SOUTH CAROLINA SUPPORT SYSTEM INSTRUCTIONAL GUIDE

Content Area	**Seventh Grade Math**

Fourth Nine Weeks

Standard/Indicators Addressed:

Standard 7-5: The student will demonstrate through the mathematical processes an understanding of how to use ratio and proportion to solve problems involving scale factors and rates and how to use one-step unit analysis to convert between and within the U.S. Customary System and the metric system.

7-5.2 Apply strategies and formulas to determine the surface area and volume of the three-dimensional shapes prism, pyramid, and cylinder. (C3)
7-5.3 Generate strategies to determine the perimeters and areas of trapezoids. (B6)

Standard 7-6: Through the process standards students will demonstrate an understanding of relationships between two populations or samples through data analysis and probability.

7-6.1 Predict the characteristics of two populations based on the analysis of sample data. (B2)
7-6.2 Organize data in box plots or circle graphs as appropriate. (B4)
7-6.3 Apply procedures to calculate the interquartile range. (C3)
7-6.4 Interpret the interquartile range for data. (B2)
7-6.5 Apply procedures to calculate the probability of mutually exclusive simple or compound events. (C3)
7-6.6 Interpret the probability of mutually exclusive simple or compound events. (B2)
7-6.7 Differentiate between experimental and theoretical probability of the same event. (A4)
7-6.8 Use the fundamental counting principle to determine the number of possible outcomes for a multistage event. (C3)
7-6.1 Predict the characteristics of two populations based on the analysis of sample data. (B2)

- **These indicators are covered in the following 3 Modules for this Nine Weeks Period.**

Module 4-1 Perimeter, Area, and Volume			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 4-1 Lesson A: **7-5.3** Generate strategies to determine the perimeters and areas of trapezoids. (B6)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations - See Appendix A	See Instructional Planning Guide Module 4-1, Introductory Lesson A See Module 4-1, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 4-1 Lesson A Assessment
Module 4-1 Lesson B: **7-5.2** Apply strategies and formulas to determine the surface area and volume of the three-dimensional shapes prism, pyramid, and cylinder. (C3)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations -See Appendix A	See Instructional Planning Guide Module 4-1, Introductory Lesson B See Module 4-1, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 4-1 Lesson B Assessment
Module 4-2 Data Collection, Representation, and Analysis			

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines
Module 4-2 Lesson A: **7-6.2** Organize data in box plots or circle graphs as appropriate. (B4) **7-6.3** Apply procedures to calculate the interquartile range. (C3) **7-6.4** Interpret the interquartile range for data. (B2)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations - See Appendix A	See Instructional Planning Guide Module 4-2, Introductory Lesson A See Module 4-2, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 4-2 Lesson A Assessment
Module 4-2 Lesson B: **7-6.1** Predict the characteristics of two populations based on the analysis of sample data. (B2)		See Instructional Planning Guide Module 4-2, Introductory Lesson B See Module 4-2, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 4-2 Lesson B Assessment

Module 4-3 Probability			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 4-3 Lesson A: **7-6.5** Apply procedures to calculate the probability of mutually exclusive simple or compound events. (C3) **7-6.6** Interpret the probability of mutually exclusive simple or compound events. (B2) **7-6.7** Differentiate between experimental and theoretical probability of the same event. (A4)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations - See Appendix A	See Instructional Planning Guide Module 4-3, Introductory Lesson A See Module 4-3, Lesson A Additional Instructional Strategies	See Instructional Planning Guide Module 4-3 Lesson A Assessment
Module 4-3 Lesson B: **7-6.8** Use the fundamental counting principle to determine the number of possible outcomes for a multistage event. (C3)		See Instructional Planning Guide Module 4-3, Introductory Lesson B See Module 4-3, Lesson B Additional Instructional Strategies	See Instructional Planning Guide Module 4-3 Lesson B Assessment

MODULE

4-1

Perimeter, Surface Area, and Volume

This module addresses the following indicators:

7-5.3 Generate strategies to determine the perimeters and areas of trapezoids. (B6)

7-5.2 Apply strategies and formulas to determine the surface area and volume of the three-dimensional shapes prism, pyramid, and cylinder. (C3)

This module contains two lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

Continuum of Knowledge

7-5.3 Generate strategies to determine the perimeters and areas of trapezoids. (B6)

- In sixth grade, students applied strategies and procedures to estimate the perimeters and areas of irregular shapes (6-5.4). Also, students in sixth grade applied strategies and procedures of combining and subdividing to find perimeters and areas of irregular shapes (6-5.5).

- In seventh grade, students generate strategies to determine the perimeters and areas of trapezoids (7-5.3). This is the first time students will be introduced to area of trapezoids.

- In eighth grade, students apply formulas to determine the perimeters and areas of trapezoids (8-5.5).

7-5.2 Apply strategies and formulas to determine the surface area and volume of the three-dimensional shapes prism, pyramid, and cylinder. (C3)

- In fourth grade one of the quadrilaterals students analyzed was trapezoid. As a result they should have familiarity with that geometric shape. However, a quick review of the characteristics of trapezoids may be necessary prior to work on perimeter and area, especially the concept of "height" of a trapezoid. Fifth grade students applied strategies and formulas to determine the volume of rectangular prisms and worked with nets – the two-dimensional representations of both rectangular prisms and cylinders. Fifth grade students applied formulas to determine the perimeters and areas of triangles, rectangles, and parallelograms. Sixth grade students generated strategies to determine the surface area of a rectangular prism and a cylinder (6-5.4). Sixth grade students focused on the perimeter and area of irregular shapes (6-5.5).

- In eighth grade, students apply formulas to determine the perimeters and areas of trapezoids (8-5.5).

Key Vocabulary/Concepts

**These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students*

*Area
*Area of Base
Lateral Area
*Base

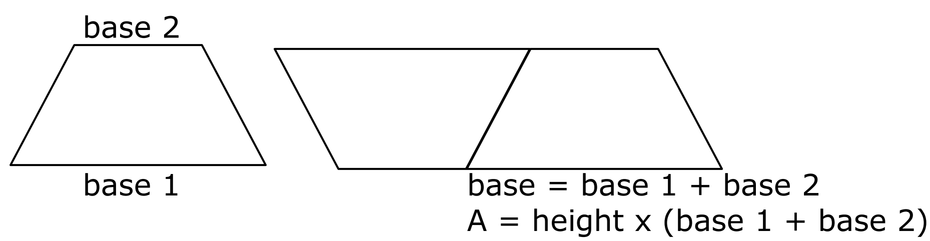
*Height
*Volume
*Surface area
Three- dimensional
*Prism
*Pyramid
*Cylinder
*Perimeter
*Trapezoid

II. Teaching the Lesson(s)

Seventh grade students should apply formulas to determine the surface area and volume of <u>prisms</u> and <u>cylinders</u>. Now for the first time they should apply strategies and formulas to determine the surface area and volume of <u>pyramids</u>. That means seventh grade students should be fluent applying formulas to find the surface area and volume of prisms, pyramids, and cylinders.

Seventh grade students should use that knowledge to generate strategies to determine the perimeters and areas of trapezoids.

When generating strategies to determine the perimeters and areas of trapezoids, students should analyze and discuss the formula for finding the area of a rectangle, A = l x w. When area formula is changed into an equivalent form of A = b x h, this form can be useful in developing the area formula for trapezoids. One strategy to determine area of a trapezoid is to have students cut out two identical trapezoids, put them together to form a parallelogram, and relate the area of the parallelogram formed to the area of the trapezoid.



Two trapezoids make a parallelogram with the same height and a base equal to the sum of bases of the trapezoid. So the formula for are of trapezoid is:

A = ½ h(base 1 + base 2)

Another strategy is to use the area formulas for a rectangle and a triangle to see why the formula for a trapezoid works.

Students should spend time investigating problem situations involving perimeters of trapezoids and be given the opportunity to discover the formulas for themselves using concrete materials and computer models. The Pythagorean Theorem is not introduced until the eighth grade. Therefore, when finding height of a trapezoid other strategies should be used.

As stated in sixth grade; "memorizing" measurement formulas becomes unnecessary when the mathematics makes sense to students and they understand the concepts.

Measurement is closely tied to many topics in geometry and algebra and should not be taught in isolation.

1. Teaching Lesson A

For this indicator, it is **essential** for students to:

- Understand the concept of perimeter
- Understand the concept of area
- Recall the characteristics of a trapezoid
- Recognize the various forms of trapezoids and rotations of the figure
- Recall and understand how to calculate areas of other shapes (i.e. rectangles, squares, parallelograms and triangles)

For this indicator, it is **not essential** for students to:

- Gain computational fluency in these procedures

a. Indicators with Taxonomy

7-5.3 Generate strategies to determine the perimeters and areas of trapezoids. (B6)

Cognitive Process Dimension: Create
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson

Display a standard trapezoid on the overhead projector or chalkboard.

Ask groups to discuss and to suggest methods for finding the area of the trapezoid shown above. As a prompt, ask, "What other shapes could you use to help you? Are there any shapes for which you already know how to find the area?" After students have had some time to discuss suggestions in their groups, ask the reporter from some groups to share their ideas.

Students may suggest several possibilities.

