SOUTH CAROLINA SUPPORT SYSTEM INSTRUCTIONAL GUIDE

Content Area	**Seventh Grade Math**

Second Nine Weeks

Standard/Indicators Addressed:

Standard: 7-2: The student will demonstrate through the mathematical processes an understanding of the representation of rational numbers, percentages, and square roots of perfect squares; the application of ratios, rates, and proportions to solve problems; accurate, efficient, and generalizable methods for operations with integers; the multiplication and division of fractions and decimals; and the inverse relationship between squaring and finding the square roots of perfect squares**.**

7-2.8 Generate strategies to add, subtract, multiply, and divide integers

Standard: 7-3: The student will demonstrate through the mathematical processes an understanding of proportional relationships.

7-3.1 Analyze geometric patterns and pattern relationships.
7-3.2 Analyze tables and graphs to describe the rate of change between and among quantities.
7-3.3 Understand slope as a constant rate of change.
7-3.4 Use inverse operations to solve two-step equations and two-step inequalities.
7-3.5 Represent on a number line the solution of a two-step inequality.
7-3.6 Represent proportional relationships with graphs, tables, and equations.
7-3.7 Classify relationships as either directly proportional, inversely proportional, or nonproportional.

*** These indicators are covered in the following 4 Modules for this Nine Weeks Period.**

Module 2-1 Integers			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-1 Lesson A: **7-2.8** Generate strategies to add, subtract, multiply, and divide integers.	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations –See Appendix A	See Instructional Planning Guide Module 2-1 Introductory Lesson A See Instructional Planning Guide Module 2-1, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 2-1 Lesson A Assessment
Module 2-1 Lesson B: **7-2.8** Generate strategies to add, subtract, multiply, and divide integers.		See Instructional Planning Guide Module 2-1, Introductory Lesson B Appendix Two See Instructional Planning Guide Module 2-1, Lesson B Additional Instructional Strategies Appendix Two	See Instructional Planning Guide Module 2-1 Lesson B Assessment
Module 2-1 Lesson C: **7-2.8** Generate strategies to add, subtract, multiply, and divide integers.		See Instructional Planning Guide Module 2-1 Introductory Lesson C Appendix Two See Instructional Planning Guide Module 2-1, Lesson C Additional Instructional Strategies Appendix Two	See Instructional Planning Guide Module 2-1 Lesson C Assessment

Module 2-1 Lesson D: **7-2.8** Generate strategies to add, subtract, multiply, and divide integers.		See Instructional Planning Guide Module 2-1, Introductory Lesson D Appendix Two See Instructional Planning Guide Module 2-1, Lesson D Additional Instructional Strategies Appendix Two	See Instructional Planning Guide Module 2-1 Lesson D Assessment
Module 2-2 Solve Mathematical Situations			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-2 Lesson A: **7-3.4** Use inverse operations to solve two-step equations and two-step inequalities.	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See	See Instructional Planning Guide Module 2-2 Introductory Lesson A See Instructional Planning Guide Module 2-2, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 2-2 Lesson A Assessment
Module 2-2 Lesson B: **7-3.4** Use inverse operations to solve two-step equations and two-step inequalities.		See Instructional Planning Guide Module 2-2, Introductory Lesson B See Instructional Planning Guide Module 2-2, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 2-2 Lesson B Assessment

Module 2-2 Lesson C: **7-3.5** Represent on a number line the solution of a two-step inequality.	Appendix A	See Instructional Planning Guide Module 2-2 Introductory Lesson C See Instructional Planning Guide Module 2-2, Lesson C Additional Instructional Strategies	See Instructional Planning Guide Module 2-2 Lesson C Assessment
Module 2-3 Equivalencies			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-3 Lesson A: **7-3.1** Analyze geometric patterns and pattern relationships.	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for	See Instructional Planning Guide Module 2-3 Introductory Lesson A See Instructional Planning Guide Module 2-3, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 2-3 Lesson A Assessment
Module 2-3 Lesson B: **7-3.6** Represent proportional relationships with graphs, tables, and equations.		See Instructional Planning Guide Module 2-3 Introductory Lesson B See Instructional Planning Guide Module 2-3, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 2-3 Lesson B Assessment

7-3.7 Classify relationships as either directly proportional, inversely proportional, or nonproportional.	School Mathematics (PSSM) Textbook Correlations – See Appendix A		

Module 2-4 Change in Various Contexts			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-4 Lesson A: **7-3.2** Analyze tables and graphs to describe the rate of change between and among quantities. **7-3.3** Understand slope as a constant rate of change. .	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 2-4 Introductory Lesson A See Instructional Planning Guide Module 2-4, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 2-4 Lesson A Assessment

MODULE

2-1

Integers

This module addresses the following indicators:

7-2.8 Generate strategies to add, subtract, multiply, and divide integers. (B6)

This module contains 4 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

7-2.8 Generate strategies to add, subtract, multiply, and divide integers. (B6)

In sixth grade, students develop a meaning of integers (6-2.2).
In seventh grade, students generate strategies to add, subtract, multiply and divide integers (7-2.8).
In eighth grade, students apply an algorithm to add, subtract, multiply and divide integers (8-2.1).

- **Key Vocabulary**

* integer
* whole number
* positive
* negative
* operations
* zero pairs

II. Teaching the Lessons

1. Teaching Lesson A

In sixth grade, students developed a conceptual understanding of an integer. Spending the time to fully explore strategies to perform integer operations will pay off in future mathematics courses for students. Students should work with concrete models and pictorial representations to build the foundation needed for eighth grade when abstract/symbolic integer operations are performed. Students should be allowed to generate algorithms for addition, subtraction, multiplication and division before introduction to traditional algorithms in eighth grade. Seventh grade students generate strategies to add, subtract, multiply and divide integers. Students should **not** be expected to perform symbolic operations with integers. To support and promote conceptual understanding of operations with integers, manipulatives should be used.

a. Indicators with Taxonomy

7-2.8 Generate strategies to add, subtract, multiply, and divide integers. (B6)

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual

b. Introductory Lesson A: Operations with Integers

PART A: Integer Operations – Addition and Subtraction

Notes: Positive and negative numbers are measured distances to the right and left of 0. It is important to remember that signed values are directed distances and not points on a line. The points on the number line are not models of integers; the directed distances are. To emphasize this for students, represent all integers with arrows, and avoid referring to the number line coordinates as "numbers." Poster board arrows of different whole-number lengths can be made in two colors, yellow pointing to the right for positive quantities and red to the left for negative quantities. The arrows help students think of integers as directed distances. A positive arrow never points left; a negative arrow never points right. Furthermore, each arrow is a quantity with both length (magnitude or absolute value) and direction (sign). These properties remain for each arrow regardless of its position on the number line. Small versions of the arrows can easily be cut from poster board for individual students to work with. Students may also use red and yellow colored pencils as they progress from the concrete to the pictorial on their way to using algorithms.

SUGGESTED LESSON PROGRESSION
A 1a.) Combining positive quantities
A 1b.) Combining negative quantities
A 2.) Zero Pairs
A 3.) Combining mixed quantities (include "zero groups" in this lesson)
A 4.) Subtracting Quantities
A 5.) "Double" Negatives

Part 1a & b: COMBINING POSITIVE QUANTITIES/COMBINING NEGATIVE QUANTITIES

Teacher Note: There are at least two big take-aways for students from this lesson. The first is that if groups of the same sign are combined, the result is a bigger group of the same sign. So, combining groups of positive quantities results in a bigger positive group; and combining groups of negatives results in a bigger negative group. Secondly, they should be able to communicate that + and – are more than operational signs; their job description has been expanded to include identifying integers as either positive or negative, as well as the negative sign meaning "the opposite of."

A, Part 1a: COMBINING POSITIVE QUANTITIES

MATERIALS:

- two-sided red/yellow counters
- red and yellow colored pencils (at least one of each per pair of students)
- students need paper and/or math notebooks to sketch/record examples

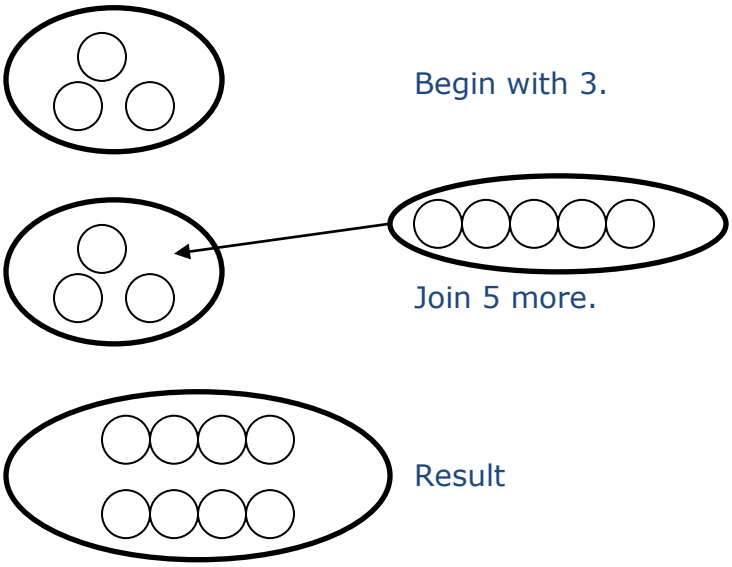
Begin by adding whole numbers. Students are familiar with the process, so using the yellow counters should help cement the concept of combining positive groups of integers. (white = yellow)

The examples shown/given in the lesson are not sufficient for students to connect the concept to an algorithm or procedure. You must either supply further examples or solicit them from the students.

All examples should be modeled for the students as they use their own counters at their desks.

All examples should also be sketched to provide a pictorial representation of the concrete model. Students should sketch their work on their own paper or in their math notebook using the colored pencils.

EX: 3 + 5



EX: 2 + 7

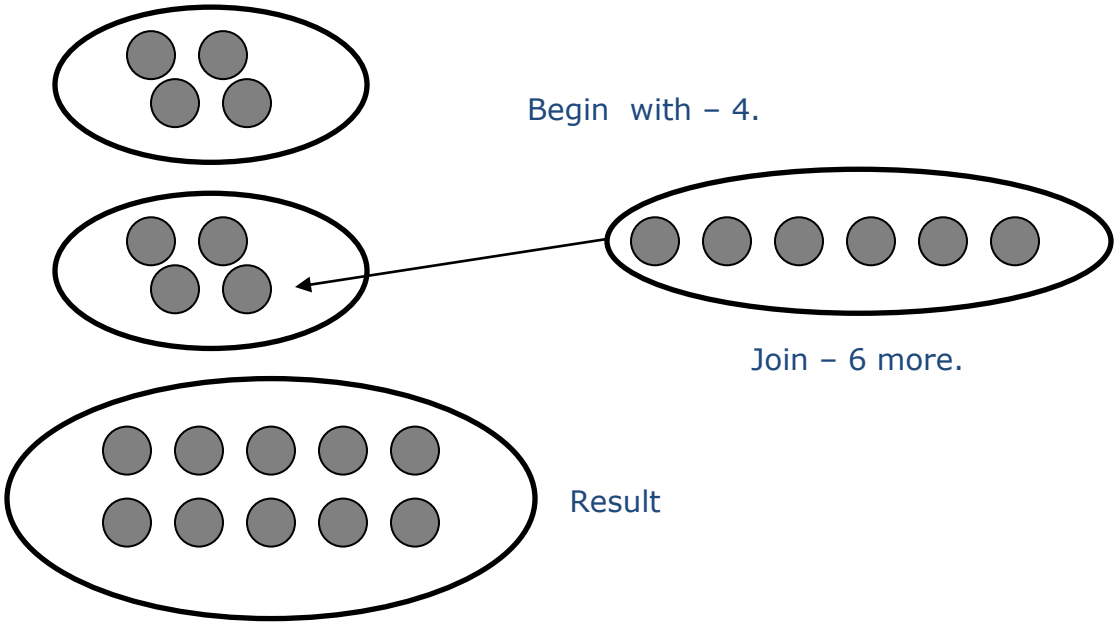
EX: 4 + 9

Model these and other examples as above.

Ask students to create their own examples. Even though it is "only adding," make sure they use the counters to build the sentences and sketch their work on their own paper or in their math notebook using yellow colored pencils.

A, Part 1b: COMBINING NEGATIVE QUANTITIES (grey = red)

EX: - 4 + - 6



EX: - 8 + - 2

EX: - 1 + - 5

Model these and other examples as above.

Ask students to create their own examples. Make sure they use the counters to build the sentences and sketch their work on their own paper or in their math notebook using yellow colored pencils.

A, Part 2: ZERO PAIRS

MATERIALS:

- two-sided red/yellow counters [white = yellow and grey = red]
- red and yellow colored pencils (at least one of each for each pair of students)
- students need paper and/or math notebooks to sketch/record examples

Ask: What happens if you combine equal groups of positive and negative integers? Let's start with +1 and – 1.

Use the counters to model 1 – 1.

How else can it be expressed (commutative property)?

1 – 1; - 1 + 1; 1 + ¯ 1

It may be useful to quickly model each expression to emphasize that they're all equal.

Ask students to give other examples of zero pairs. Model them using the counters. Sketch your work on the overhead or board. Students should be working with their own counters and sketching their work. They need to come to the conclusion that equal groups of positives and negatives form zero pairs. Other ways to consider it: they "zero out" or "cancel each other out."

