SOUTH CAROLINA SUPPORT SYSTEM INSTRUCTIONAL GUIDE

Content Area	**Seventh Grade Math**
	Third Nine Weeks

Standards/Indicators Addressed:

Standard 7-4: The student will demonstrate through the mathematical processes an understanding of proportional reasoning, tessellations, the use of geometric properties to make deductive arguments, the results of the intersection of geometric shapes in a plane, and the relationships among angles formed when a transversal intersects two parallel lines.

- 7-4.1 Analyze geometric properties and the relationships among the properties of triangles, congruence, similarity, and transformations to make deductive arguments. (B4)
- 7-4.2 Explain the results of the intersection of two or more geometric shapes in a plane. (B2)
- 7-4.3 Illustrate the cross section of a solid. (C2)
- 7-4.4 Translate between two- and three-dimensional representations of compound figures. (B2)
- 7-4.5 Analyze the congruent and supplementary relationships—specifically, alternate interior, alternate exterior, corresponding, and adjacent—of the angles formed by parallel lines and a transversal. (B4)
- 7-4.6 Compare the areas of similar shapes and the areas of congruent shapes. (B2)
- 7-4.7 Explain the proportional relationship among attributes of similar shapes. (B2)
- 7-4.8 Apply proportional reasoning to find missing attributes of similar shapes. (C3)
- 7-4.9 Create tessellations with transformations. (B6)
- 7-4.10 Explain the relationship of the angle measurements among shapes that tessellate. (B2)

Standard 7-5: The student will demonstrate through the mathematical processes an understanding of how to use ratio and proportion to solve problems involving scale factors and rates and how to use one-step unit analysis to convert between and within the U.S. Customary System and the metric system.

- 7-5.4 Recall equivalencies associated with length, mass and weight, and liquid volume: 1 square yard = 9 square feet, 1 cubic meter = 1 million cubic centimeters, 1 kilometer = $\frac{5}{8}$ mile, 1 inch = 2.54 centimeters; 1 kg = 2.2 pounds; and 1.06 quarts = 1 liter. (A1)

7-5.5 Use one step unit analysis to convert between and within the US Customary System and the metric system. (C3)

*** These indicators are covered in the following 5 Modules for this Nine Weeks Period.**

Module 3-1 Equivalencies			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 3-1 Lesson A: 7-5.4 Recall equivalencies associated with length, mass and weight, and liquid volume.	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A	See Instructional Planning Guide Module 3-1 Introductory Lesson A	See Instructional Planning Guide Module 3-1 Lesson A Assessing the Lesson
Module 3-2 Conversions			
Module 3-2 Lesson A: 7-5.5 Use one step unit analysis to convert between and within the US Customary System and the metric system. (C3)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle	See Instructional Planning Guide Module 3-2, Introductory Lesson A See Module 3-2, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 3-2 Lesson A Assessment

	NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – See Appendix A		

Module 3-3 Plane Geometry			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 3-3 Lesson A: **7-4.2** Explain the results of the intersection of two or more geometric shapes in a plane. (B2) **7-4.3** Illustrate the cross section of a solid. (C2) **7-4.4** Translate between two- and three-dimensional representations of compound figures. (B2)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations -See Appendix A	See Instructional Planning Guide Module 3-3, Introductory Lesson A See Module 3-3, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 3-3 Lesson A Assessment
Module 3-3 Lesson B: **7-4.5** Analyze the congruent and supplementary relationships—specifically, alternate interior, alternate exterior,		See Instructional Planning Guide Module 3-3, Introductory Lesson B See Module 3-3, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 3-3 Lesson A Assessment

corresponding, and adjacent—of the angles formed by parallel lines and a transversal. (B4)			

Module 3-4 Plane and Transformational Geometry			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 3-4 Lesson A: **7-4.9** Create tessellations with transformations. (B6) **7-4.10** Explain the relationship of the angle measurements among shapes that tessellate. (B2)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM)	See Instructional Planning Guide Module 3-4, Introductory Lesson A See Module 3-4, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 3-4 Lesson A Assessment

Module 3-4 Lesson B **7-4.1** Analyze geometric properties and the relationships among the properties of triangles, congruence, similarity, and transformations to make deductive arguments. (B4)	Textbook Correlations - See Appendix A	See Instructional Planning Guide Module 3-4, <u>Introductory Lesson B</u> See Module 3-4, Lesson B <u>Additional Instructional Strategies</u>	See Instructional Planning Guide Module 3-4 <u>Lesson B Assessment</u>
Module 3-5 Proportional Reasoning			

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines
Module 3-5 Lesson A: **7-4.6** Compare the areas of similar shapes and the areas of congruent shapes. (B2) **7-4.7** Explain the proportional relationship among attributes of similar shapes. (B2) **7-4.8** Apply proportional reasoning to find missing attributes of similar shapes. (C3)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations - See Appendix A	See Instructional Planning Guide Module 3-5, Introductory Lesson A See Module 3-5 Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 3-5 Lesson A Assessment

MODULE

3-1

Equivalencies

This module addresses the following indicators:

7-5.4 Recall equivalencies associated with length, mass and weight, and liquid volume: 1 square yard = 9 square feet, 1 cubic meter = 1 million cubic centimeters, 1 kilometer = $\frac{5}{8}$ mile, 1 inch = 2.54 centimeters; 1 kg = 2.2 pounds; and 1.06 quarts = 1 liter.

This module contains one lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

1. Continuum of Knowledge

7-5.4 Recall equivalencies associated with length, mass and weight, and liquid volume: 1 square yard = 9 square feet, 1 cubic meter = 1 million cubic centimeters, 1 kilometer = $\frac{5}{8}$ mile, 1 inch = 2.54 centimeters; 1 kg = 2.2 pounds; and 1.06 quarts = 1 liter.

- In fourth grade, students recalled basic conversion facts within the customary system for length, weight and capacity (4-5.8). They used equivalencies to convert units of length, weight, and capacity within the Customary System (4-5.3). In fifth grade, students focus on the metric system recalling facts (5-5.8) and converting within the metric system (5-5.3).

- In seventh grade, students will recall equivalencies associated with length, mass and weight, and liquid volume (7-5.4). Also students use one step unit analysis to convert within and between the U.S. Customary System and metric system (7-5.5). In eighth grade, students use multi-step unit analysis to convert within and between the Customary and metric systems (8-5.7).

1. Key Concepts/Key Terms

*Units and Equivalencies from the indicator
*Equivalent expressions
Cubic meter
Square Yard

**These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students*

II. Teaching the Lesson

1. Teaching Lesson A

For this indicator, it is **essential** for students to be able to:

- Recall measurement facts listed in the indicator
- Explore real world model of each to support retention
- Generate common references for these equivalencies.

For this indicator, it is **not essential** for students to:

- Do any problems for this indicator

a. Indicators with Taxonomy

7-5.4 Recall equivalencies associated with length, mass and weight, and liquid volume: 1 square yard = 9 square feet, 1 cubic meter = 1 million cubic centimeters, 1 kilometer =⅝ mile, 1 inch = 2.54 centimeters; 1 kg = 2.2 pounds*; and 1.06 quarts = 1 liter. (A1)*

Cognitive Process Dimension: Remember
Knowledge Dimension: Factual

b. Introductory Lesson –

Materials:

4 x 6 (or smaller) index cards
Note: It is recommended that more than one set of cards be made so that multiple groups of students can play at the same time. If so, then colored index cards should be used. If cards are accidentally mixed, then it will be easy to see that all "yellow" go together, all "green" go together, etc. without taking time to sort the cards.

- Permanent Marker
- Laminate to preserve
- Gallon size plastic storage bag or similar storage container

Introductory Lesson: "Measurement Concentration"

Write one-half of each equivalency on an index card with a permanent marker. Laminate all index cards to preserve. Place the entire set of cards into a gallon size plastic bag. Write "Equivalencies Concentration" on the front of the bag. NOTE: It would be a good idea to include the equivalencies required by other "Equivalencies" indicators in previous grades. This makes the game more interesting and challenging and helps with recall.

To Play:
One or more players

Shuffle/mix all "equivalency cards" and place them face down. The cards can be placed face down in a random, square, triangle, rectangle, etc. pattern. The players make that decision. Once all cards are face down, player one turns over any one card of their choice, reads the card, and announces what they need for a match. For example, if a card stating "1 square yard" is turned over, then the player announces "1 square yard = 9 square feet. I am looking for 9 square feet". That player then turns over a second card of their choice. If the two cards form an equivalency, the player takes those two cards, and has earned another turn. That player continues to turn over cards, read and announce until no match is made. When no match is made, the two revealed cards are turned back over and the next player takes a turn. If any player neglects/forgets to read and announce they loose a turn. The players who are waiting for a turn must "concentrate" on which cards were revealed, where the cards are located face down, and then use that information to make matches when it is their turn to play. After all cards are revealed or time has been called, the player with the most matches wins. Note: If through observation it is determined that students are not verbalizing the equivalencies prior to turning over a card, then students could be required to write the equivalencies in order to earn play privileges.

c. Misconceptions/Common Errors –

Students have difficulty knowing when to use appropriate units of measure such as linear units, square units or cubic units.

d. Additional Instructional Strategies –

- Connections as well as associations should be made with familiar equivalencies such as from linear units of measure to square and cubic units of measure. For example, there are 3 feet in 1 yard so to find the number of square feet in 1 square yard, have students draw a square with side lengths of 3 feet (1 yard); then find the area of the square. The area will be 9 square feet which is equivalent to 1 square yard.

- Allow students to participate in hands-on activities that visually demonstrate what each measurement looks like. For example, one section on their index finger is about 1 inch. Also have student feel the weight of something that weighs 2.2 pounds, etc. Memory cards and games can be used to support retention.

- The use of pictures (visual images) to recall equivalencies is another strategy to teach this indicator. For example, rulers with both customary and metric units can help with the relationship between cm and inches. By examining the ruler students will be able to see that 2.54 cm = 1 inch.

- The strategies above are used to provide visual images to develop measurement equivalencies. These visual images will help student be able to recall the required equivalencies.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

The following Web sites are possible resources:

This site has lesson ideas for the teacher
http://desktoppub.about.com/cs/intermediate/a/basicmetric_2.htm

This site has conversion charts that may be printed out for students
http://www.sawmillsoftware.com/activeserverpages/worksheets/measurementconversions.asp

Lesson on Conversions
http://www.eduref.org/Virtual/Lessons/Mathematics/Measurement/MEA0007.html

Good conversions :)
http://www.convert-me.com/en/

Good Table
http://www.mathleague.com/help/metric/metric.htm

SCETV (you may need to create a pass code-follow prompts)
http://www.oneplacesc.org

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f. *Assessing the <u>Lesson</u>*

Assessment can be addressed through questioning, "observation" as students play games such as *Concentration*, or through a formal "fill in the blank" type of assessment.

III.Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-5.4 Recall equivalencies associated with length, mass and weight, and liquid volume: 1 square yard = 9 square feet, 1 cubic meter = 1 million cubic centimeters, 1 kilometer =⅝ mile, 1 inch = 2.54 centimeters; 1 kg = 2.2 pounds*; and 1.06 quarts = 1 liter. (A1)*

The objective of this indicator is recall, which is in the "factual knowledge" cell of the Revised Taxonomy. Although the focus of the indicator is to recall, integrating hands-on activities and real world examples help to support retention. The learning progression to **recall** requires students to <u>understand</u> that each fact is a symbolic form (in and cm or pounds and kilogram) that represents the same relationship or quantity (7-1.4). Students explore hands-on or real world example, where appropriate, to generalize the connection among a variety of representational forms (7-1.7) of these equivalencies. They should develop personal and meaning strategies for recalling these facts.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Mike wants to make a large square pizza. If his pizza is 1 square yard, what would it measure in square feet?

2. George walked 1 kilometer before he got tired and had to stop. How many miles did he walk?

3. Grace wants to convert 35 inches to centimeters. Which equivalence measure does she need to remember?
   a. 1 inch = 1.54 cm
   b. 1 inch = 2.14 cm
   c. 1 inch = 2.54 cm
   d. 1 inch = 3.14 cm

4. A 3-year old picked up an exercise weight weighing 1 kilogram. How many pounds did he pick up?

MODULE

3-2

Conversions

This module addresses the following indicators:

7-5.5 Use one step unit analysis to convert between and within the U.S. Customary System and the metric system. (C3)

This module contains one lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

- **Continuum of Knowledge**

7-5.5 Use one step unit analysis to convert between and within the U.S. Customary System and the metric system. (C3)

- In fourth grade, students recalled basic conversion facts within the customary system for length, weight and capacity (4-5.8). They used equivalencies to convert units of length, weight, and capacity within the Customary System (4-5.3). In fifth grade, students focus on the metric system recalling facts (5-5.8) and converting within the metric system (5-5.3).

- In seventh grade, students will recall equivalencies associated with length, mass and weight, and liquid volume (7-5.4). Also students use one step unit analysis to convert within and between the U.S. Customary System and metric system (7-5.5). In eighth grade, students use multi-step unit analysis to convert within and between the Customary and metric systems (8-5.7).

