What Are My Chances NOW?

Lesson Overview

In this lesson, students will determine and analyze events and outcomes for compound events. Experiments will be based on both independent and dependent events. Students will extend their learning by designing their own games and simulations based on both independent and dependent events of compound probabilities.

Alignment

Standard/Indicator Addressed

SCCCR Math 7.DSP.8 Extend the concepts of simple events to investigate compound events.

a. Understand that the probability of a compound event is between 0 and 1.
b. Identify the outcomes in a sample space using organized lists, tables, and tree diagrams.
c. Determine probabilities of compound events using organized lists, tables, and tree diagrams.
d. Design and use simulations to collect data and determine probabilities.
e. Compare theoretical and experimental probabilities for compound events.

Standards for Mathematical Practice (as appropriate)

Standard 1: Make sense of problems and persevere in solving them.

a. Relate a problem to prior knowledge.
b. Recognize there may be multiple entry points to a problem and more than one path to a solution.

Standard 2: Reason both contextually and abstractly.

a. Make sense of quantities and their relationships in mathematical and real-world situations.

Standard 3: Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others.

a. Construct and justify a solution to a problem.

b. Compare and discuss the validity of various reasoning strategies.
c. Make conjectures and explore their validity.
d. Reflect on and provide thoughtful responses to the reasoning of others.

Standard 6: Communicate mathematically and approach mathematical situations with precision.

a. Express numerical answers with the degree of precision appropriate for the context of a situation.
b. Represent numbers in an appropriate form according to the context of the situation.
c. Use appropriate and precise mathematical language

Science and Engineering Practices (as appropriate)

S.1A.4: Analyze and interpret data.

S.1A.5: Use mathematics and computational thinking.

S.1A.6: Construct explanations.

S.1A.8: Obtain, evaluate, and communicate information.

Connections

Active Learning Strategies (for Purposeful Reading, Meaningful Writing, and Productive Dialogue)

Elbow Partners
Pairs Squared
Graphic Organizers

Content Area Connections

The mathematics in this lesson supports Science content by developing student understanding of compound probability. This is necessary for students to better understand and work with concepts in Standard 7.L.4.

Lesson Plan

Time Required –Four 60-minute class periods

Disciplinary Vocabulary – random, chance(s), probability, ratio, percentage, likelihood (unlikely, neither likely nor unlikely, likely), outcomes, simple event, compound event, theoretical probability, experimental probability, probability notation [P(H|T)]

Materials Needed:

For Day One:

- Engage slides
- 1 quarter, 1 penny, and 1 4-region spinner for each pair of students
- Rock, Paper, Scissors rules and task cards (Handout 1)

For Days Two and Three:

- Rock, Paper, Scissors, Water rules and task cards (Handout 2)
- blank spinners
- regular dice
- 8-sided dice
- 10-sided dice
- coins
- cards
- Design a Game handout (Handout 3)

For Day Four:

- paper bag
- color tiles
    - 1 yellow
    - 2 blue
    - 3 green
    - 4 red
- colored pencils to match tiles (one set for each pair of students – optional)
- You PROBABLY Remember handout (Handout 4)
- Design a Simulation task sheet (Handout 5)
- slips of paper or index cards labeled INDEPENDENT and DEPENDENT (enough for each group of four students to randomly draw one of the cards or slips)

Formative Assessment Strategies:

Student dialogue, exit tickets, design of games and simulations

Computational Thinking:

- **Logically organizing and analyzing data:** Students use their graphic organizers to collect data from playing the games of chance. They use that data to explore theoretical and experimental probability.
- **Representing data through abstractions such as models and simulations:** The games of chance serve as simulations of compound events to help students develop understanding of theoretical and experimental probability of independent and dependent compound events, as well as compare theoretical and experimental probability of both independent and dependent compound probability.
- **Confidence in dealing with complexity:** The simulations in this lesson build on the examples of simple probability from an earlier lesson. Students must use prior experience with those games of chance in concert with the simulations to help establish a foundation for better mathematical understanding of compound events.
- **Persistence in working with difficult problems:** Many students struggle with using fractions, ratios, and percentages. This lesson provides the context and opportunity for students to review and refine those skills.
- **The ability to communicate and work with others to achieve a common goal or solution:** Students must work together as partners and in small groups to complete the analyze the more complex games and combine their data for a whole class snapshot of the results of the games. Students must also design a simulation that illustrates independent and dependent compound probability.

