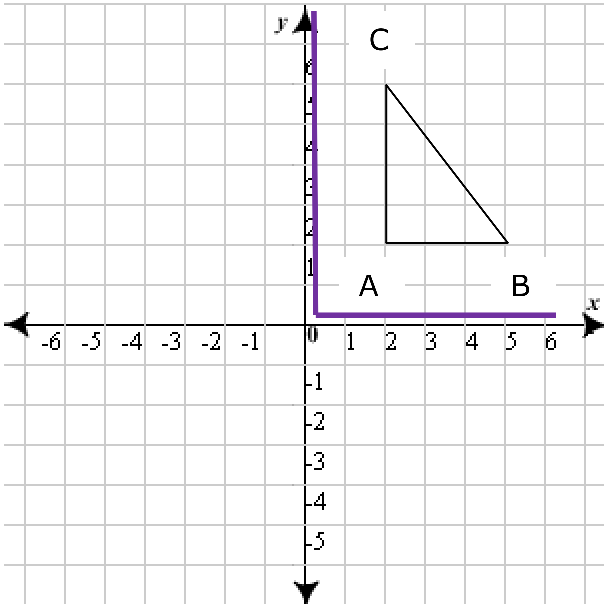
ΔABC: A (2, 2), B (5, 2), C (2, 6)

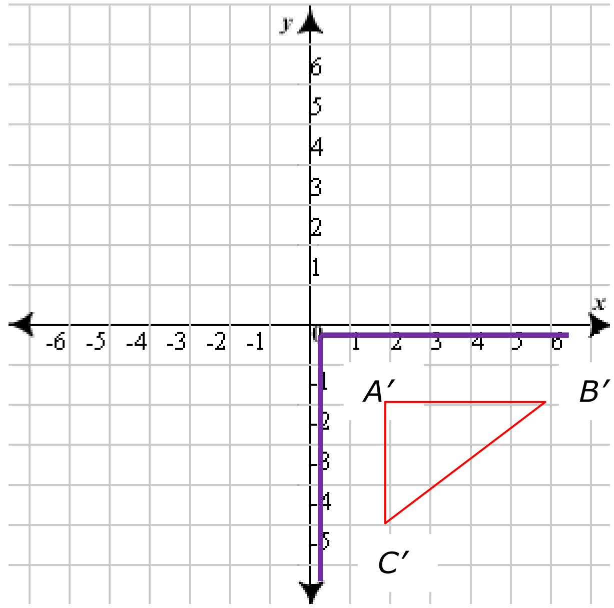
Rotate ΔABC 90^0 clockwise around point A. Think about tilting the purple "L" so that it rotates 90^0, taking the triangle with it. Point A stays the same, while points B and C relocate.

- This is a good place to use the patty paper or a cut-out shape that can be physically rotated.
- If you know the location of one point, you can count units to find the locations of the others. The size and shape don't change, so the line segments that make up the sides are the same number of units, and the angles formed by the sides are the same.

ΔA′B′C′: A′ (2, 2), B′ (2, -1), C′ (6, 2)

A figure may be rotated using a point outside the shape as the point of rotation. In this example, the origin is used.





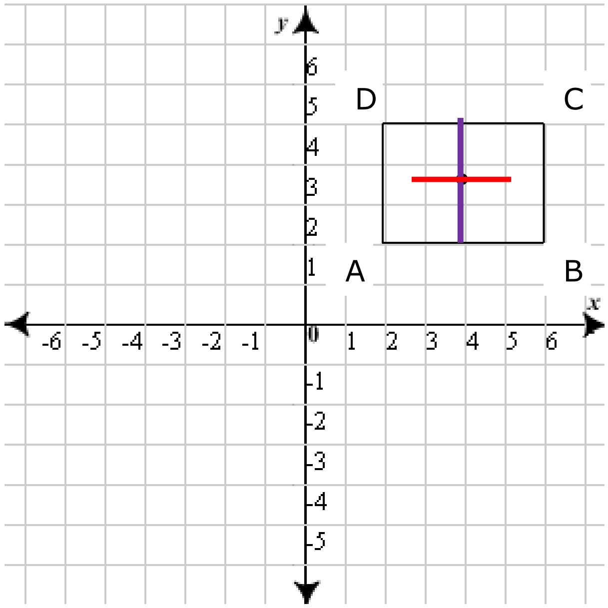
ΔABC: A (2, 2), B (5, 2), C (2, 6)