Using any of the methods that they identify, have students determine the area of a trapezoid with bases of 24 cm and 10 cm and with legs of 15 cm and 13 cm. Allow all groups an opportunity to determine the area, and be sure to get class consensus on what the area is. It is important that all groups determine the area correctly.

c. Misconceptions/Common Errors

A common error made by students when using formulas comes from no conceptual understanding of the meaning of height in geometric figures. Before using formulas involving height, students should discuss the meaning of height of a geometric figure and be able to identify where a height could be measured.

d. Additional Instructional Strategies/Differentiation

- After giving students a picture of a trapezoid with various lengths, students can work with partners to explore possible strategies to calculate the area of a trapezoid. The teacher will serve as a facilitator, asking questions to get students to find the area or perimeter. Then after some time of exploration, and students have their strategies, they will need to test their strategy on various trapezoids to see if works. Students will share with the class how they determined the perimeter and area of the trapezoid. Students may recall that from their sixth grade experience of sub-dividing an irregular shape into several known shapes. This may be a strategy that they could try.

- Following estimating perimeters and areas of trapezoids, give students a lab investigation to discover the formula for finding area of trapezoids. Have students cut out two identical trapezoids, put them together to form a parallelogram, and relate the area of the parallelogram formed to the area of the trapezoid. Have students generate problems for other students to solve, and then the students will exchange problems for partners to solve.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Area Tool for Trapezoids: http://illuminations.nctm.org/ActivityDetail.aspx?id=108

f. Assessing the Lesson

Generating strategies means that the concept is being introduced for the first time. Instructional and assessment activities should foster conceptual understanding with concrete and pictorial models only.

2. Teaching Lesson B

For this indicator, it is **essential** for student to:

- Understand the concept of surface area as the areas of all the faces
- Understand the concept of height of a solid
- Be able to fluently calculate with fractions, decimals, and exponents.
- Know the estimated equivalent of pi as 3.14 and $\frac{22}{7}$.
- Be able to calculate circumference
- Be able to find the area of squares, rectangles, triangles, trapezoids.
- Understand the concept of volume—cubic units

For this indicator, it is **not essential** for students to:

- Calculate using non-equilateral triangles, pentagons, hexagons, etc.

a. Indicators with Taxonomy

7-5.2 Apply strategies and formulas to determine the surface area and volume of the three-dimensional shapes prism, pyramid, and cylinder. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson

Construct rectangles with Unifix or multi-link cubes. Calculate area. Construct rectangular prisms using these rectangles as bases. Calculate volume. Students develop relationship between area of base and volume of prisms.

c. Misconceptions/Common Errors

Students often confuse the concept of surface area and volume. Teachers need to build conceptual knowledge for these two concepts through using hands-on models and visual explorations. For example, nets with grids on the faces could be used to build the definition of surface area, and unifix cubes could be used to conceptualize the definition of volume.

The indicator does not specify the types of prisms, pyramids and cylinders; therefore, they experiences should include a variety of examples i.e. rectangular prism, triangular prism, triangular pyramids, etc.

d. Additional Instructional Strategies/**Differentiation**

- Review the characteristics of each solid and allow students time to investigate the role the height plays in the calculation of volume and surface area.
- Use models, nets, cubes and other hands-on explorations to investigate the concept of surface area and volume.
- Explore various investigations using manipulatives (unifix cubes, blocks, etc) to discover the concept of the formulas for surface area and volume.
- Use 3-D models that have capabilities to pour water/rice/beans/etc. into them to show the relationships among volume formulas.
- *Counting on Frank* by Rod Clement is a great picture book that offers scenarios for students to explore and apply their knowledge and understanding of volume. Teachers could extend the book and connect it to surface area with their students.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L203 (*Counting on Frank* lesson plan)
- Students should have opportunity to set-up and solve multiple problems and problem situations. They also need to have opportunities to use a calculator to explore more challenging problems.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

http://standards.nctm.org/document/eexamples/chap6/6.3/index.htm

f. Assessing the Lesson

As the indicator indicates, apply strategies and formulas means the concept is introduced for first time. The goal must be to progress to fluency.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-5.3 Generate strategies to determine the perimeters and areas of trapezoids. (B6)

The objective of this indicator is to generate strategies, which is in the "create conceptual" knowledge cell of the Revised Taxonomy table. To create means to reorganize elements (area of perimeter of other polygons) into a new structure (trapezoids). The learning progression to **generate** requires students to recall formulas for area of squares, rectangles, parallelograms, and triangles. They recall the definition of height of a geometric shape and understand how to locate the height. Students work cooperatively to generate strategies for determining the area of a trapezoid. Students share their strategies with the class and generalize mathematical statements (7-1.5) about their strategy using clear and correct spoken words to communicate their understanding (7-1.6). Students evaluate their strategies using a variety of trapezoids to prove or disprove their statement (7-1.2). After discussing students' strategies, students may revise their mathematical statement based on their observations. Students apply their strategies and a formula to determine the area and perimeters of trapezoids through various problem situations and generate problems for peers to solve (7-1.1).

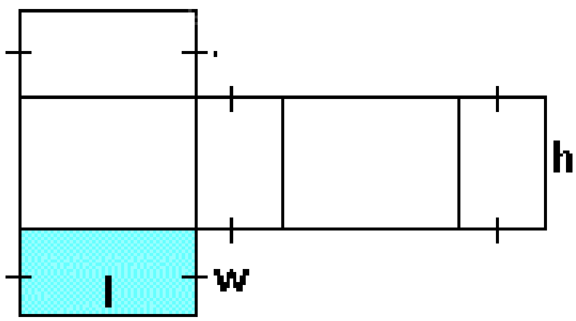
7-5.2 Apply strategies and formulas to determine the surface area and volume of the three-dimensional shapes prism, pyramid, and cylinder. (C3)

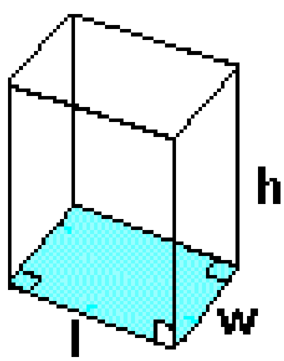
The objective of this indicator is to apply, which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies to solve a problem or problem situation. Although the focus is to gain computational fluency with calculating surface area and volume of prisms, pyramids, and cylinders, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression for **finding surface area** requires students to recall the characteristics of the faces of rectangular and triangular prisms, rectangular and triangular pyramids, and cylinders. They also need to understand area of rectangles, circles, squares, and triangles, as well as, circumference of circles. Students use their conceptual knowledge of area to translate their understanding of concrete representations to the symbolic form and be able to apply their understanding to a 3-D picture of each type of solid (rectangular prism, square pyramid, triangular pyramid with equilateral base, and cylinders). Students need to be able to generalize and communicate the formula for surface area based on investigations/hands-on activities to see relationships for the symbols in the formulas. Students should also use correct and clearly written or spoken words to explain their reasoning for their answers (7-1.6). Students should apply these procedures in context as opposed to only rote computational exercises and generalize connections among representational forms and real world situations (7-1.7).

The learning progression for **volume of 3-D figures** requires students to recall the characteristics of the faces of rectangular and triangular prisms, square and triangular pyramids, and cylinders. They understand that the concept of volume is the number of cubic units to fill a space. Students generalize and communicate the formula for volume based on investigations/hands-on activities to see relationships among the solids and connections to the symbols in the formulas (7-1.4). Students should also use correct and clearly written or spoken words to explain their reasoning for their answers (7-1.6). Students should apply these procedures in context as opposed to only rote computational exercises and generalize connections among representational forms and real world situations (7-1.7).

The following examples of possible assessment strategies may be modified as necessary to meet students/teacher needs. These examples are not derived from nor associated with any standardized testing.

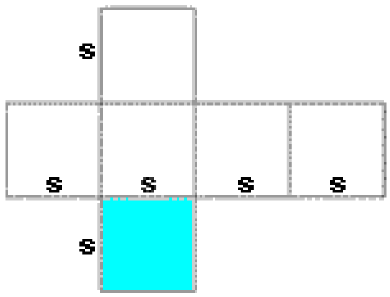
1. Here is a net of the rectangular prism pictured below. The length (l) = 7cm, the width (w) = 3cm and the height (h) = 3.5cm. Explain how you would find the surface area of this rectangular prism.



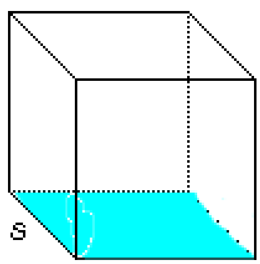


2. Use the formula $V = lwh$ to find the volume of the rectangular prism in question 1.

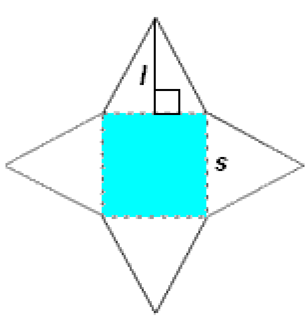
3. The sides of this cube equals 8 inches. What is the surface area of this cube?