- ***Key Concepts/Key Vocabulary***

Unit Analysis
*Equivalencies
Dimensional Analysis
One-step Unit Analysis
Unit cancellation (cancelling units)
*Mathematical Notation/Symbols: Abbreviations for length, weight/mass, and liquid volume for U.S. customary and metric system units

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lesson

For this indicator, it is **essential** for students to:

- Understand proportional reasoning
- Multiplying fractions
- Simplify expression
- Understand that each equivalency when written as a fraction is a form of one
- Use equivalencies to convert between systems
- Set up ratios to convert using unit analysis

For this indicator, it is **not essential** for students to:

- Memorize the equivalencies
- Do more than a one-step unit-analysis problems (8th grade)

In seventh grade students are asked to convert a quantity expressed in one set of units to another equivalent quantity expressed in a different set of units. In other words, using unit (dimensional) analysis means that you will keep the units of measure throughout the problem.

For example, if there are 2.54 cm to 1 inch, how many centimeters are in a foot? By writing each in fraction format (horizontally), students can see that all units cancel except the final ones in the answer.

$$\frac{2.54\text{ cm}}{1\ \cancel{\text{in.}}} \times \frac{12\ \cancel{\text{in.}}}{1\text{ ft.}} = \frac{30.48\text{ cm}}{1\text{ ft.}} = 30.48\text{ cm in a foot.}$$

Another example: 10 yd to feet

$$\frac{10\ \cancel{\text{yd}}}{1} \times \frac{3\text{ ft}}{1\ \cancel{\text{yd}}} = \frac{10 \times 3\text{ ft}}{1} = 30\text{ ft}$$

In other words, you can cancel units just like you do factors when dealing with fractions. This is called "unit cancellation".

Teacher Notes: In science unit analysis is typically called dimensional analysis. Therefore, students should be familiar with both *unit* and *dimensional analysis* terms.

1. <u>*Teaching Lesson A*</u>

a. *Indicators with Taxonomy*

7-5.5 Use one step unit analysis to convert between and within the U.S. Customary System and the metric system. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. *Introductory Lesson –*

Note: This indicator should be based on indicator 7-5.4. The equivalencies which students should recall in 7-5.4 could serve as the basis for beginning a discussion on one-step unit analysis.

Tell the students the following story or give them a copy to read:
After school while waiting for the bus you are watching some repairmen begin to patch a small crack in the concrete sidewalk. You hear one say they need six

quarts of water for the small amount of concrete mix that is needed. However, the only measuring tool they have is a 1 liter Pepsi bottle. You can tell they need help determining how many 1 liter bottles of water they should use. They look over at you and ask for your help. How many 1 liter bottles of water should they use to equal 6 quarts of water?

Allow the students to work in pairs or small groups to find an answer. Note: As you move around the room if groups are struggling, remind them of the equivalencies they are memorizing for indicator 7-5.4.

Ask students to share strategies. Make certain that students express their answer in terms of liters and indicate that it will take 5 whole liters and seven tenths of another OR almost 6 liters.

Ask students to keep the strategies in their mind because you will come back to them shortly.

Next write the following on the board:

A. $8 \times 1 = 8$

B. $\frac{6}{1} \times \frac{3}{3} =$

$^{2}\frac{\cancel{6}}{1} \times \frac{3}{\cancel{3}}{}_{1} =$

$\frac{6}{1}$

Ask how equations A and B (above) are similar. (Response: Both use the multiplicative identity element – students may respond that in both instances something is multiplied times one.)

Remind students that any time something is multiplied by one- the result is equal to the original amount.

Leave the above examples on the board and now add the following example:
How many square feet are in 3 square yards?

$\frac{3 \text{ sq yds}}{1} \times \frac{9 \text{ sq ft}}{1 \text{ sq yd}}$

$\frac{3\ \cancel{\text{sq yds}}}{1} \times \frac{9 \text{ sq ft}}{1\ \cancel{\text{sq yd}}}$ Tell students- units can be cancelled just like when working with fractions.

$= 27$ sq ft

Ask students to think about the strategies they heard for solving the concrete/water mixture problem and challenge them to find another way to solve the problem using the "anything times one = anything" and cancellation rules you just demonstrated on the board.

Move about the room and provide prompting questions. Allow students to share correct strategies.

$$\frac{6\ \cancel{\text{quarts}}}{1} \times \frac{1\ \text{liter}}{1.06\ \cancel{\text{quarts}}} =$$

$$\frac{6\ \text{liters}}{1.06} =$$

5.7 liters

Tell students that this strategy is called one-step *unit analysis* or *dimensional analysis* and can be used to convert both within and between the US Customary System and metric units of measurement – time, mass and weight, length, liquid volume, etc. Next year they will build on this strategy by using two-steps or more than two different units.

It is important that students understand that unit analysis is related to other mathematics they have previously learned and that it is not something entirely new they have to accommodate in their memory bank. By introducing the concept in this exploratory manner, students will have a basis for understanding rather than simply repeating a process that appears to be random.

Multiple opportunities to use one-step unit analysis will be necessary. Opportunities should be in context and should include conversions within and between the US Customary System and the metric system.

c. Misconceptions/Common Errors

Students may have trouble with placing their units in the correct position of the fraction to get their units to "cancel".

d. Additional Instructional Strategies / Differentiation

- In the non-essentials, it states that students do not need to memorize the equivalencies in order to meet this indicator. Although it would be beneficial for students to recall the equivalencies, it is not essential because the focus of the indicator is on the **process of unit analysis** not on memorizing facts. The work of students should be on gaining computational fluency with the process of setting up and simplifying the problem. Students will become frustrated because they can't recall the equivalencies and will be distracted from the central focus.

- One strategy that may be used to address this issue is to give students an assessment on recalling their facts (focus on retention) and a separate assessment where students are given the equivalencies and asked to do conversions (focus on conversions).

- In science, unit analysis is typically called dimensional analysis; therefore, students should be familiar with both terms.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

The following Web sites are possible resources:

Good tutorial sites that can be used for absent or struggling students:
http://hotmath.com/hotmath_help/topics/unit-analysis.html

http://www.purplemath.com/modules/units.htm

http://mathforum.org/library/drmath/view/62176.html

SCETV (user may need to create a pass code- follow prompts)
http://www.Oneplacesc.org

Using manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. Assessing the Lesson

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

- During classroom instruction teacher questioning is a critical assessment strategy.
- Ask students to compare in writing, one-step unit analysis and multiplying fractions.

- Ask students what the typical mistake is that most students make when setting up unit analysis problems; and why it is important to set up problems a certain way.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7 -5.5 Use one-step unit analysis to convert between and within the U.S. Customary System and the metric system.

The objective of this indicator is use, which is the "apply procedural" knowledge cell of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem. The focus is to gain computational fluency with conversions within and between the US Customary and metric system. The learning progression to **use** requires students to understand the relationship between the equivalencies and a form of one. Students explore teacher generated examples to investigate what happens to the ratios when they are multiplied (what cancels or doesn't cancel). They generalize mathematical statements (7-1.5) summarizing a strategy that can be used to set up and simplify a problem. Students use their understanding of proportional reasoning to explain and justify their answers using correct and clearly written or spoken words. They also understand the original measurement and the converted measurement are different symbolic forms of the same quantity (7-1.4).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. A sewing pattern calls for fabric in square feet. 29.25 square feet are needed. How many square yards should be bought?

2. 4 cubic meters = ______ million cubic centimeters.

3. If Sally won a 6 kilometer race, how many miles did she run?

4. 10 inches = _____ centimeters.

5. A cat weighs 8.8 pounds, but medicine is prescribed based on weight in kilograms. Convert 8.8 pounds to kilograms.

6. 3.71 quarts = _____ liters.

MODULE

3-3

Plane Geometry

This module addresses the following indicators:

7-4.2 Explain the results of the intersection of two or more geometric shapes in a plane. (B2)
7-4.3 Illustrate the cross section of a solid. (C2)
7-4.4 Translate between two- and three-dimensional representations of compound figures. (B2)
7-4.5 Analyze the congruent and supplementary relationships—specifically, alternate interior, alternate exterior, corresponding, and adjacent—of the angles formed by parallel lines and a transversal. (B4)

This module contains four lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

1. Continuum of Knowledge

7-4.2 Explain the results of the intersection of two or more geometric shapes in a plane. (B2)

- In third grade, students classified lines and line segments as either parallel, perpendicular, or intersecting (3-4.3).
- In eighth grade, students use ordered pairs, equations, intercepts and intersections to locate points and lines in a coordinate plane (8-4.2).

7-4.3 Illustrate the cross section of a solid. (C2)

- In fifth grade, students translated between two-dimensional representations and three-dimensional objects (5-4.4).

7-4.4 Translate between two- and three-dimensional representations of compound figures. (B2)

- In fifth grade, students explored methods for translating between two-dimensional representations and three-dimensional objects (5-4.4).

7-4.5 Analyze the congruent and supplementary relationships—specifically, alternate interior, alternate exterior, corresponding, and adjacent—of the angles formed by parallel lines and a transversal. (B4)

- In third grade, students classified lines and line segments as either parallel, perpendicular or intersecting lines (3-4.3).

2. Key Concepts/Key Terms

Transversal line
*Alternate exterior angles
*Alternate interior angles
Corresponding angles
Adjacent angles
*Cross Section
Compound Figure
*Intersection
Geometric shapes
*Three-dimensional
*Two-dimensional
Isometric
*Vertical plane
*Horizontal plane
*Solid
View Point
Illustrate

Perspective
Translate
*Cube
*Sphere
*Cylinder
*Prism
*Pyramid
*Cone
*Rectangle
*Square
*Triangle
*Circle
*Parallel
*Congruent
*Supplementary
*Perpendicular

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lessons

1. Teaching Lesson A

For this indicator, it is **essential** for students to:
- Understand the meaning of intersection
- Understand the result of a line crossing another line is a point
- Understand the result of a line crossing a circle is two points
- Explain the results from the intersection of any 2 figures that they have studied.
- Understand the orientation of the shapes effect the number of intersections.

For this indicator, it is **not essential** for students to:

a. Indicators with Taxonomy

7-4.2 Explain the results of the intersection of two or more geometric shapes in a plane. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson – "Intersecting Shapes"

Materials: string or yarn, round stickers or clay

Literature Connections:
Shape Up by David Adler

Introductory Lesson: "Intersecting shapes":

Begin lesson with a discussion of the word intersection, connecting it to roads. Ask the students why traffic accidents often happen at an intersection. (It is where the two roads intersect). Guide students to define the word intersection while they use string to model an intersection. Discuss what happens when the string crosses. Tell students to put a sticker (or ball of clay) on the place where the strings cross and guide them to see that a point if formed at the intersection. Make a diagram of the string/sticker on the board, labeling the lines and point that were made. Ask the students to try to make the string cross at more than one point. Allow students to use the string to make shapes on their desks while using stickers to mark the points at the intersections. Create a chart on the board that shows what happens when different shapes intersect. Challenge students to discover the maximum number of points that can be made with each shape as they change the orientations of the shapes. Close the lesson by having students make cause and effect statements about what happens when different shapes overlap in a plane. "The point of intersection of _____ is ______ because ____."

c. Misconceptions/Common Errors –

- It may be difficult for students to visualize the orientation of shapes.

d. Additional Instructional Strategies/Differentiation –

- Students use grid paper or geoboards to model the results of intersecting geometric shapes on a plane. Begin by using two triangles, and then vary the shapes. Have students discuss their observations. Students record observations on a table. Have them organize the results and discuss their findings with the class. Have students use the findings from the activities to make predictions about all intersecting geometric shapes in a plane. Extend their exploration by asking them to explain the results of shapes in two planes intersecting.

- Using models and pictures will aid student as they try to visualize these intersections. For example, a pen and piece of paper may be used to demonstrate what happens when a plane and a line intersect.

e. Technology-

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and the virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Pattern Blocks: http://mason.gmu.edu/~mmankus/Handson/pbinfo.htm

f. Assessing the Lesson

Use the student-generated cause and effect statements to assess student understanding of this lesson.

2. Teaching Lesson B

For this indicator, it is **essential** for students to:

- Understand and visualize horizontal and vertical cross planes
- Determine the cross sections for cubes, prisms, sphere and cylinders, cones and pyramids
- Understand the connection between the three dimensional object and its two dimensional cross section (sphere – circle, rectangular prism – rectangle, etc..)
- Explore cross sections using concrete and pictorial models

For this indicator, it is **not essential** for students to:

- Focus on cross sections that are not vertical or horizontal. No cross sections cut at a slant.

b. Indicators with Taxonomy

7-4.3 Illustrate the cross section of a solid. (C2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson – "Cross-Sections"

Materials:
Cardstock paper, Clay, dental floss

Introductory Lesson: "Cross-Sections"

First, students will create planes. The teacher will model instructions while students follow. Give students a piece of card stock (one-fourth of a piece for each student will work). Have students fold their paper in half and cut it. Place the two halves on top of each other and cut a slit half way through both pieces. Put the two pieces of paper together at the slits. This should make two intersecting planes. Review the following definitions: horizontal plane, vertical plane, intersection. Make and cut shapes with floss.
Understand the connection between the three dimensional object and its two dimensional cross section (sphere – circle, rectangular prism – rectangle, etc..)
Explore cross sections using concrete and pictorial models

f. Misconceptions/Common Errors –

- Students often cannot visualize three- dimensional shapes from two-dimensional drawings

g. Additional Instructional Strategies/Differentiation –

- Tell students that you are going to "deconstruct" the layers of a three-dimensional object. Build a **rectangle** using interlocking cubes. Have students illustrate on isometric dot paper the original rectangle and a view of a horizontal or cross section that makes up the rectangle. Then, have them illustrate a vertical cross section.