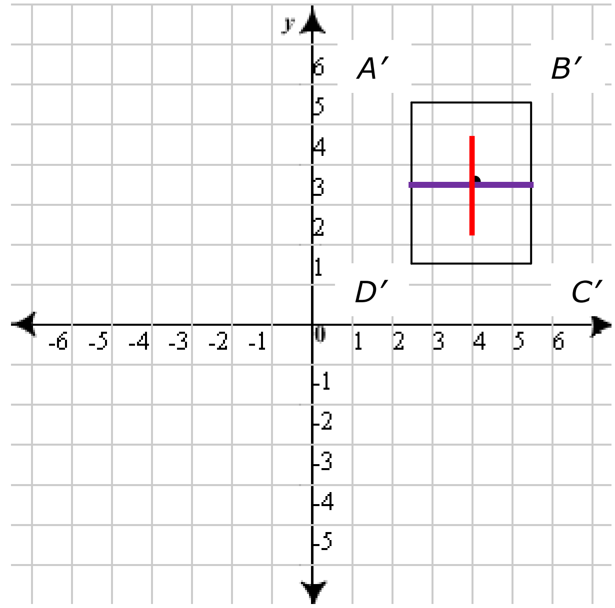
Rotate ΔABC 90^0 clockwise around the origin. It's the same idea as before, but around a different point. Again, think about tilting the purple "L" so that it rotates 90^0, taking the triangle with it.

- This is a good place to use the patty paper or a cut-out shape that can be physically rotated.
- Find point A first. Then count to find points B and C.

ΔA′B′C′: A′ (2, -2), B′ (6, -2), C′ (2, - 5)

A figure may be rotated using a point inside the shape as the point of rotation.





The pictures don't always look right when drawn on the computer. Do this with paper, pencil, and a cut-out for yourself.

□ABCD: A (2, 2), B (6, 2), C (6, 5), D (2, 5)

Find the center point by drawing the diagonals and marking the intersection. Then put in the "plus sign" as a reference. It's two-colored so that it the rotation will be noticeable.

Think about turning the "plus sign" 90^0. As the plus sign rotates, the rectangle moves with it. The vertices may not "land" exactly on intersections.

- This is a good place to use the patty paper or a cut-out shape that can be physically rotated.
- Find point A first. Then place points B, C, and D.

□A′B′C′: A′ (2½, 5½), B′ (5½, 5½), C′ (5½, 1½), D′ (2½, 1½)

c. *Misconceptions/Common Errors*

- Students often use the terms translation and transformation interchangeably.
- Students may think that the point of rotation has to be on the figure but it doesn't have to be.

- Students find it hard to figure out the degree of rotation and often cannot identify the point of rotation.

d. *Additional Instructional Strategies/Differentiation*

- Archimedes' Puzzle (Illuminations lesson) Explores symmetry and transformations. http://illuminations.nctm.org/LessonDetail.aspx?id=L720
- Paper Quilts (Illuminations lesson) Explores translations, reflections, rotations, and line symmetry. http://illuminations.nctm.org/LessonDetail.aspx?id=U104
- Using the coordinate system, students will draw three polygons, labeling their vertices and listing the ordered pairs that identify them. Students will decide which transformation to apply to each polygon and describe the movement that occurs for each one, labeling new vertices, and ordered pairs.
- Have students manipulate pairs of pattern blocks to show each of the three transformations on a coordinate plane or grid board.
- Have students create a quilt square from a 6 in. by 6 in. square piece of paper. They will design the square to include parallel/perpendicular lines, similar/congruent figures, and an example of a reflection, rotation, and a translation. Once the square is complete, have students complete a written explanation of their work using correct math terminology.