A, Part 3: COMBINING MIXED QUANTITIES

MATERIALS:

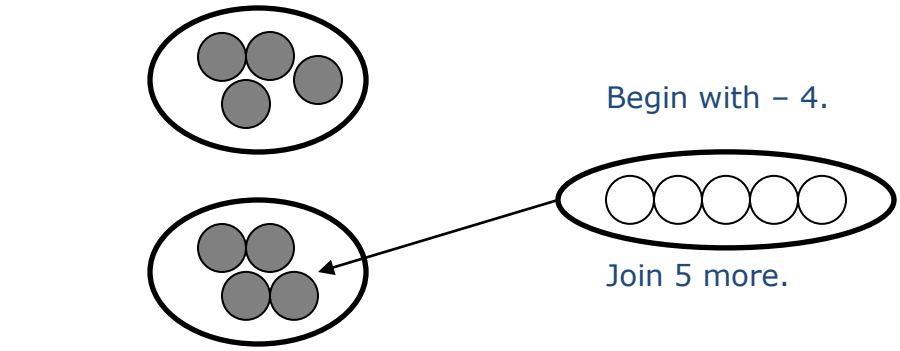
- two-sided red/yellow counters [white = yellow and grey = red]
- red and yellow colored pencils (at least one of each for each pair of students)
- students need paper and/or math notebooks to sketch/record examples

The examples shown/given in the lesson are not sufficient for students to connect the concept to an algorithm or procedure. You must either supply further examples or solicit them from the students.

All examples should be modeled for the students as they use their own counters at their desks.

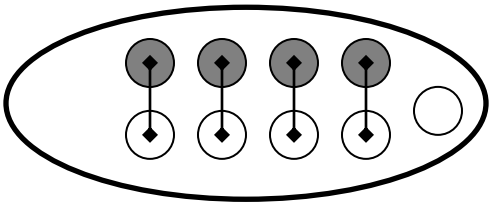
All examples should also be sketched to provide a pictorial representation of the concrete model. Students should sketch their work on their own paper or in their math notebook using the colored pencils.

EX: - 4 + 5

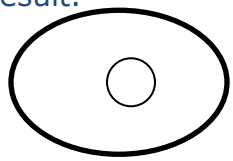


Begin with – 4.

Join 5 more.



Cancel pairs of + and – ("zero pairs") to get result.



EX: 7 + - 4

EX: - 3 + 5

Model these and other examples as above.

Ask students to create their own examples. Make sure they use the counters to build the sentences and sketch their work on their own paper or in their math notebook using yellow colored pencils.

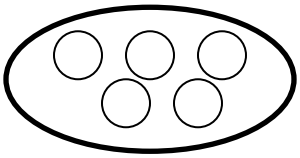
A, Part 4: SUBTRACTING QUANTITIES

MATERIALS:

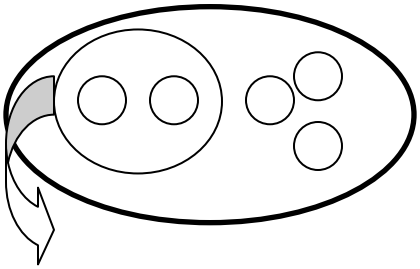
- two-sided red/yellow counters [white = yellow and grey = red]
- red and yellow colored pencils (at least one of each for each pair of students)
- students need paper and/or math notebooks to sketch/record examples

> *The examples shown/given in the lesson are not sufficient for students to connect the concept to an algorithm or procedure. You must either supply further examples or solicit them from the students.*
>
> *All examples should be modeled for the students as they use their own counters at their desks.*
>
> *All examples should also be sketched to provide a pictorial representation of the concrete model. Students should sketch their work on their own paper or in their math notebook using the colored pencils.*

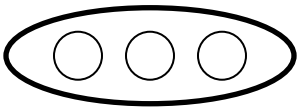
EX: 5 – 2



Begin with 5.



Remove 2 for the result.



Result

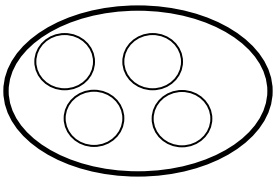
EX: 7 - 4

EX: 2 - 1

Model these and other examples as above.

Ask students to create their own examples. Make sure they use the counters to build the sentences and sketch their work on their own paper or in their math notebook using yellow colored pencils. Ask students to create their own examples. Make sure they use the counters to build the sentences and sketch their work on their own paper or in their math notebook using yellow colored pencils.

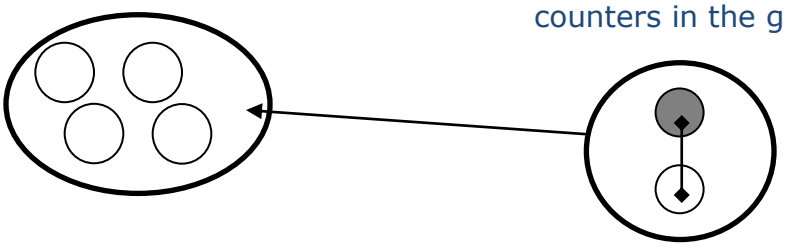
EX: 4 – 5

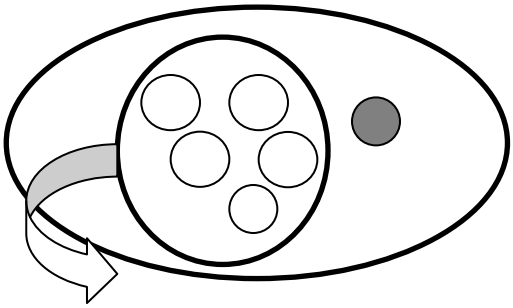


Begin with 4.

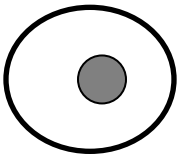
You want to remove 5, but there are only 4 positive

counters in group.
Add 1 zero pair so that there are 5 positive counters in the group.





Remove 5 for the result.



2 - 7

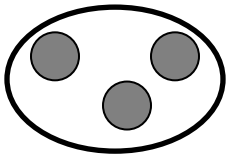
1 - 4

Model these and other examples as above.

Ask the students to give examples (one at a time) and model those. Students should put the examples in the empty boxes on the recording sheet, use the counters, and sketch their work. When sufficient examples have been modeled and solved (use your professional judgment), ask the students to generalize what happens when combining mixed groups of integers. You should hear something like this: Find the difference between the two groups; the larger group determines the sign of the solution. Again, more mathematically, find the difference in the absolute values and the answer takes the sign of the greater value.

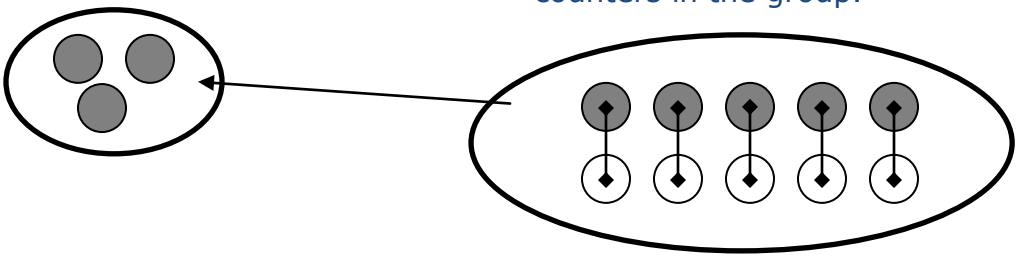
EX:

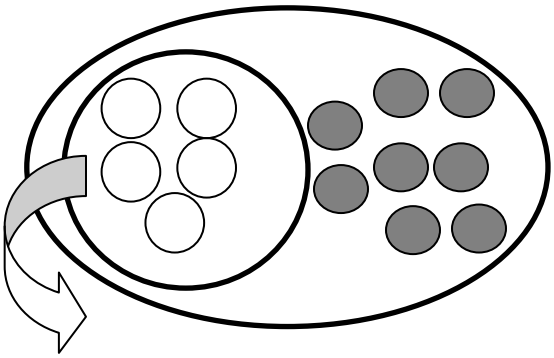
- 3 – 5 =



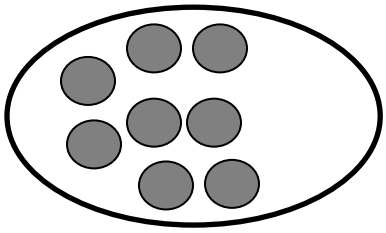
Begin with – 3.

You want to remove 5, but there are only negative counters in group.
Add 5 zero pairs so that there are 5 positive counters in the group.





Remove 5 for the result.



- 2 – 7
- 4 – 9

Model these and other examples as above.

Ask students to create their own examples. Again, insist that students use the counters to build the sentences and sketch their work on their own paper or in their math notebook using red colored pencils.

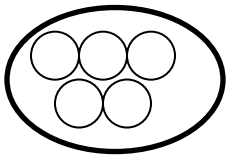
When students have a collection of examples of combining both positive quantities and negative quantities, ask them to generalize a "rule" for combining these quantities. They should be able to see that

they can add the groups and keep the sign. More mathematically speaking, add the absolute values and keep the sign of the two groups. This leads to the idea that an integer takes the sign immediately preceding it. Thus – 4 + - 9 has the same value as – 4 – 9.

A, Part 5: DOUBLE NEGATIVES

Suggestion: Begin by subtracting from positive integers.

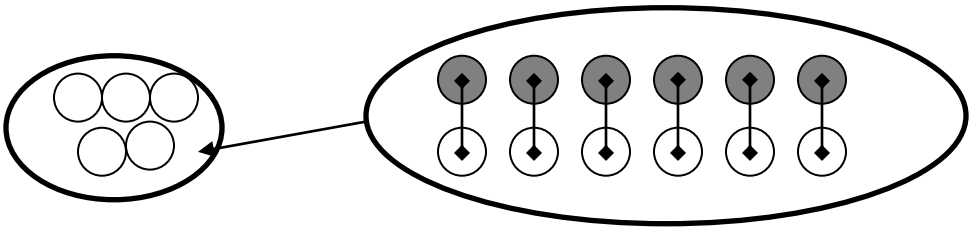
5 – ¯ 6

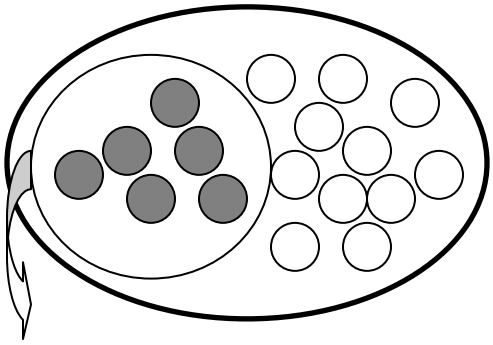


Begin with 5.

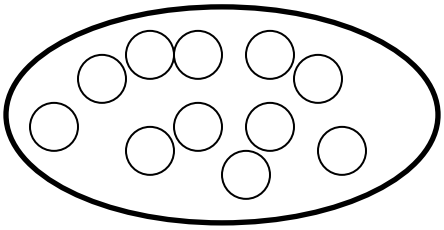
You want to remove ¯6, but there are no negative counters in group.

Add 6 zero groups so that there are 6 negative counters.





Remove ¯ 6 for the result.



EX: 3 - ¯ 4

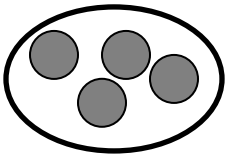
EX: 8 - ¯ 5

Model these and other examples as above.

At this point, ask students to make a generalization about what to expect when subtracting a negative from a positive. They should recognize that subtracting the negative gives the same result as adding the opposite. Subtracting $^{-}6$ from 5 has the same result as adding $^{+}6$ to 5. This shows clearly in the models and sketches.

Before modeling the next example, ask students if they think their generalization about adding the opposite will hold true when subtracting a negative from a negative.

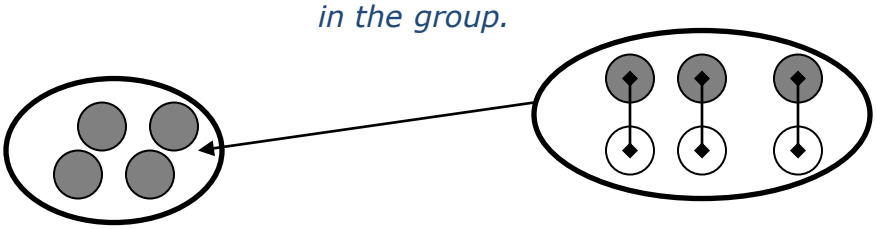
EX: - 4 – - 7

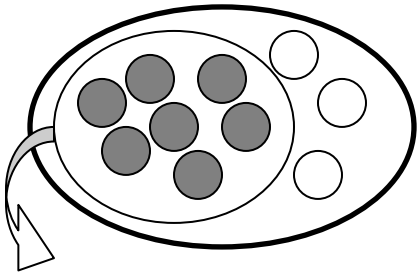


Begin with $^{-}4$.

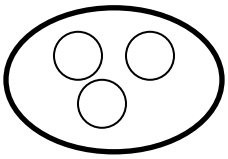
You want to remove $^{-}7$, but there are only 4 negative counters available.

Add 3 zero groups so that there are 7 negative counters in the group.





Remove $^{-}7$ for the result.



So…is $^{-}4 - {}^{-}7$ the same as $^{-}4 + 7$? It is INDEED! Let's see if it works with other examples.

EX: $^{-}9 - {}^{-}3$

EX: $^{-}10 - {}^{-}5$

This is usually the hardest process for students to generalize. They need to recognize that subtracting the negative gives the result of adding the opposite. It may take modeling and sketching many expressions.