- Discuss the relationship between the horizontal and vertical cross sections. *When engaging in such a discussion, it is important for students to understand that the concrete models and pictorial illustrations are the result of the intersection of a plane in a segment of the geometric shape. (This stays true to the definition of a cross section – the intersection of a plane and a geometric solid.)*

- Have students perform the following activity. First, have them make a cube using modeling clay. Then, have them slice the cube using dental floss or fishing line. Once they are comfortable with "slicing" have them try to make the following cross sections by slicing a cube. A couple of the shapes are impossible to make. Discuss what makes them impossible.

Square	Equilateral triangle	Rectangle (not a square)
Triangle (not equilateral)	Pentagon	Hexagon
Octagon	Parallelogram (not rectangle)	Circle

- It may be helpful for students to think of cross sections as "deconstructing" the layers of a three-dimensional object. For example, to illustrate the cross section of a rectangle, students may build a rectangle using interlocking cubes.

Then the students might illustrate on isometric dot paper the original rectangle and a view of one of the "layers" or cross sections that make up the rectangle. The ability to do this will enable students to develop, justify, and understand formulas (such as area, surface area, volume, etc.) that are used in regards to two- and three-dimensional figures.

- Another strategy involves using real world objects. For example, an orange can be used for a sphere, a small rectangular cake can be used for the rectangular prism and a stack of cookies can be used to illustrate a cylinder.
- When engaging in discussion, it is important for students to understand that the concrete models and pictorial illustrations are the result of the intersection of a plane in a section of the geometric shape. (This stays true to the definition of a cross-section- the intersection of a plane and a geometric solid.)
- Getting Solid with Shapes
  http://www.indianastandards.org/files/math/math_8_4_3.pdf

e. Technology-

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Pattern Blocks: http://mason.gmu.edu/~mmankus/Handson/pbinfo.htm

g. Assessing the Lesson

To assess understanding of this lesson, ask students to identify the possible two-dimensional shapes that might be formed from cross-sections of a sphere, a cylinder, or a cone.

3. Teaching Lesson C

For this indicator, it is **essential** for students to:

- Recall the characteristics of two and three dimensional shapes (see Key Concepts)
- Identify from concrete and pictorial models the view points that give the figure its overall shape.
- Identify from concrete and pictorial models the overall shape when provided with the perspective (front, top and side views)

For this indicator, it is **not essential** for students to:

- Illustrate the nets for a given three-dimensional shape.
- Construct a three-dimensional shape when given its net.

c. Indicators with Taxonomy

7-4.4 Translate between two- and three-dimensional representations of compound figures. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson – "Two and Three-dimensional Representations"

Materials: Various containers (cereal boxes, oatmeal box), magazines with pictures, 3-d shapes (cube, prism, sphere, cylinder, cone, pyramid), stacking cubes

Literature Connections:

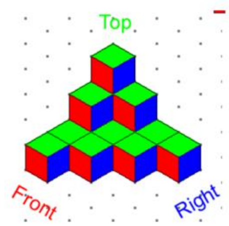
Introductory Lesson: "Two and Three-dimensional Representations":

Place an object that has obvious differences on all sides in the middle of the class, where students will have different views. Ask the students to describe what they see. Discuss the differences due to the differing perspectives and guide them to name the varying perspectives (front, side, top views). Have students work together to observe various objects from the different perspectives and either describe or draw the different views. Challenge students to make predictions about the views, then check to see if they are correct.

Review the properties of 3-dimensional shapes including cubes, prisms, sphere and cylinders, cones and pyramids. As a class, have students complete a chart showing the different perspectives of each. Next, ask students what would happen if we were to stack shapes to make compound shapes. Allow students to build compound figures and explore the results of changing the views. Pairs of students can build compound figures and trade with other groups, making predictions and checking them. Have the students look at magazines and draw the different perspectives. Close by asking the students how the 3-dimensional shapes are related to 2-dimensional shapes.

c. *Misconceptions/Common Errors –*

- Students may think that compound figures are constructed of different shapes. Clarify that compound figures may be composed by repeating any given shape (stacks of cubes). For example,



d. Additional Instructional Strategies/Differentiation –

- Ask students how formulas (such as area, surface area, volume, etc.) that are used in regards to two- and three-dimensional figures relate to their figure.

- Extensive modeling with concrete objects needs to be done in order for the students to develop a mental picture of compound three-dimensional shapes and the two-dimensional viewpoints that give the figure it's overall shape and vice versa.

- Using Cubes and Isometric drawings
  http://illuminations.nctm.org/LessonDetail.aspx?id=U166

- Guess The View
  http://www.fi.uu.nl/toepassingen/00198/toepassing_wisweb.en.html

e. Technology-

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No suggestions for use of technology or websites are included at this time.

f. Assessing the <u>Lesson</u>

Assess understanding of this lesson through an exit ticket question such as: "What are some ways that three-dimensional shapes are related to two-dimensional shapes?"

II. Teaching Lesson D

For this indicator, it is **essential** for students to:

- Understand the term congruent angles.
- Understand the term supplementary angles.
- Recall what two parallel lines cut by a transversal looks like
- Understand and identify the terms alternate exterior, alternate interior, corresponding, and adjacent in regards to the angles formed by parallel lines and a transversal.
- Find the value of a missing angle

For this indicator, it is **not essential** for students to:

- Use multiple sets of parallel lines and transversals in one drawing.

a. Indicators with Taxonomy

7-4.5 Analyze the congruent and supplementary relationships—specifically, alternate interior, alternate exterior, corresponding, and adjacent—of the angles formed by parallel lines and a transversal. (B4)

Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson –

Materials:
Cardboard strips
brads

Literature Connections:

Introductory Lesson: "Angle Relationships"

Groups of students create models of two parallel lines cut by a transversal using cardboard strips and brads. Attach the pieces of cardboard together at the intersections using the brads.

Students measure the angles formed by the intersecting lines to make conjectures about the measures of the angles formed by a transversal. Groups share their conjectures and defend their hypotheses about the relationships between the alternate interior, alternate exterior, corresponding, and adjacent angles.

c. Misconceptions/Common Errors –

- Students will think that angles are congruent when there are no parallel lines present. Emphasize that angles (alternate exterior, alternate interior, corresponding, and adjacent angles) can only be congruent when they are formed by parallel lines being intersected by a transversal.

- Students may be confused when the transversal is slanted differently. Exploring examples where the transversal is increasing from left to right and decreasing from left to right will help with this confusion.

d. Additional Instructional Strategies/Differentiation –

- Students need to formulate their own conclusions in regards to the angles formed by two parallel lines and a transversal through guided investigation before the formal definitions are introduced. Label the angles 1,2, 3, 4, etc… They measure the angles formed by these lines, and from their findings, make conjectures and draw conclusions about which angles are congruent(a concept taught in fifth grade) and which angles are supplementary(a concept taught in sixth grade). Once the students have a good understanding of which angles are congruent and/or supplementary, then the formal definitions can be introduced.
- Review and practice measuring angles with a protractor in a figure similar to the one given in the example.
- Group discussion about angles in the classroom and the real world. Students could make a list of the angles they see around their house, school, or neighborhood. *Quadrant C*
- Ask the students to use a geoboard to create parallel lines and transversals. They point out the corresponding angles and explain why they correspond. Do the same with vertical angles.

e. Technology-

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and the virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

This is a modified version of the activity “Angles Formed by Parallel Lines” by Karen Droga Campe found on the Texas Instruments Activity Exchange web site. http://education.ti.com/educationportal/activityexchange

f. Assessing the Lesson

Assess student understanding through questioning and dialogue as they formulate conjectures related to the angle relationships identified.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-4.2 Explain the results of the intersection of two or more geometric shapes in a plane. (B2)

The objective of this indicator is to explain which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To explain is to construct a cause and a sentence structured like "The point of intersection of _____ is ______ because ____." The learning progression to **explain** requires students to recall the characteristic of points, lines and planes. They explore concrete and pictorial models of the intersection of geometric shapes and analyze them to determine where and how the shapes intersect. Students use deductive reasoning (7-1.3) to generalize mathematical statements (7-1.5) related to the intersection of certain shapes (line and circle, line and plane, circle and circle, etc…). They use correct and clearly written or spoken words to communicate their understanding (7-1.6).

7-4.3 Illustrate the cross section of a solid. (C2)

The objective of this indicator is to illustrate, which is in the "understand procedural" knowledge cell of the Revised Taxonomy. To illustrate is to find specific example or illustrate of a concept; therefore, students explore a variety of example of cross section in order to understand how cross sections are found. The learning progression to **illustrate** requires students to recall the characteristics of cubes, rectangular prisms, cylinders and spheres. They analyze a variety of solids that are orientated vertically and model them with concrete objects in order to develop a mental picture of these three dimensional shapes. Students explore the result of cutting horizontal and vertical sections and generalize mathematical statements (7-1.5) about the relationship between the solid and its cross section. The students pose follow-up questions to the teachers and their classmates in order to prove or disprove their conjectures (7-1.2). They generalize connection among a variety of representational forms (concrete, pictorial and real world) to deepen their conceptual understand of this relationship. Students use correct and clearly written and spoken words to communicate their understanding (7-1.6).

7-4.4 Translate between two- and three-dimensional representations of compound figures. (B2)

The objective of this indicator is to translate, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To translate is to change from one form to another; therefore, students develop a conceptual understanding of the relationship between two dimensional compound shapes and their three dimensional representation. The learning progression to **translate** requires students to recall the characteristics of two and three-dimensional shapes and their two-dimensional nets. Students use concrete and pictorial models to explore how two-dimensional shapes work together and predict their perspectives using inductive and deductive reasoning (7-1.3). Students explore front, top and side views. Students generalize connections (7-1.7) between two dimensional and three dimensional representations using correct and clearly written words to communicate (7-1.6) their understanding. They use that understanding to translate between two and three-dimensional representations of compound figures.

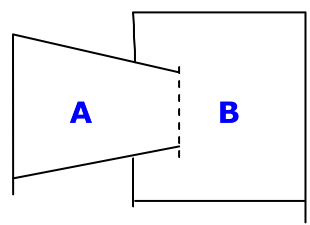
7-4.5 Analyze the congruent and supplementary relationships—specifically, alternate interior, alternate exterior, corresponding, and adjacent—of the angles formed by parallel lines and a transversal. (B4)

The objective of this indicator is to analyze, which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To analyze means to break down materials into its constituent parts and determine how the parts relate to one another and the overall structure; therefore, students determine the relationship between and among angles in the parallel line/transversal diagram. The learning progression to **analyze** requires students to recall the diagram illustrating two parallel lines cut by a transversal. Given a diagram, students explore the relationship by measuring angles. They generalize connections (7-1.7) between angles using correct and clearly written or spoken words (7-1.6). They explore a variety of examples to evaluate their conjectures (7-1.2) using inductive and deductive reasoning (7-1.3). They use these connections to generalize mathematical statements (7-1.5) that summarize the relationship between angles. At this point, students connect these relationships to the terms alternate interior, alternate exterior, corresponding and adjacent angles.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. What occurs when two or more geometric shapes intersect in a plane?

2. The point(s) of intersection of Shapes A & B ____________because ______________.



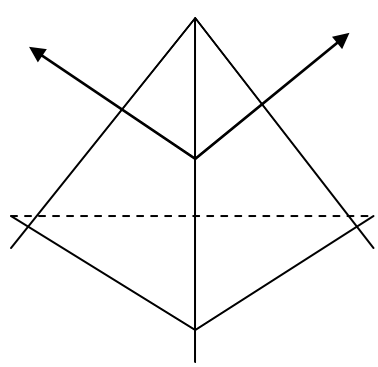
(answer: a line because multiple points intersect)

3. What are some examples of ways shapes intersect in a plane? Provide an illustration and explain your reasoning.

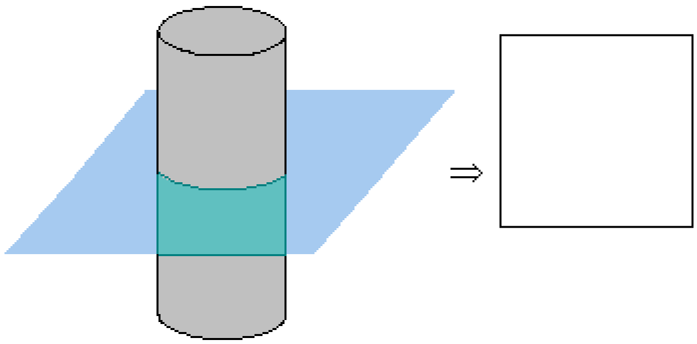
4. When a 3-D solid is sliced, what kind of figure is formed on the surface of the slice faces? (answer: 2-D) How do you know?

5. If a cube is sliced horizontally, what shape will the cross section show? How do you know? If the cube is sliced vertically, what shape will the cross section show? How do you know?

