SOUTH CAROLINA SUPPORT SYSTEM INSTRUCTIONAL GUIDE

Content Area	**Sixth Grade Math**

Fourth Nine Weeks

Standards/Indicators Addressed:

Standard 6-2 The student will demonstrate through the mathematical processes an understanding of the concepts of whole-number percentages, integers, and ratio and rate; the addition and subtraction of fractions; accurate, efficient, and generalizable methods of multiplying and dividing fractions and decimals; and the use of exponential notation to represent whole numbers.

6-2.6* Understand the relationship between ratio/rate and multiplication/division. (B2)
(revisited here, introduced in 1st nine weeks)

Standard: 6-5 The student will demonstrate through the mathematical processes an understanding of surface area; the perimeter and area of irregular shapes; the relationships among the circumference, diameter, and radius of a circle; the use of proportions to determine unit rates; and the use of scale to determine distance.

6-5.6* Use proportions to determine unit rates. (C3) ***(revisited here, introduced in 1st nine weeks)***
6-5.7* Use a scale to determine distance. (C3) ***(revisited here, introduced in 1st nine weeks) Connects to 2.6***

Standard 6-6 The student will demonstrate through the mathematical processes an understanding of the relationships within one population or sample.

6-6.1* Predict the characteristics of one population based on the analysis of sample data. (B2)
6-6.2* Organize data in frequency tables, histograms, or stem-and-leaf plots as appropriate. (B4)
6-6.3* Analyze which measure of central tendency (mean, median, or mode) is the most appropriate for a given purpose. (B4)
6-6.4* Use theoretical probability to determine the sample space and probability for one- and two-stage events such as tree diagrams, models, lists, charts, and pictures. (C3)
6-6.5* Apply procedures to calculate the probability of complementary events. (C3)

* These indicators are covered in the following 5 Modules for this Nine Weeks Period.
Teaching time should be adjusted to allow for sufficient learning experiences in each of the modules.

Module 1-3 Operations: Multiplication and Division
*****revisited here, introduced in 1st nine weeks*****

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines
****Module 1-3** **Lesson A:** **6-2.6** Understand the relationship between ratio/rate and multiplication/division. (B2)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations - See Appendix A	Introduced in 1st nine weeks. See Module 1-3 , Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 1-3, Lesson A Assessment

Module 1-5 Proportional Reasoning *****revisited here, introduced in 1st nine weeks*****			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
****Module 1-5** **Lesson A:** **6-5.6** Use proportions to determine unit rates. (C3)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations - See Appendix A	Introduced in 1st nine weeks. See Instructional Planning Guide Module 1-5, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 1-5 Lesson A: Assessment
Module 1-5 Lesson B: **6-5.7** Use a scale to determine distance. (C3)		Introduced in 1st nine weeks. See Instructional Planning Guide Module 1-5, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module Lesson B Assessment

Module 4-1 Data Collection and Representation			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 4-1 Lesson A: **6-6.2** Organize data in frequency tables, histograms, or stem-and-leaf plots as appropriate. (B4)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for	See Instructional Planning Guide Module 4-1, Introductory Lesson A See Instructional Planning Guide Module 4-1, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 4-1 Lesson A: Assessment

	School Mathematics (PSSM) Textbook Correlations - See Appendix A		
Module 4-2 Data Analysis			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 4-2 Lesson A: **6-6.3** Analyze which measure of central tendency (mean, median, or mode) is the most appropriate for a given purpose. (B4)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School	See Instructional Planning Guide Module 4-2, Introductory Lesson A See Instructional Planning Guide Module 4-2, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 4-2, Lesson A: Assessment

Module 4-2 Lesson B: **6-6.1** Predict the characteristics of one population based on the analysis of sample data. (B2)	Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations - See Appendix A	See Instructional Planning Guide Module 4-2, Introductory Lesson B See Instructional Planning Guide Module 14, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 4-2, Lesson B: Assessment
Module 4-3 Probability			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 4-3 Lesson A: **6-6.4** Use theoretical probability to determine the sample space and probability for one- and two-stage events such as tree diagrams, models, lists, charts, and pictures.	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School	See Instructional Planning Guide Module 4-3, Introductory Lesson A See Instructional Planning Guide Module 4-3, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 4-3, Lesson A: Assessment

(C3)	Mathematics Developmentally 6th Edition, John Van de Walle		
Module 4-3 Lesson B: **6-6.5** Apply procedures to calculate the probability of complementary events. (C3)	NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations - See Appendix A	See Instructional Planning Guide Module 4-3, Introductory Lesson B See Instructional Planning Guide Module 4-3, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 4-3, Lesson B: Assessment

MODULE

4-1

This module addresses the following indicators:

6-6.2 Organize data in frequency tables, histograms, or stem-and-leaf plots as appropriate. (B4)

This module contains one lesson. This lesson are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

6-6.2 Organize data in frequency tables, histograms, or stem-and-leaf plots as appropriate.