4. This picture represents the net above in question 3. How many 4 inch cube boxes will fit into this cube?



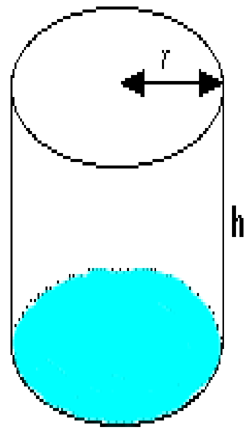
5. The net below is of a square pyramid. If the area of one of the triangles is 9sq. cm, and the length of the side of the base is 6cm, what is the surface area of the square pyramid?



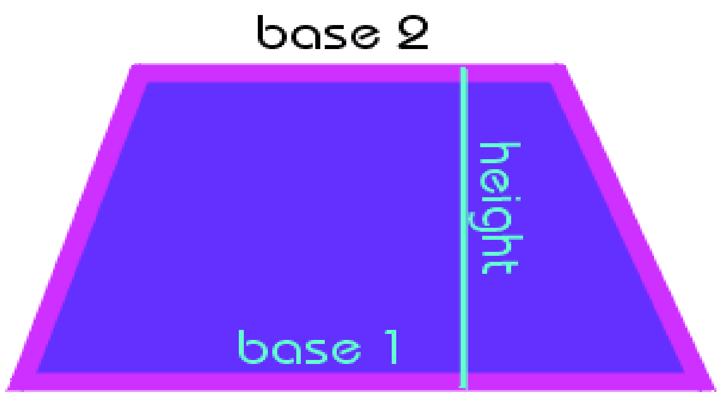
6. If the radius of the base of this cylinder (soda can) is 4cm and the height is 8cm, how much soda will this can hold?

$$V = Bh$$

$$V = \pi r^2 h$$

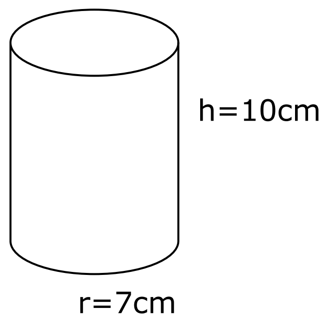


7. What strategies would you use to find the area of this trapezoid?

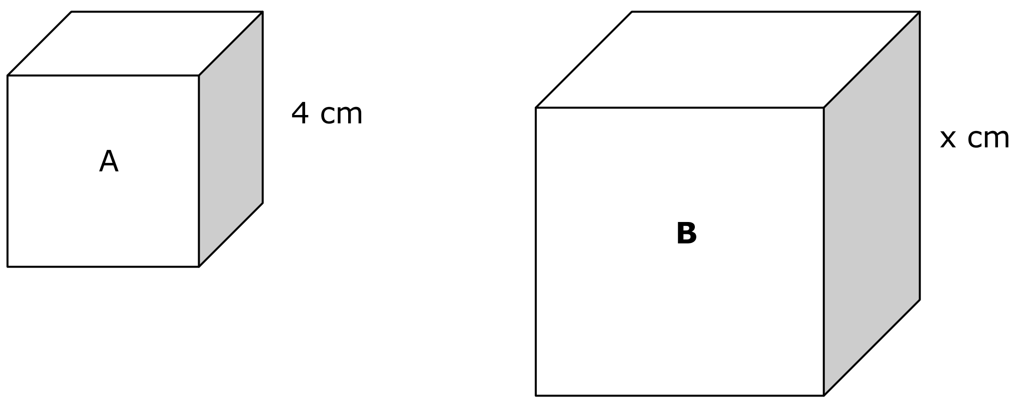


8. What do you need to know to find the perimeter of the trapezoid above in question 7?

9. What is a.) the surface area and b.) volume of a cylinder with a height of 10cm and a radius of 7cm? c) Explain how to find the surface area using the formulas for the area of a circle and the area of a rectangle. (Use pi = 22/7)



10. Cube A below has a side of length 4 cm, find the total surface area of cube A.



11. Cube B is proportional to Cube A in the figures above in question 10.
   a) Use a scale factor of 3 to find the corresponding edge "x". ________
   a) What is the total surface area of cube B? _________
   b) If both cubes are constructed from 1 cm blocks and the edge of cube A has a scale factor of 3 to construct cube B, how many 1 cm blocks would be required to build cube B? ________
   c) Cube B would require how many times as many 1 cm blocks to construct as Cube A? _________
   e) What is the ratio of the surface area of cube A to cube B?

MODULE

4-2

Data Collection, Representation, and Analysis

This module addresses the following indicators:

7-6.1 Predict the characteristics of two populations based on the analysis of sample data. (B2)
7-6.2 Organize data in box plots or circle graphs as appropriate. (B4)
7-6.3 Apply procedures to calculate the interquartile range. (C3)
7-6.4 Interpret the interquartile range for data. (B2)

This module contains three lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

Continuum of Knowledge

7-6.1 Predict the characteristics of two populations based on the analysis of sample data. (B2)

- In sixth grade, students predicted the characteristics of one population based on the analysis of sample data (6-6.1).

- In eighth grade, students will generalize the relationship between two sets of data by using scatterplots and lines of best fit (8-6.1).

7-6.2 Organize data in box plots or circle graphs as appropriate. (B4)

- In fifth grade, students applied procedures to calculate the measures of central tendency [median] (5-6.3). Fifth grade students used a protractor to measure angles from 0 to 180 degrees (5-5.2). In sixth grade, students analyzed which measure of central tendency is the most appropriate for a given purpose (6-6.3) and organized data in frequency tables, histograms, or stem-and-leaf plots as appropriate (6-6.2).

- In eighth grade, students interpret graphic and tabular data representations by using range and the measures of central tendency (8-6.8). They will also organize data in matrices or scatterplots as appropriate (8-6.2).

7-6.3 Apply procedures to calculate the interquartile range. (C3)

- In third grade students first applied a procedure to find the range of a data set (3-6.1).

- In eighth grade students will interpret graphic and tabular data representations by using range and the measures of central tendency (8-6.8).

7-6.4 Interpret the interquartile range for data. (B2)

- In third grade, students applied a procedure to find the range of a data set (3-6.1)

- In eighth grade, students interpret graphic and tabular data representations by using range and the measures of central tendency (8-6.8).

Key Concepts/Key Terms

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

*Box-and-whisker plot (box plot)
*Upper Quartile
*Upper Extreme
*Lower Quartile
*Lower Extreme
*Interquartile Range
*Five Number Summary
*Median
Outlier
*Circle Graph
Central Angle
*Degrees
*Percents
*Population
*Data
Analyze
*Sample data
*Test data
*Random Sample
Control Sample
*Shape of the data

II. Teaching the Lessons

1. <u>Teaching Lesson A: "Teacher Ages"</u>

7-6.2 Organize data in box plots or circle graphs as appropriate. (B4)

For this indicator, it is **<u>essential</u>** for students to:

- Understand the purpose of box plots and circle graphs
- Determine the Five Number Summary (median, lower and upper extremes, lower and upper quartiles)
- Recognize an outlier marked by an * on a box and whisker plot summary.
- Understand that 100% of a circle is 360°.
- Use proportions to determine the angle size of each central angle based on a percent.
- Utilize a protractor to measure angles less than or greater than 180°.
- Understand equivalent ratios.
- Develop the steps in constructing a circle graph as well as carry out the steps (Write each category as a fraction and write each fraction as a percent. Express each category as a decimal, as a percent, and determine the degrees of each central angle.)

- Use proportions to solve for the unknown.

For this indicator, it is **not essential** for students to:

- Determine the mean of the data sets.
- Calculate the parameters of an outlier.

7-6.3 Apply procedures to calculate the interquartile range. (C3)

For this indicator, it is **essential** for students to:

- Understand the meaning of interquartile range
- Understand that quartiles are three numbers that divide an ordered set of data into four equal sized subsets.
- Understand that 25% of the data falls between two successive quartiles
- Determine the median of a set of data.
- Interpret the upper and lower quartiles of a set of data or from a box and whisker plot
- Use the quartiles to calculate the interquartile range.

For this indicator, it is **not essential** for students to:

- Make a box-and-whisker plot

7-6.4 Interpret the interquartile range for data. (B2)

For this indicator, it is **essential** for students to:

- Understand the meaning of interquartile range
- Use appropriate mathematical language to interpret the meaning of the interquartile range in real world situations

For this indicator, it is **not essential** for students to:

- Make a box-and-whisker plot

a. Indicators with Taxonomy

7-6.2 Organize data in box plots or circle graphs as appropriate. (B4)

Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

7-6.3 Apply procedures to calculate the interquartile range. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

7-6.4 Interpret the interquartile range for data. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson –"Teacher Ages"

Materials Needed:

Centimeter Grid Paper
Data set: For example, some ages of teachers such as:
24,62,30,40,31,32,33,36,37,38,39,40,46,48,55,57,37,31,26,32,54,55,35,25,41,
35,60
Transparency of Number line on grid
Blank Transparencies

Introductory Lesson: "Teacher Ages"

Any set of data the students collect can be used for this exercise. Test grades for each class or comparing one class to another (no one has to know how each student scored, simply the list of test scores) would be of interest to students. The answers to the questions for this lesson are based on the teacher's age data set above. NOTE: Box plots can be drawn vertically or horizontally.

Tell students that today they are going to learn about a new way to represent data – a box-and-whisker plot; some people call it a box plot for short. A box plot is a visual way to easily see the median and range of a data set.