e. ***Technology-***

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and the virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- *Tesselmania* and *Geometers Sketchpad* are great software programs to use when having students explore transformations and symmetry.
- National Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/category_g_3_t_3.html
- Interactive Transformations (Choose a specific type or combinations) http://www.mathsnet.net/transform/
- Alphabet Geometry with mini-movies http://www.misterteacher.com/abc.html
- Interactive tool with SC Mathematics standards; click on the indicator you are addressing. http://www.shodor.org/interactivate/standards/organization/19/

f. Assessing the Lesson

Use probing questions throughout the lesson as students are manipulating their materials through a series of transformations. Look for correct vocabulary usage. Also, as an exit slip, have students explain how transformations affect the location of the original polygon in the coordinate plane.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

6-4.1 Represent with ordered pairs of integers the location of points in a coordinate grid. (C2)

The objective of this indicator is to represent which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To represent means to change from one form to another; therefore, students gain an understanding of coordinates by translating them from numerical form (coordinate) to graphical form (grid). The learning progression to **represent** requires students to understand the meaning of integers. They recall and understand the structure of the coordinate plane (grid). Students use correct and clearly written or spoken words to communicate the meaning 6-1.6) of a coordinate by identify the values of x and y. They use a strategy to plot points and explain their strategy to their classmates and teachers. Students also analyze a graph to determine the coordinates. They use their understanding of quadrants to justify why a coordinate as certain signs. They evaluate their explanations and pose follow-up questions to prove or disprove their answers (6-1.2). Students then engage in repeated practice to support retention.

6-4.2 Apply strategies and procedures to find the coordinates of the missing vertex of a square, rectangle, or right triangle when given the coordinates of the polygon's other vertices. (C3)

The objective of this indicator is to apply, which is in the "apply procedural" knowledge cell of the Revised Taxonomy table. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus of the indicator is to apply, the learning progressions should include strategies that integrate conceptual and procedural knowledge. The learning progression to **apply** requires students to understand integers and the properties of the squares, rectangles and the right triangles. Students generalize mathematical statements related to the relationship between and among coordinates (6-1.5) such as the x values for two coordinates on the same vertical side are the same but the y values are different. As students explore a variety of examples, they use inductive and deductive reasoning to formulate conjectures (6-1.3) and evaluate these conjectures by posing follow-up questions to prove or disprove their them (6-1.2). Students use their understanding of these relationships to generate and solve complex problems. They use correct and clearly written or spoken notation to communicate their answers.

6-4.3 Generalize the relationship between line symmetry and rotational symmetry for two-dimensional shapes. (B2)

The objective of this indicator is to generalize which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge of rotational and line symmetry should be explored using a variety of examples. The learning progression to **generalize** requires students to recall and understand the meaning of line symmetry and rotational symmetry. Students experiment with rotating concrete models and generate descriptions and mathematical statements about their observations. Students use inductive and deductive reasoning to formulate mathematical arguments about the relationship between the two types of symmetry. They explain and justify their answers using correct and clearly written or spoken words to communicate their understanding of this relationship (6-1.6).

6-4.4 Construct two-dimensional shapes with line or rotational symmetry. (B6)

The objective of this indicator is to construct, which is in the "create conceptual" knowledge cell of the Revised Taxonomy table. To construct means to put elements together to form a coherent or functional whole; therefore, students show their conceptual knowledge of line and rotational symmetry by creating shapes. The learning progression to **construct** requires students to recall and understand characteristics of two dimensional shapes. Students analyze two-dimensional shapes to identify pattern relationships between shapes that have line or rotational symmetry. They use inductive and deductive reasoning to formulate mathematical arguments explaining the similarities and differences between two- dimensional shapes with line or rotational symmetry (6-1.3). They use this understanding to construct shapes and use correct and clearly written or spoken words and notations to explain how they constructed their shapes (6-1.6).