6. Examine the illustration below. What will the cross section show?

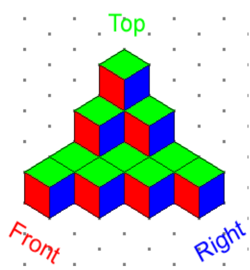


7. Draw a picture of what the cylinder becomes once cut along the cross section show below. Explain how you know.

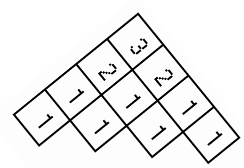


8. What connections can you make between a three-dimensional object and its two-dimensional cross section?

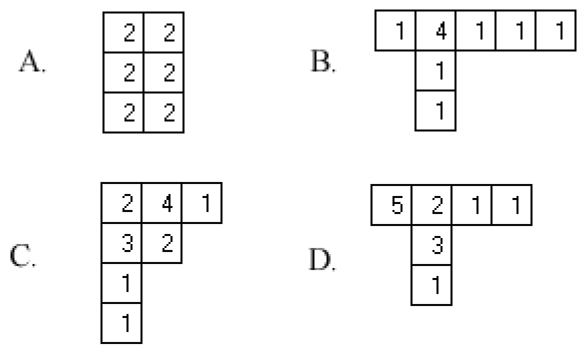
9. Draw the base of the isometric drawing below and indicate using numerals how many blocks are stacked on the base?



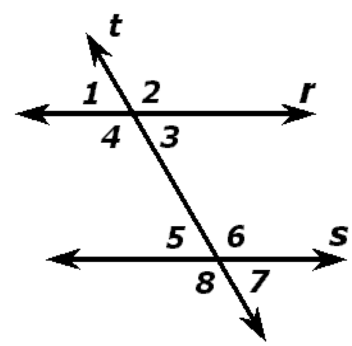
Answer



10. Draw (using isometric paper) or demonstrate (using actual cubes) the following four 3-D shapes.



11. If *r* is parallel to *s and angle 1 = 60 degrees* then find the measures of the other seven angles in the figure below.



Answer:

Angle 2 *= 120 degrees since it is supplementary to angle 1. Supplementary angles are any two angles whose sum is 180 degrees.*

Angle 3 *= 60 degrees since Angle 1 and Angle 3 are vertical angles. Vertical angles are two nonadjacent angles formed by two intersecting lines.*

Angle 4 *= 120 degrees since it is supplementary to angle 1.*

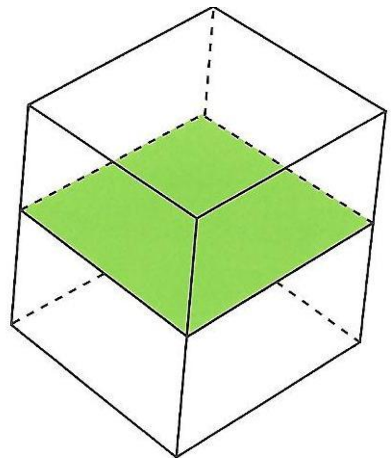
Angle 5 *= angle 1 by the rule of a transversal intersecting 2 parallel lines*

Angle 6 *= angle 2,*

Angle 7 *= angle 3, and*

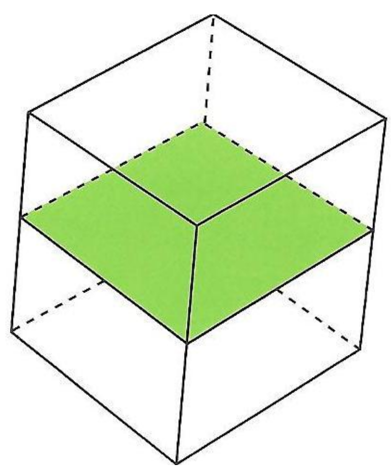
Angle 8 *= angle 4 by the rule of a transversal intersecting 2parallel lines*

12. Identify the two dimensional polygon shown in the cross section of the cube below:

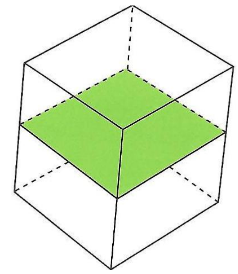


a.) parallelogram
b.) triangle
c.) square
d.) rhombus

13. If the cube has a side dimension of 4, what is the area of the figure in the cross section?

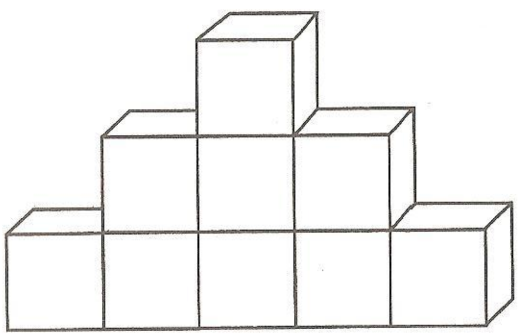


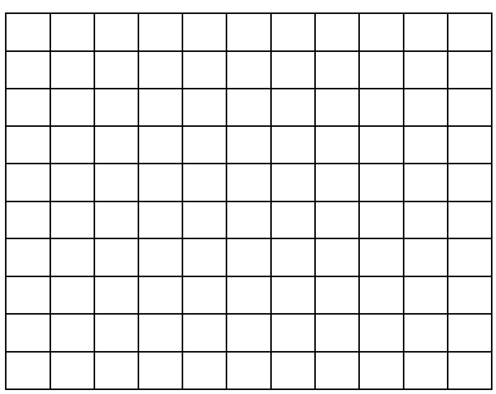
14. Two geometric prisms are formed when Greg sliced the cube into halves. What are these geometric prisms called?



a.) cubes
b.) rectangles
c.) rectangular prisms
d.) triangular prisms

15. Sketch the top view of Figure B below.

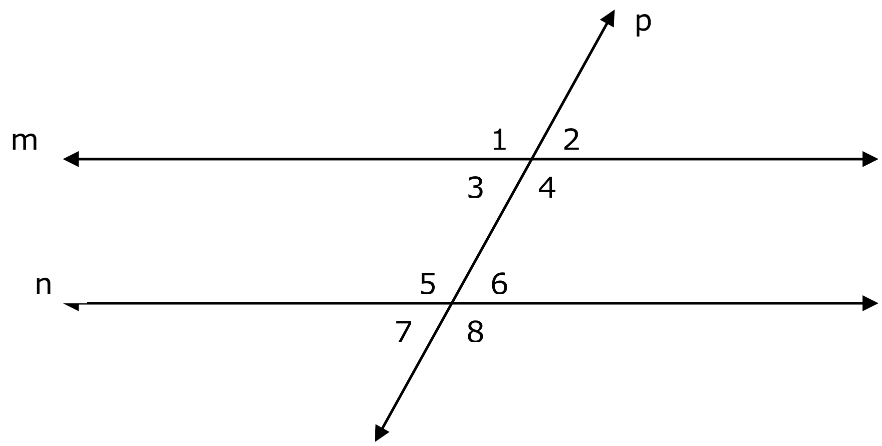




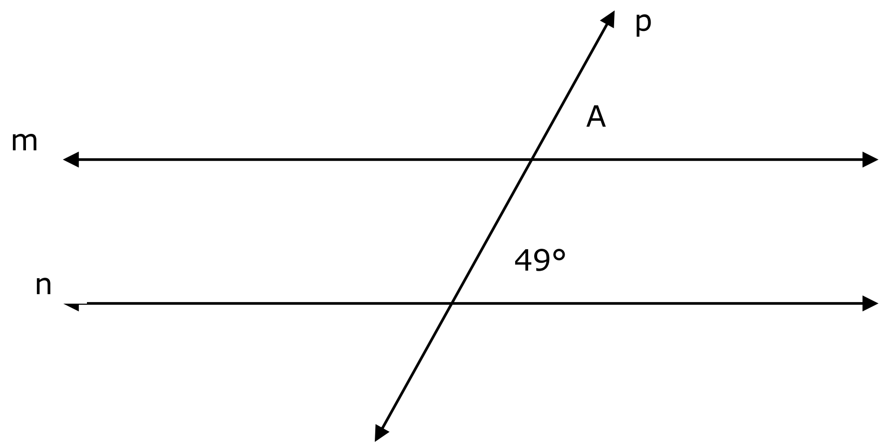
16. Lines *m* and *n* do not intersect. Line *p* intersects line *m* to form a right angle. Which of the following is NOT true?
    a. Lines *p* and *n* are perpendicular.
    b. Lines *m* and *n* are parallel.
    c. Lines *p* and *m* are perpendicular.
    d. All three statements are true.

(Source: NCTM, Mathematics Assessment Sampler: Grades 6-8)

17. Line m is parallel to line n. Line p intersects line m and n. If angle 2 has a measure of 51 degrees, find the measure of
    a.) angle 3
    b.) angle 7
    c.) angle 8



18. In the figure above, line m and n are parallel. Line p intersects line m and n. What is the measure of angle A?
a) 45 ° b) 149° c) 131° d) 49°



MODULE

3-4

Transformational Geometry

This module addresses the following indicators:

7-4.9 Create tessellations with transformations. (B6)
7-4.10 Explain the relationship of the angle measurements among shapes that tessellate. (B2)
7-4.1 Analyze geometric properties and the relationships among the properties of triangles, congruence, similarity, and transformations to make deductive arguments. (B4)

This module contains two lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

Continuum of Knowledge

7-4.9 Create tessellations with transformations. (B6)

- In sixth grade, students identified the transformations used to move a polygon from one location to another on the coordinate plane (6-4.5). Students also explained how the transformation would affect the original location of the polygon in the coordinate plane (6-4.6).

7-4.10 Explain the relationship of the angle measurements among shapes that tessellate. (B2)

- In sixth grade, students identified the transformations used to move a polygon from one location to another on the coordinate plane (6-4.5). Students also explained how the transformation would affect the original location of the polygon in the coordinate plane (6-4.6).

7-4.1 Analyze geometric properties and the relationships among the properties of triangles, congruence, similarity, and transformations to make deductive arguments. (B4)

- In fifth grade, students classified shapes as congruent (5-4.3). In sixth grade, they identify the transformations used to move a polygon from one location to another in the coordinate plane (6-4.5) and classified shapes as similar (6-4.8).
- In eighth grade, students apply a dilation to a square, rectangle or right triangle in a coordinate plane (8-4.3).

Key Concepts/Key Terms

*Tessellation
*Transformation
*Congruency
*Similarity
*Ratios
Deductive
Conjectures
*Proportional
*Rotation
*Translation
*Reflections
*Vertex
*Regular polygons
Sum of the angle
*Interior angle

*Plane

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lessons

1. Teaching Lesson A

7-4.9 *Create tessellations with transformations. (B6)*

For this indicator, it is **essential** for students to:

- Use transformations (translations, reflections, and rotations) to change a polygon to a new shape by cutting sections out of one or two of the sides and using a transformation to place this section on the other side.
- Use the transformed polygon as a stencil to create their own tessellation and explain how they transformed the polygon (i.e. identify the transformation used to create the stencil).
- Identify the transformation used to create a given tessellation.
- Create tessellations using combinations of regular polygons and rectangles.

For this indicator, it is **not essential** for students to:

- Create a tessellation using non-regular polygons (other than rectangles)

7-4.10 Explain the relationship of the angle measurements among shapes that tessellate. (B2)

For this indicator, it is **essential** for students to:

- Find the measure of an interior angle of a polygon
- Identify if a regular polygon will form a tessellation by itself or if it can be used in a combination to create a tessellation.
- Understand that in order for regular polygons to tessellate, the sum of the measure of the angles surrounding appoint (at a vertex) must 360 degrees.
- Use the formula $\frac{(n-2)180}{n}$ to find the measure of one interior angle.

  (n-2)180 will tell the students the sum off all the angles. *N*= number of sides.
- Know that tessellations cover a plane without overlaps or gaps using congruent figures or a combination of congruent figures.

For this indicator, it is **not essential** for students to:

- Identify if a non-regular polygon will form a tessellation.

a. Indicators with Taxonomy

7-4.9 Create tessellations with transformations. (B6)

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual Knowledge

7-4.10 Explain the relationship of the angle measurements among shapes that tessellate. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson: "Tessellating Polygons"

Materials:

pattern blocks
graph paper
scissors
tape
markers

Introductory Lesson: "Tessellating Polygons"

Let students arrange a variety of regular polygons on graph paper using lines to help keep form. (Polydrons can also be used as a manipulative, as well as traced figures reproduced and cut from paper.)

Make a chart and record the interior angle degrees of regular polygons (pattern blocks). The object is for students to discover that in order for regular polygons to tessellate the sum of the angles must be 360 degrees (using a combination of different regular polygons or just several of the same regular polygon). When an arrangement of polygons are placed side by side to cover a surface without overlaps or gaps, creating a convergence of angles meeting at one vertex that equals 360 degrees, a tessellation will occur.

Use this information to help create a tessellation.