- In third grade, students organize data in tables, bar graphs, and dot plots (3-6.2). In fourth grade, students interpreted data in tables, line graph, bar graph an double bar graphs whose scale increment are greater than or equal to 1 (4-6.2).
- In seventh grade students will organize data in box plots and circle graphs (7-6.2).

- **Key Concepts/Key Terms**

*Frequency Tables
*Histograms
*Stem-and-leaf plots
*Data
*Data analysis

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lesson

1. Teaching Lesson A

Stem-and-leaf plots are a popular form of representation in which numeric data are plotted by using the actual numerals in the data set to form a graph. In stem-and-leaf plots, all of the data are maintained. Thus, it is an efficient method of ordering data and individual elements of data can be easily identified. Students need to understand the importance of putting the stems and leaves in numeric order. If students are comparing a double stem and leaf plot, make sure they know how to read the numbers correctly. In addition, stress the importance of creating a key when creating a stem-and-leaf plot.

A histogram is a form of a bar graph in which the categories are consecutive, equal intervals along a numeric scale. The height or length of each bar is determined by the number of data elements falling into that particular interval and the bars are drawn without any spaces between them.

Teachers need to help students to connect data analysis with content outside mathematics – science and social studies. They need to guide students to the understanding that data analysis is a process that helps make sense of a situation.

6-6.2 Organize data in frequency tables, histograms, or stem-and-leaf plots as appropriate. (B4)

For this indicator, it is **essential** for students to:

- Understand the advantages and disadvantages of each type of graph
- Understand the most appropriate representation is based the questions that the data is supposed to answer
- Understand how to structure each type of graph
- Organize data in order from least to greatest.
- Determine equal intervals

For this indicator, it is **not essential** for students to:

- Analyze or make conclusions regarding the data given.
- Do calculations of central tendency.

a. Indicators with Taxonomy

6-6.2 Organize data in frequency tables, histograms, or stem-and-leaf plots as appropriate. (B4)

Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

Materials Needed:
Chart paper or poster board
Markers

Introductory Lesson: "Data About Us"

Determine a class survey question around which to collect data. Survey questions should measure numerical data so that the results can be displayed in frequency distributions, histograms, and stem and leaf plots.

Possible questions for exploration are:
"What are the ages of people in our families?"
"How old are teachers in our school?"

"How many hours a week do students spend on the internet?"
"How many hours a week do students spend sleeping?"
'How many hours a week do you watch television?"
"How many pairs of shoes do you have?"
"How many songs do you have on your iPod?"
"How many minutes a day do you spend texting?"
"How many M&Ms are in a fun-size bag of M&Ms?"
"How many seconds can a typical 6^{th} grader hold their breath?"

Use one of the questions above or create one of your own.
Once data are collected, demonstrate the organization of example data into each of the three types of graphical displays (histogram, frequency distribution, stem and leaf plot.) Groups of 2-3 students should organize the data they have collected into the three types of graphical displays.

Once data are organized into the three types of graphical displays, pose questions to the class and ask them which of the displays would best be used to answer each question.

Students should determine that different types of data displays provide different information and that the type of display used should be determined by the questions one hopes to answer with the data. They should also recognize the connections between the three types of displays and the similarities between data displayed in frequency distributions, stem and leaf plots and histograms.

c. Misconceptions/Common Errors

- Students use histograms as bar graphs in general. In fact they are special bar graphs where bars are used to display numerical data that have been organized into equal intervals. The entire range is covered in the interval with no overlapping. This is why a student is told not to put spaces between the bars unless the data represents zero.
- Some students may question the width of their bars when they make the histogram. Students need to understand that the width needs to be the same for all bars since the bars represent the same interval length.
- Some students have trouble with the height of the bars and the vertical scale for the bars. It is a good point for students to look at the data that they have grouped together prior to graphing to see what would be the largest value overall. They could then use this to set their scale prior to actually graphing the bars.
- Technology may be used to help students focus on the graph and its message. Graphing calculators produce histograms without much difficulty. They allows for the size of the interval to be specified and easily changed.

- Students may forget to put the data in numerical order when creating a stem and leaf plot.

d. Additional Instructional Strategies

- This is the first time students are introduced to those forms of data representation. As a result, students will need sufficient experiences so that they are able to make a determination as to which form of representation is appropriate for different purposes.