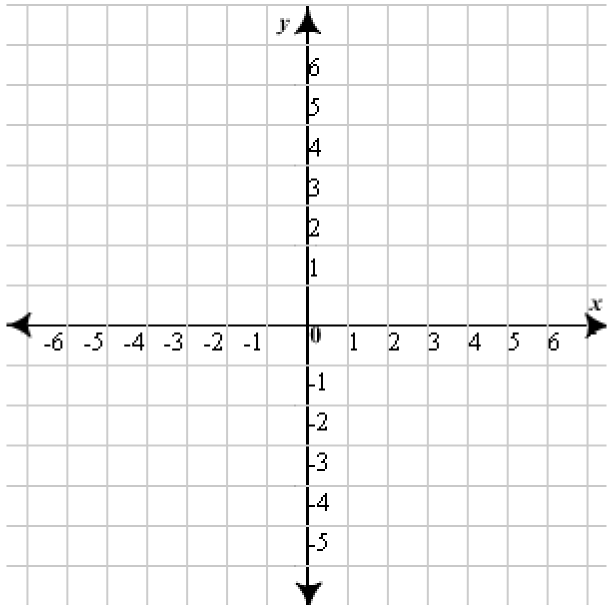
6-4.5 Identify the transformation(s) used to move a polygon from one location to another in the coordinate plane. (A1)

The objective of this indicator is to identify which is in the "remember factual" knowledge cell of the Revised Taxonomy. To identify is to locate knowledge in long term memory. Although the focus of the indicator is to remember, hands-on activities build conceptual knowledge and support retention. The learning progression to **identify** requires students to recall the meaning of transformation, translation, rotation and reflection. They explore these transformations using concrete models, pictorial models and real world examples to generalize mathematical statements (6-1.5) about the relationships between transformed shapes. Students use these relationships to identify transformations when given two polygons. They explain and justify their answers using correct and clearly written or spoken words (6-1.6).

6-4.6 Explain how transformations affect the location of the original polygon in the coordinate plane. (B2)

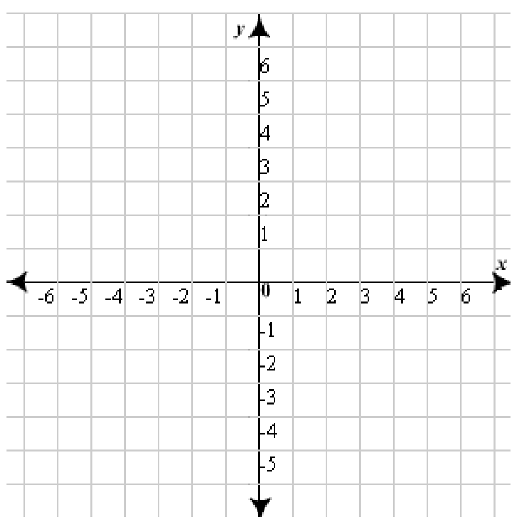
The objective of this indicator is explain which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To explain is to construct a caluse and effect models; therefore, as students explain they use the structure "the transformation ______ because _____." The learning progression to **explain** requires students to recall and understand the meaning of transformation. They *recognize* the relationships among rotations, reflections, and translations. Students explore and generate examples of transformation and generalize connections (6-1.7) of real world situations where transformations are needed. Using their understanding, students *evaluate* their explanations of the effect of transformation by posing questions to prove or disprove their reasoning (6-1.2). They use correct and clearly written or spoken words and notation to communicate their reasoning (6-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

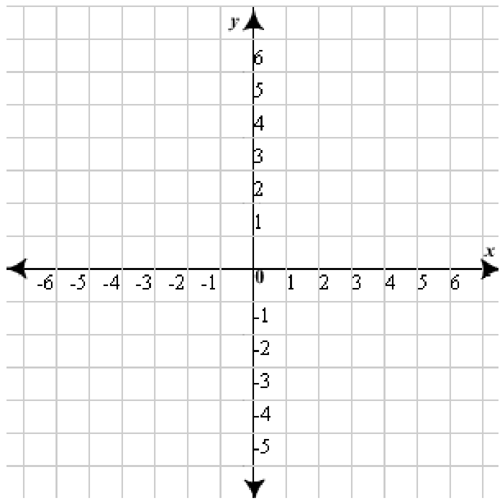


- Plot the following points on the coordinate grid above. Also, label each quadrant on the grid.
    - A (-5, -2)
    - B (5, -2)
    - C (4, -3)
    - D (0, 6)
    - E (0, -6)
    - F (-1, 0)
    - G (-6, -5)

2. Given the ordered pairs (-1, 2), (5, 2), and (5, 5), what might be the 4th ordered pair of the rectangle? Graph the figure.