(Examples)

Triangle	Rectangle	Hexagon	Octagon	
60 degrees	90 degrees	120 degrees	135 degrees	
4		1		60 + 60 + 60 + 60 + 120 = 360
6				60 x 6 = 360
	1		2	90 + 135 + 135 = 360

When a combination is discovered that will tessellate, the student should sketch or trace it on graph paper. Next, record the angle measures that created the tessellation. Have students create their own portfolio of 'discovered' patterns of regular polygons that tessellate. Move about the class asking if students notice anything about the polygons that will tessellate compared to the ones that will not. Close the lesson by discussing the relationship among angles that tessellate using a cause and effect model.

c. Misconceptions/Common Errors –

- Students may sometimes get confused differentiating between congruency and similarity. It is very important for students to be given the opportunity to investigate the similarities and differences between similar and congruent triangles.

d. Additional Instructional Strategies/Differentiation –

- This is the students' first introduction to tessellations. Tessellations (the covering of a plane without overlaps or gaps) are formed by transformations such as translations, reflections, and rotations. The emphasis should be on the transformations used to create the tessellation and the relationship among the polygons that can be used to tessellate (or tile) a plane (piece of paper).
- A great investigational lesson would use a square (sticky note size or smaller) or any regular polygon and let the students transform the polygon using a translation, reflection, or rotation. Students will change one or two sides of the square and then use this transformed square as a stencil to create a tessellation. The student needs to explain and illustrate how the polygon was transformed and explain how this affected the final tessellation. A guided lesson for this would be essential, tell the students to create a tessellation using each type instead of having them try to create all in one day. Students can also add details inside their transformed image to make their tessellation appear more Escher like.

- Students should also be able to identify what transformation was used to create a tessellation from a given example. Escher examples may be used but can get complicated and may be hard to identify.
- One strategy to teach finding the sum of angles and the measure of one angle of a regular polygon is by using the chart A below. Another strategy is to use Chart B. Both of these strategies will require guidance from the teacher. The objective is for students to discover that in order for regular (all sides are the same length) polygons to tessellate, the sum of the measures of the angles surrounding a point (at a vertex) must be 360°. This sum can be attained by using a combination of different regular polygons or just several of the same polygon. Using regular polygon manipulatives and the following chart may be helpful.

Chart A:

Regular Polygon - # 0f sides	# of Triangles created from 1 vertex Formula:____________	Sum of degrees for all angles of the polygon Formula:___________	# in degrees in 1interior angle of the polygon. Formula:__________	Will this polygon form a pure tessellation?
Triangle				
Quadrilateral				
Pentagon				
Hexagon				
Heptagon/Septagon				
Octagon				
Nonagon				
Decagon				
Dodecagon				
20-gon				
30-gon				

Chart B:

Shape	Triangle	Square	Hexagon	Octagon	
Measure of one interior angle	60°	90°	120°	135°	Combinations that total 360° and therefore will tessellate.
Number of each shape Used	6				60 (6) = 360
	4		1		60 (4) + 120 = 360
		1		2	90 (1)+ 135 (2) = 360
			3		120 (3) = 360
	1	2	1		60(1)+90(2) +120(1) = 360

e. Technology-

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and the virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- Congruency
  http://nlvm.usu.edu/en/nav/frames_asid_165_g_3_t_3.html?open=instructions&from=category_g_3_t_3.html

- Congruency, Similarity, and Symmetry
  http://standards.nctm.org/document/eexamples/chap6/6.4/part3.htm

f. Assessing the Lesson

Assess student understanding in this lesson through questioning during the discovery phase of the lesson and through the student portfolios of "discovered" patterns of regular polygons.

2. <u>Teaching Lesson B</u>

7-4.1 For this indicator, it is **<u>essential</u>** for students to:

- Recall the properties of polygons
- Understand the properties of congruency.
- Understand the properties of similarity.
- Perform transformations (reflection, rotation, translation)
- Use deductive reasoning to move from general to specific statements about triangles, congruence, similarity and transformation

For this indicator, it is **<u>not essential</u>** for students to:

- Prove congruency and similarity using formal methods of proofs.
- Find measures of missing sides and angles of triangles based on congruency and similarity.

a. Indicators with Taxonomy

7-4.1 Analyze geometric properties and the relationships among the properties of triangles, congruence, similarity, and transformations to make deductive arguments. (B4)

Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

***b. Introductory Lesson* – "Congruency in Triangles"**

Materials:

Literature Connections:

Introductory Lesson: "Congruency in Triangles"

Give students triangles that are congruent and have then compare the side lengths and angles. Have them to discuss and come up with the properties of congruent triangles and similar triangles. Make sure to conclude the discussion with a teacher led demonstration/discussion of SAS, SSS, ASA, AAS HL. Also make sure students understand that the corresponding angles are congruent in similar triangles. (proportional sides will be addressed in the next unit)

Ask students which transformations you could use to prove that 2 triangles are congruent or similar.

Use different triangles to perform transformations (translations, reflections, and rotations). Discuss what happens to the angles and sides when the transformation occurs. Have students form conjectures (such as the angle measure will always stay the same) and test them.

c. Misconceptions/Common Errors –

- Students may sometimes get confused differentiating between congruency and similarity. It is very important for students to be given the opportunity to investigate the similarities and differences between similar and congruent triangles.

d. Additional Instructional Strategies/Differentiation –

- The wording of the indicator implies that students make deductive arguments about the concepts of congruency, similarity and transformations using polygons other than triangles. Students analyze triangles, as well, to make deductive arguments about their properties and relationships. A primary focus is for students to begin to think more formally about the concepts of congruency, similarity and transformations and how they might be used when proving relationships.
- Review the difference between congruent and similar. Have students draw several pairs of congruent and similar triangles on graph paper. Students list the corresponding sides and corresponding angles of each pair of triangles.
- Draw pairs of similar triangles on index cards and label the dimensions. Have students find the scale factor of similarity. Vary the scale factors – use whole numbers, common fractions, and decimals.

- After the investigations, students work in cooperative groups to write generalizations about corresponding sides and corresponding angles in both congruent and similar triangles.

e. Technology-

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and the virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

No suggestions for use of technology or websites are included at this time.

f. Assessing the <u>Lesson</u>

Assess student understanding of congruent triangles through questioning as they share their conjectures with the class.

III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-4.9 Create tessellations with transformations. (B6)

The objective of this indicator is to <u>create</u>, which is in the "conceptual knowledge" of the Revised Taxonomy. To create is to put together elements to form a new, coherent whole or to make an original product; therefore, students use their understanding of transformations to create tessellations. The learning progression to **create** requires students to <u>recall</u> the definitions of translation, reflection, and rotations and to <u>identify</u> when they are being used. Students <u>explore</u> various strategies that allow them to develop an understanding of how to create a tessellation using the various transformations. Students <u>apply</u> their conceptual knowledge of transformations to create tessellations. Students also <u>identify</u> the transformations used to create a given tessellation. Students should use these procedures in context by examining real world examples (7-1.7) such as tiles or patterns as opposed to only creating tessellations from polygons. They <u>use</u> correct and clearly written or spoken words to communicate their understanding of the transformations used to create the tessellation (7-1.6).

7-4.10 Explain the relationship of the angle measurements among shapes that tessellate. (B2)

The objective of this indicator is to explain, which is in the "understand conceptual" of the Revised Taxonomy. To explain is to construct a cause and effect model; therefore, students summarize relationships among angles that tessellate using that model. The learning progression to **explain** requires students to recall the meaning of tessellate and how to create a tessellation. Students also recall the characteristics of regular polygons and how to find the measure of one interior angle of a polygon. They use this knowledge to explore relationships among the angles of a variety of regular polygons. They generalize connections (7-1.7) and generalize mathematical statements (7-1.5) explaining those connections. They use correct and clearly written or spoken words to communicate their understanding of these relationships to their classmates and teacher (7-1.6).

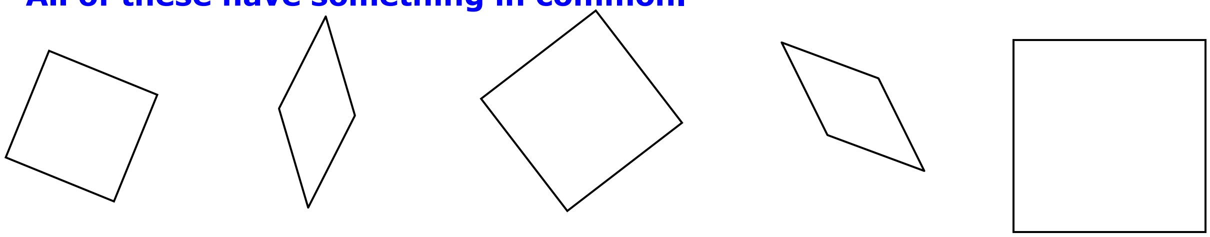
7-4.1 Analyze geometric properties and the relationships among the properties of triangles, congruence, similarity, and transformations to make deductive arguments. (B4)

The objective of this indicator is to analyze, which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge of similarity, congruency, transformation and triangles should include exploring a variety of polygons. The learning progression to **analyze** requires students to compare congruent and similar polygons to understand their similarities and differences. Students use inductive and deductive reasoning to generalize connections among these shapes. They generalize mathematical statements (7-1.5) about these connections and pose follow-up questions to prove or disprove their conjectures (7-1.2). They explain and justify their deductive arguments using correct and clearly written or spoken words or notation to communicate (7-1.6) their findings to their peers and teacher.

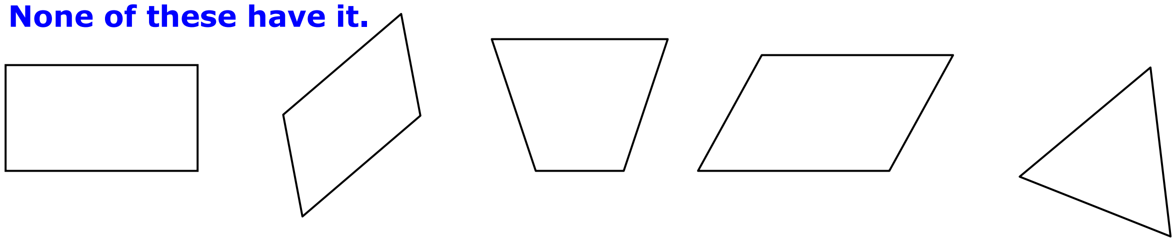
The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. *Source: Adapted from Teaching Student-Centered Mathematics Grades 5-8, John A. Van de Walle and LouAnn H. Lovin, Pearson, 2006, page 197.*

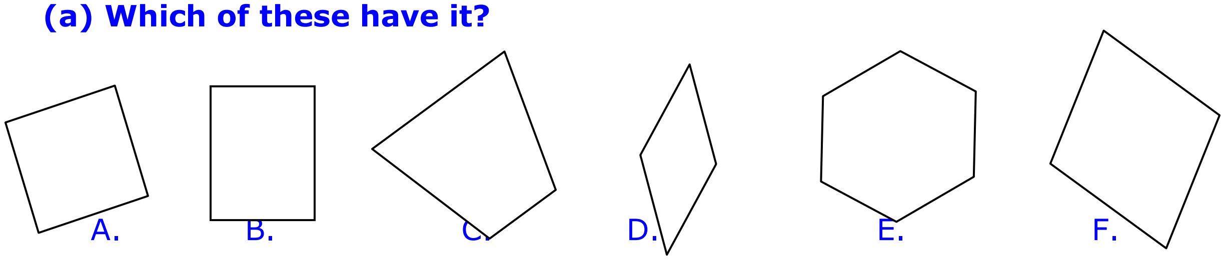
All of these have something in common.



None of these have it.



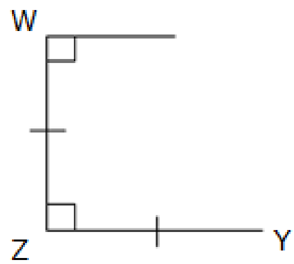
(a) Which of these have it?



(b) Explain the common characteristic(s).

(c) If the polygon does not have the characteristic(s), identify the missing characteristic from each polygon from that polygon?

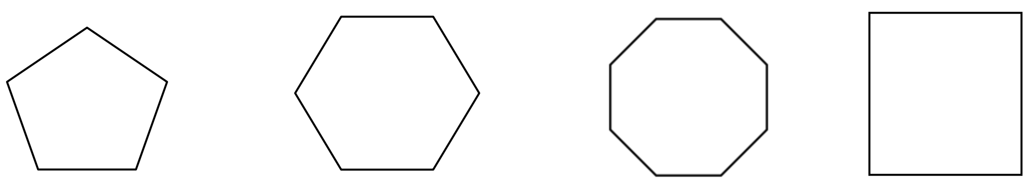
2. If no other sides or angles are congruent, which best describes the figure? How do you know?



Solution: *The completed figure would be a trapezoid. The two right Angles require the opposite sides to be parallel, so no other sides are parallel. Only one pair of adjacent sides is congruent.*

Source: Question 2 adapted from Mathematics Assessment Sampler Grades 6-8: (Reston,VA: The National Council of Teachers of Mathematics, 2005, P.111).