- Connect data analysis with content outside mathematics – science and social studies guide students to the understanding that data analysis is a process that helps make sense of a situation. Opportunities will naturally arise in all subject areas. Use these opportunities to allow students to collect data related to what they are studying and represent the data in appropriate graphs.

e. Technology

- NCTM - Exploring Histograms - The interactive data analysis tool in this investigation allows students to enter their own sets of data, create a histogram and examine how various statistical functions such as mean, mode, and median are affected by the choice of data. http://illuminations.nctm.org/LessonDetail.aspx?ID=L449

- NCTM – "There is a Difference: Bar Graphs vs. Histograms" Lesson Plan using data about Presidents to explore the differences between bar graphs and histograms. http://illuminations.nctm.org/LessonDetail.aspx?id=L812

- Overview of Frequency Distribution, Histograms, and Stem and Leaf Plots: http://www.purplemath.com/modules/stemleaf.htm

- Stem and Leaf Plotter-This interactive data analysis tool allows students to enter their own sets of data and create a stem and leaf plot of the data.

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Student understanding of this lesson is assessed through questioning and listening through the class dialogue. Group graphical displays may also be used as artifacts of student understanding of how to create the graphical displays.

Students should be given multiple opportunities to display data in multiple ways and to use data to answer questions.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

6-6.2 Organize data in frequency tables, histograms, or stem-and-leaf plots as appropriate. (B4)

The objective of this indicator is to organize which is in the "analyze conceptual" knowledge cell of the Revised Bloom's Taxonomy. Conceptual knowledge is knowledge of interrelationships among basic elements (frequency tables, histograms and stem-and-leaf plots) within a larger structure (data analysis) that enable them to function together. The learning progression to **organize** requires students to understand the structure and purpose for each type of graph. Students compare each type of graph and discuss the data based on the advantages and disadvantages of each. To deepen conceptual understanding, students may generate questions that could be answered by the data display in each type of graph. They explain and justify their answers using correct and clearly written or spoken words (6-1.6)

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Which set of data is represented by the stem and leaf plot?

2	0
3	2 2
4	0 1 2
5	2 5
6	0 1 1 7

key:
2 | 0 = 20

A. 20, 32, 40, 41, 42, 52, 55, 60, 61, 67
B. 2, 0, 3, 2, 2, 4, 0, 1, 2, 5, 2, 5, 6, 0, 1, 1, 7
C. 20, 32, 32, 40, 41, 42, 52, 55, 60, 61, 61, 67

2. What intervals for the x and y axes would be appropriate given the data:

X	Y
0	0
1	500
2	1000
3	1500
4	2000
5	2500

3. Given the data sets, which type of graph would be best (histogram or stem and leaf plot)?

A. 45,94,85,72,91,102,96,98,91,100,101,48,64,65
B. 40,40,40,51,53,57,61,64,64
C. 0,10,20,45,92,100,125,128,210,215,292,350,400,450,480,500,525

MODULE

4-2

This module addresses the following indicators:

6-6.1 Predict the characteristics of one population based on the analysis of sample data. (B2)
6-6.3 Analyze which measure of central tendency (mean, median, or mode) is the most appropriate for a given purpose. (B4)

This module contains two lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

6-6.1 Predict the characteristics of one population based on the analysis of sample data.

- In third grade students analyzed dot plots and bar graphs to make predictions about populations (3-6.4).

- In seventh grade, students predict the characteristics of two populations based on the analysis of sample data (7-6.1).

6-6.3 Analyze which measure of central tendency (mean, median, or mode) is the most appropriate for a given purpose.

- In fifth grade, students first applied procedures to calculate the measures of central tendency of mean, median, and mode (5-6.3).

- In seventh grade, students apply procedures to calculate the interquartile range (7-6.3) and interpret the interquartile range for data (7-6.4). In eighth grade, students interpret graphic and tabular data representations by using range and the measures of central tendency (8-6.8).

- **Key Concepts/Key Terms**

  *Range
  *Data
  Sample
  Population
  *Measures of Central tendency (mean, median, mode)
  *Predict
  Graphic Organizer
  Outliers

  ** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lessons

1. Teaching Lesson A: "Report Card Time"

Students are often just told how to find the measures of central tendency and directed when to use each one. Students should be given different situations and determine which of these measures best communicate information in the given situation. Students need to look at each measure and determine how they compare to each other. Students should also be able to determine which measure best represents the data with the respect to the context in which it is presented. This indictor lends itself for students to apply reasoning skills to determine which measure or measures best represent the data as well as communication skills to explain and defend the reasoning used.