<u>TEACHER NOTE</u>: Contextual problems need to be done here for students to relate the concept to.

c. Misconceptions/Common Errors –

- Students may misunderstand zero pairs.
- When using a number line to explore operations, students could misinterpret direction on a number line.

d. Additional Instructional Strategies –

Please see the technology link below for using a number line rather than counters.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

http://www.mathguide.com/lessons/Integers.html

f. Assessing the Lesson

Model and draw each of the sentences below. Be sure to include the solution!

a) $^{-}2 - 7 =$ ☐

b) $5 - 7 =$ ☐

c) $^{-}3 + 12 =$ ☐

d) $4 - {}^{-}8 =$ ☐

2. Teaching Lesson B – Multiplying Integers

a. Indicators with Taxonomy

7-2.8 Generate strategies to add, subtract, multiply, and divide integers. (B6)

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual

b. Introductory Lesson –Multiplying Integers

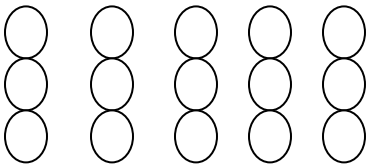
Materials needed:

- 2 color counters (in these notes, red is gray and yellow is white)
- colored pencils
- paper
- overhead counters or some way for you to model as they work (Smart Board, Promethean, etc.)

Students should be paired so they can discuss the math they are doing. Remind students that positive integers don't have to have the + sign to indicate that they are positive. You may choose whether or not to use the sign until students "believe" you.

1. Start by asking students why the following expression is true. Then ask them to model it with their counters. *Be sure they are recording their work so they can return to it later to make some generalizations.*

$$3 + 3 + 3 + 3 + 3 = 5 \bullet 3$$



Add 5 groups of 3 positive counters → total of $^{+}15$

This should help them remember that multiplication is the shortcut for repeated addition.

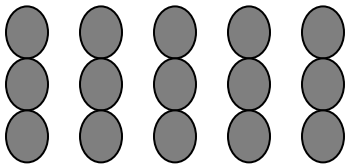
2. Ask students whether or not they think the following expression is true. Have them use their counters to help their thinking – yes or no is not sufficient.

$$^{-}3 + {}^{-}3 + {}^{-}3 + {}^{-}3 + {}^{-}3 = 5\,(^{-}3)$$

Move around the room as students work. If pairs seem stuck, direct their attention to the previous expression and ask them if they can make any connections. Ask students to share their solutions.

Possible model:

$^{-}3 + {}^{-}3 + {}^{-}3 + {}^{-}3 + {}^{-}3 = 5\,(^{-}3)$

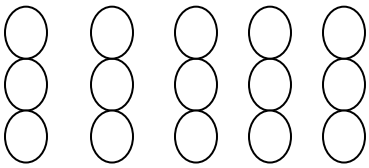


Add 5 groups of 3 negative counters → total of $^{-}15$

3. Compare the two expressions and their models.

(A)

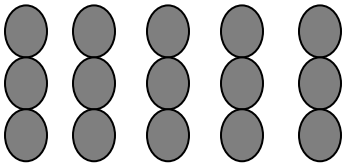
$3 + 3 + 3 + 3 + 3 = 5 \bullet 3$



Add 5 groups of 3 positive counters → total of $^{+}15$

(B)

$^{-}3 + {}^{-}3 + {}^{-}3 + {}^{-}3 + {}^{-}3 = 5 \bullet {}^{-}3$



Add 5 groups of 3 negative counters → total of $^{-}15$

Have students draw a T-chart and label the columns A and B (to correspond to the two expressions). Then ask them to use the chart to compare them.

Expression A **$5 \bullet 3$**	**Expression B** **$5 \bullet {}^{-}3$**
• both of the factors are positive • there are 5 groups of 3 positive counters • the product is positive	• one factor is positive and one is negative • there are 5 groups of 3 negative counters • the product is negative

One set of examples is NOT sufficient. You may either make a list of pairs of expressions for students to model or allow them to make their own. Either way, they need to be convinced that what is true for this pair is true for any pair. Students need to come to the conclusion that the product of 2 positive integers is positive; the product of a positive and a negative integer is negative. DON'T just give them the rule. Let them figure it out.

NOTE: Examples for pairs of examples should follow this form:

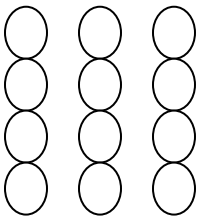
(+ factor) (+ factor) = positive product

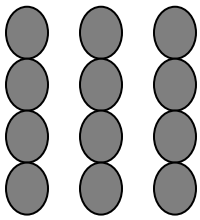
(+ factor) (- factor) = negative product [good opportunity to revisit the commutative property here]

4. This is a little tricky. Students need to have a good grasp of the previous part of this lesson before moving onto this part.

Begin by restating what they've already proved true.

$3 \bullet 4$ means, "3 groups of 4 positive counters"	$3 \bullet {}^{-}4$ means, "3 groups of 4 negative counters"



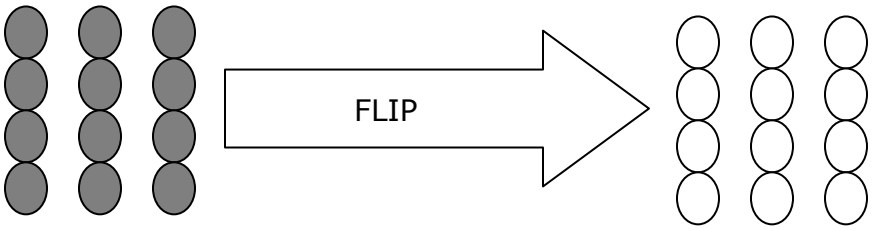


So, what about this one? ${}^{-}3 \bullet {}^{-}4$

Remember from the earlier discussions of integers that a – sign can be read as "the opposite of." So, ${}^{-}3$ can also be read as "the opposite of 3."

Thus, ${}^{-}3 \bullet {}^{-}4$ is the same as the opposite of $3 \bullet {}^{-}4$.

So, build $3 \bullet {}^{-}4$ AND take the opposite.



${}^{-}3 \bullet {}^{-}4 = {}^{+}12$

One example is NOT sufficient. You may either make a list of expressions for students to model or allow them to make their own. Either way, they need to be convinced that what is true for this expression is true for any expression. Students need to come to the conclusion that the product of 2 negative integers is positive. DON'T just give them the rule. Let them figure it out.

NOTE: Examples should have 2 negative factors.

Generalizations students need to make about multiplying integers from building and drawing models:

- ***2 positive factors yield a positive product***
- ***a positive factor and a negative factor yield a negative product***
- ***2 negative factors yield a positive product (TRICKY, TRICKY!!)***

c. Misconceptions/Common Errors –

- Students may misunderstand zero pairs.
- When using a number line to explore operations, students could misinterpret direction on a number line.

d. Additional Instructional Strategies –

Please see the technology link below for using a number line rather than counters.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

http://www.mathguide.com/lessons/Integers.html

f. Assessing the Lesson

Formative assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

3. Teaching Lesson C – Dividing Integers

a. Indicators with Taxonomy

7-2.8 Generate strategies to add, subtract, multiply, and divide integers.

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual

b. Introductory Lesson – Dividing Integers

Materials needed:

- student notes from multiplying integers
- paper
- pencil

Pair students so they can talk about the math they are doing.

Begin by asking students to recall the work they did modeling multiplication of integers. There were three generalizations they made as they worked with counters to model and draw. Ask for and record them on a class chart.

- positive x positive = positive
- negative x positive = negative
- negative x negative = positive

Ask students what the inverse of multiplication is. [division]

***Use "fact families" to help students apply the generalizations they made about multiplying integers to dividing integers.*

You will need more than these examples. You should also concentrate on one kind of family at a time to start.

Tell students that the complete expression gives the signs of the terms. Move around the room as students work, checking answers and asking questions.

Possible questions and statements to guide thinking for any of the "families."

- How do you know your signs are correct?
- Rewrite your division sentence as a multiplication sentence with a missing factor.

POSITIVE Fact Family	MIXED Fact Family	NEGATIVE Fact Family
$8 \bullet 4 = 32$	$2 \bullet {}^{-}9 = {}^{-}18$	${}^{-}5 \bullet {}^{-}6 = 30$
$4 \bullet \square = 32$	$\square \bullet 2 = {}^{-}18$	$\square \bullet {}^{-}6 = 30$
$32 \div \square = 8$	$\square \div {}^{-}9 = 2$	$30 \div {}^{-}6 = \square$
$\square \div 8 = 4$	${}^{-}18 \div \square = {}^{-}9$	$\square \div {}^{-}5 = {}^{-}6$

c. Misconceptions/Common Errors

- Students may misunderstand zero pairs.
- When using a number line to explore operations, students could misinterpret direction on a number line.

d. Additional Instructional Strategies –

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Once you have completed all four operations with your students, you can let students practice working with integers on the following Internet site. http://funbrain.com/linejump/index.html

f. Assessing the Lesson

Formative assessment is embedded within the lesson through questioning and observation; however, other formative assessment strategies should be employed.

Exit slip:

Explain why the generalizations for multiplication of integers are also true for division of integers.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-2.8 Generate strategies to add, subtract, multiply, and divide integers. (B6)

The objective of this indicator is to generate which is in the "conceptual knowledge" of the Revised Taxonomy. To create is to put together elements to form a new, coherent whole or to make an original product. Conceptual knowledge is not bound by specific examples. The learning progression to **generate** requires students to recall and understand the concept of integers. Students explore problem situations (story problems) and explore various strategies to solve those problems by applying their conceptual knowledge of integers. Students translate their understanding of concrete and/or pictorial representations by generalizing connections between their models and real world situations (7-1.7). Students should use these procedures in context as opposed to only rote computational exercises and use correct and clearly written or spoken words to communicate about these significant mathematical tasks (7-1.6). Students formulate questions to prove or disprove their methods (7-1.2) and generate mathematical statements (7-

1.5) about these operations. They should evaluate the reasonableness of their answers using appropriate estimation strategies.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. In golf, the goal is to have the lowest score among all the players. Listed below are the scores of four players. Which player has the best score?

   Player A: -8 Player B: -12 Player C: 6 Player D: 2

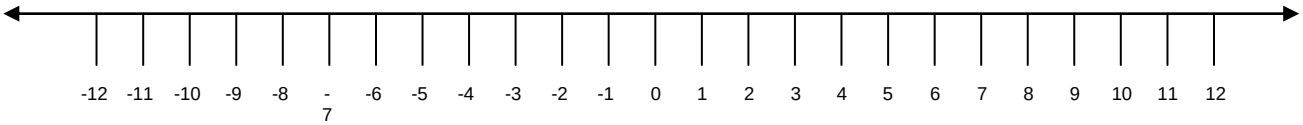
   a) Player A b) Player B c) Player C d) Player D

2. What is the value of -3 – (- 6)?

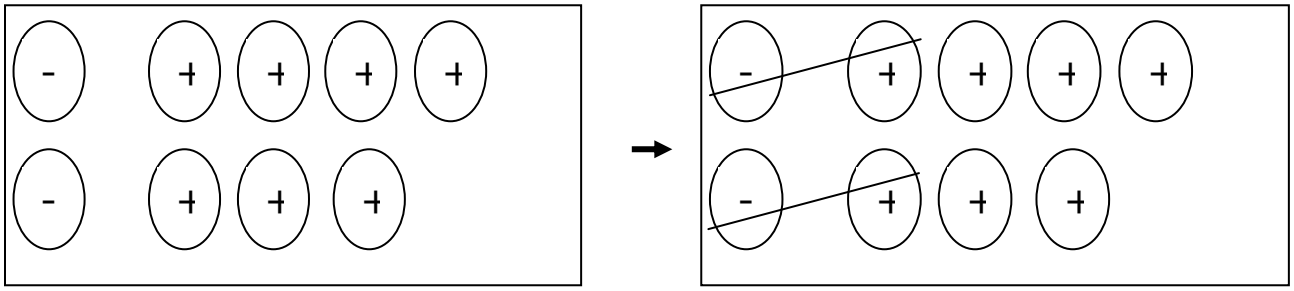
3. The low temperature on Saturday was -9º F. The high temperature on Sunday was 14 degrees warmer than the low temperature. What was the high temperature on Sunday?

4. In 1940, the surface area of the Dead Sea was about 980 square kilometers. From 1940 to 2001, the average rate of change in surface area was about - 6 square kilometers per year. What was the surface area of the Dead Sea in 2001?

5. Use the number line below to show a method to solve -6 + 9 = x.



6. The model below demonstrates which equation?

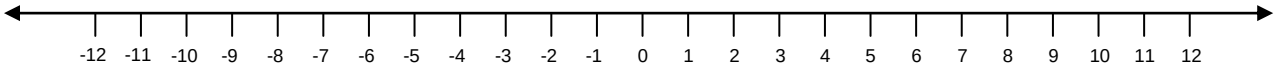


a. – 2 – 7 = 5
b. – 2 – 7 = 9
c. – 2 + 7 = 5
d. – 2 + 7 = 9

7. If there are 10 positive counters and 6 negative counters, how many zero pairs can be created?

8. Use models to show 5 – 8.

9. Use the number line below to find the sum of 4 + (-7).



10. What does -6 ÷ 2 mean?

MODULE

2-2

Solve Mathematical Situations

This module addresses the following indicators:

7-3.4 Use inverse operations to solve two-step equations and two-step inequalities. **(C3)**

7-3.5 Represent on a number line the solution of a two-step inequality. **(B2)**

This module contains 3 lessons. These Lessons are **INTRODUCTORY ONLY.** Lessons in S³ begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

7-3.4 Use inverse operations to solve two-step equations and two-step inequalities. **(C3)**

- In sixth grade, students use inverse operations to solve one-step equations that involve only whole numbers (6-3.5). They have not yet had any instruction in solving inequalities. The foundation begins to be built in the sixth grade with simple one step equations (with whole numbers only).
- In eighth grade, students apply a procedure to solve multi-step equations (8-3.4). In Algebra I, students carry out a procedure to solve linear equations for one variable algebraically (EA-4.7)

7-3.5 Represent on a number line the solution of a two-step inequality. **(B2)**

- **Key Vocabulary/Key Terms:** * These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.
  Equation*
  Expression*
  Inequality*
  Inverse*
  Inverse operations*
  Multiplicative inverse*
  Additive inverse*
  Two-step equations*
  Two-step inequalities*
  Solution*

II. Teaching the Lesson(s)

In grade six, students use inverse operations to solve one-step equations that involve only whole numbers. Although sixth grade students do represent algebraic relationships with variables in simple inequalities, they have not yet had any instruction in solving inequalities. As grade eight students prepare for Algebra I, a strong foundation in solving equations is a necessity. The foundation begins to be built in grade six with simple one step equations (with whole numbers only), transitions to one and two-step equations (with rational numbers) and inequalities in grade seven, and the process continues into grade eight with the focus on solving inequalities and multi-step equations.