Ask students what they think about when they hear the terms *box and whiskers?* Allow a student to come to the overhead/board and draw what comes to mind. (It might be a silly box with whiskers but that is a visual to help students begin to make a connection to the formal name.) Leave that up and tell students you'll come back to that drawing at the end of today's lesson.

Show the teacher's ages data set. Either gather real data or use the data above.

First, have students make a number line on grid paper long enough to record teacher ages (the grids ensure even intervals).

Since students have had previous experience with stem-and-leaf plots, ask them to display the data in that format on a separate sheet of notebook paper or in their journal. (A QUICK review may be necessary.)

Stem-and-Leaf Plot Answer:

Teachers' Ages

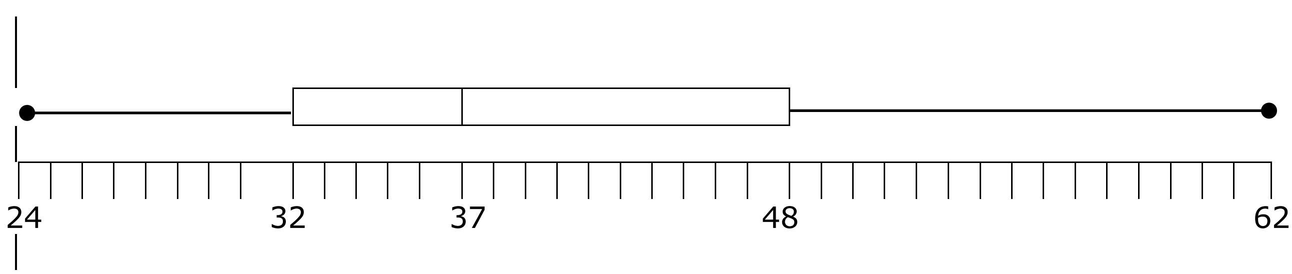
2	4, 5, 6
3	0, 1, 1, 2, 2, 3, 5, 5, 6, 7, 7, 8, 9
4	0, 0, 1, 6, 8,
5	4, 5, 5, 7
6	0, 2

Tell the students to find the median three times on the data set - first the median of all the data and second the median of each "half" above and below the central median, to mark the medians on the data set and on the number line. (The median of all the data is 37; the median of the lower half is 32 and the median of the upper half is 48 – NOTE the use of *upper half* and *lower half* refer to the position of the data in numerical order NOT the upper part of the stem-and-leaf plot.) Allow time for students to mark and record the medians. Allow students to compare with a partner.

Next have students make two rectangles "the box" that encloses all three medians. (See sample below. Box is around 32, 37, and 48.) Ask students to talk with a partner about how many parts the number line has now been divided into. (Four parts/quartiles.) Allow students to share their reasoning and demonstrate on the overhead. (Cover your teacher made grid transparency with a clear transparency so several students can explain and no time is wasted cleaning your original.)

Explain to students that the data is now divided into quartiles. Ask what the word *quartiles* makes them think of mathematically. (quarters, fourths, etc.) Say, "Since you know the data is divided into quarters, what can you tell me about the data contained in the box?" (The box contains the middle half of the data.)

Point to the lower extreme (24) and tell students that is what it is called. Label your box plot and tell students to do the same. Ask students, "Why is it called the *lower extreme* when it is at the upper part of the stem and leaf plot?" (Because lower extreme refers to the data set values not the location.) Do the same for the *upper extreme* (62).



Point to 32 and tell students that is called the "first quartile" or "lower quartile". Label your graph and have students do the same. Point to 37 and remind students that is the median of the set of data. Tell them it is sometimes referred to as the "second quartile". Label your graph with both "median" and "second quartile" and tell students to do the same. Point to 48 and ask students what they think it is called. (Third quartile or Upper quartile) Label your graph and tell students to do the same. To help students remember the vocabulary, ask them to look at the labels and discuss with a partner how the labels are related. (Lower, Median, Upper OR First, Median, Third) Allow students to share their thinking.

Refer students to the drawing on the board (when a student drew their interpretation of box and whiskers). Ask students to look at their box plot and talk with a partner about what is missing (whiskers) and where they think the missing pieces should go? (Extend from the box to the extremes.) Say, "We said the data is divided into quarters/quartiles. How are the whiskers related to that?" (The whiskers cover the upper and lower fourths of the graph.) (Common Student error – Students may think that the longer the half of the box and/or the longer the whiskers, the more data points that are included. Each half of the box and each whisker represent one-fourth of the data, no matter how the lengths compare. The length of the box and/or whiskers is an indicator of the spread of the data, not the quantity of data points.)

Ask students for the range of the data. (60-24 = 36)

Say, "While the whole data set has a range, it is sometimes useful to know the range of the data within the "box". This is the last feature to our graph. What is the range of the data within the box? (48-32 = 16) This is called the *inter-quartile range*." Label your graph and ask students to do the same.

Ask students, "What does 16 represent?" (half of the teachers' ages fall within a 16 year range)

c. Misconceptions/Common Errors –

- The most common error when organizing into a box and whisker plots consists of failing to place a data set in least to greatest order prior to determining the five number summary.
- Students may not understand that each quartile refers to one-fourth of the number of items, not one-fourth of the range.
- Students may calculate the range as the interquartile range.
- Interquartile range may be unusually difficult for some students to grasp as it is a much higher order and more abstract concept.
- Students may not understand the number of data elements between successive quartiles depends on the size of the data set but the percent of the data between the successive quartiles is always 25%.

d. Additional Instructional Strategies –

- Human Box and Whisker Plot: http://www.learnnc.org/lp/pages/3767 - Students will learn how to construct box and whisker plots as they actively participate in being a part of one based upon their heights. As an extension of the lesson, students will learn how to interpret a graph of this type.
- Location, Location, Location: Students will utilize simple grid paper strips to interpret or calculate the median, quartiles and interquartile range of data sets: http://public.doe.k12.ga.us/DWPreview.aspx?WID=89&obj=101363&mode=1

- Using NBA Statistics for Box and Whisker – Students use information from NBA statistics to make and compare box and whisker plots. http://illuminations.nctm.org/LessonDetail.aspx?id=L737

e. Technology

Box Plotter: Create a customized box plot with your own data, or display a box plot of an included set of data.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=77

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding

f. Assessing the Lesson

Assess student understanding through answers to questions about the creation of the box plot during the lesson. Student box plots of teacher ages may be used as artifacts of their work.

2. Teaching Lesson B: "Edible Circles"

For this indicator, it is **essential** for students to:

- Understand the purpose of box plots and circle graphs
- Determine the Five Number Summary (median, lower and upper extremes, lower and upper quartiles)
- Recognize an outlier marked by an * on a box and whisker plot summary.
- Understand that 100% of a circle is 360°.
- Use proportions to determine the angle size of each central angle based on a percent.
- Utilize a protractor to measure angles less than or greater than 180°.
- Understand equivalent ratios.
- Develop the steps in constructing a circle graph as well as carry out the steps (Write each category as a fraction and write each fraction as a percent. Express each category as a decimal, as a percent, and determine the degrees of each central angle.)
- Use proportions to solve for the unknown.

For this indicator, it is **not essential** for students to:

- Determine the mean of the data sets.
- Calculate the parameters of an outlier.

a. Indicators with Taxonomy

7-6.2 Organize data in box plots or circle graphs as appropriate. (B4)

Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson: "Edibles Circles"

Materials Needed:
Bag of M&M's or Skittles (or any candy with multiple color pieces)
Compass
Straight-edge

Literature Connections:

Introductory Lesson: "Edible Circles"

Have the students sort all the M&M's in the bag by color. Each students should create a circle using all the M&M's from the bag, keeping like colors together on the edge of the circle.

Use the compass to draw a circle the same size as the one created using the M&M's. On the circle, make a tick mark to indicate the separation of colors. Draw a radius between each new color of M&M to the center of the circle.

To determine the percentage of each color of M&M, have the students divide the number of M&M's of a specific color by the total number of M&M's in their bag. Ask the students, "What does the percentage mean about your M&M's? What is the relationship between the number of M&M's and the percentages? What do you notice about the area of the circle that corresponds to each color? What are you learning about representing data using circle graphs?"

c. Misconceptions/Common Errors –

- Students should have practice using a protractor and compass to create circles and measure angles. The knowledge of equivalent ratios is crucial.
- The most common error when organizing data into a circle graph is using the percent calculations as the degrees of central angles.

d. Additional Instructional Strategies –

Students collect data on how they spend their 24 hours in a day and graph their results on a circle graph. Let students compare/contrast their graphs and their data sets. For example, everyone's SCHOOL angle will be the same because all students spend the same number of hours in school each day. However, the SLEEP angles will vary depending on the hours of sleep a students gets each night. Students check each other's data sets with their circle graphs for reasonableness.

e. Technology

Create a Graph—Enter data to create and print multiple types of graphs.
http://nces.ed.gov/nceskids/createagraph/default.aspx

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding

f. Assessing the Lesson

Assess student understanding by listening to their responses to the questions at the end of the lesson: ""What does the percentage mean about your M&M's? What is the relationship between the number of M&M's and the percentages? What do you notice about the area of the circle that corresponds to each color? What are you learning about representing data using circle graphs?" These questions may also be used for a written response on an exit ticket or in a math notebook.