Each of the measures of central tendency has strengths that make it a better choice to represent a set of data in various situations. Students should learn to select the most appropriate average to describe a typical item or a data set. After investigating how each measure of central tendency is affected by the distribution of the data, students can make more informed decisions about which average to use for a given purpose.

- For sets of data with no very low or very high numbers, **mean** works well.
- For sets of data with a couple of points much higher or lower than most of the others, **median** may be a good choice.
- For sets of data with many identical data points, **mode** may be a better description.

6-6.3 Analyze which measure of central tendency (mean, median, or mode) is the most appropriate for a given purpose. (B4)

For this indicator, it is **essential** for students to:

- Compute each central tendency
- Compare the central tendencies (advantages and disadvantages)
- Determine how each measure of central tendency may influence conclusions
- Understand that for sets of data with no very low or very high numbers, **mean** works well.
- Understand that for sets of data with a couple of points much higher or lower than most of the others, **median** may be a good choice.
- Understand that for sets of data with many identical data points, **mode** may be a better description.

For this indicator, it is **not essential** for students to:

- Determine range.

a. Indicators with Taxonomy

6-6.3 Analyze which measure of central tendency (mean, median, or mode) is the most appropriate for a given purpose. (B4)

Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson A: "Report Card Time"

Students are familiar with the procedures for calculating mean, median, and mode from fifth grade, but may need to refresh their knowledge before instruction on this indicator.

Provide students with a list of grades for a fictional student in the class, such as 80, 85, 80, 75, 60, 80, 90, 85. Ask students to calculate the mean, median and mode of the data. Ask "Which measure most accurately describes this student's performance in class? If you were him/her, which measure would you want used to calculate your grade for your report card? Why?"

Ask: "Knowing that students sometimes have bad days, I decide as the teacher to drop the lowest grade. How might this change effect each of the measures of central tendency?" Have students make predictions about how this change might affect the mean, median, and mode. Students should share their predictions and their reasoning with one another. After dialogue about their predictions, have students compare their predictions with the calculated statistics.

Repeat this process with several other scenarios. For example, "What might happen if this student makes a 100 on the next test?" "Suppose the mean of this student's grades is 84 after the next test, what grade did he/she make on the new test?" "How does making a zero on the next text change the mean? The median? The mode?"

c. Misconceptions/Common Errors

- Students frequently forget to order the data before finding the median.
- It is a misconception that *average* is a single number which represents the idea of a *typical* value when the average might be the value of the mean, median, or mode. Typically, students will choose mean as the *average*.

d. Additional Instructional Strategies/Differentiation

When to Use Mean, Median, or Mode

Scale of Measurement	Measure of Central Tendency
Nominal (categorical such as sex or race)	Mode
Ordinal (such as salary categories)	Median (sometimes mode)
Interval	Symmetrical Data – mean Skewed Data – median
Ratio	Symmetrical Data – mean Skewed Data – median

- Have students work in cooperative groups to collect the following data from their classmates:
  - heights
  - ages (in months)
  - pulse rates
  - distance from home to school
  - hours per day spent watching TV
  - hours per day spent studying
  - number of consecutive sit-ups they can do

  Then, have students find the mean, median, mode, and range of each set of data and explain which central tendency is most appropriate for each set of data.(A5)

- NCTM – Building Height – Students use clinometers to measure the height of a building. The class will compare measurements, talk about the variation in their results, and select the best measure of central tendency to report the most accurate height.
  http://illuminations.nctm.org/LessonDetail.aspx?id=L764

e. Technology

- NCTM - Exploring Histograms The interactive data analysis tool in this investigation allows students to create their own sets of data and examine how various statistical functions such as mean, mode, and median are affected by the choice of data.
  http://illuminations.nctm.org/LessonDetail.aspx?ID=L449

- NCTM – "Averages and *The Phantom Tollbooth*" – Students explore the concept of averages through children's literature.
  http://illuminations.nctm.org/LessonDetail.aspx?id=L204

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Give student the following descriptions of possible data sets. Their task is to decide which average (mean, median, or mode) is the most appropriate average to describe a typical item in each of the following data sets.

- Size of shoes sold in a shoe store
- Salaries in a large company
- Salaries in a small company
- Cost of houses in your local area
- Test scores for a student
- Size of cereal boxes in a grocery store
- Cost of hotel rooms at a particular hotel
- A person's monthly electric bill for a year

It may be helpful to suggest to students to generate sample data sets for each situation. Students should be ready to justify their choice of an average.