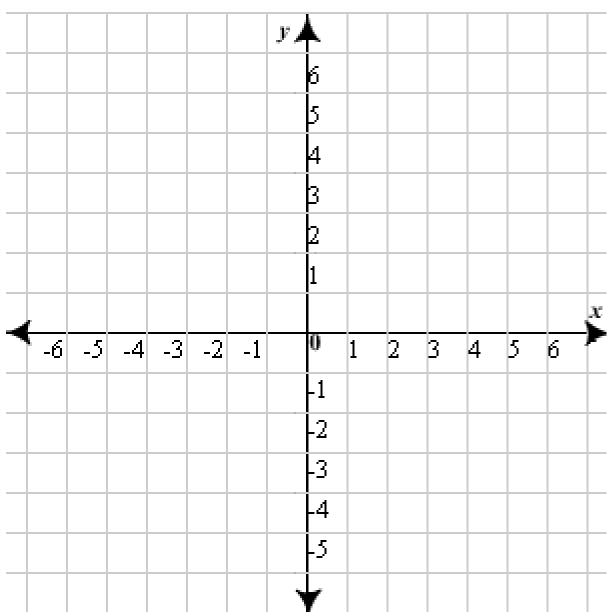


3. Given the ordered pairs (0, 0) and (4, 4) what could be another ordered pair that would form a right triangle? Graph the figure.

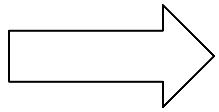


4. Given the square formed by the ordered pairs (2, 2), (2, 2), (-2, -2), and (2, -2). How many ordered pairs would have to be changed to form another rectangle that is not a square?

What is one possible solution? Sketch the graph.



5. Does this figure have line symmetry? Does it have rotational symmetry? Explain you answer.



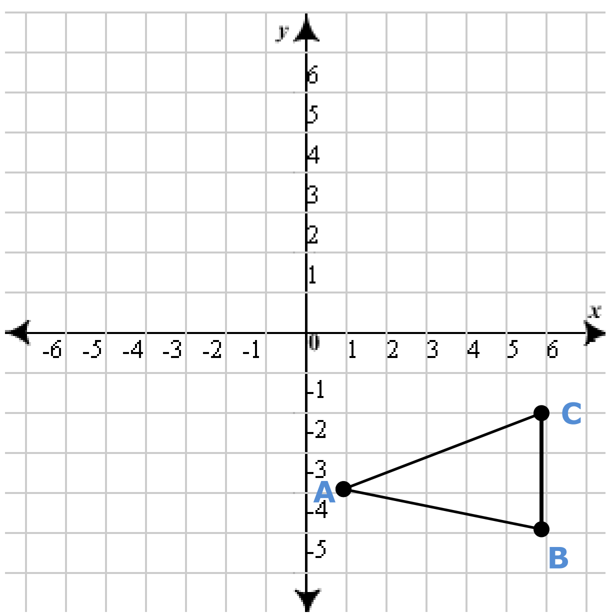
6. Construct a two-dimensional shape that has both line and rotational symmetry.

7. Construct a two-dimensional shape with line symmetry and no rotational symmetry.

8. Construct a two-dimensional figure with rotational symmetry and no line symmetry

Use the graph to answer questions 9 – 13.

The coordinates of the vertices of ΔABC are A (1, -4), B(7, -5), C(7, -2)



9. Identify the transformation, if the coordinates of ΔA′B′C′ are A′(-1, -4), B′(-6, -5), C′(-6, -2).

   A. Reflection
   B. Rotation
   C. Translation
   D. Dilation

10. In what ways did the transformation in problem #9, affect the location of ΔABC?

11. Identify the transformation of the coordinates if the vertices of A′B′C′ are A′(1, 1), B′(6, 0), C′(6, 3).
    A. Reflection
    B. Rotation
    C. Translation
    D. Dilation

12. In what ways did the transformation in problem # 11 affect the location of ΔABC?

13. If ΔABC is rotated 90^{o} counter clockwise with point A as the center of rotation, what is the affect of the transformation on point A?