3. Which of the following two shapes can be combined to form a tessellation? Justify your answer.

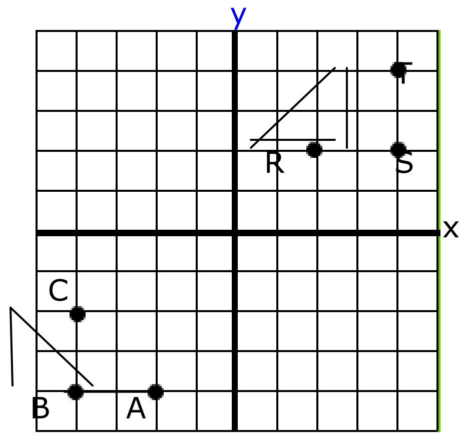


A. Pentagon B. Hexagon C. Octagon D. Square

Solution: *The Octagon and the Square can be combined to form a tessellation. Students should be able to explain the relationship of the angle measurements among shapes that tessellate.*

Source: Question 3 adapted from Mathematics Assessment Sampler Grades 6-8: (Reston,VA: The National Council of Teachers of Mathematics, 2005, P.129).

4. ΔABC and ΔRST are shown on the grid below.



Which of the following transformations will map abc onto rst.

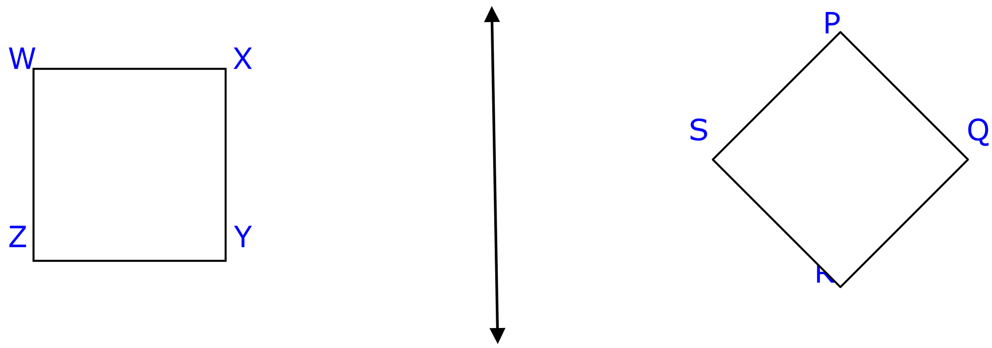
a. Reflect ΔABC over the y-axis, and shift up 6 spaces.
b. Reflect ΔABC over the x-axis, and shift up 6 spaces.
c. Reflect ΔABC over the y-axis, and shift down 6 spaces.
d. Reflect ΔABC over the y-axis, reflect over the x-axis, and shift down 4 spaces.

Solution: *The correct response is a.*

Source: Question 4 adapted from Mathematics Assessment Sampler Grades 6-8: (Reston,VA: The National Council of Teachers of Mathematics, 2005, P. 131).

***You can create more examples for flips, slides, etc.**

5. Describe the transformations that would move square PQRS onto square WXYZ.

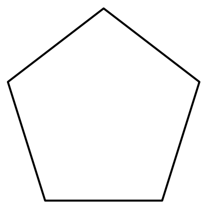


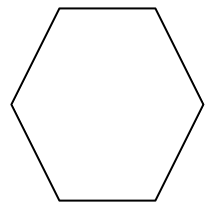
Solution: *Answers may vary.*

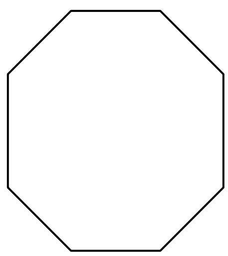
Source: Question 5 adapted from Mathematics Assessment Sampler Grades 6-8: (Reston,VA: The National Council of Teachers of Mathematics, 2005, P.132).

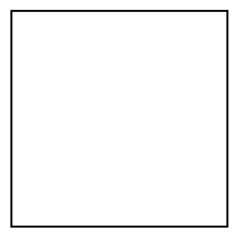
6. Which of the following two shapes can be combined to form a tessellation?

Pentagon Hexagon









Octagon Square

a) the pentagon and hexagon
b) the octagon and the square
c) the pentagon and the octagon
d) the hexagon and the square

(Source: NCTM, Mathematics Assessment Sampler: Grades 6-8)

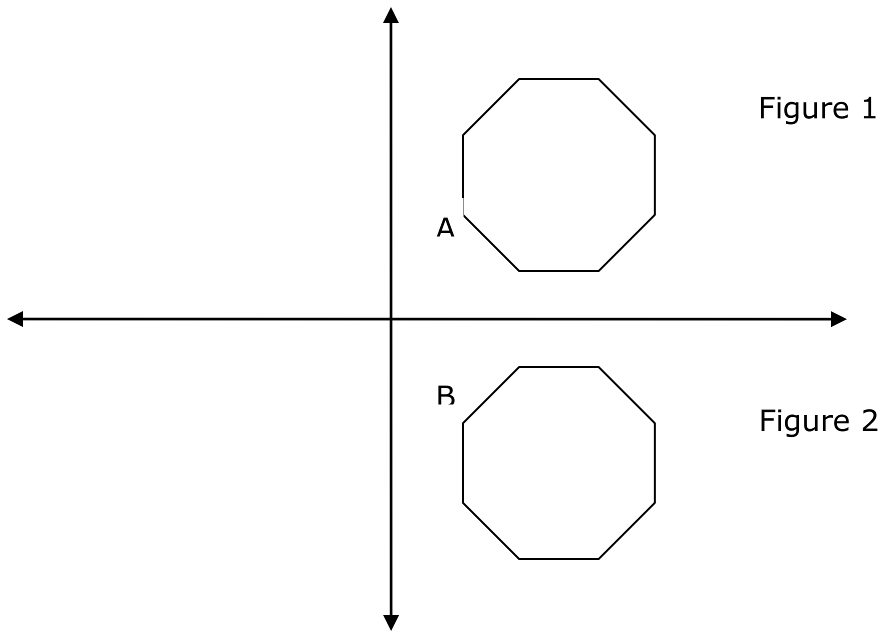
7. Use the pattern blocks to help you investigate which regular polygons tessellate. Find the measure of each interior angle of the regular polygons in the set. How many of the polygons of the same shape can you place around a point without an overlap? Record your findings in the chart below. DO NOT USE A PROTRACTOR.

Polygon	**Sketch**	**Measure of interior angle**	**How many polygons around a point?**	**Gap or overlap?**
triangle		60^o	6	no gap
square		________	________	________
pentagon		________	________	________
hexagon		________	________	________
octagon		________	________	________
decagon		________	________	________
dodecagon		________	________	________

What conjectures can you make as a result of this investigation of regular polygons?

8. Which is most likely the type of transformation that takes place from Figure 1 to Figure 2?

a) Rotation about the origin
b) Dilation
c) Reflection across the x-axis
d) Reflection across the y-axis



9. Using the figure above, if the coordinates of Point A are (1,2), what are the coordinates of point B?

a) (1, 2) b) (2,1) c(1,-2) d(-2,1)

10. What are the coordinates of the point (3,1) after a 180° rotation clockwise about the origin?

a) (3, 1) b) (3,-1) c (-1,-3) d (-3,-1)

MODULE

3-5

Proportional Reasoning

This module addresses the following indicators:

7-4.6 Compare the areas of similar shapes and the areas of congruent shapes. (B2)
7-4.7 Explain the proportional relationship among attributes of similar shapes. (B2)
7-4.8 Apply proportional reasoning to find missing attributes of similar shapes. (C3)

This module contains one lesson. This lesson is **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

Continuum of Knowledge

7-4.6 Compare the areas of similar shapes and the areas of congruent shapes. (B2)

- In sixth grade, students compared the angles, side lengths, and perimeters of similar shapes (6-4.7)
- In eighth grade, students apply dilations to squares and other polygons to form similar shapes on the coordinate system (8-4.4).

7-4.7 Explain the proportional relationship among attributes of similar shapes. (B2)

- In sixth grade, students compared the angles, side lengths, and perimeters of similar shapes (6-4.7) and classified shapes as similar (6-4.8).
- In eighth grade, students apply dilations to squares and other polygons to form similar shapes on the coordinate system (8-4.4). These standards will also be addressed again in high school geometry.

7-4.8 Apply proportional reasoning to find missing attributes of similar shapes. (C3)

- In sixth grade, students compared the angles, side lengths, and perimeters of similar shapes (6-4.7) and classified shapes as similar (6-4.8).
- In eighth grade, students apply dilations to squares and other polygons to form similar shapes on the coordinate system (8-4.4). These standards will also be addressed again in high school geometry.

Key Concepts/Key Terms

*Similar shapes
*Congruent shapes
*Corresponding sides
*Corresponding angles
Proportional reasoning
Proportional relationship
Proportion
Cross products

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

II. Teaching the Lesson(s)

1. Teaching Lesson A

7-4.6 Compare the areas of similar shapes and the areas of congruent shapes. (B2)

For this indicator, it is **essential** for students to:

- Identify the corresponding angles and sides of similar and congruent figures.
- Know the difference between a similar and congruent figure.
- Know that the area of congruent shapes is equal while the area of similar shapes is proportional but not equal.
- Use proportional reasoning to describe relationships between the areas of similar shapes
- Understand how increasing the side lengths affect the area- Example if you triple the side length the area is 9 times the original area.

For this indicator, it is **not essential** for students to:

- To find the length of a missing side using a proportion.
- To find the measure of a missing angle.

7-4.7 Explain the proportional relationship among attributes of similar shapes. (B2)

For this indicator, it is **essential** for students to:

- Identify the proportional relationships of polygons.
- Set up proportions using corresponding sides.
- Understand that cross products of a proportion should be equal to prove that two polygons are similar.
- Understand that sides that correspond are connected by corresponding angles that are congruent.
- Understand that corresponding angle measures are congruent while corresponding sides are proportional

For this indicator, it is **not essential** for students to:

- To find the side length of a missing side
- To find the measure of a missing angle

7-4.8 Apply proportional reasoning to find missing attributes of similar shapes. (C3)

For this indicator, it is **essential** for students to:

- Understand the characteristics of similar shapes
- Determine corresponding relationships between angles and sides
- Solve a proportion
- Understand proportional reasoning

- Use proportions to find the measure of a missing side length.
- Use proportions to find the measure of a missing angle

For this indicator, it is **not essential** for students to:

- To find the measure of a missing angle based on the sum of the angles in the polygon.

a. Indicators with Taxonomy

7-4.6 Compare the areas of similar shapes and the areas of congruent shapes. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

7-4.7 Explain the proportional relationship among attributes of similar shapes. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

7-4.8 Apply proportional reasoning to find missing attributes of similar shapes. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson –

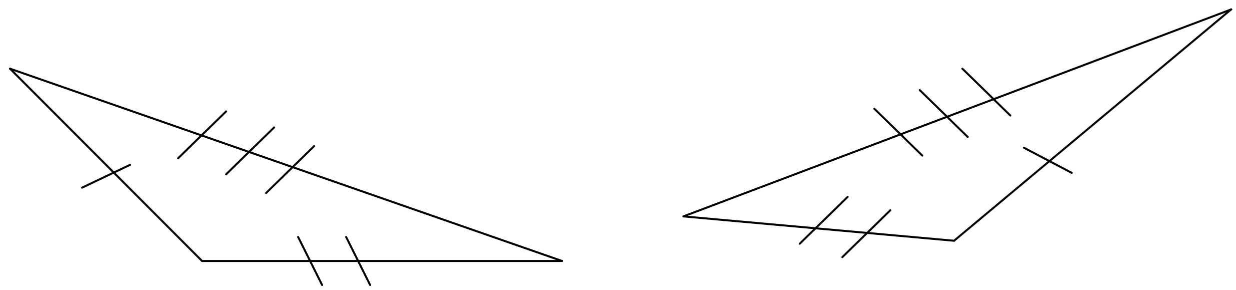
Materials:
Assorted triangles cut from cardstock (see sample below)
Scissors
Protractors
Rulers (with inches)
Colored Pencils (optional)
Calculators (recommended)

Literature Connections:

Introductory Lesson: "Similar Triangles"

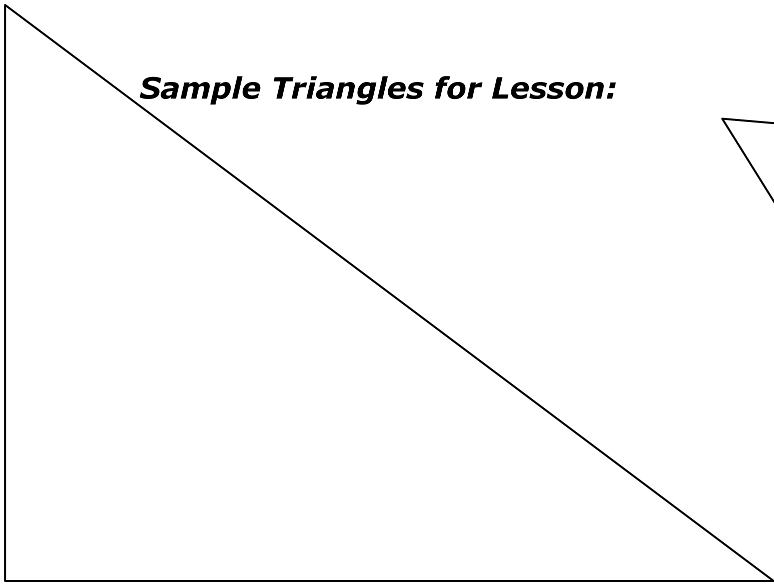
- Students should work in groups of 3 or 4. Have them sort the triangles into groups based on similar characteristics. Each group should share with the class how they grouped the triangles.
- Following the discussion, ask students what similar triangles are: same shape, different size. Now tell the students to group the similar triangles together.