2. Teaching Lesson B : "Lunch Choices"

6-6.1 Predict the characteristics of one population based on the analysis of sample data. (B2)

For this indicator, it is **essential** for students to:

- Make predictions from data from varying formats.
- Translate data to a graph or a picture
- Understand that the prediction from the sample data is an estimation for the population
- Make predictions based on the shape of the data (central tendency, spread of the data and outliers)
- Observe trends in the data
- Justify their predictions using mathematical reasoning
- Use terms like most, between, at least etc… to describe characteristics.

For this indicator, it is **not essential** for students to:

- Analyze data by comparing two data sets to each other.

a. Indicators with Taxonomy

6-6.1 Predict the characteristics of one population based on the analysis of sample data. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson B– "Lunch Choices"

As students return to class from lunch, students poll a sample of their peers on their lunch choices to collect data to answer the question "What did you eat for lunch today?" (If the class is in the morning, students may poll peers on their lunch the previous day or on their breakfast choices.)

Use the sample data collected to make predictions about the lunch choices of the entire school population.

Possible questions:

- If there are 2000 students in the entire school, how many would you expect to have eaten pizza for lunch yesterday? (Substitute appropriate food.)
- If the cafeteria staff asked how much of each dish to prepare for lunch next week, what would you suggest to them?
- Use your sample data to write a proposal for the cafeteria staff to help them minimize food waste in the school.

c. Misconceptions/Common Errors

- Students may assume that they are finding exact predictions when they are actually guessing.

d. Additional Instructional Strategies/Differentiation –

- Whenever data is collected in the classroom, ask the students what the data tells them and what predictions they can make from the data.
- Having students analyze real world data that is relevant to their lives is an excellent strategy to build conceptual understanding and engage students. For example, you or your students can collect data about what students eat for lunch and predict what that data may or may not mean for the entire 6^{th} grade population.

- Have students poll people in their neighborhoods for information. Then have the students predict how their peers will poll on the same information based on their data. (A5)
- Have students predict the outcome of the rolling of a pair of dice based on 100 previous rolls. For example, in 100 previous rolls, the following totals came up:

Total on Pair of Dice	**Number of Times Rolled**		**Total on Pair of Dice**	**Number of Times Rolled**
1	*0		7	22
2	3		8	14
3	4		9	12
4	6		10	6
5	12		11	4
6	14		12	3
*Cannot roll 1 with two dice. Lowest possible roll is 2. Highest possible roll is 12.				

Have students make a prediction of the next 100 rolls. Do the experiment again and compare results. (A5)

- Put ten colored tiles in a bag (two red, one blue, four green, and three yellow). Have each student pull a tile from the bag. Go around the room two or three times, recording the colors pulled. Have students predict the number of each color after 10 pulls, 25 pulls, 50 pulls, etc. *(A5)*

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

No suggestions for use of technology or websites are included at this time.

f. Assessing the <u>Lesson</u>

According to a survey of 30 students, 13 prefer chicken and 17 prefer hotdogs. If 750 students will be attending the picnic, how many will prefer each selection? Explain.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

6-6.1 Predict the characteristics of one population based on the analysis of sample data.

The objective of this indicator is to predict, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To predict means to draw logical conclusion from presented information. The learning progression to **predict** requires students analyze a set of data and generate conjectures about the population. They understand that it is sometimes difficult to do so from just analyzing the numbers. They translate their data to another form (graph or picture). They understand that each is a distinct symbolic form that represent the same relationship (6-1.4) and generalize connections (6-1.7) among these representational forms. They make observations about the shapes and proximity of the data in order to make reasonable predictions based on those observations. They use inductive and deductive reasoning (6-1.3). Students explore a variety of real world situations (6-1.7) and summarize their predictions using correct and clearly written or spoken words to communicate their understanding (6-1.6).

6-6.3 Analyze which measure of central tendency (mean, median, or mode) is the most appropriate for a given purpose.

The objective of this indicator is to *analyze*, which is in the "analyze procedural" knowledge cell of the Revised Taxonomy. Analyze requires student to break material into its constituent parts and to determine how the parts relate to one another and to an overall structure or purpose. Procedural knowledge is tied to knowledge of criteria for determining when to use appropriate procedures or steps. The learning progression to **analyze** requires students to understand the differences between the mean, median and mode and how they compare to each other. Students should differentiate between varying situations where mean, median, or mode is the preferred measure of central tendency to use when describing data. Students determine which measure best represents the data with respect to the context in which it is presented. Although, student work with central tendency will be limited to relationships within one population or sample, they are exposed to problem situations with deconstructing (determining point of view) where bias or values influences the choice of central tendency in a sample or population. Students apply reasoning skills to evaluate their conjectures and pose questions to prove or disprove their conjectures (6 – 1.2) using correct and clearly written or spoken words (6–1.6) Students also use deductive reasoning to reach a conclusion from known facts (6-1.5).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Given the data set, calculate mean, median and mode. Determine which is the best measure of central tendency. Explain.