See sixth grade Algebra Indicator 6-3.2: (Apply order of operations to simplify whole-number expressions) for information on prior knowledge for order of operations.

A more in-depth look at the concept of equality/inequality began in the 6th grade and continues throughout seventh and eighth grade.

Teaching Lesson A: Solve two-step equations

Seventh grade will be the first time that solving any type of inequality (both one-step and two-step) is introduced and the first time to be exposed to solving two-step equations.

If students have a solid foundation with the concepts of equality/inequality and that understanding is applied to solving equations and inequalities, the notion of "balancing" both sides of an equation or inequality should not present a problem for students.

Note that students in grade seven are to use inverse operations to solve equations and inequalities. A connection should be made to order of operations: when solving equations or inequalities (particularly two steps), we proceed in isolating the variable by doing the order of operations in reverse order.

In seventh grade, students should also understand that solutions to inequalities can be written as an inequality, in set notation, or graphed on a number line. It is important to distinguish between the meaning of < verses ≤ (less than vs. less than or equal to) and > verses ≥ (greater than vs. greater than or equal to), particularly in regards to their graphs.

For this indicator, it is **essential** for students to**:**

- Understand the concept of inverse operations
- Understand the concepts of equivalency and inequality
- Use substitution to check solutions of two-step equations and inequalities
- Distinguish between the meaning of < verses ≤ and > verses ≥, particularly in regards to their graphs
- Use concrete and pictorial models to solve equations

For this indicator, it is **not essential** for students to:

- Solve equations or inequalities that require operational use with negative integers

[Caution should be exercised when introducing solving inequalities that include negative numbers. The tendency is to simply tell students to reverse the inequality symbol when multiplying or dividing by a negative number. However, without understanding why, a student will soon forget that "rule" of inequalities. It is important that students understand "why" the sign is reversed when multiplying or dividing by a negative number.]

***a.* Indicators with Taxonomy**

7-3.4 Use inverse operations to solve two-step equations and two-step inequalities.

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. **Introductory Lesson: Solve two-step equations**

Warm-up exercises: How do one and two-step equations differ?
Give several problems for students to solve that involve solving linear one-step equations. (e.g.)

m + 17 = 95, **3n = 21,** **16 = 4p,** **x÷3 = 18**

1. Review inverse operations. (The inverse operation of addition is subtraction, etc.)

2. Review the order of operations involved when using inverse operations. Addition and/or subtraction are completed first, and then multiplication and/or division are completed next.

3. Demonstrate the value of algebra tiles using a set of overhead algebra tiles.

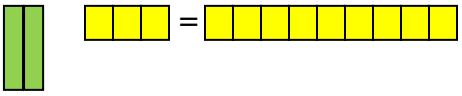
e.g.  = x  =1  = -1

NOTE: If algebra tiles are not available, a small plastic cup works well for the variable and chips may be used for the integers.

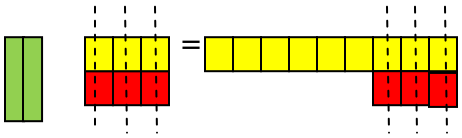
Have students draw these (algebra tiles and values) at the top of their paper for reference during class work.

Introduce and model several two-step linear equations using the algebra tiles.
For example:

2x + 3 = 9

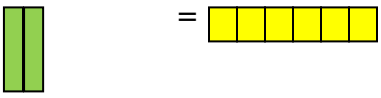


2x + 3 - 3 = 9 - 3 both sides

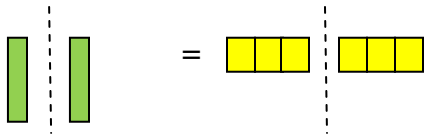


Subtract three from

2x = 6

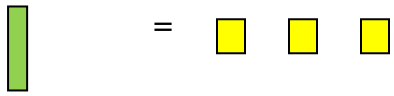


$\frac{2x}{2} = \frac{6}{2}$ groups



Divide each side into 2

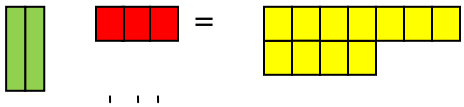
x = 3 groups



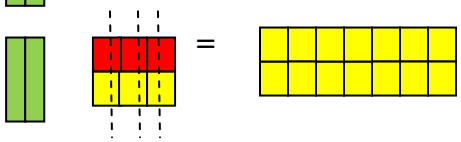
, x = 3 items in each of 2

4. Show another example such as 2m - 3 = 11.

2 m - 3 = 11

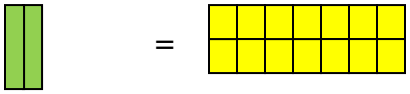


2x -3 +3 = 11+3

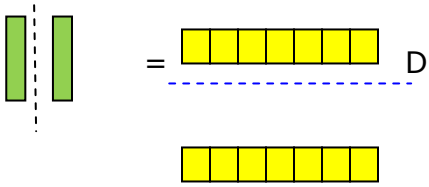


Add three to both sides

2x = 14



$\frac{2x}{2} = \frac{14}{2}$ groups



Divide each side into 2 equal

x = 7

x = 7 items in each of the 2 groups

5. You will not be able to model every single type of two-step equation. Just use the manipulatives to help explain why you add and subtract first, and then multiply and divide. Your students must see and understand that when solving equations you use Order of Operations in *reverse* to find the variable. Solving equations is, trying to undo a finished equation. Therefore, start "undoing" with the operations of addition and subtraction and move back through the order of operations. (Note: Refer to the Order of Operations table method described in S³- 6th grade module #2-2, for a visual reference to support the concept of operating in reverse through the order of operations.)

6. Group students and assign at least five, two-step linear equations for them to solve using algebra tiles (or a drawn visual representation of the

algebra tiles) and the correct algorithm. As students begin, remind them to note which operation they should undo first when solving each equation.

7. Have each group demonstrate one solution with algebra tiles for the class.

8. Afterwards, have the students solve the equations without the manipulatives. Use the algorithm to demonstrate how to solve two-step equations.

9. Using a form of a Kagan strategy called Rally Table, students might play "Pass the Folder". Cooperative teams of 4 are given one piece of paper in a folder and one pencil (or a white board w/ dry erase marker, etc). Set a time limit. Provide each team a problem to solve. One team member will begin the process by writing the first step to solve the two-step equation. He/she passes the written work to the next team member. The second team member checks the work of the first, and adds the next step to the problem. The equation continues to go around the team as each student checks the prior work and adds a step. When the time limit expires, all pencils are placed on the team tables. The teams take turns sharing their steps/solutions with the rest of the class.

c. Misconceptions/Common Errors –

Student may perform the inverse operation to only one side of the equation.
Student may use an incorrect inverse operation.
Students may become confused when given problems with the variable on the right side of the equal sign, such as $25 = 3x + 7$.
Students may undo the multiplication or division first, and then undo the addition or subtraction.
Some students may try to simplify across the equal sign. For example, $x + 7 = 7$; the student may add 7 to the right side and get $x = 14$.

d. Additional Instructional Strategies –

"Secret Numbers"
Materials- For each group of three students you need: 8 small containers, 80 countable objects, 1 code sheet prepared by the teacher

Review how to solve equations. Then explain that today they will learn how to write equations by playing a game. In this game, each team will try to solve for the other teams' secret numbers. This strategy can be used at any time during instruction.
(I have found it most useful when I introduce solving algebraic equations.)

Procedure:
Divide the class into groups of two or three.
Instruct each group to select a recorder to keep an account of the events beginning in step 6.
Distribute 8 containers and 80 counters to each group.

Each group is assigned a different letter of the alphabet. Each group's 8 containers are labeled with the lowercase form of that alphabet letter. (If the same lesson is taught repeatedly, the same containers can be used over and over.)
Each group chooses a "secret number" between one and ten and reports it to the teacher.
The teacher keeps a record of all "secret numbers" on a code sheet.
Have each group place their "secret number" of counters in each of their containers.
Each group will now have 8 containers; each of which contains the same amount of counters and are labeled with the same letter of the alphabet.

Discuss ways to express the total number of counters in all 8 containers.
For example: m+m+m+m+m+m+m+m or x+x+x+x+x+x+x+x.
Build on that idea: 8m or 8x.
Have each group exchange some, but not all containers with another group.
For example, 3 m's are exchanged for 3 x's. Each group records its holdings in the following manner: m+m+m+m+m+x+x+x or 5m + 3x and x+x+x+x+x+m+m+m or 5x + 3m.

Each group confers with the teacher who checks the code sheet to tell them the total number of counters their group is holding. For example, the first group has 5m + 3x counters. The teacher tells them they have 22 counters.

Discuss if necessary how to write an equation to express the total number of counters. For example, 5m + 3x = 22.

Each group has written an equation to describe their 'holdings' and should solve for the unknown variable. Students continue to trade until they have discovered each group's "secret number" or until time has run out.

Encourage students to keep their group's solutions a secret so that each group can make the discoveries on their own.

Return to whole group. Explain that they should now be familiar with how to write algebraic expressions and solve algebraic equations that represent real objects.

e. Technology

Virtual Manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictoral representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

http://illuminations.nctm.org/LessonDetail.aspx?ID=26
http://nlvm.usu.edu/en/nav/frames_asid_189_g_3_t_2.html?open=activities&from=grade_g_3.html (virtual algebra tiles)

f. Assessing the Lesson

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized test.

- Assign a two step equation for students to trade with a partner. They should show their solution by using models and by showing their work step-by-step.
- In a two-step equation which operation do you 'undo' first?
- Have students write a step-by-step explanation for solving an equation such as
  $4x - 7 = 13$.
- How do you check an equation to see if your solution is correct?

2. Teaching Lesson B

***a.* Indicators with Taxonomy**

7-3.4 Use inverse operations to solve two-step equations and two-step inequalities.

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

***b.* Introductory Lesson: Represent real-life situations using inequalities**

Complete a lesson on inequalities modeled after **Teaching Lesson A** (algebra tiles). Additionally, ask students to name several numbers that are possible solutions. Have them substitute these in the inequality to check.

***c.* Misconceptions/Common Errors –**

Student may perform the inverse operation to only one side of the equation.
Student may use an incorrect inverse operation.
Students may become confused when given problems with the variable on the right side of the equal sign, such as $25 = 3x + 7$.
Students may undo the multiplication or division first, and then undo the addition or subtraction.
Some students may try to simplify across the equal sign. For example, $x + 7 = 7$; the student may add 7 to the right side and get $x = 14$.

d. **Additional Instructional Strategies**

Number of players: 2
Materials: Index cards, spinner, counters

Copy onto index cards, the numbers 0 through 30- one number per card.
Shuffle the cards and place them face down in a pile.
Label the sections of a spinner with the following inequalities:

$3x - 5 > 10$ $17 + c < 25$ $4p > 16$ $d/2 + 15 < 20$
$2f + 60 > 75$ $m - 42 > 21$

*Create a large copy the game board below.

Both partners place a counter in the first square of a row. Decide which partner will go first.
In turn, each partner selects an equation card and spins the spinner.
You may only advance one square on each turn, *if* the number on the card is part of the inequality's solution set.
The winner is the first partner to reach the tenth square

e. **Technology**

Virtual Manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictoral representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

http://illuminations.nctm.org/LessonDetail.aspx?ID=26

f. **Assessing the Lesson**

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized test.

Solve the inequality.

1. $0.6 + 3p > 1.8$
   a. $p > 0.4$
   b. $p > 0.8$
   c. $p > 4$
   d. $p > 40$

2. $m \div 2 - 1 < 6$
   a. $m < 10$
   b. $m > 10$
   c. $m < 14$
   d. $m > 14$

3. Teaching Lesson C

Now that students understand how to solve two step inequalities they should be ready to begin graphing solutions on a number line. Remind students that in the sixth grade and since, they have had various experiences with integers on a number line.

For this indicator, it is **essential** for students to**:**

- Distinguish between the meaning of < verses ≤ and > verses ≥
- Understand the inequalities have several solutions unlike equations that only have one
- Understand the meaning of open and closed circles
- Understand how to determine the direction in which to shade
- Translate from graph to inequality and inequality to graph
- Check values from the graph by using substitution

For this indicator, it is **not essential** for students to:

- Determine solutions and represent on a number line the solutions to multi-step inequalities

a. Indicators with Taxonomy

7-3.5 Represent on a number line the solution of a two-step inequality. (B2)

Cognitive Dimension: Understand
Knowledge Dimension: Conceptual

b. Introductory Lesson: Graphing solutions on a number line

Graphing inequalities on number lines is done to represent the solution to inequalities. It provides a visual representation of the answer. Graphing inequalities is a simple process. See the examples below. Adapted from: http://library.thinkquest.org/20991/alg/NGraphing.html#Inequalities

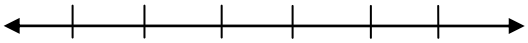
Part 1- Drawing the Number Line: How do you graph the solutions to a two-step inequality using a number line?

1- Draw a straight horizontal line.