3. Teaching Lesson C: "Backpack Weight"

For this indicator, it is **essential** for students to:

- Make predictions from data in varying formats.
- Translate data to a graph or a picture
- Understand that the prediction from the sample data is an estimation for the population
- Make predictions based on the shape of the data (central tendency, spread of the data and outliers)
- Compare the distribution of the data (shape)
- Observe trends in the data
- Understand that the differences in spread of the two data sets are important in comparing the sets
- Justify their predictions using mathematical reasoning

For this indicator, it is **not essential** for students to:

- Analyze data in scatter plots.
- Compare data sets by using summary statistics.
- Compare data sets that are in different representational forms

a. Indicators with Taxonomy

7-6.1 Predict the characteristics of two populations based on the analysis of sample data. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson – "Backpack Weight"

Materials Needed:
Student backpacks

Literature Connections:

Introductory Lesson: "Backpack Weight":

Students weigh their backpacks, sorting data by male and female.

Use data analysis to make predictions about which gender carries the most weight in their backpacks.

Sample question: "Based on the data from our class, do boys or girls carry more weight in their backpacks?

c. Misconceptions/Common Errors –

- Students may assume that they are making predictions when they are actually guessing.
- Students do not make their predictions based on mathematical reasoning.

d. Additional Instructional Strategies –

- Having students analyze real world data that is relevant to their lives is an excellent strategy to build conceptual understanding and engage students. Students could plan and design experiments to collect data. An example could be data on sneaker prices from two stores. Students analyze the data and make inferences and predictions related to the expected cost of a pair of sneaker.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding

f. Assessing the Lesson

Assess student understanding by questioning during the dialogue comparing backpack weights. Students should choose appropriate data analysis measures to compare male and female backpack weights.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-6.1 Predict the characteristics of two populations based on the analysis of sample data. (B2)

The objective of this indicator is to predict, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To predict means to draw logical conclusion from presented information. The learning progression to **predict** requires students analyze two sets of data and generate conjectures about the population. They understand that it is sometimes difficult to do so from just analyzing the numbers. They translate their data to another form (graph or picture). They understand that each is a distinct symbolic form that represents the same relationship (7-1.4) and generalize connections (7-1.7) among these representational forms. They make observations about the shapes and proximity of the data in order to make reasonable comparisons and predictions based on those observations. They use inductive and deductive reasoning (7-1.3). Students explore a variety of real world situations (7-1.7) and summarize their predictions using correct and clearly written or spoken words to communicate their understanding (7-1.6).

7-6.2 Organize data in box plots or circle graphs as appropriate. (B4)

The objective of this indicator is to organize, which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To organize is to determine how elements fit or function within a structure. The learning progression to **organize** requires students to generalize mathematical statements (7-1.5) about the purposes for using box and whisker plots and circle graphs (7-1.6). They determine a five number summary in order to analyze or create a box and whisker plot. They also determine how the parts (percents and angle degrees) of a circle graph relate to the whole in order to create a circle graph from data. Students should be able to *describe* the procedures to *organize* data *using* correct and clearly spoken words and mathematical notation (7-1.6). They should *use* deductive **reasoning to reach a conclusion from known facts (7-1.5)**.

7-6.3 Apply procedures to calculate the interquartile range. (C3)

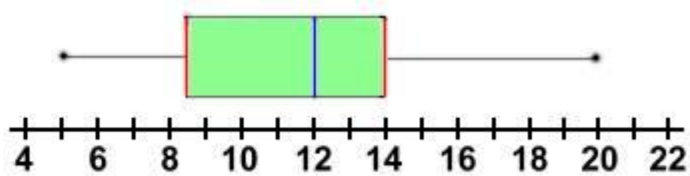
The objective of this indicator is to apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Procedural knowledge is based on criteria for using skills, algorithms, techniques, and methods in familiar or unfamiliar tasks. The learning progression to **apply** requires students to understand the meaning and purpose of interquartile range. They use deductive **reasoning (7-1.5) to** demonstrate an understanding of the relationships between the upper and lower quartiles**.** Students must *use* correct and clearly written or spoken words and notation to *communicate* their understanding of how to calculate the interquartile range of a pair of box and whisker plots from data (7-1.6).

7-6.4 Interpret the interquartile range for data. (B2)

The objective of this indicator is to interpret, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To interpret involves changing from one form of representation to another (e.g. numerical to verbal). The learning progression to **interpret** requires students to recall and understand the meaning of interquartile range. Students use their understanding analyze data in a variety of forms. They use inductive and deductive **reasoning (7-1.5) to reach a conclusion and** explain how the interquartile range impacts the data. Students **explain and justify their answers using** correct and clearly written or spoken words (7-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Graph F: Quiz Scores



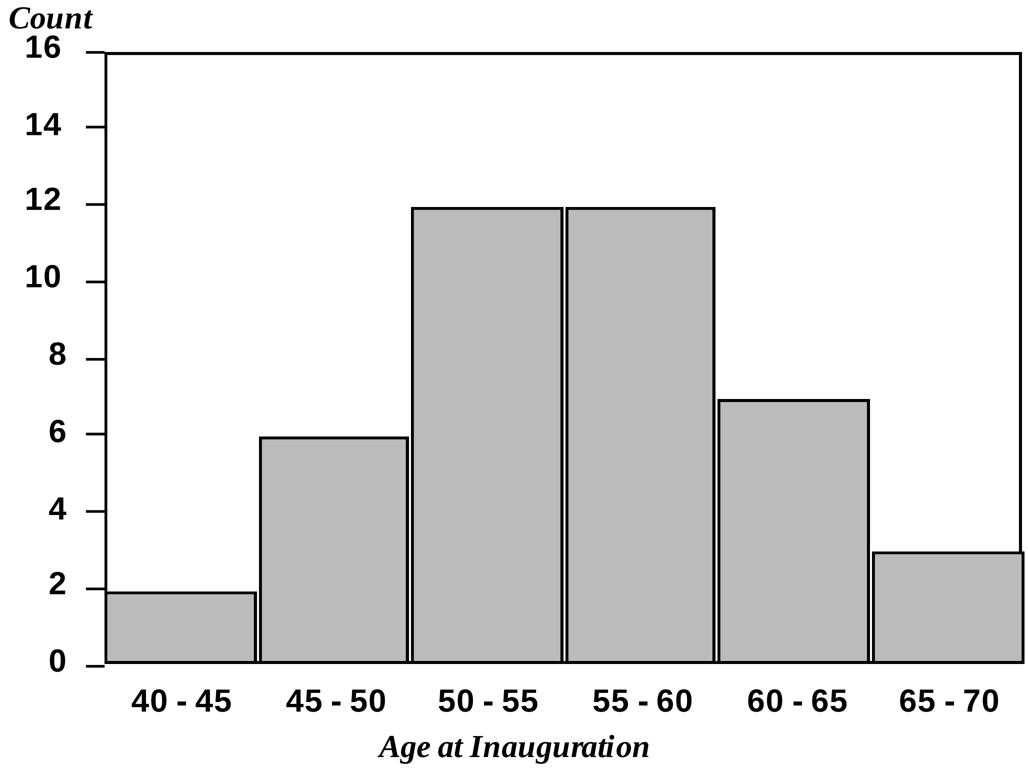
The box plot represents the quiz scores of students. Which of the statements could be true of the test scores earned by the class?

a. Three-fourths of the class had a test score lower than 12 points.
b. Half of the class scored between 8.5 and 12.
c. Three fourths of the class had a test score of 8.5 or above.
d. Half of the class scored between 14 and 20.

2. Refer to the graph of Student Quiz Scores above, what is the interquartile range of the scores?
   a. 5
   b. 12
   c. 15
   d. 2

3. Given data below, how many presidents were between the age of 50 and 60 when inaugurated?

Age at inauguration and the count of the first 42 US Presidents (from Washington to Clinton).

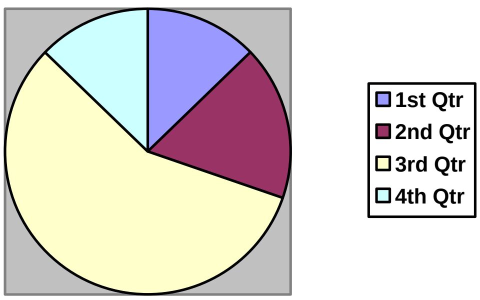


(Source: NCTM Assessment Sampler)

4. Sue found this data in an almanac.

Crop	Area planted (in acres)
Wheat	55,000
Corn	32,500
Potatoes	30,000
Lettuce	40,000
Carrots	22,500
Total	**180,000**

She needs to use the data above to make a circle graph. What is the best estimate of the number of degrees that should be in the central angle of the section that represents wheat?



- ○ 60º
- ○ 65º
- ○ 80º
- ○ 110º

(Source: Released PACT items)

5. Look at the table below. It shows Ian's bowling scores for 6 games. What is the difference in the mean score and the median score of Ian's six bowling scores?

Game	Score
1	145
2	130
3	120
4	135
5	125
6	140

What is his median bowling score?