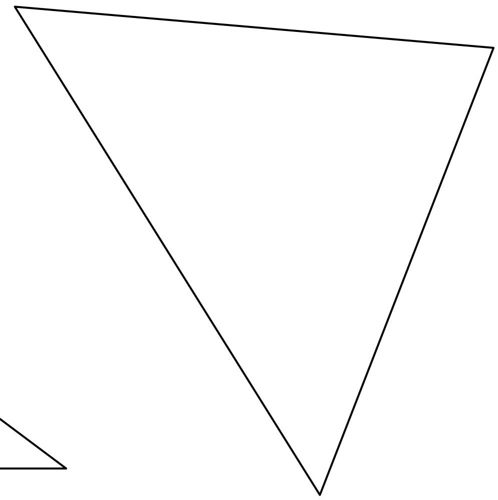
- Guide the students through labeling the triangles in the following way: 1a, 1b, 1c, 2a, 2b, 2c, 3a, 3b, 3c, with the number being the similar groups: 1 – acute triangles, 2 – right triangles, 3 – obtuse triangles, and the letter being the size: a – smallest, b – middle size, c – largest.
- Students should match the corresponding angles for each group of similar triangles. Ask: "What do you notice about the angles in similar triangles?"
- Allow students to test the conjectures they make about the angles of the similar triangles by measuring the angles with a protractor.
- Next, students will discover what a scale factor is. Discuss the concept of corresponding sides with the students. Have them label the corresponding sides of each set of triangles. They can use different colored pencils to mark the corresponding sides or they can mark them using slashes as shown below:

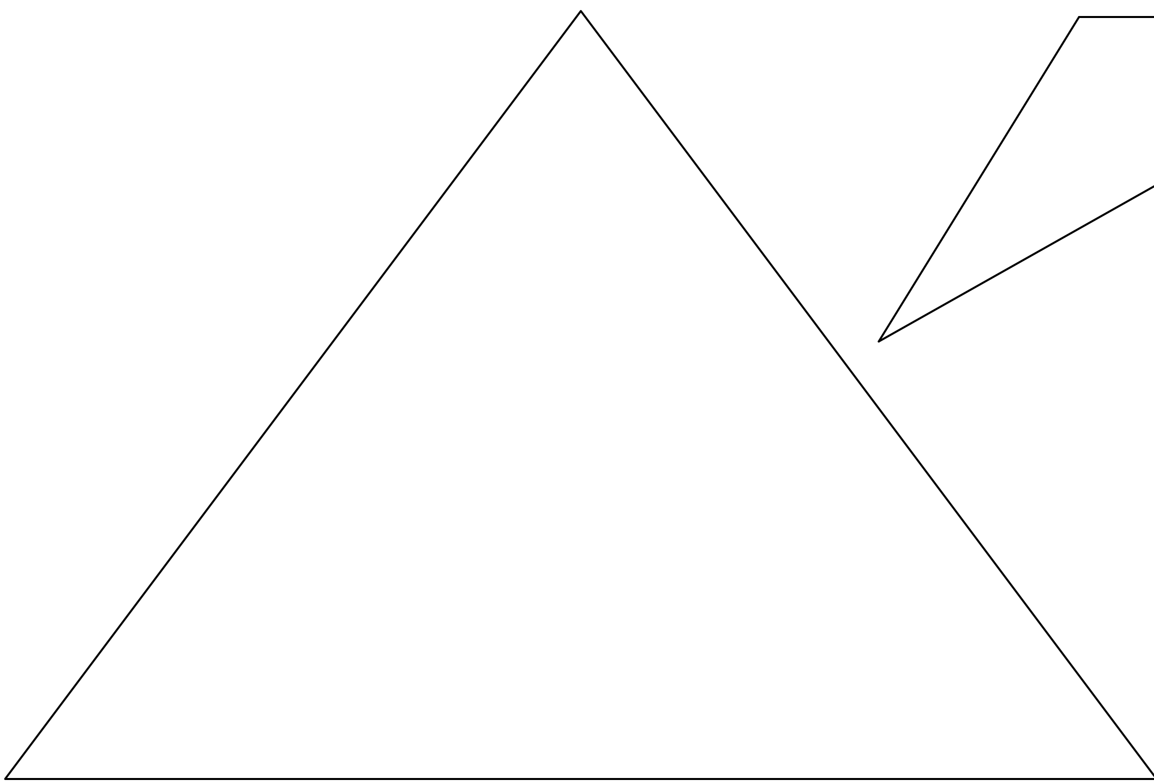


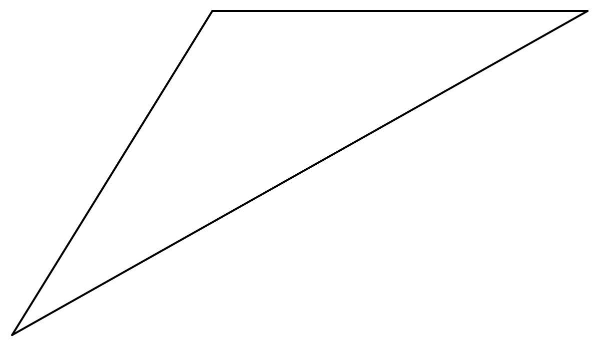
- Each group will create a table showing the ratio of the sides of pairs of similar triangles. Use the data to make a conjecture about the relationship between the sides of similar triangles.

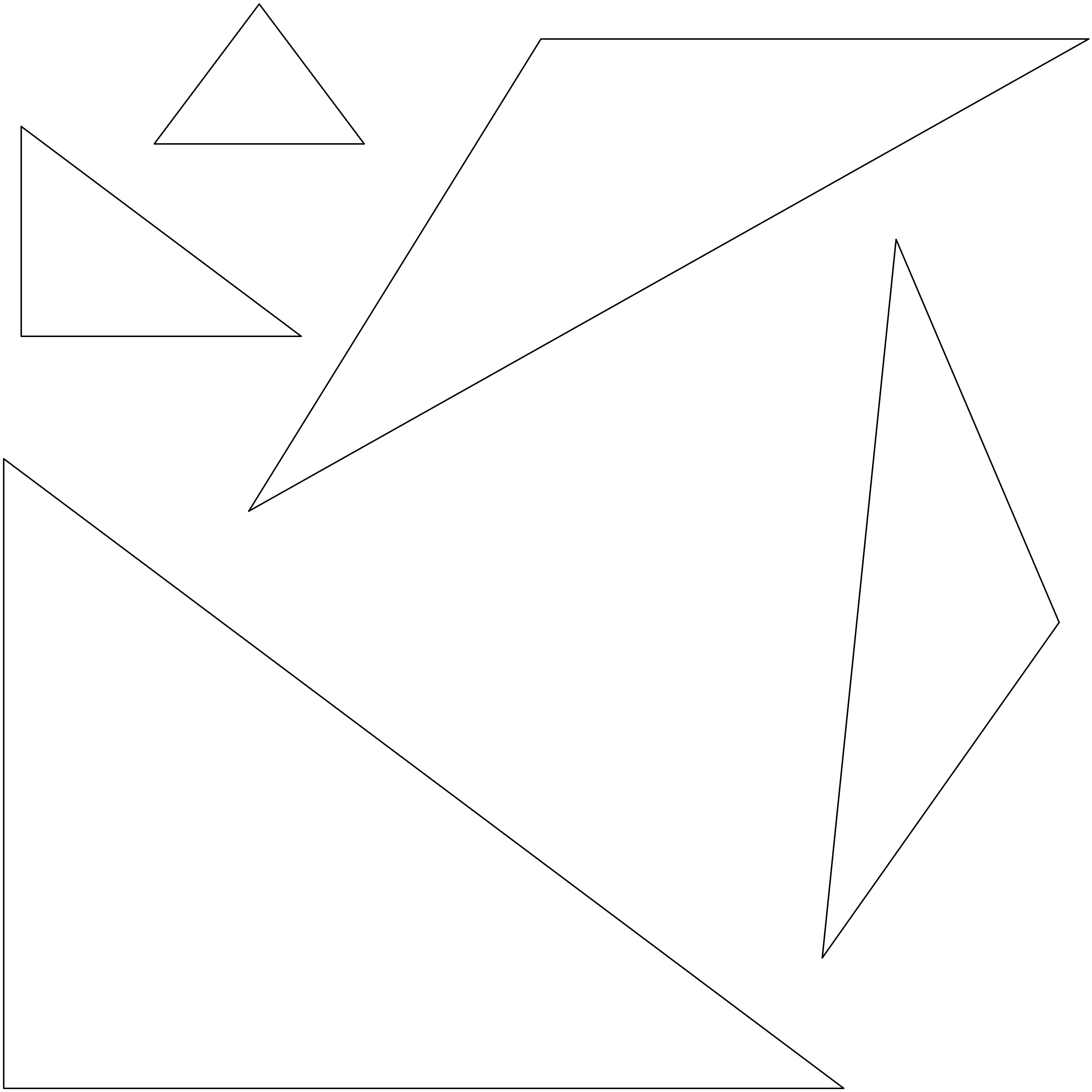
Sample Triangles for Lesson:











c. Misconceptions/Common Errors –

- Students often think the increase of the side length will be the same for the increase in the area. They do not make the connection that if the side length is multiplied by 3 or tripled, then the area will be increase by the square of that number or in this case will be nine times the original area.
- Students may forget that they should have a correspondence between sides and angles.
- Students often think that all shapes have the same orientation. Students do not focus on the concept that corresponding sides are connected by the corresponding angles.

d. Additional Instructional Strategies/ Differentiation –

- Re-teach: Have students to build shapes using pattern blocks. Have them to build a similar shape using the same type of pattern blocks and then find the proportional relationship between the number of pattern blocks used (or the area). Have students to draw similar shapes on graph paper and find the areas. Have them to find the proportional relationships between the areas. Have students use different size manipulatives such as straws, or tiles to build figures. Measure sides and predict the measures of other sides using proportions before measuring
- Students will compare the areas of similar shapes and the areas of congruent shapes. Students should be given the opportunity to discover that the areas of congruent shapes are equal whereas the areas of similar shapes are not.
- Although not required, this indicator may be extended to allow students to discover and understand that the area of the larger of two similar shapes will be the area of the smaller shape multiplied by the square of the scale factor needed to create the larger similar shape. This may be difficult for students to initially conclude; therefore, numerous examples should be done to help students see this relationship.
- The use of isometric dot paper is a great strategy to show the students the relationships between similar shapes and area.
- It is extremely important that students have made the connections in regards to similar figures that corresponding angle measures are congruent while corresponding sides are proportional.
- Seventh grade students must have an in-depth understanding of when and how to apply proportional reasoning to solve various types of problems. Although students set up a proportion to find missing attributes, they should also communicate how they would solve the problem using proportional reasoning. A sample explanation could be "since this side is twice as big as its corresponding side in order to keep the proportional relationship the other side must be twice as much too so it equals 8."

e. Technology-

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and the virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

No suggestions for use of technology or websites are included at this time.

f. Assessing the Lesson

As an exit slip, have students write definitions of the following terms in their own words: similar figures, corresponding angles, corresponding sides, and scale factor.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-4.6 Compare the areas of similar shapes and the areas of congruent shapes. (B2)

The objective of this indicator is compare, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning; therefore, students develop a conceptual understanding of the area of similar and congruent shapes using a variety of examples. The learning progression to **compare** requires students to recall the relationships among attributes of similar and congruent shapes. Using a variety of concrete and pictorial examples, students explore relationships among the area of similar and congruent shapes. They also explore how the area is affected by changes in the dimensions. Students generalize connections (7-1.7) and generalize mathematical statements (7-1.5) based on their observations. They use these generalizations to compare areas of a variety of shapes.

7-4.7 Explain the proportional relationship among attributes of similar shapes. (B2)

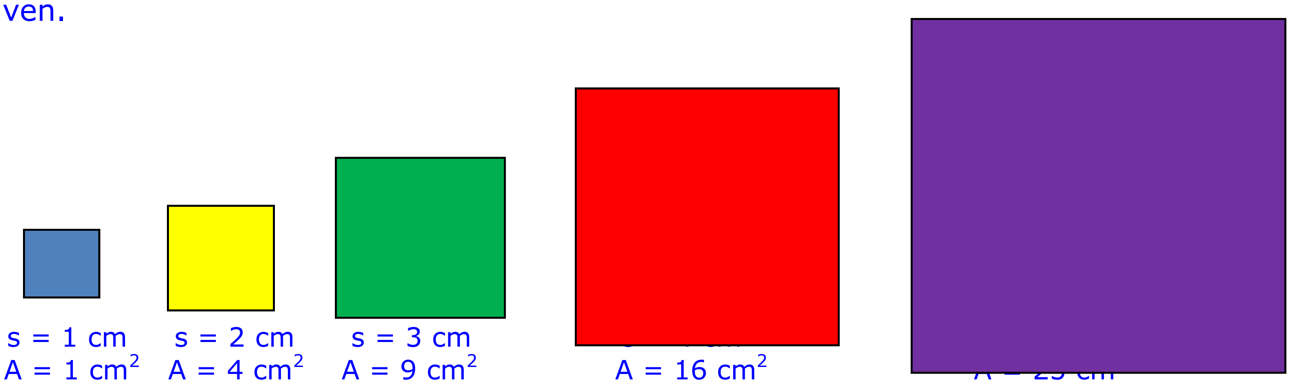
The objective of this indicator is to explain which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To explain means to construct a cause and effect model; therefore, students should show their conceptual understanding of these proportional relationships is that model. The learning progression to **explain** requires students to recall and understand the characteristics of similar shapes. Students analyze shapes and create a correspondence between angles and sides. They use these correspondences to set up possible proportional relationships. Students use their understanding of the cross product to evaluate their conjectures then explain and justify their answers using correct and clearly written or spoken words (7-1.6).