   10 students ride the same bus home from school each day. Students are on the bus different times due to where their stops are in the route. The chart shows the number of minutes students are on the bus each afternoon.

Student	Number of minutes
Mary	8
Jane	8
Bobby	10
Billy	15
Leigh	15
Jeannie	22
Kim	22
Tom	22
George	28
Chris	72

2. Would you use mean, median or mode to determine the best measure of central tendency for the following data sets?

   A. 3,3,4,5,6,7,7,9,15,107
   (Answers: accept mode or median…but not mean due to the outlier)

   B. 15,15,18,19,19,19,25,28,32,37
   (Answers: accept any of the choices)

   C. 10,15,19,20,30,34,38,41,41,41
   (Answers: accept mean or median…but not mode due to the mode being the highest number)

3. What prediction could you make about temperatures based on the tables?

Temperatures in City A

Time	Temperature
12pm	87 degrees
2pm	89 degrees
4pm	87 degrees
6pm	80 degrees
8pm	72 degrees
10pm	68 degrees
12am	55 degrees

Temperatures in City B

Time	Temperature
12pm	62 degrees
2pm	64 degrees
4pm	60 degrees
6pm	58 degrees
8pm	52 degrees
10pm	47 degrees
12am	42 degrees

MODULE

4-3

This module addresses the following indicators:

6-6.4 Use theoretical probability to determine the sample space and probability for one- and two-stage events such as tree diagrams, models, lists, charts, and pictures. (C3)

6-6.5 Apply procedures to calculate the probability of complementary events. (C3)

This module contains two lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

Planning the Module

- **Continuum of Knowledge**

6-6.4 Use theoretical probability to determine the sample space and probability for one- and two-stage events such as tree diagrams, models, lists, charts, and pictures.

- In grade five students were formally introduced to representing the probability of a single-stage event in words and fractions (5-6.5).

- In seventh grade, students apply procedures to calculate (7-6.5) and interpret (7-6.6) the probability for mutually exclusive simple and compound events. In eighth grade, they apply procedures to calculate (8-6.5) and interpret (8-6.6) probability for two dependent events.

6-6.5 Apply procedures to calculate the probability of complementary events.

- In fifth grade, students represented the probability of a single stage event as a fraction and in words (5-6.5) and concluded why the probabilities of the outcomes of an experiment must equal one. (5-6.6)

- In seventh grade, students apply procedures to calculate (7-6.5) and interpret (7-6.6) the probability of mutually exclusive events as well as compound events. In eighth grade students will apply procedures to calculate (8-6.5) and interpret (8-6.6) the probability of dependent events.

- **Key Concepts/Key Terms**

  *Probability
  *Sample Space
  *Theoretical probability
  *Outcome
  Combinations
  *Event
  Compound event
  *Complementary Events
  Complement
  Population
  Sample
  Possible outcomes
  *Mutually Exclusive Events
  *Experimental Probability

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lessons

1. Teaching Lesson A-"Coins and Cubes"

For this indicator, it is **essential** for students to:

- Understand the concept of theoretical probability
- Multiply fractions for the probabilities of two-stage events.
- Construct tree diagrams, lists, models, charts and picture as appropriate
- Interpret probability notation. For example: P(blue, green)
- Understand which representation will be the most effective way to create the sample space

For this indicator, it is **not essential** for students to:

- Find the probability of events that are not depicted as tree diagrams, models, lists, charts, or pictures.

a. Indicators with Taxonomy

6-6.4 Use theoretical probability to determine the sample space and probability for one- and two-stage events such as tree diagrams, models, lists, charts, and pictures. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson A–"Coins and Cubes"

Introduce the problem: "How many possible outcomes are there if Ted tosses two coins and draws a cube from a bag that contains 1 blue, 1 green and 1 red centimeter cube?"

Distribute two-color counters, centimeter cubes and bags. Explain that an ***event*** is the result or outcome of an experiment, such as tails on a coin toss, and that a ***compound event*** is two or more events combined.

Allow students to experiment with the coins and cubes to determine all of the possible outcomes from tossing two coins and drawing a cube from a bag.

Facilitate dialogue to create a class list of all possible outcomes. Ask: "How might we organize our work so that we find all of the possible outcomes in a more efficient way?" Teacher should guide student dialogue to ensure methods such as tree diagrams, charts, lists, models, and pictures are identified by the group. If no student suggests these methods, teacher should introduce them to the students as possible methods.