6. Given Set A and Set B below, what is the difference in the medians? *(Taken from NCTM Navigations Data Analysis)*

Set A Heights of Students

13	8 8 8 9
14	1 2 7 7 7
15	0 0 1 1 1 2 2 2 2 2 3 3 6 6 7 8
16	
17	1

Set B Heights of Basketball Players

18	0 3 5
19	0 2 5 7 8 8
20	0 0 2 3 5 5 5 5 7
21	0 0 0 1 4 5

What groups would be affected most by changing or adding a number?

MODULE

4-3

Probability

This module addresses the following indicators:

7-6.5 Apply procedures to calculate the probability of mutually exclusive simple or compound events. (C3)
7-6.6 Interpret the probability of mutually exclusive simple or compound events. (B2)
7-6.7 Differentiate between experimental and theoretical probability of the same event. (A4)
7-6.8 Use the fundamental counting principle to determine the number of possible outcomes for a multistage event. (C3)

This module contains three lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation students need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

Continuum of Knowledge

7-6.5 Apply procedures to calculate the probability of mutually exclusive simple or compound events. (C3)

- In sixth grade, students use theoretical probability to determine the sample space and probability for one and two stage events (6-6.4) and applied procedures to calculate the probability of complementary events (6-6.5).
- In eighth grade, students use theoretical and experimental probability to make inferences and convincing arguments about an event or events (8-6.3). They also apply procedures to calculate the probability of two dependent events (8-6.4) as well as interpret the probability for two dependent events (8-6.5)

7-6.6 Interpret the probability of mutually exclusive simple or compound events. (B2)

- In sixth grade, students use theoretical probability to determine the sample space and probability for one and two stage events (6-6.4) and applied procedures to calculate the probability of complementary events (6-6.5).
- In eighth grade, students use theoretical and experimental probability to make inferences and convincing arguments about an event or events (8-6.3). They also apply procedures to calculate the probability of two dependent events (8-6.4) as well as interpret the probability for two dependent events (8-6.5)

7-6.7 Differentiate between experimental and theoretical probability of the same event. (A4)

- In sixth grade, students used theoretical probability to determine the sample space and probability for one- and two-stage events using tree diagrams, models, lists, charts, and pictures (6-6.4).
- In eighth grade students will use theoretical and experimental probability to make inferences and convincing arguments about an event or events (8-6.3)

7-6.8 Use the fundamental counting principle to determine the number of possible outcomes for a multistage event. (C3)

- In sixth grade, students use theoretical probability to determine the sample space and probability for one and two stage events (6-6.4) and applied procedures to calculate the probability of complementary events (6-6.5).
- In eighth grade, students use theoretical and experimental probability to make inferences and convincing arguments about an event or events (8-6.3). They also apply procedures to calculate the probability of two dependent events (8-6.4) as well as interpret the probability for two dependent events (8-6.5)

Key Concepts/Key Terms

** These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.*

*Probability
*Complementary Events
*Complement
*Population
*Sample
*Sample Space
*Possible Outcomes
*Experimental Probability
*Theoretical Probability
Mutually Exclusive
Not Mutually Exclusive
Independent Event
Compound Event
Simulation
Fundamental Counting Principle
Multi-stage event

II. Teaching the Lesson(s)

For this indicator, it is **essential** for students to:

- Understand the characteristics of theoretical probability
- Understand the characteristics of experimental probability
- Understand the purpose of a simulation
- Understand that experimental probability is the background for examining theoretical probability
- Understand that the experimental approach has its limitations because it is impossible to perform an infinite number of trials
- Understand the relationship between a simulation and experimental probability
- Understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of the outcomes in which the event occurs

For this indicator, it is **not essential** for students to:

- Find the probability of compound events.

a. Indicators with Taxonomy

7-6.7 Differentiate between experimental and theoretical probability of the same event. (A4)

Cognitive Process Dimension: Analyze
Knowledge Dimension: Factual Knowledge

b. Introductory Lesson – "What are My Chances?"

Materials Needed:

- Dice
- Pennies or other coins
- Several Decks of Cards
- Calculators
- "What are My Chances?" Activity Sheet

Suggested Literature Connections:

- ***Do You Wanna Bet? Your Chance to Find Out About Probability***
  by Jean Cushman, Houghton Mifflin, 1991
  Two boys become involved in everyday situations that involve probability.
- ***Pigs at Odds: Fun With Math and Games***
  by Amy Axelrod, Sharon McGinley-Nally, Simon & Schuster, 2000
  This book explores probability in the environment of a carnival complete with many games.

Introductory Lesson: "What are My Chances?"

Pairs of students will conduct 5 experiments. Set up 5 stations, 1 for each game of chance, before class. Pairs can rotate through the room, so that everyone gets a chance to conduct each of the experiments.

Show students a coin, and tell them they will be running experiments today and exploring probabilities. Explain that "theoretical probability' is the probability we expect by using mathematics to calculate the chances. Write the formula for probability on the board:

$$P(\text{Event}) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$$

Discuss with students what the formula means. Use questioning to guide students in calculating the theoretical probability for sample results for each of the experiments. For example, the probability of heads when you toss a coin, the probability of rolling a 3 when you roll a die, the probability of choosing a red card from a deck of cards, the probability of choosing a diamond from a deck of cards, and the probability of choosing the K of diamonds from a deck of cards.

Ask elbow partners to talk together to develop a definition for "theoretical probability". Discuss the student explanations for theoretical probability and help them understand that theoretical probability is the likeliness of an event happening based on all the possible outcomes.

Ask: "So, if this is what "theoretical probability" means, what might "experimental probability" mean?"

Use the same procedure to develop a definition of "experimental probability" as was used to develop the definition for "theoretical probability".

Use the coin as an example. it five times and record the number of heads. If it comes up heads 3 times, ask students what the experimental probability of getting heads is based on the experiment (the experimental probability). After this discussion, tell students they will be conducting several experiments to compare theoretical probability to experimental probability.

Distribute the What Are My Chances? activity sheet to students so they can follow along as you briefly explain what they will be doing for each experiment. Go through a single trial of each of the experiments and clarify any questions students may have.

Each pair of students should conduct each of the 5 experiments. As students conduct their experiments, you can tie in the theoretical probability by asking students questions like these as a reminder:

- What is the likelihood of getting a heads?
- What is the likelihood of getting a 4 on the die?
- What is the likelihood of picking a red card?
- What is the likelihood of picking a diamond?
- What is the likelihood of picking the 5 of diamonds?

Lead students in dialogue about their experimental results and how they compare with the theoretical probability. Explain that theoretical probability is the probability we might expect over a long period of time with many, many trials. The more trials we conduct, the closer the experimental probability will be to the theoretical probability.

You can give several examples of times where small numbers are not good predictors of large numbers results:

- Would it be fair to give a report card grade based on 1 test? or 1 assignment?
- Would it be accurate to conclude that a coin will always come up heads after flipping it once?
- If 50% of students in a class said they like country music, do you think that means 50% of students in the whole school like country music?
- Could you assume that if a person throws a basketball once and makes a basket from half court, then they are a good shooter?

Combine the data from the entire class to get a large sample of data and compare the experimental probability with a large number of trials to the theoretical probability.
Adapted fro NCTM Illuminations:

http://illuminations.nctm.org/LessonDetail.aspx?ID=L789)

What Are My Chances?

NAME______________________________

You will be evaluating games of chance to help you understand probability. For each game of chance, predict what will be the most frequent outcome. Then run the experiment 10 times. For each trial, record the actual outcome in the Result row. If this matches your predicted outcome, put a check mark in the Prediction row.

1. Flip a Coin
Prediction for most frequent outcome: Heads Tails

RESULT										
PREDICTION										

2. Roll 1 Die
Prediction for most frequent outcome: 1 2 3 4 5 6

RESULT										
PREDICTION										

3. Pick a Card Color
Prediction for most frequent outcome: Red Black

RESULT										
PREDICTION										

4. Pick a Card Suit
Prediction for most frequent outcome: Clubs (♣) Spades (♠) Diamonds (♦) Hearts (♥)

RESULT										
PREDICTION										

5. Pick an Exact Card
Prediction for most frequent outcome: __________ (e.g., 3♥)

RESULT										
PREDICTION										

NCTM ILLUMINATIONS Resources for Teaching Math

6. In which game of chance were your predictions most accurate?

7. Complete the table below with the probability for each event. Use the results from your experiments above to calculate the experimental probabilities.

Game of Chance	Event	Experimental Probability	Theoretical Probability
Flip a Coin	Heads		
Roll 1 Die	6		
Pick a Card Color	Red		
Pick a Card Suit	Diamonds		
Pick an Exact Card	5 of Diamonds		

8. Compare the theoretical and experimental probabilities for each game of chance. Were you close in any of the experiments?

9. Collect data from the entire class for the probability of an event matching the predicted event (**Note:** This works even if different groups predicted different outcomes.) Record the number of correctly predicted trials and the experimental probability of each. Since each group performed 10 trials for each game, the number of trials will be 10 × the number of groups.