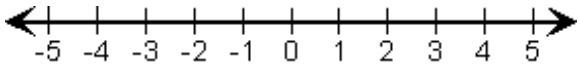
_____

2- Draw arrows on both ends of the horizontal line, pointing outward. The arrows represent infinity.

3- Draw short vertical lines at equal intervals. The number of lines (or tick marks) and intervals depends on the numbers being graphed



4- Write the numbers in the vicinity of the number being graphed below the horizontal line, one number for each tick.

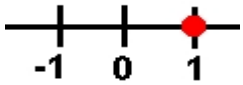


The following graphing examples adapted from ehow.com**.** http://www.ehow.com/how_4425481_graph-inequalities-number-line.html

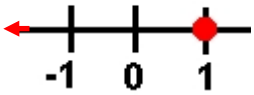
Part 2- Graphing Inequalities with Equal to or Greater / Less Than

1- Figure out what the inequality solution requires. (x is equal to or greater than 1)

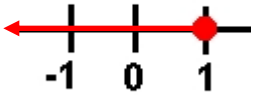
2- Draw a solid dot on the number line where the number being graphed is represented. The solid dot represents the starting point for the inequality solution
and it is part of the solution.



3- Draw an arrow on the number line pointing in the direction of the inequality to infinity.



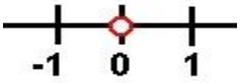
4- Draw a line, thicker than the horizontal line, from the dot to the arrow. This represents all of the numbers that fulfill the inequality. (complete number line by adding the arrow on the other end to signify infinity.



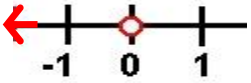
Part 3- Graphing Inequalities with Greater / Less Than

1. Figure out what the inequality solution requires. (x is less than 1)

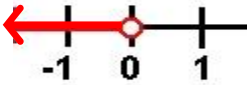
2. Draw an open dot on the number line where the number being graphed is represented. The open dot represents the starting point for the inequality solution but is not part of the solution.



3. Draw an arrow on the number line pointing in the direction of the inequality to infinity.



4. Draw a line, thicker than the horizontal line, from the dot to the arrow. This represents all of the numbers that fulfill the inequality.



c. Misconceptions/Common Errors –

Students may solve inequalities but fail to graph solutions correctly because they are unclear about which direction to shade. This is an issue especially with problems where the variable is on the right side: For example, $-5 < x$.

When students have trouble with two-step inequalities, it is often because they do not understand two-step equations.

d. Additional Instructional Strategies –

This strategy will introduce students to "open" and "closed" notations when graphing solutions. Have students write "open" and "closed" at the top of their notes and place the appropriate signs beneath each. Draw examples of graphs for students. Remind students also that the number line goes on indefinitely in each direction. Remember to

question students to name several integers that will make the solution true. For example, if $x < 5$ is the solution, students may substitute 4, 0, or -3 into the inequality to prove the solution true. Have students work in cooperative pairs to solve and graph the solutions to two step inequalities. They must agree on a solution before moving to the next problem.

Sketch several graphs on the overhead that show the solution sets of several inequalities. Have students state the corresponding inequalities.

Explain to your partner the difference in using an open and a closed circle when graphing the solution of an inequality.

e. Technology

Virtual Manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictoral representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Student support:
http://www.algebra.com/algebra/homework/Number-Line/NumberLine.faq.question.115140.html

f. Assessing the Lesson

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized test.

EXIT SLIP:

- What part of solving and graphing inequalities do you find the most difficult? Why?
- Explain to your partner the difference in using an open and a closed circle when graphing the solution of an inequality.
- Solve each inequality. Then show the solution on a number line.
  $x + 5 \leq 21$
  $5c - 21 < 9$
  $Z \div 4 + 8 < 16$
  $4f - 17 \geq 25$

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-3.4 Use inverse operations to solve two-step equations and two-step inequalities. **(C3)**

The objective of this indicator is use which is in the "apply procedural" of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus is to gain computational fluency with solving two step equations and inequalities, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **use** requires students to explore the concepts of equivalency, inequality and variables using concrete models such as balance scales. Student use this understanding of balance to analyze a variety of examples of simple two step equations and inequalities. Students use inductive reasoning (6-1.3) to generalize connections (6-1.7) among types of equations/inequalities and generate mathematical statements (6-1.5) related to how these equations can be solved. They use concrete manipulatives and pictorial to support their conceptual understanding. Student engage in meaningful practice to support retention of these processed and check their answers.

7-3.5 Represent on a number line the solution of a two-step inequality. **(B2)**

The objective of this indicator is to represent which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To represent is to translate from one form to another and to understand is to construct meaning; therefore, students are constructing meaning between the graphical representation and algebraic representation of solution to inequalities. The learning progression to **represent** requires students to understand the meaning of inequality symbols and open/closed circles. Students understand that the solution set to an inequality consists of multiple solutions. They explore this concept through the use of concrete models such as a balance scale. They use this understanding to graph the given inequality on a number line. They recognize that the graph and the inequality are equivalent symbolic representations of the same relationship (7-1.4). Students then evaluate their graph to determine if it is correct by verifying that solutions are in the shaded region. Students explain and justify their answers using correct and clearly written and spoken words (7-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Ask students to identify and correct the error in the following solution of the equation 3 x - 7 = 22.

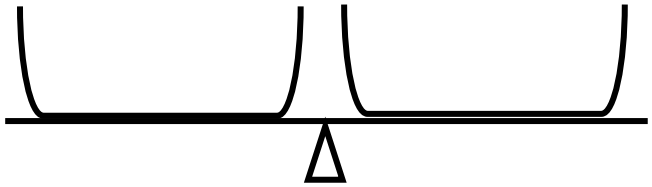
$$3x - 7 = 21$$

$$\frac{3x}{3} - 7 = \frac{21}{3}$$

$$x - 7 = 7$$

$$x = 14$$

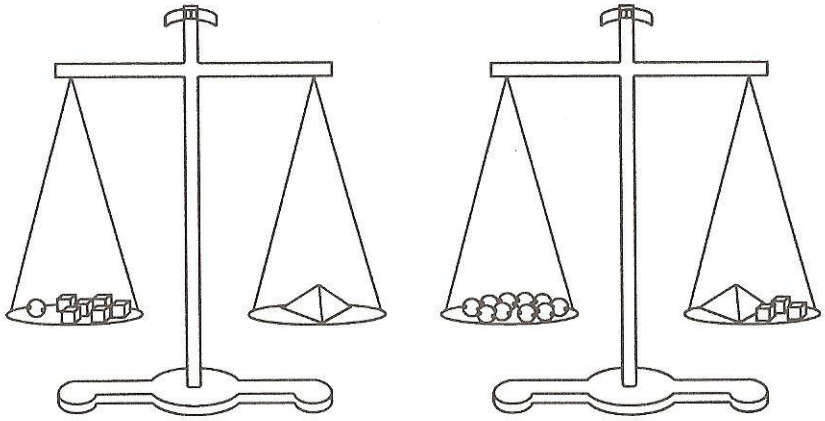
2. Have students write a step-by-step explanation for solving an equation such as 4x – 7 = 13. Use the balance scale below.



3. Solve the equation 3y + 6 = 42.

4. Solve 60 = n ÷ 4 – 20.

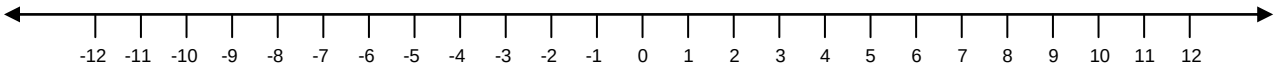
5. Use the balances below.
Jane set up the following scales to illustrate an equation.



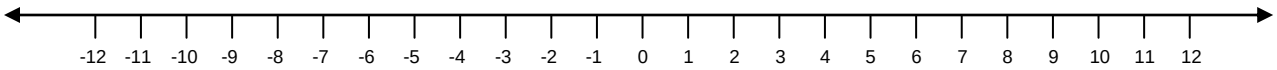
How many spheres would two pyramids weigh?

8. Solve each inequality. Then show the solution on the number line below.

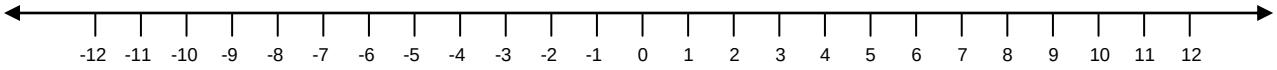
$x + 5 \leq 21$



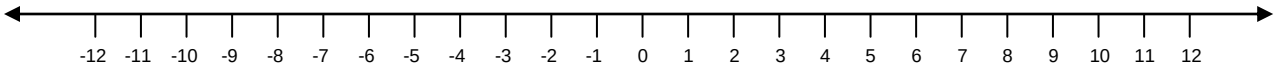
$5c - 21 < 9$



$z/4 + 8 < 16$



$4f - 17 \geq 25$



9. What is the value of *x*:
$-18x \div 3 \geq 12$

a) $x = 2$ b) $x \geq -2$ c) $x \leq 2$ d) $x \leq -2$

10. Explain the steps you would use to solve the equation $5y + 2 = 17$.

MODULE

2-3

Patterns, Relationships, and Functions
and
Representations, Properties, and Proportional Reasoning

This module addresses the following indicators:

7-3.1 Analyze geometric patterns and pattern relationships
7-3.7 Classify relationships as either directly proportional, inversely proportional, or nonproportional.
7-3.6 Represent proportional relationships with graphs, tables, and equations. (B2)

This module contains 2 lessons. These Lessons are **INTRODUCTORY ONLY.** Lessons in S3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

7-3.1 *Analyze geometric patterns and pattern relationships.*

In third grade, students created numeric pattern that involve whole-number operations (3-3.1). In fourth grade, analyzed numeric, nonnumeric, and repeating patterns involving all operations and decimal patterns through hundredths (4-3.1). In fifth grade, students represented numeric, algebraic and geometric pattern in words, symbols, algebraic expressions and algebraic equations (5-3.1). In sixth grade, they analyzed numeric and algebraic pattern and pattern relationships (6-3.1).

In eighth grade, students represent algebraic relationship with equations and inequalities (8-3.2).

7-3.6 *Represent proportional relationships with graphs, tables, and equations.*

In sixth grade, students used proportions to determine unit rates (6-5.6).
In eighth grade, students translate among verbal, graphic, tabular and algebraic representations of linear functions (8-3.1).

7-3.7 *Classify relationships as directly proportional, inversely proportional, or non-proportional.*

In sixth grade, students used proportions to determine unit rates (6-5.6).
In eighth grade, students translate among verbal, graphic, tabular and algebraic representations of linear functions (8-3.1).

- **Key Concepts/Key Terms**

  * These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

  Proportional relationships*
  Directly Proportional*
  Inversely Proportional*
  Nonproportional*
  Multiple representations (table, graph and equation)*
  Geometric patterns*
  Arithmetic patterns*
  Figurate Numbers
  Triangular numbers
  Square numbers
  Sequence*

II. Teaching the Lesson(s)

Beginning in fourth grade students interpreted data in tables, line graphs, bar graphs, and double bar graphs with scale increments greater than one. Building
on prior knowledge, seventh grade students will represent relationships through graphs, tables, and equations.
In sixth grade students determined whether two ratios were equivalent and used proportions to determine units rates.

From the study of patterns seventh grade students should advance to classifying relationships as:

(a) directly proportional (when one quantity always changes by the same factor as another, $^{y}/_{x}$ = k or y= kx, where k is a constant);
(b) inversely proportional (when one quantity decreases by the same factor as the other increases, xy= k or y= $^{k}/_{x}$ where k is a constant); or
(c) nonproportional.

Teacher Note: When using two ordered pairs (x_1, y_1) and (x_2, y_2), teachers may refer to these alternative forms:

Directly proportional is $\frac{y_1}{x_1} = \frac{y_2}{x_2}$. Inversely proportional is $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

An overall curriculum theme seen throughout the seventh grade Algebra strand is the concept of a constant rate of change (slope) and the tables, equations, and graphs that result from a relationship that has a constant rate of change. It is important that students be given ample opportunities to discover the connection between direct proportionality and the table, equation, and graph this relationship produces; as it leads to the gentle introduction of slope (Module 2-4) and then linear functions in the eighth grade. As students graph directly proportional relationships, they should be able to identify the unit rate as the slope of the related line.

Representing inversely proportional relationships in tables, equations, and graphs allows students to understand that not all tables and equations produce similar graphs and that slope only exists when there is a "constant" rate of change. This understanding is important as the focus in eighth grade will be on the table, equations, and graphs derived from linear functions.

1. Teaching Lesson A:

Students have used patterns all their lives. They began to learn about and study patterns as early as kindergarten. In kindergarten through second grade, students progress from identifying patterns to translating patterns into rules. The emphasis on the creation of numeric patterns begins in third grade, and in fourth and fifth grades, students will transition to analyzing patterns, then representing these patterns in words, expressions, and equations. Middle school continues this study of patterns by placing

emphasis on numeric and algebraic patterns in the sixth grade, with the seventh grade focusing on geometric patterns.

For this indicator, it is **essential** for students to**:**

- Understand the meaning of geometric pattern
- Compare the magnitude of numbers
- Understand the concept of growing patterns

For this indicator, it is **not essential** for students to:

- None noted

a. Indicators with Taxonomy

7-3.1 Analyze geometric patterns and pattern relationships. (B4)

Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson: Explore Geometric patterns

Teacher lesson notes:
Some texts define geometric sequences and "figurate numbers" differently. Because the focus of this indicator is geometric patterns, we will make no distinction other than to explain how texts see them differently. The explanation is provided to alert you as a teacher that student learning opportunities should include all types without students having to identify the various types. Also, some texts label geometric patterns as growing patterns.