Have students use the sample space to answer questions about probability, such as "What is the probability that Ted gets two heads and a red cube?"

c. Misconceptions/Common Errors

Students sometimes misinterpret the term sample space. It is a list of <u>all</u> possible outcomes. An event is one of these outcomes.

Many students have misconceptions about the outcomes of real events in life, basing predictions on what they <u>believe</u> should happen, rather than on <u>real data</u>. Studying probability will help students develop critical thinking skills and interpret the probability of events that happen in their lives.

Students may struggle with words like "*at least, or, and*, etc…" For example, getting *at least* two head from a toss of four coins.

d. Additional Instructional Strategies –

- Provide compound events involving spinners, number cubes, coins, and sacks of color tiles or centimeter cubes with 2 to 5 single events in the compound event. Have students model and find the possible outcomes of the compound event.

- Have student pairs use color tiles or centimeter cubes to model and find combinations involving real-world situations, such as combinations of hats, gloves, and coats, or various menu items. Name items so that students can determine the number of choices, such as red, blue, and brown hats, or wool, fleece, and leather gloves.

- Theoretical probability is the ratio of the number of favorable outcomes to the total number of possible outcomes. For example, the theoretical probability of a coin landing on tails is

$$P(\text{heads}) = \frac{\text{number of sides with tails}}{\text{number of sides}} = \frac{1}{2}.$$

- Define sample space as the set of all possible outcomes of an event. Introduce several ways to create a sample space: (A5)

- Make a list. Have students list all of the possibilities of tossing 2 coins, then 3 coins, and 4 coins.

- Make a table. Have students list all of the possible sums from rolling 2 dice.

- Draw a picture. Give students 2 spinners and have them draw all of the possible spins.

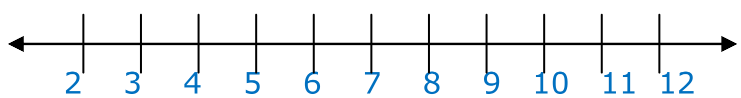
- Draw a tree diagram. Put several colors of tiles in a bag and have students draw a tree diagram to show all possible ways to pull 2 tiles at a time.

- Write each letter of the alphabet on a card and put cards in a bag. Have students list the sample space for pulling a letter that their name contains.

- Have students see you put 3 red and 1 blue tile in a bag. Show students how to write the probabilities for pulling each color: P(red) = $\frac{3}{4}$ and P(blue) = $\frac{1}{4}$ Have students pull tiles from the bag 20 times, without looking, and record the results. Compare the simulated experiment to the theoretical probability.

- Play a dice game where 2 dice are rolled and the sums are recorded.

  Have students predict which sum they think will be rolled the most often by placing 12 chips on the following number line:



  The teacher will roll two 2 dice and the students will remove a chip each time one of the marked numbers is rolled. The winner is the first student to remove all of his/her chips. Discuss the "best-placed" chips after the game.

e. Technology

NCTM - Stick or Switch - Lesson based on the “Monty Hall Problem”: A student picks one of three doors in the hopes of winning the prize. The host, who knows the door behind which the prize is hidden, opens one of the two remaining doors. When no prize is revealed, the host asks if the student wishes to "stick or switch." Which choice gives you the best chance to win?
http://illuminations.nctm.org/LessonDetail.aspx?ID=L377

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives.

Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

Assess student understanding through questioning and listening to the dialogue about the sample space and the probabilities of different events.

Lakeview Diner serves hamburgers, hotdogs, and chicken fingers. You can choose from fries, slaw, and salad. Draw a tree diagram to show the sample space.

2. Teaching Lesson B: "Picking Ducks"

For this indicator, it is **essential** for students to:

- Understanding the meaning of complementary events
- Generate the complementary event when given a simple event. For example, if the simple event is rolling an even number, students should state that the complementary event is rolling an odd number.
- Find the probabilities of a simple event and its complement
- Find the probability of the complement given the probability of the simple event. For example, if the probability that a child speaks Spanish is 7/20 then what is the probability that a child does not speak Spanish. 1 –
- Interpret probability notation. Ex. P(head or tails) when tossing a coin.
- Understand that the word "or" in probability notation indicates addition.
- Add and subtract fractions with like denominators when determining probability of the complements of events.
- Understand that the sum of the probabilities of all the outcomes in a sample space is 1; therefore, the sum of the complementary events is 1.

For this indicator, it is **not essential** for students to:

- Use the word "mutually exclusive." (7^{th} grade)

a. Indicators with Taxonomy

6-6.5 Apply procedures to calculate the probability of complementary events. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson B – "Picking Ducks"

Introduce the problem: "Erica picks a prize from a bag filled with 3 blue ducks, 2 yellow ducks, 6 green ducks, and 1 red duck. What is the probability that Erica picks a yellow duck and what is the probability that she does not pick a yellow duck? What is the probability that Erica picks either a blue duck or a green duck?"