Game of Chance	# of Correct Predictions	Experimental Probability
Flip a Coin		
Roll 1 Die		
Pick a Card Color		
Pick a Card Suit		
Pick an Exact Card		

10. Are the experimental probability different in Questions 7 and 9? Why or why not?

11. How do the theoretical probabilities in Question 7 compare to the experimental probabilities in Question 9? What do you think would happen if even more trials were added?

c. Misconceptions/Common Errors –

- When a probability experiment has very few attempts or outcomes, the result can be deceptive. Computer simulations may help students avoid or overcome erroneous probabilistic thinking. Simulations afford students access to relatively large samples that can be generated quickly and modified easily." (NCTM 2000, p254) Using large samples, the distribution is more likely to be close to the actual distribution. When simulations are used, you will need to help students understand what the simulation data represent and how they relate to the problem situation.
- Students may confuse what has happened with what may happen.
- Students may believe that because an event has recently happened it has a high probability of reoccurring.
- Students may use too small a sample size when using experimental relative observed frequencies as a measure of probability.
- Students may make the assumption that outcomes are equally likely (in order to calculate theoretical probabilities) when they are not.
- Students may assume that the theoretical probability and observed relative frequency of an event will be the same.
- Students may allot a probability greater than one to an event.
- Students may fail to systematically list possible outcomes and thereby miss counting some outcomes or count some outcomes more than once.
- Students may fail to understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of the outcomes in which the events occurs.

d. Additional Instructional Strategies –

- In order for students to develop an understanding of the concepts of theoretical and experimental probabilities, instructional strategies should elaborate those concepts. Students should understand that a simulation is a procedure for answering questions about a real problem by conducting an experiment that closely resembles the real situation. As students think about and discuss the real problem and the factors that make the real problem more complex, they should be reminded that a simulation is an approximation of the real problem and that with a simulation some of the factors are eliminated such as any possible danger, complexity of the problem, or length of time necessary to solve the problem.

- Modeling with concrete objects such as spinners, dice, coins, cards, or marbles in a bag needs to be done in order for the students to develop a mental picture of the probabilities of experimental events as compared to the theoretical probability.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- Probability – http://nlvm.usu.edu/en/nav/category_g_3_t_5.html Spinners - National Library of Virtual Manipulatives
- Boxing Up: Students explore the relationship between theoretical and experimental probabilities using an interactive tool known as a "box model." A "box model" is a statistical device that can be used to simulate standard probability experiments such as flipping a coin or rolling a die. http://illuminations.nctm.org/LessonDetail.aspx?id=L448

f. Assessing the Lesson

Assess student understanding of experimental and theoretical probability through questioning and listening to student responses during class dialogue. Use a strategy such as randomly choosing popsicle sticks with student names written on them to encourage all children to participate in the dialogue.

2. Teaching Lesson B

7-6.5 Apply procedures to calculate the probability of mutually exclusive simple or compound events. (C3)

For this indicator, it is **essential** for students to:

- Add and subtract fractions with like denominators when determining probability of the complements of events.
- Find the probabilities of a simple event and its complement and describe the relationship between the two.
- Interpret probability notation. Ex. P(head or tails) when tossing a coin.
- Multiply fractions when determining the probability of simple compound events.
- Explain that the word “or” in a probability situation indicates addition.
- Determine that the sum of the probabilities of all the outcomes in a sample space is 1; therefore, the sum of the complementary events is 1.
- Multiply fractions when determining the probability of simple compound events.
- Explain that the word “then” in a probability situation indicates multiplication.

For this indicator, it is **not essential** for students to:

- Calculate the probability of dependent events (8^{th} grade).

7-6.6 Interpret the probability of mutually exclusive simple or compound events. (B2)

For this indicator, it is **essential** for students to:

- Interpret probability notation. Ex. P(head or tails) when tossing a coin.
- Interpret that the word "or" in a probability situation indicates addition.
- Determine that the sum of the probabilities of all the outcomes in a sample space is 1; therefore, the sum of the complementary events is 1.
- Interpret that the word "then" in a probability situation indicates multiplication.

For this indicator, it is **not essential** for students to:

- Interpret the probability of dependent events (8^{th} grade).

a. Indicators with Taxonomy

7-6.5 Apply procedures to calculate the probability of mutually exclusive simple or compound events. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

7-6.6 Interpret the probability of mutually exclusive simple or compound events. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson –

Materials:
Coins (4 pennies per student)

Literature Connections:

- ***Do You Wanna Bet? Your Chance to Find Out About Probability***
  by Jean Cushman, Houghton Mifflin, 1991
  Two boys become involved in everyday situations that involve probability.
- ***Pigs at Odds: Fun With Math and Games***
  by Amy Axelrod, Sharon McGinley-Nally, Simon & Schuster, 2000
  This book explores probability in the environment of a carnival complete with many games

Introductory Lesson – "Tossing Coins"

Ask students to make predictions of what might happen if someone tosses four coins. What do they predict the probability of getting two heads in a toss of four coins might be? Students should share their predictions and defend them to the group. List student predictions on the board or on chart paper. All students should choose a prediction, one of their own or one they agree with that another student has offered.

Pairs of students perform 25 trials of the experiment of tossing four coins and keeping track of the results. After their trials, they should join their data with another pair to calculate the experimental probability of two heads after 50 trials.

Compile the class data into a whole group data table to maximize the number of experimental trials. (The more trials that are conducted, the closer the experimental probability is to the theoretical probability.)

As a group, compare the results with the predictions made at the beginning of the lesson. Reconcile differences between predictions and experimental results.

Groups of 4 students use the results of the experiment to make conjectures about a formula for calculating theoretical probability for mutually exclusive compound events such as tossing 4 coins.

c. Misconceptions/Common Errors –

- When a probability experiment has very few attempts or outcomes, the result can be deceptive. Computer simulations may help students avoid or overcome erroneous probabilistic thinking. Simulations afford students access to relatively large samples that can be generated quickly and modified easily." (NCTM 2000, p254) Using large samples, the distribution is more likely to be close to the actual distribution. When simulations are used, you will need to help students understand what the simulation data represent and how they relate to the problem situation.
- Students may tend to misinterpret problem situations involving the use of the word "NOT" when determining the probability of the complement of an event.
- One misconception students have revolves around the concepts of sample space, sample size, compound events and simple events. By not having an innate understanding of sample space, a student may falsely assign similar probabilities to two different events.
- Students may confuse what has happened with what may happen.
- Students may believe that because an event has recently happened it has a high probability of reoccurring.
- Students may use too small a sample size when using experimental relative observed frequencies as a measure of probability.
- Students may make the assumption that outcomes are equally likely (in order to calculate theoretical probabilities) when they are not.

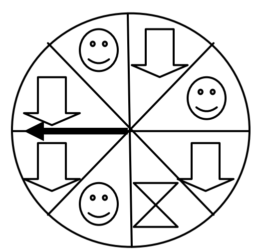
- Students may assume that the theoretical probability and observed relative frequency of an event will be the same.
- Students may allot a probability greater than one to an event.
- Students may fail to systematically list possible outcomes and thereby miss counting some outcomes or count some outcomes more than once.
- Students may fail to understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of the outcomes in which the events occurs.

d. Additional Instructional Strategies –

- Modeling with concrete objects such as spinners, cards, or marbles in a bag needs to be done in order for the students to develop a mental picture of the probabilities of complementary events.

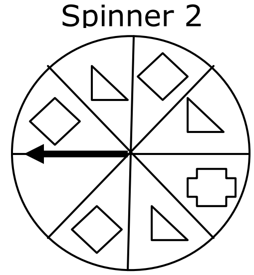
What is the complement of:
P (☺ or ⧗)
using spinner 1?

Spinner 1



Find the compound probability:
P (☺ then ◺)
using spinner 1 and 2?

Spinner 2



e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

- National Library of Virtual Manipulatives –Assorted online applications to simulate traditional probability problems such as coin tossing and spinning a spinner. http://nlvm.usu.edu/en/nav/category_g_3_t_5.html

- Monty Hall, Let's Make a Deal: http://www.stat.sc.edu/~west/javahtml/LetsMakeaDeal.html - This applet can be used to help introduce the experimental probability than the other remaining door. Have each student commit to an answer. Tell them that by the end of class they will know enough to solve the problem. For a concrete example, provide each pair of students with cups and a small item (such as a coin, button, eraser) to simulate the Monty Hall problem. Combine data from each pair to get the sum of the classes' data on a spreadsheet.

f. Assessing the Lesson

Assess understanding of this lesson by asking groups to predict the probability of another multi-stage event, such as rolling an even sum when two die are tossed or getting two blues out of three spins on a spinner.

3. Teaching Lesson C

For this indicator, it is **essential** for students to:

- Understand the Fundamental Counting principle
- Use a method for recording outcomes such as lists, tree diagram, etc..
- Understand that it does not matter with which event they begin, the number of outcomes is the same because of the commutative property of multiplication

For this indicator, it is **not essential** for students to:

- None noted

a. Indicators with Taxonomy

7-6.8 Use the fundamental counting principle to determine the number of possible outcomes for a multistage event. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson –
Materials:

Suggested Literature Connections:

- ***Do You Wanna Bet? Your Chance to Find Out About Probability***
  by Jean Cushman, Houghton Mifflin, 1991
  Two boys become involved in everyday situations that involve probability.
- ***Pigs at Odds: Fun With Math and Games***
  by Amy Axelrod, Sharon McGinley-Nally, Simon & Schuster, 2000

This book explores probability in the environment of a carnival complete with games.