Geometric sequence – Each successive term is determined by multiplying the preceding term by a fixed number, the ratio. Take a family tree for example. You start with one couple who each has two parents. Each of those have two more parents (8 grandparents for the original couple), etc. In table form the sequence becomes:

Step/Number of the term	Term
1 (original couple)	$2 = 2^1$
2 (4 parents)	$4 = 2^2$
3 (8 grandparents)	$8 = 2^3$
4 (16 great grandparents)	$16 = 2^4$

So, following this pattern/sequence the nth term would be 2^n.

Figurate numbers – Typically we think of we think of triangular or square numbers – representations can be arranged in the shape of triangles or squares. There are figurate numbers for each polygon and many interesting patterns can be found by exploring the relationships. Take square numbers for example:

* 1 star	* * * * 4 stars	* * * * * * * * * 9 stars

In table form the pattern becomes:

Step/Number of the term	Term
1	1^2
2	2^2
3	3^2

So, following this pattern/sequence the nth term would be n^2 .

The number of stars does not depend on the previous number of stars, therefore it is not geometric.
The difference between the number of stars is not constant, and therefore it is not arithmetic.

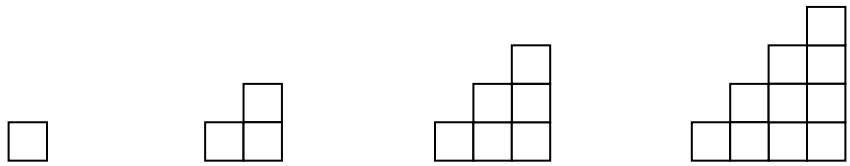
Lesson: Explore Geometric patterns
Review of prior knowledge: Have students draw or build examples of growing patterns. Circulate about the class asking student's to share the reasoning that supports their examples. Look for patterns that use addition or multiplication to find the next term. Have students share their patterns – and identify which patterns used are either addition or multiplication. As students share, implement vocabulary such as "number of term (or step)" and "term". Ask students, "What are some differences you've noticed about the examples we've shared? Listen for explanations that reference the way terms were determined in a specific pattern (addition or multiplication). If there are no student patterns that use multiplication, be prepared to provide an example before asking about the difference.

Introduction: Demonstrate a three or four step pattern. (You may decide to use the star pattern shown above. If so, provide materials, i.e., grid paper, blocks, toothpicks, etc.) Ask students to extend the pattern and write their justification for the extension. Students should work in small

groups to share their thinking. When appropriate, groups will share their patterns and justifications for their extensions.

Ask students how they might determine the 100th term of a pattern? After some discussion, explain the need to identify how patterns are determined and understand the applications needed to create the next term. Connect this to the present work and explain that they have begun working with **geometric patterns** – patterns that rely on multiplication – including exponents they used in the first nine weeks.

Tell students about visiting an art exhibit where you saw a sculpture entitled "Stairs to the Stars". The artist created steps by placing a series of concrete blocks in a pattern that looked like this.



After viewing the exhibit you wondered how many blocks would be needed if the artist made a series of 50 stacked blocks rather than the series of 4 parts to the sculpture.

Challenge the students to determine how many blocks would be needed in the 50th part *without* drawing all the parts between 4 and 50. If students are struggling, you might suggest that they make a table or chart and look for relationships/patterns.
(Have cubes/manipulatives available for students to visualize their thinking with something concrete.)

Additional learning opportunities will be needed by students.

c. Misconceptions/Common Errors

Students are accustomed to looking for repeating or growing patterns that involve simple addition – either performing the operation of addition to get the next step or adding another shape/figure, etc. When observing the class as they tackle this problem, look for students who may not have a solid understanding of those simple relationships; and are thus not able to move forward to more complex thinking.

d. Additional Instructional Strategies

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual Manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictoral representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Possible resources:
Interactivate
http://www.shodor.org/interactivate/discussions/Pattern/
Virtual Manipulatives
http://NLVM.usu.edu/en/nav/vlibrary.html

f. Assessing the Lesson

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

- Use teacher observation and questioning during the introductory lesson.
- Use a ticket out check method - Show students a geometric pattern with at least three terms. Ask students to extend the pattern and explain their reasoning. Their response is a ticket out.
  Note: The verb in the indicator is "analyze" and the verb in this assessment is "extend". Students will need to "analyze" prior to extending. Therefore the strategy is appropriate for the indicator.

2. Teaching Lesson B

7-3.7 Classify relationships as either directly proportional, inversely proportional, or nonproportional. (B2)

For this indicator, it is **essential** for students to know**:**

- Understand the meaning directly proportional
- Recognize examples of
- Inversely Proportional
- Non-proportional – A relationship exists between quantities but the relationship is neither directly proportional nor inversely proportional.

For this indicator, it is **not essential** for students to:

- None noted

7-3.6 Represent proportional relationships with graphs, tables, and equations. (B2)

For this indicator, it is **essential** for students to**:**

- Represent directly proportional relationships
- Identify the unit rate as the slope of the related line
- Represent inversely proportional relationships
- Recognize if a relationship is directly, inversely or not proportional
- Understand the relationship between the multiple representations (graph, table and equation) of proportional relationships

For this indicator, it is **not essential** for students to:

- Represent non-proportional relationships

a. Indicators with Taxonomy

7-3.7 Classify relationships as either directly proportional, inversely proportional, or nonproportional. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

7-3.7 Represent proportional relationships with graphs, tables, and equations. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson for Directly Proportional

NOTE: In this lesson students will be asked to create T-Charts and Graphs. These should be done individually because the charts and graphs will be used in the next module of the nine weeks. They may compare and share but each student will need his/her own T-Chart and Graph for future use. If students are absent, they should be required to make up this portion of the lesson so they will have the necessary material for later use.

*Tell the students that David, the son of a friend of yours, is getting a part-time job to earn money during the holidays. He is making $7.50 per hour. Ask students to make a T-Chart showing how much David will make for each hour earned if he works up to ten hours. Remind students to label the T-Chart and title it "David's Earnings". Ask students to compare their charts for possible errors.

Using grid paper, students should graph the data with each line on the X axis valued at increments of one- to represent the number of hours worked; and each line on the Y axis valued at increments of $7.50- to represent the amount of money earned. Remind students to label the graph and title it

"David's Earnings". Circulate and ask probing questions to monitor student's work. Example questions might be:

"Can the dots be connected on the graph? If so, why is that possible? If not, why not?" (Yes they can be connected because David may work part of an hour and the points not actually shown on the graph would tell how much he would make if he worked times less than whole hours.)

"Describe the pattern on either the T-Chart or the graph." (The more hours worked, the money is earned. So, there is a relationship between hours worked and money earned; as one increases so does the other.)

"Could there ever be information on this graph below zero on the X or Y axis – why or why not?" (No, because you cannot work negative hours nor be paid negative amounts).

Have students set the chart and graph aside momentarily.

Introductory Lesson for Inversely Proportional

New Scenario: David is saving money for a train ticket to Washington, D.C. to visit his friend, Cameron, who moved away during the summer. David knows that it is 240 miles from his home to D.C. If the train travels at a constant rate of speed of 20 mph it will take 12 hours for the trip. At 60 mph it will take 4 hours; at 80 mph it will take 3 hours; at 120 mph it will take 2 hours, etc. Ask students to document the mph and time to travel the 240 miles on a T-chart. Remind students to label the T-chart and title it "David's 240 Miles Trip".

Using grid paper, students should graph the data with each line on the X axis valued at increments of one- to represent the number of hours needed to travel 240 miles, and each line on the Y axis valued at increments of 20- to represent the rate (number of miles per hour). Remind students to label the graph and title it "David's 240 Miles Trip". Circulate and ask probing questions to monitor student's work. Example questions might be:
"Can the dots be connected on the graph? If so, why is that possible? If not, why not?" (Yes they can be connected because time continues as miles are covered.)

"Describe the pattern on either the T-Chart or the graph." (The faster the train travels the less hours are needed to go 240 miles. So, there is a relationship between speed and distance traveled. One quantity increases by the same factor that the other decreases.)

"Could there ever be information on this graph below zero on the X or Y axis – why or why not?" (No, because you cannot go back in time nor travel negative miles.)

Working in small groups or pairs, have the students compare the T-Charts and the graphs, and record their observations. Allow time for whole class

share-out and discussion. Show the definitions for "Directly Proportional" and "Inversely Proportional" on the board/overhead. You may use:

Directly Proportional – As one quantity increases, the other quantity increases by the same constant factor. The two quantities must have the same **constant ratio** to be directly proportional.

Inversely Proportional – As one quantity increases, the other quantity decreases by the same constant factor.

Ask students to determine which definition matches what chart and graph, and justify their choice.

Note: Some definitions may say the quantities change by reciprocal factors. That definition is also correct because for one quantity to decrease proportionally, the reciprocal factor must be applied.
*Additionally, there may be references to the term, indirectly proportional. However, the two terms, inversely proportional and indirectly proportional are not exact synonyms and should not be used interchangeably.

Introductory Lesson for Nonproportional
Not all relationships are either directly proportional or inversely proportional – even if at first it appears they fit the definition. (See Examples A and B- adapted from Algebra To Go, Great Source Publishing, 2000.)

Example A: You are 14 and your sister is half your age, 7. Next year you will be 15 and your sister will be 8, more than half your age. While both ages increase, the factor does not remain constant. Thus, there is no direct proportionality.
Note: It would be best to state the first sentence of this example and have students make a T-Chart and a graph of the ages for the next six years. Remind students to label and title T-Chart, "Ages". Then ask students how this example is like and is different from the other T-Charts and graphs. (Both factors increase but not by a constant factor). You may also want to ask students which definition appears to fit at first glance, yet does not meet the requirements.

Example B: The longer a candle burns, the shorter it gets. Does it get shorter at the same rate? No, after one hour of burning, a 10-inch candle may be only 8-inches tall. However, after two hours of burning it may be 6 inches tall. While the burning time increased and the height decreased, the height did not decrease by a constant factor. Therefore, there is no inverse proportional relationship.

Explain to students that the two examples they just discussed are called nonproportional. Add a third definition to the board…

Nonproportional – A relationship exists between quantities but the relationship is neither directly proportional nor inversely proportional.

Ask students to provide examples of each of the three types of relationships and justify their reasoning.

IMPORTANT: You may either ask students to save the T-Charts and graphs they made OR you may collect and hold for when you move to the introductory lesson on indicators 7-3.2 and 7-3.3, which follows this module.

Before moving to representing proportional relationships with equations, students will need multiple opportunities to classify, make T-Charts, and graph proportional and nonproportional relationships.

c. Misconceptions/Common Errors

A common misconception is to think two quantities are directly proportional simply because they increase at a constant rate. In fact, it is the ratio of the quantities that must remain constant.

d. Additional Instructional Strategies

While additional instructional strategies are needed, no suggestions are included at this time.

e. Technology

Virtual Manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictoral representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Suggested internet resources
A good Web site resource for teachers and students on proportionality
http://www.wikihow.com/Determine-Whether-Two-Variables-Are-Directly-Proportional
Web site with a variety of lessons and ideas
http://mathforum.org/library/drmath/sets/select/dm_direct_indirect.html
SCETV
http://oneplacesc.org
Interactivate
http://www.shodor.org/interactivate/search?ss=GRAPH+EQUATIONS
Virtual Manipulatives
http://NLVM.usu.edu/en/nav/vlibrary.html

f. Assessing the Lesson

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

- Use a ticket out check method - Show graphs depicting directly proportional, inversely proportional and nonproportional relationships, labeled A,B,C. Have students put their name on a sheet of paper, identify graphs, and turn in their response as a ticket out of the class.
- Use a ticket out check method – Show graphs depicting directly proportional, inversely proportional or nonproportional relationships. Ask students to select a graph, identify the relationship and justify their identification. Have students turn in their response as a ticket out of the class.
- Use either of the above mentioned strategies but substitute a chart or table for the graph.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-3.1 Analyze geometric patterns and pattern relationships

The objective of this indicator is analyze, which is in the "understand conceptual" knowledge of the Revised Bloom's Taxonomy. To understand is to construct meaning. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge of geometric patterns and pattern relationships should include a variety of examples. The learning progression to **analyze** requires student to recall and understand the meaning of geometric patterns. Students distinguish between relevant and irrelevant numbers in mathematical word problems, compare the magnitude of numbers and compare the relationship among objects in order to generate a mathematical statement (7-1.5) that can be used to predict the next element in the pattern. Students use correct and clearly written or spoken words, variables, and notations to communicate their understanding (7-1.6).

7-3.7 Classify relationships as either directly proportional, inversely proportional, or nonproportional.

The objective of this indicator is to classify which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning and conceptual knowledge is not bound by specific examples; therefore, students should gain a conceptual understanding of proportionality in order to classify any shape. The learning progression to **classify** requires students to understand the characteristics of directly

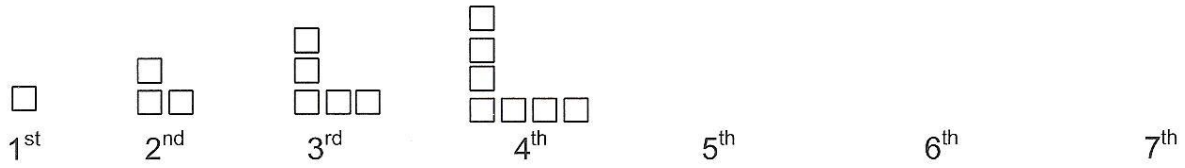
proportional, inversely proportional and non-proportional relationship represented as graphs, tables and equations. Students analyze these representations and compare them against known characteristics. As students analyze these relationships, they categorize them as directly, inversely or non-proportional. Students explain and justify their answers using correct and clearly written and spoken words, variables and notation to communicate their understanding (7-1.6).