Distribute color tiles to the students. Students should use color tiles to model the ducks and answer the questions in the problem.

- What is the probability that Erica picks a yellow duck?
- What is the probability that Erica does not pick a yellow duck? (This is known as the "complement" of drawing a yellow tile. Share this vocabulary with students.)
- What is the probability that Erica picks a blue duck? What is the complement of "Erica picks a blue duck"? What is the probability that Erica does not pick a blue duck?
- What do you notice about the probabilities of complementary events?

c. Misconceptions/Common Errors –

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies –

- Give spinners to pairs of students. Each student formulates a probability question, one about complementary events, such as what is the probability of spinning an 8 and what is the probability of not spinning an 8, and the other about mutually exclusive events. Students trade questions, find the probabilities by inspecting their spinners, and then discuss their results.
- Have pairs of students use polyhedral dice to determine the probabilities of complementary and mutually exclusive events. For example, students find the probabilities of rolling and not rolling a multiple of 3 and the probability of rolling a prime number or a 4.
- Write the vowels a, e, i, o, u on five small slips of paper of the same size. Turn the papers over and mix them up. Draw a vowel from the stack. Ask the class to identify two events in this game that are complements. Add several consonants to the stack, discuss complements, and have students prove that an event and its complement equal 1.
- Place three different colored tiles in a basket and draw one out. Discuss the complement of the event. Repeat this process several times until the idea of complements is understood.
- Have students make a poster of several complementary events. Draw, color, and discuss why they are considered complementary of each other. Show how the events equal 1.

e. Technology

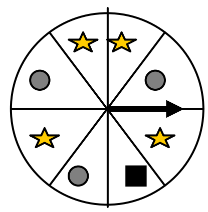
Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

No suggestions for use of technology or websites are included at this time.

f. Assessing the <u>Lesson</u>

Use the spinner to answer the question.

What is the complement of P(■ or ●)? Explain how you found the answer. (A5)



III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

6-6.4 Use theoretical probability to determine the sample space and probability for one- and two-stage events such as tree diagrams, models, lists, charts, and pictures.

The objective of this indicator is to <u>use</u> which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is to use, the learning progression should integrate experience that build the student's conceptual understanding of theoretical probability as well computational fluency with constructing representation of sample space. The learning progression to **use** requires students to <u>understand</u> the meaning of sample space and an event. Given an event, students <u>make</u> and <u>justify</u> a prediction of the probability of the event. Students <u>evaluate</u> their conjectures (6-1.2) by <u>creating</u> the sample space (the set of all possible outcomes) for one-and two- stage events and making decisions about what form of representation is best for the situation. Students <u>develop</u> probability-based thinking by performing actual experiments, recording and discussing the results and using the results as evidence for drawing conclusions. They <u>use</u> correct and clearly written or spoken words to communicate their understanding (6-1.6).

6-6.5 Apply procedures to calculate the probability of complementary events.

The objective of this indicator is to apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. To apply is means to carry out a procedure on a familiar task or use a procedural with an unfamiliar task; therefore, student's experiences should extend beyond familiar tasks such as cards, dice and coins. The learning progression to **apply** requires students to understand the meaning of complementary events and sample space. Students generate examples of complementary events to demonstrate understanding of the concept. They use their understanding of the relationship between complementary events to find the probability of one of the simple events. They explain and justify their answers using correct and clearly spoken words and notation (6-1.6).

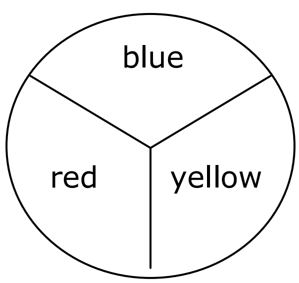
The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. How many times would you expect to get Heads, then Heads (Heads on the first toss and heads on the second toss) when tossing a coin 2 times?

   How would you express this as a probability?

2. Create a tree diagram for the following event:

   Spin the spinner once, then roll the number cube once.



3. What is the sample space for rolling a die once?

4. What is the probability you will draw a blue OR a red chip out of a bag containing 4 red, 5 blue and 3 yellow chips?

5. If the probability of event A is $\frac{3}{5}$ and the probability of event A and the probability of event B are complementary, what is the probability of

event B?

6. If you have a bag of yellow and red marbles, why are the probability of drawing a yellow marble and the probability of not drawing a yellow marble complementary?