7-4.8 Apply proportional reasoning to find missing attributes of similar shapes. (C3)

The objective of this indicator is to apply, which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is to apply, the learning progression should integrate strategies to support conceptual as well as procedural knowledge of proportional reasoning. The learning progression to **apply** requires students to recall the characteristics of similar shapes. They also recall and understand how determine corresponding parts of similar shapes. Students use their understanding of the relationships between corresponding parts to describe proportional relationships. They describe these relationships verbally using proportional reasoning and symbolically using a proportion with an unknown. Students evaluate their answers (7-1.2) and use correct and clearly written or spoken words (7-1.6) to justify their answers using their understanding of proportional reasoning.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Below is a representation of several similar squares. The area of each square is given.



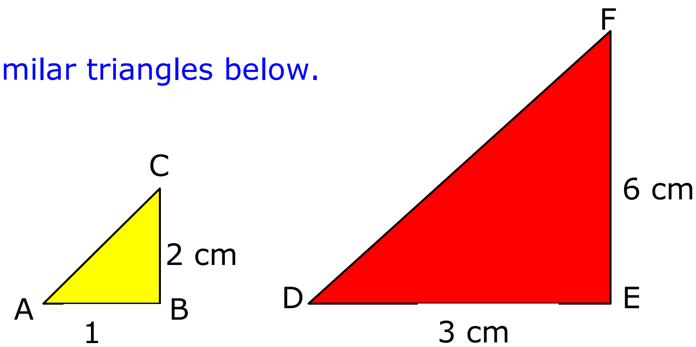
s = 1 cm, A = 1 cm^2 s = 2 cm, A = 4 cm^2 s = 3 cm, A = 9 cm^2 A = 16 cm^2

a. What patterns and relationships do you notice between the side lengths and the areas of the squares?

b. In what ways is the area of each square affected by the change in the side lengths? Provide an example to justify your reasoning.

c. In what ways do the areas of the blue and red square compare? In what ways do the areas of the blue and purple square compare? Explain your reasoning.

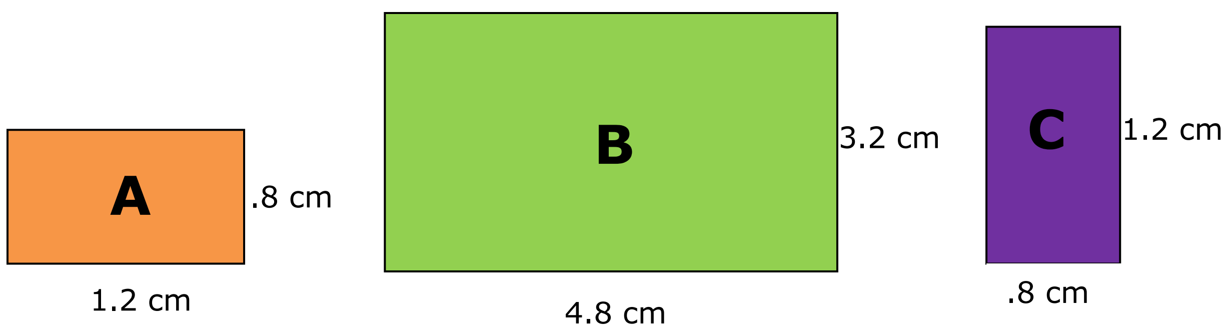
2. You are given the similar triangles below.



a. Identify all corresponding angles and side lengths.

b. The side length of the red triangle is triple the side length of the yellow triangle. What is the relationship between the areas of these triangles? Explain your reasoning.

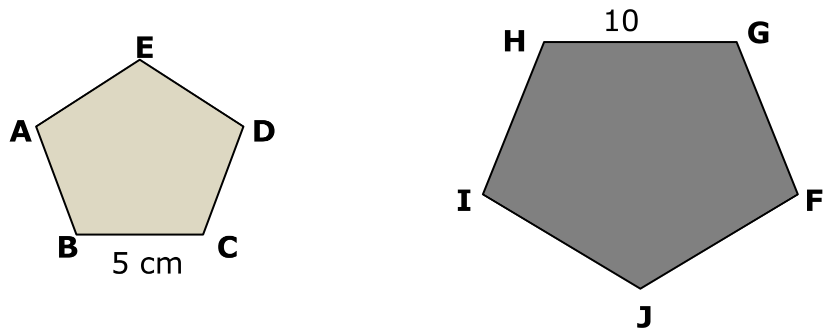
3. Compare the side lengths *and* areas of the following shapes and make a statement to summarize your findings:



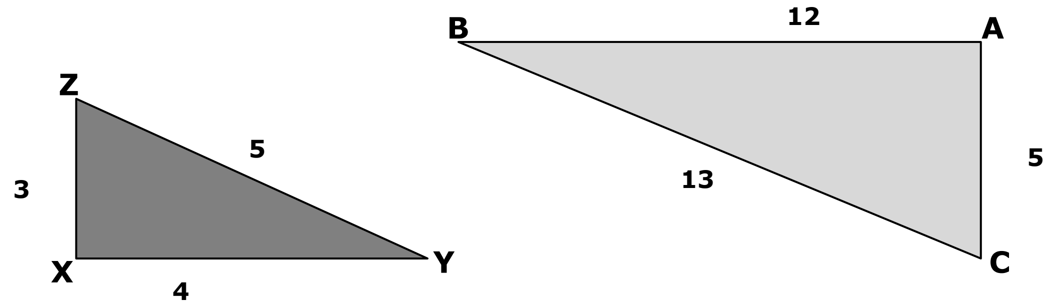
a. Shape A and shape B.

b. Shape A and shape C.

4. Using the shapes below, explain why these regular pentagons are similar using cross products.



5. Use the right triangles below to do the following:



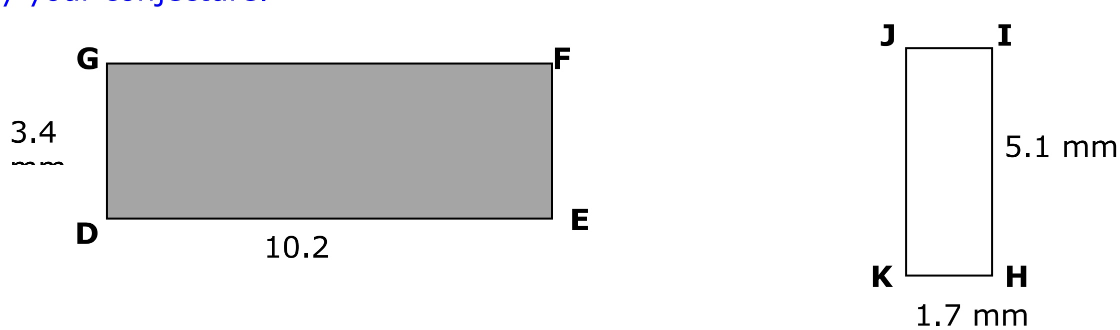
a. Explain the relationship between the angles

b. Explain the relationship between the sides

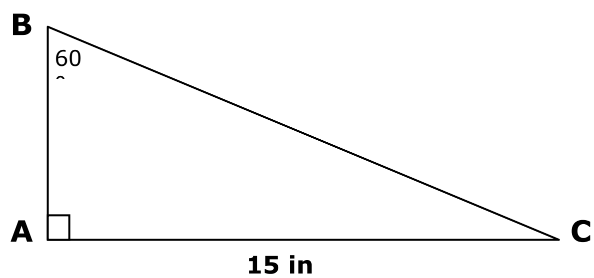
c. Set up proportions using corresponding sides

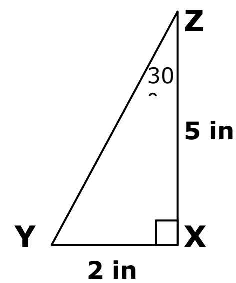
d. Show that the polygons are similar using cross products

6. Make a conjecture about the shapes below. Use cross products to explain and justify your conjecture.



7. Using the similar right triangles below, complete the following tasks:



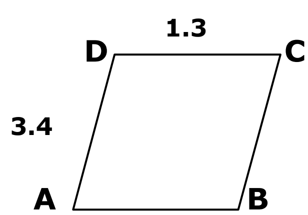


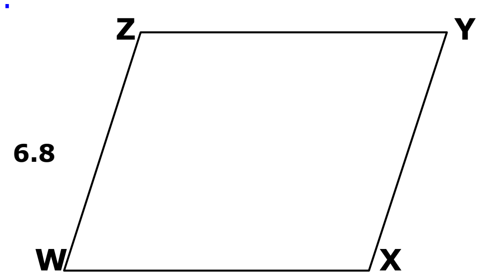
a. Describe the relationships between the sides and angle measures.

b. Find the measure of angle A, angle Y and angle C. Explain your reasoning.

c. Find the length of side AB using proportional reasoning. Justify your result.

8. Use the similar equilateral triangles below to:



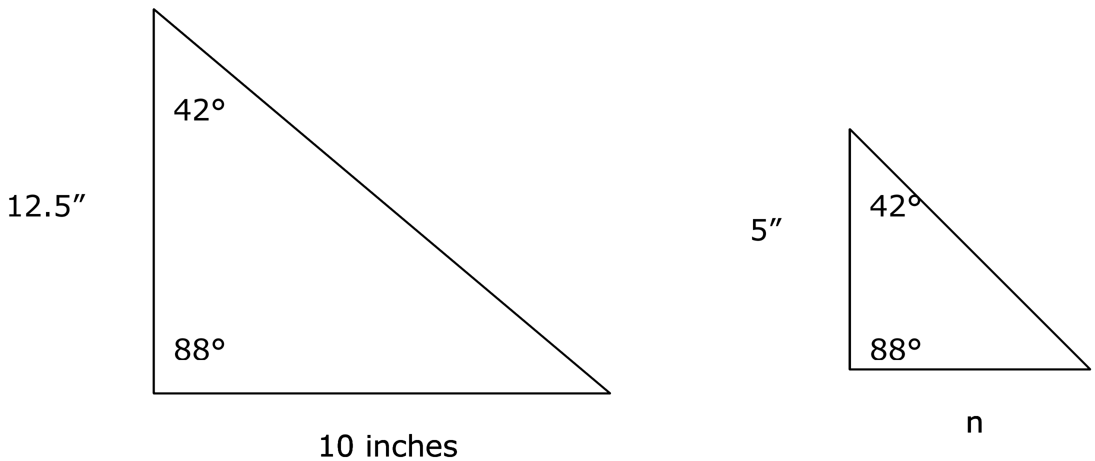


a. Identify all corresponding angle and side lengths

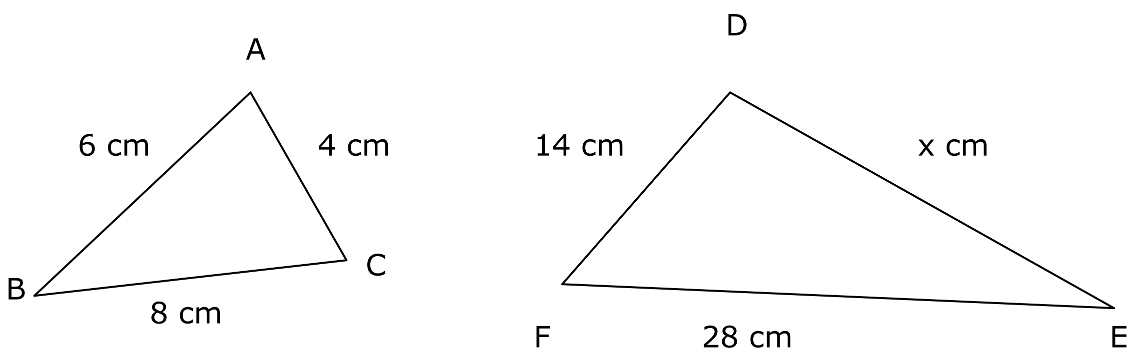
b. Describe all proportional relationships

c. Using symbolic representation and cross products, find the missing length of side ZY. Evaluate your result. Explain your process and your reasoning.

9. The triangles shown below are similar. What is the value of **n**?



10. ΔABC ~ ΔDEF. Find the value of x



11. A building casts a shadow of 32ft. A 6ft basketball player standing next to the building cast a 2.5ft shadow. What is the height in feet of the building?