7-3.6 Represent proportional relationships with graphs, tables, and equations. (B2)

The objective of this indicator is to represent which is in the understand conceptual knowledge cell of the Revised Taxonomy. To represent means to translate from one form to another and to understand is to construct meaning; therefore, students construct meaning for proportional relationships by translating among graph, tables, and equations. The learning progression to **represent** requires students to recall and understand the meaning of proportional relationships. Students analyze teacher generated examples and generalize mathematical statements about proportionality. They use deductive reasoning to move from general statements to specific statements about directly proportional and inversely proportional relationships. They evaluate their conjectures (7-1.2) by applying their understanding to other examples and explain and justify their answers to their classmates and teacher. Students recognize and understand that the representations (graphs, tables, and equations) are equivalent symbolic representations of the same relationship (7-1.4). They progress to generating and solving more complex problem (7-1.1).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Look figures below. Find a pattern in the sequence of figures. How many tiles would be needed to build the 10th figure?



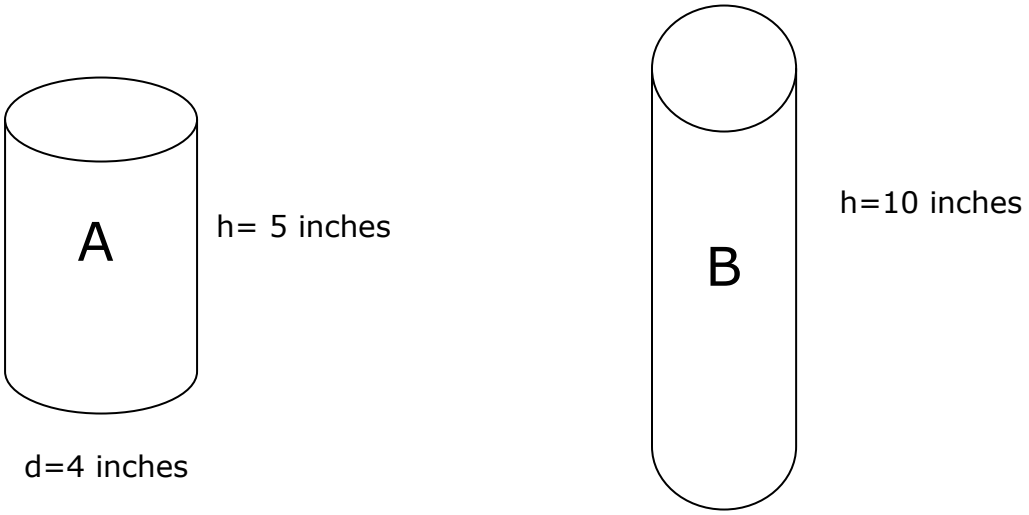
2. *Give the next term in the sequence: 1, 0.5, 0.25, …*

   a.) 0.2
   b.) 0.125
   c.) -0.25
   d. 0.1

3. Look at the values in the table below. What is the value of y in the 90th term of the sequence below?

x	*1*	*2*	*3*	*4*	*5*	*…*	*10*	*11*	*…*	*90*
y	*1*	*3*	*5*	*7*	*9*	*…*	*19*	*21*	*…*	*n*

4. Cylinder A, with a diameter of 4 inches and a height of 5 inches, is proportional to cylinder B with a diameter of 2 inches and a height of 10 inches. If cylinder A holds 24 ounces of soup, how much does cylinder B hold?



Pictures not drawn to scale

MODULE

2-4

Change in Various Contexts

This module addresses the following indicators:

7-3.2 Analyze tables and graphs to describe the rate of change between and among quantities. (B4)

7-3.3 Understand slope as a constant rate of change. (A2)

This module contains 1 lessons. These Lessons are **INTRODUCTORY ONLY.** Lessons in S³ begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

Continuum of Knowledge

7-3.2 Analyze tables and graphs to describe the rate of change between and among quantities.

- In fifth grade, students analyzed change over time (5-3.2).
- In seventh grade, students analyze tables and graphs to describe rate of change and understand slope as a constant rate of change (7-3.2).
- In eighth grade, students identify the slope of a linear equation from a graph, equation and/or table (8-3.7)
- The focus of the indicator is on students building a conceptual understanding of rate of change. This conceptual understanding is foundational for their future studies in Algebra I and is a major concept in Calculus.

7-3.3 Understand slope as a constant rate of change.

- In fifth grade, students analyzed change over time (5-3.2).
- In seventh grade, students analyze tables and graphs to describe rate of change and understand slope as a constant rate of change (7-3.2).
- In eighth grade, students identify the slope of a linear equation from a graph, equation and/or table (8-3.7)

Key Concept/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

Rate of Change*
Slope *
Independent*
Dependent*
Independent variable*
Dependent variable*
Horizontal change*
Vertical change*
Steepness*

II. Teaching the Lesson

1. Teaching Lesson A: Graphing Quantity to determine Slope

As early as first grade, students begin to classify change over time as quantitative or qualitative. Then they move to illustrate situations that show change over time- as increasing or decreasing in grades 3 and 4

respectively. By the end of the fifth grade, students are analyzing change over time. In seventh grade students should analyze tables and

graphs to describe the rate of change and understand slope as a constant rate of change.

As stated earlier, patterns continue to be explored in the seventh grade, but the focus becomes more symbolic. Seventh grade students should examine patterns in tables and graphs, describe the change among quantities, and connect their observations to a rate of change to determine if the rate of change is or is not constant. Students should be provided with opportunities to discover that the rate of change and slope are one in the same. Once this observation is made, students can use this understanding to solve problems as they analyze tables and graphs.

For the indicator **7-3.2**, it is **essential** for students to**:**

- Examine patterns in tables and graphs
- Describe the change among quantities
- Determine if the rate of change is constant or not.
- Represent rate of change as "vertical change divided by horizontal change"
- Represent the rate of change as "change in the dependent variable divided by change in the independent variable"
- Discover that the rate of change and slope are one in the same.

For the indicator **7-3.2**, it is **not essential** for students to:

- Calculate slope using a formula

For this indicator **7-3.3**, it is **essential** for students to**:**

- Understand unit rates
- Plot points
- Generate a table of values
- Recognize a pattern of constant change
- Know the relationship between rate of change and the steepness of the line
- Make connections that an increasing line is a positive slope and a decreasing line is a negative slope
- The relationship between the rate of the change of the table and the rise/run of the graph
- Understand that the graph of the line has a constant rate of change

For this indicator **7-3.3**, it is **not essential** for students to:

- Use a formula to compute slope
- Compute the slope from a line using rise over run triangles

a. Indicators with Taxonomy

7-3.2 Analyze tables and graphs to describe the rate of change between and among quantities. (B4)

Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

7-3.3 Understand slope as a constant rate of change. (A2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Remember

b. Introductory Lesson: Graphing Quantity to determine Slope

In the previous module students created T-Charts and Graphs based on David, the son of your friend and on ages. If you took these up, pass them back to students. If students saved the material ask them to retrieve it.

Ask students to refresh your memory about the T-Charts and Graphs. Ask what characteristics distinguished proportional relationships from nonproportional relationships? (Changes are based on a constant factor.)

Tell students that David got a new job and is now earning $15.00 per hour. Ask students to make a T-Chart showing how much David will make for each hour earned if he works up to ten hours. Remind students to label the T-Chart and title it "David's New Earnings". Ask students to compare the new charts with each other for possible errors.

Next, give students grid paper and ask them to graph the data with each line on the X axis valued at increments of one- for hours worked and each line on the Y axis valued at increments of $15.00- for money earned. Remind them to label the graph and title it "David's New Earnings". Circulate among the students and ask probing questions as they work.

Ask students to compare the graph of "David's Earnings" with "David's New Earnings". Again, ask students if the dots on the graph can be connected? Why or why not? Ask students to connect the dots on both graphs. What do they notice? If a student does not mention the steepness of the line on the graph ask, "What do you notice about the "slant" of the line on the graph? (The slant is steeper on the graph where he makes more money.)

Tell students to pretend that the graphs of David's Earnings" and David's New Earnings" reflect millions of dollars earned by a corporation over a number of years. Which graph do you think stockholders/owners would rather see and why? (The steeper the line, the greater the earnings over time.)

Another way to refer to steepness is "slope". Ask students what they think of when they hear the word "slope". Allow time for answers. (If students do not suggest stairs as an example of something they might think about when describing the word slope, then point that out. Draw a few stairs on the board. Ask students- when they think about how steep stairs can be, if they think about the "dip" for each step?: or if they think about the total slant of the stairs? Draw a straight line touching the steps to illustrate your point. Typically we think of the slant or slope of the total stairs from highest to lowest. So, even though hills and stairs may have "dips" the steepness is determined by the degree of slant of a straight line, and not the "dips".

Ask students to compare "David's Earning", "David's New Earnings" and "Ages" graphs. What do they notice about the lines in those graphs? (Ages is not a straight line.)

Explain that in mathematics, the steepness of a line is referred to as "slope". Then, place this or a similar definition on the board: Slope – A constant rate of change

Next, place this or a similar definition on the board: Graph – A picture of the change in one quantity in terms of the other quantity.

Ask students to mentally relate the definitions to David's graphs, and share their thinking with a partner. Next, allow students to share their thinking with the whole class. (The point is to have students focus on the fact that a graph represents relationships and when there is a constant change in the relationship the graph of the line has slope.)

Ask students to again pretend that David's graphs are a picture of business earnings in the millions of dollars. Would stockholders/owners want to know that a graph of their earnings had slope? Or, how much slope and why? (They would want to know how much slope- because that is an indication of the amount of earnings.)

Slope is determined by how much one quantity changes compared to the other quantity. In the case of David's graphs, his initial rate of change was $7.50 – in other words for each hour worked he made $7.50 – each time a new point was added to the graph his earnings changed by $7.50. For his new earnings the rate of change was $15.00 – each time a new point was added to the graph his earnings changed by $15.00. Let's look at some other graphs and talk about the slope or rate of change.

Show students examples of other graphs and guide them through a rise over run discussion. Follow this by independent student work on determining slope. Be certain to include examples of graphs that do not have a constant rate of change. Next, move to work with tables.

c. Misconceptions/Common Errors

- A common student mistake is to place the horizontal (run) quantity over the vertical (rise) quantity when computing rate of change. This is an indication that they lack conceptual understanding.
- Students may misunderstand that change must be constant.
- Students mistakenly write slope as "change in x divided by change in y"

d. Additional Instructional Strategies

Navigating through Algebra Grades 6-8 "Stories to Graphs and Graphs to Stories": Students translate verbal description of distance-time relationship to graph form and vice versa. An emphasis is placed on the student explaining what is happening during a certain interval of time. Giving students pictures of different shaped bottles, and pictures of what the graphs would look like if the bottles were filled with water at a constant rate: and asking the students to match the graphs and bottles will provide insight into their understanding of graphical and contextual representations. Such an activity can be found on page 288 of Elementary and Middle School Mathematics, Sixth Edition, John A Van De Walle.

Enrichment student lessons involving slope can be found with these links:
(Note: Teachers should explore the lessons first to determine student academic level appropriateness.)
http://www.terragon.com/tkobrien/algebra/topics/slope/slope.html

http://www.analyzemath.com/Slope/Slope.html

Having an understanding of the multiple representations is essential for students building their conceptual understanding of slope. Students should examine changes in the table of values and the graph simultaneously to understand that both representations should convey the same meaning. This creates that connection between the steepness of the line and the rate: because as the rate changes in the table, the steepness changes as well.

e. Technology

Virtual Manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictoral representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Suggested resources:

A tool such as a CBR (Calculator Based Ranger by Texas Instrument) is a great way to generate a lot of interest and make learning about slope a fun experience for students. Activities for the CBR at the middle level can be found online at this address:
http://education.ti.com/educationportal/sites/US/nonProductSingle/activitybook_cbr_cbl_73_datacollection.html

This site contains information beyond the level of these 7th grade indicators, making it a good site for teacher reference:
http://mathforum.org/library/drmath/sets/select/dm_slope.html

SCETV
http://www.Oneplacesc.org

Interactivate
http://www.showdor.org/interactivate/

Virtual Manipulatives
http://NLVM.usu.edu/en/nav/vlibrary.html

Note: The first part of each of the Web based lessons listed under "Additional Instructional Strategies" above would be appropriate resources for introducing the concept of slope.

f. Assessing the Lesson

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

- During the lesson student observation and questioning is a way to determine if students are on target.
- Since indicator 7-3.3 is at the remember level of Blooms, students could complete the sentence "Slope is a _____". The students written response would serve as a ticket out.
- Since indicator 7-3.2 is at the analyze level of Blooms, a simple graph or table could be provided to students and they could be required to identify the depicted rate of change.

III. Assessing the Module

Assessment Guidelines

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-3.2 Analyze tables and graphs to describe the rate of change between and among quantities. (B4)

The objective of this indicator is to analyze which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To analyze means to break down materials into parts (vertical change and horizontal change) and determine how the parts related to one another and the overall structure (graph or table). The learning progression to **analyze** requires students to understand the meaning of the terms independent and dependent variable and find each from a table or a graph. As students explore real world data,

they determine the change for each and use their prior knowledge of rate to create a verbal description of the observed changes. They then translate their description to numerical form indicating appropriate units. Students should evaluate their answers and pose questions to prove or disprove their reasoning (7-1.2). Students generalize mathematical statements about rate of change and use that understand to solve other problems that model physical, social and mathematical phenomena (7-1.1).