Introductory Lesson: "False Advertising"

Tell students that you've decided to boycott a neighborhood restaurant due to false advertising. The restaurant claims that there are over 200 combinations from which to choose for an appetizer trio sampler, but you think that is just not true.

The restaurant has 9 different menu items: Boneless Buffalo Wings, Fried Shrimp, Spinach and Artichoke Dip, Cheeseburger Sliders, Steak Quesadillas, Buffalo Chicken Wings, Mozzarella Sticks, Mini Tacos and Spicy Cheese Dip.

For an appetizer trio sampler, you may choose any 3 items from the list. Repeats are allowed.

Ask: "How many combinations are possible? Is my claim of false advertising justified?"

Groups of students work to determine all of the possible combinations of appetizer trio samplers available and write a letter to the management of Applebee's supporting my boycott or defending their advertising claim of over 200 combinations.

c. Misconceptions/Common Errors –

Students may not understand the number of outcomes will be the same regardless of which event they begin with.

d. Additional Instructional Strategies –

If ice cream comes in either a cup or a cone and comes in three different flavors, what are the possible outcomes? (Six)

```
             / chocolate cup
   / chocolate <
  /           \ chocolate cone
 /
/             / strawberry cup
<-- strawberry <
\             \ strawberry cone
 \
  \           / vanilla cup
   \ vanilla  <
              \ vanilla cone
```

But if students start with a cup or cone, there will still be six outcomes.

```
           / chocolate cup
          /
   / cup <-- strawberry cup
  /       \
 /         \ vanilla cup
/
<
\
 \          / chocolate cone
  \        /
   \ cone <-- strawberry cone
           \
            \ vanilla cone
```

e. *Technology*

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

f. *Assessing the Lesson*

Assess understanding of the lesson by questioning and dialogue during the inquiry into the advertising claim. Letters to management may be used as artifacts of learning.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

7-6.5 Apply procedures to calculate the probability of mutually exclusive simple or compound events. (C3)

The objective of this indicator is to apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Procedural knowledge may be applied to a familiar task or to an unfamiliar task; therefore, the student's procedural knowledge of probability should include various "real-world" type situations (7-1.7) including but not limited to the use or the simulation of the use of dice, cards, coins, etc. The learning progression to **apply** requires students to understand the meaning of mutually exclusive, simple and compound. They calculate the probability of the complement of an event and compound events in a real world context . They generalize connections (7-1.7) with these real world situations by using deductive **reasoning to** interpret the meaning of their answers. Students use correct and clearly spoken words and mathematical notation (7-

1.6) to communicate their understanding. Students engage in ample opportunities to calculate, evaluate, and interpret the probability of mutually exclusive simple or compound (combination of at least two simple events) events (7-1.2).

7-6.6 Interpret the probability of mutually exclusive simple or compound events. (B2)

The objective of this indicator is to interpret, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To interpret is to change from one form (numerical) to another form (verbal). The learning progression to **interpret** requires students to understand the meaning of mutually exclusive, simple or compound events. Given a real world problem and a probability, students use inductive and deductive reasoning (7-1.3) and summarize observations. They use their understanding of the characteristics of these types of events to generalize mathematical statements (7-1.5) about their observations. Students use correct and clearly written or spoken words to communicate their understanding.

7-6.7 Differentiate between experimental and theoretical probability of the same event. (A4)

The objective of this indicator is to differentiate which is in the "analyze procedural" knowledge cell of the Revised Taxonomy. To differentiate is to distinguish relevant from irrelevant parts or important from unimportant parts of presented material. The learning progression to **differentiate** requires students to recall the meaning of experimental and theoretical probability. Students explore these concepts through real world applications in order to generalize connections between probability and real world situations (8-1.7). Based on their observations, they generalize mathematical statements using deductive and inductive reasoning (8-1.5). They understand the role of simulations and use correct and clearly written or spoken words to communicate their understanding (8-1.6). They use their understanding to distinguish similarities and difference between the two types and probability.

7-6.8 Use the fundamental counting principle to determine the number of possible outcomes for a multistage event. (C3)

The objective of this indicator is to use which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Procedural knowledge is the knowledge of steps and the criteria of when to use those steps. The learning progression to **use** requires students to recall the meaning of outcomes and how to construct tree diagrams, lists, etc… Students use their prior knowledge of single and two stage events to generalize connections (7-1.7) with multistage events. They generate and solve complex problems (7-1.1) and explain and justify their answers using correct and clearly written or spoken words (7-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

**From NCTM Mathematics Assessment Sampler Grades 6-8*

1. Suppose that A represents the following event:
   A = No students were absent from study hall today.

   Explain what the complement of A represents.

   (Answer: The complement is the even that one or more students were absent from study hall today.)

2. The table below summarizes by gender the number of musicians in an orchestra who wear glasses.

	Men	Women	Total
Glasses	32	3	35
No Glasses	56	39	95
Total	88	42	130

If A member of the orchestra is chosen at random.

a) What is the probability that the person wears glasses? Explain your reasoning.

   (Answer: The probability that the person wears glasses is 35/130 = 0.27 or 27%)

b) If you were told that person was a man, would you change your answer to part a? Explain your reasoning.

   (Answer: Yes. If the person is known to be a man, the probability is 32/88 = 0.364 for about 36 %.)

3. A government agency was interested in how the description of workers in the United States changed from 1985 to 1995. The table below shows the relative frequencies of new workers from 1985 to 1995. For example, 0.15 means that 15/100 were white men.

	White	Nonwhite	Immigrant	Total
Men	0.15	0.07	0.13	0.35
Women	0.42	0.14	0.09	0.65
Total	0.57	0.21	0.22	1.00

What is the probability that a randomly selected new worker is –

a. an immigrant? (Answer: 0.22)
b. a woman? (Answer: 0.65)
c. an immigrant and a woman? (Answer: 0.09)
d. an immigrant or a woman? (Answer: 0.22 + 0.65 – 0.09= 0.78; the woman immigrant is counted twice.)
e. White or nonwhite? (Answer:0.57 + 0.21 = 0.78)
f. a man or nonwhite? (Answer:0.35 + 0.21 – 0.07 = 0.49; the nonwhite man is counted twice.)
g. nonwhite or an immigrant? (Answer:0.21 + 0.22 = 0.43)

Following items (#4 and #5) are from NCTM Navigations Series Grades 6-8 Probability

4. Imagine that one of two blue cubes has a "1" on it and the other blue cube has a "2" on it; think of these cubes as B_1 and B_2, so B_1 and B_2 are different cubes. Imagine that one of two red cubes has a "1" on it and the other red cube as a "2" on it; think of these cubes as R_1 and R_2, so R_1 and R_2 are also different cubes.

a) List the possible combinations of two cubes that could be drawn from the cup. Use the designations B_1, B_2, R_1, and R_2 for the four cubes. (Answer:The six equally likely outcomes are B_1B_2; B_1R_1; B_1R_2; B_2R_1; B_2R_2; R_1R_2)

b) Determine the theoretical probabilities of "same" and "different". Do more of the combinations correspond to "same" or "different"? (Answer:The probability of "different" is 0.67 and the probability of "same" is 0.33. More combinations correspond to "different"; four are "different" and two are "same".)

5. Suppose that you have to take a true-or-false test with five questions and you have forgotten to study. You take the test, but you have to guess on each question.

a) Since there are only two choices for each question (true or false), what is the probability that you will guess the correct answer for the 1st question? 2nd question? 3rd question? 4th question? 5th question?

b) Using C for a correct guess and W for a wrong guess, list all the possible outcomes of answering the five questions on the test. What is the total number of outcomes? How might you determine the number of outcomes without listing all of them?

c) If you are truly guessing, what is the probability associated with each of the eight outcomes? Explain your answer.

d) If 70 percent is the lowest passing grade, what is the probability that you will pass the test by guessing?

e) How would your answers change for parts a,b,c, and d if the test was three questions with three multiple choice options for each answer??

6. Given 24 marbles in a bag, the probability of drawing yellow is $\frac{2}{3}$. How many marbles are yellow?

7. Shannon has shirts and slacks in the following colors in her closet. If she selects her shirt and slacks at random, what is the probability that she selects a red shirt and brown slacks?

Shirts	Slacks
Red	Black
Green	Brown
Blue	Grey
Yellow	

8. Kito has 10 cards labeled 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Kito randomly draws one of the cards. Find the theoretical probability Kito will draw an even numbered card?

9. Marge wants to create a 4 digit password code for her phone from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9. Which expression can she use to find the number of possible 4-digit pin numbers she can make?

a.) 1+2+3+4+5+6+7+8+9 b.) 9x8x7x6
c,) 4X3X2X1 d.) 9x8x7x6x5x4x3x2x1

10. Look at the table below. It shows Ian's bowling scores for 6 games. What is the mean score of Ian's six bowling scores?

Game	Score
1	145
2	130
3	120
4	135
5	125
6	140

11. Charles aims his bow and arrow at the center of the square target and shoots 5 arrows. Only two of the arrows hit the target. What is the probability that he will score a number greater than 4?

1	2
4	3

12. The table below shows the plan selection of 100 phone customers. What is the probability that one phone customer will select either plan C or plan D?

Plan	Number of Customers
A	12
B	24
C	26
D	14
E	24

a) 1/2 b) 2/5 c) 64/100 d) 3/5