7-3.3 Understand slope as a constant rate of change. (A2)

The objective of this indicator is to understand which is in the understand conceptual knowledge cell of the Revised Taxonomy. To understand is to construct meaning; therefore, students will construct meaning related to the relationship between the steepness of a line and the rate of change. The learning progression to **understand** requires students to understand the meaning of rate of change and understand a process for determining the rate of change. Students explore the relationship between the changes in the independent/dependent variables in a table of values and changes in the steepness of a line by examining these representations simultaneously. Students generalize connections (7-1.7) between these forms and generate mathematical statements (7-1.5) to summarize these connections using correct and clearly written or spoken words (7-1.6)

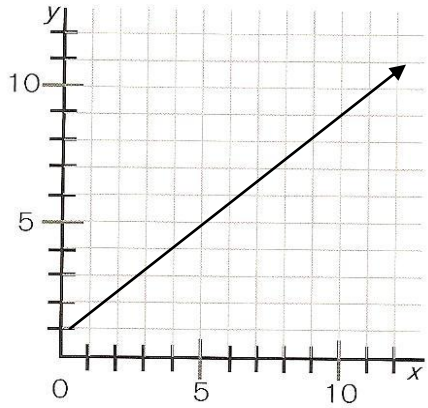
The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Jim wants to plant the largest rectangular garden possible in his back yard using 37 feet of fence. He has decided the garden should be in the shape of a square. Complete the table below to show the largest garden plot Jim can make using his 37 foot fence.

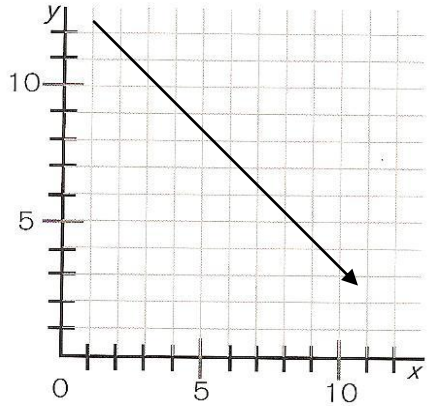
Side Length	Perimeter	Area
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Using the data in your table above, which list illustrates a constant rate of change in, the perimeter or the area? _____

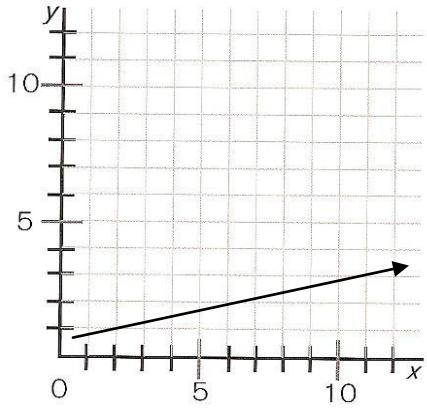
2. The number of days required to build a bridge decreases as the number of workers increases. Which of the graphs below could be used to represent this relationship?



Graph A

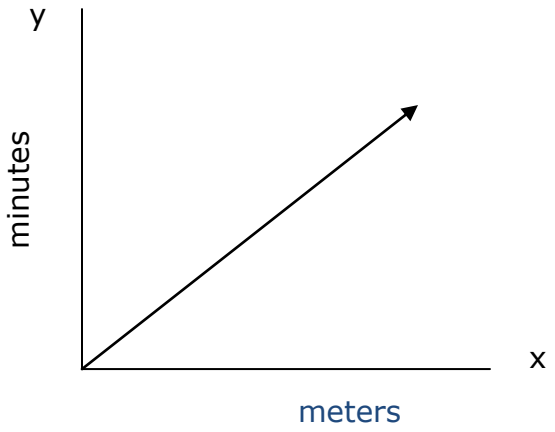


Graph B



Graph C

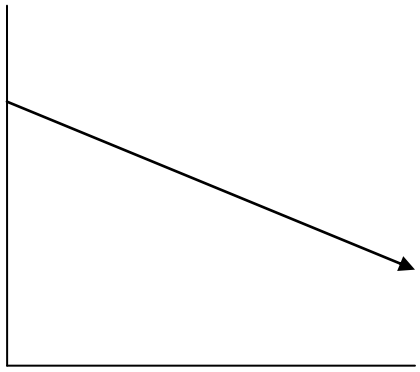
3. Jamie drew the graph below to demonstrate the elevation he climbed up the side of a mountain.



Which statement could describe Jamie's graph?

a) Jamie's elevation increased 5 meters every five minutes.
b) Jamie's elevation increased 10 meters every minute.
c) Jamie's elevation increased 15 meters every 10 minutes
d) Jamie's elevation increased 20 meters every minute.

4. Which situation below best illustrates the graph below?



a) The height of a child from age six to twelve
b) The volume of a balloon as it is being filled with air
c) The amount of gasoline in a truck's tank during a five-hour non-stop trip
d) The volume of water in the local University's swimming pool as it is being filled

5. Complete the table below to find n. Then use the table to answer questions 7 – 9.

X	*1*	*2*	*3*	*4*	*5*	*6*
Y	*1*	*4*	*9*	*16*	*25*	*n*

6. The value of n in the table above is represented by
a) 2x6 b) 6^2 c) 6+6 d) 36/2

7. Write an equation for the nth term in the table: _____

8. The graph of the values in the table above represents a constant rate of change?
   A. True
   B. False
   C. Cannot tell

9. TJ and Chris are walking to school. TJ walks 1.5 meters per second and Chris walks at 2 meters per second. TJ lives 20 meters closer to the school. They both leave for school at the same time. Complete the table below.

Time in Seconds	TJ's Distance in Meters	Chris's Distance in Meters
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

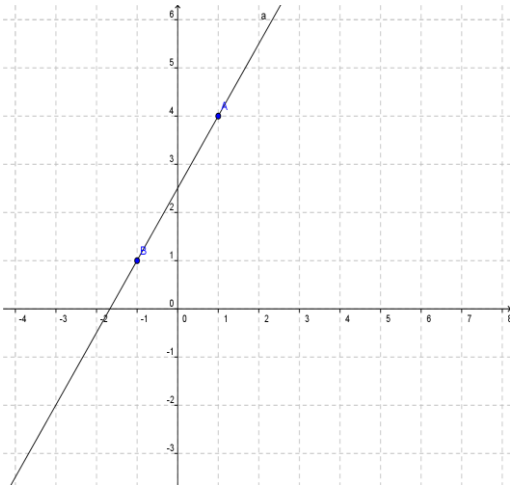
How long will it take before they meet up with each other and can walk the rest of the way together?

After how many seconds will Chris have walked twice as far as TJ?

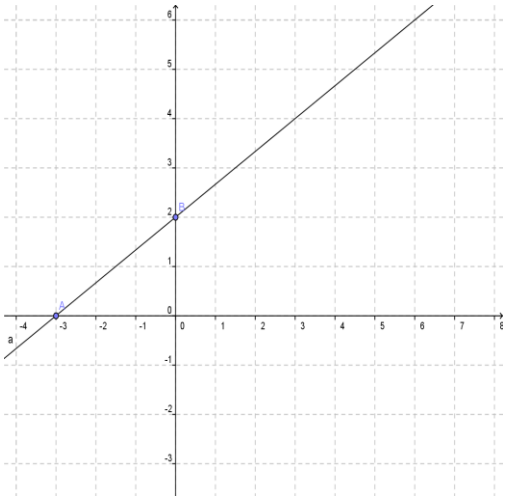
Sketch a graph in the table below to show the distance traveled during the first 10 seconds by Chris and TJ. Be sure to label your axis.

10. Write two equations and find the distance traveled by Chris and TJ
    a.) after 1 minute _____
    b.) after 30 minutes _____

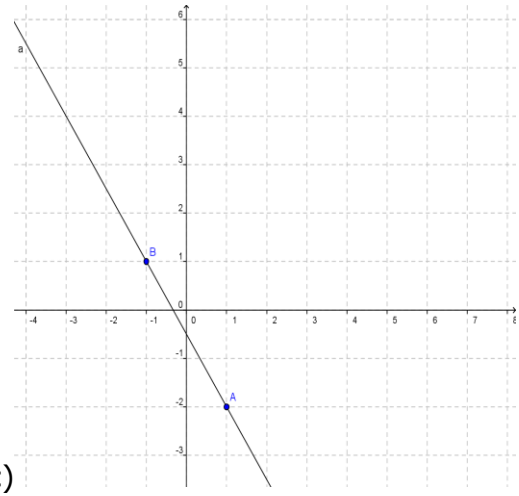
11. Which of the following graphs has a slope of 3/2?
    a.) A b) B c) C d) D



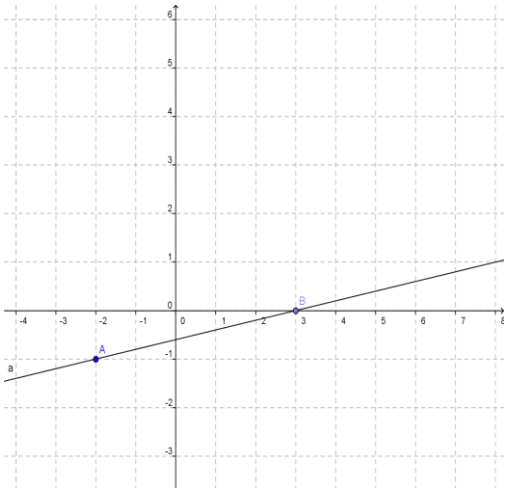
a)



b)

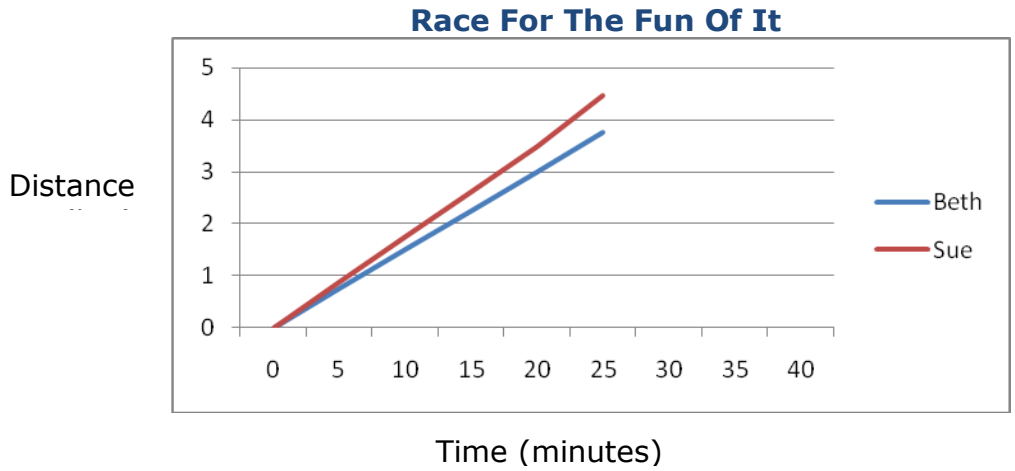


c)



d)

12. Sue and Beth have entered a 5K race, "Race For The Fun Of It". A graph that represents the race can be found below.



- What is the relationship between Beth and Sue's rate of change?
- Describe Beth's rate of change.
- Identify the dependent and independent variables.
- Represent Beth's rate of change as "change in dependent variable divided by change in independent variable".
- Who do you predict will win this race? Give reasons for your prediction.

13. Dave decided to begin ride his bicycle to school every morning. Dave's mom bought him a bicycle computer so he could monitor the distance and time he traveled. Below is a table that details Dave's trip to school.

Time (hours)	**Distance (miles)**
0	0
.1	1
.15	1.5
.2	2
.25	2.5

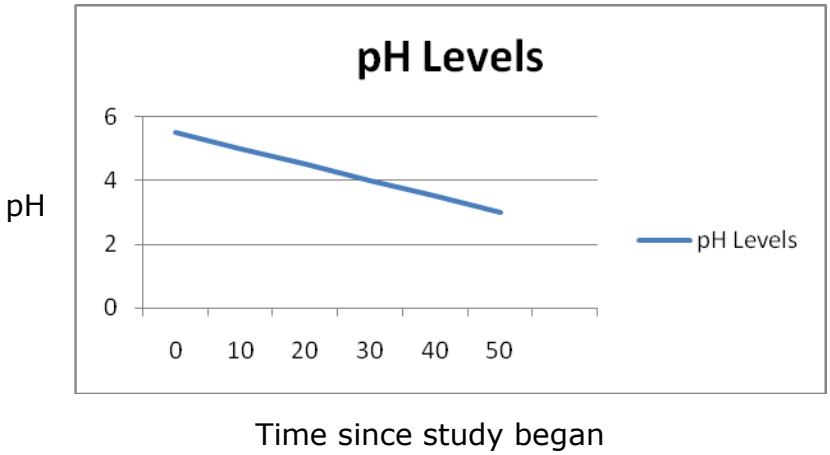
- Create a graph to represent Dave's biking rate.
- Describe Dave's rate of change.
- Represent Dave's rate of change. In what ways does this rate of change relate to the independent and dependent variables?
- Explain why time is the independent variable.

14. John has a clear plastic cup filled with 2 cm of water. John begins to drop marbles in the cup, one at a time, and observes what happens. Below is a table that represents the date he collected.

Number of marbles	0	1	2	3	4	5
Height of water (cm)	2	3.2	4.4	5,6	6.8	7

- Describe the patterns and relationships you notice in the table.
- What happens to the water as you add marbles to the cup? How would you represent this change?
- Describe the relationship between the independent and dependent variables.
- What is the change in the dependent variable? What is the change in the independent variable? Use this information to represent the rate of change of the water.
- Create a graph to represent John's data. What is the relationship between the rate of change in water height and the steepness of the line? Is this rate of change positive or negative? How do you know?
- Is the change in water height a constant rate of change? How do you know?

15. For several years, scientists have studied pH levels in rain in a California forest. The graph below displays their findings.



- What connections do you notice between the steepness of the line and the rate of change of the independent and dependent variables?
- Represent the rate of change of the pH levels. Explain your reasoning. Is this rate of change constant? How do you know?