Misconceptions:

- When a probability experiment has very few attempts or outcomes, the result can be deceptive. Computer simulations may help students avoid or overcome erroneous probabilistic thinking. Simulations afford students access to relatively large samples that can be generated quickly and modified easily. Using large samples, the experimental distribution is more likely to be close to the theoretical distribution. When simulations are used, you will need to help students understand what the simulation data represent and how they relate to the problem situation.
- Students may confuse what has happened with what may happen.
- Students may believe that because an event has recently happened it has a high

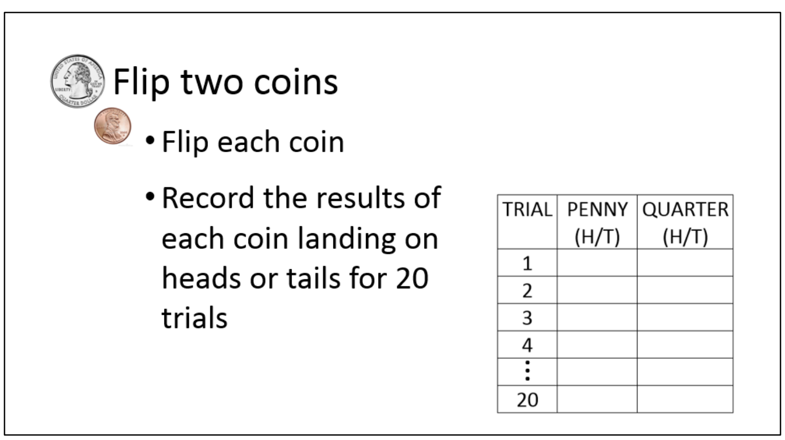
probability of reoccurring.

- Students may use too small a sample size when using experimental relative observed frequencies as a measure of probability.
- Students may make the assumption that outcomes are equally likely (in order to calculate theoretical probabilities) when they are not.
- Students may think the probability of an event is greater than 1, rather than $0 \le p \le 1$.
- Students may think that when representing the probability in fraction form, they write the number of probable outcomes over the number of not possible outcomes rather than the number of favorable outcomes over the total number of outcomes in fraction form.
- Students may not realize the probability of an outcome can be renamed into a simplified ratio.
- Students may confuse the words “and” and “or” when solving for the probability of compound events or compound independent events in problem situations.
- Students may not accurately reflect the total possible outcomes of an event, which may be affected by the outcome of another event, specifically when the items are not replaced to the original set.
- Students may not be able to distinguish between experimental and theoretical probability.

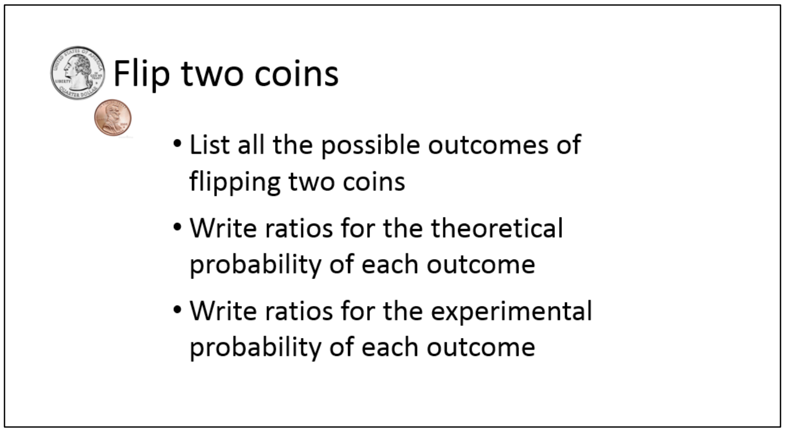
DAY ONE

Engage

- Students work with Elbow Partners.
- After students are seated in pairs, display the following slide:



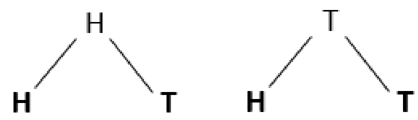
- Set a timer for 60 seconds. When time is called, display the following slide:



- Have pairs form groups of four to discuss their answers.
- Circulate as students work to ask guiding questions and check student progress. Take note of students who make lists, tables, or tree diagrams.
- Bring students together to discuss whole group. Ask for volunteers or use popsicle sticks to choose students to share.
    - All possible outcomes (sample space). Try to get an example of each of these: list, table, or tree diagram. If one of those is missing, offer it as another possibility.
    - Theoretical probabilities
    - Experimental probabilities
    - Where do these fit on a number line between 0 and 1?
    - Experimental probability: will vary – ask students to share some of theirs
    - Why do different pairs have different experimental probabilities?
    - How do the theoretical probabilities relate to the experimental probabilities in the trials you conducted?

- FLIPPING TWO COINS representations

  tree diagram



list

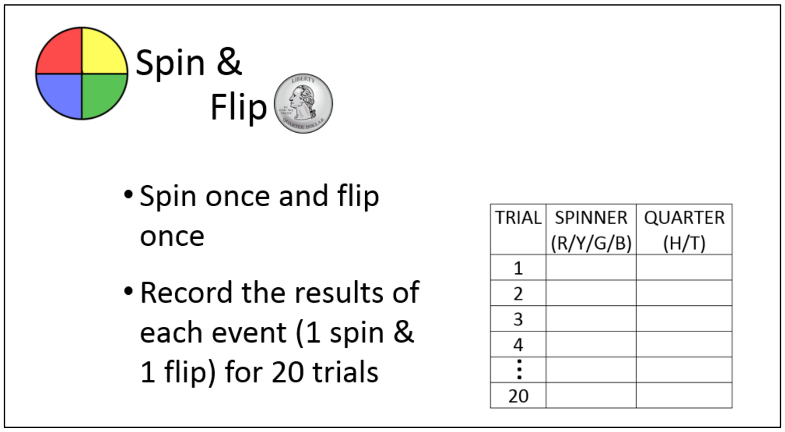
QH,PH or HH
QH,PT or HT
QT,PH or TH
QT,PT or TT

table/matrix

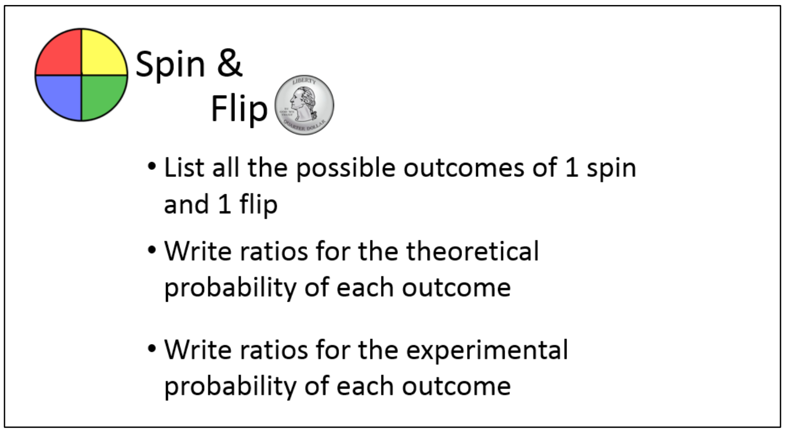
	Quarter heads	**Quarter tails**
Penny heads	**HH**	**HT**
Penny tails	**TH**	**TT**

NOTE: This table is very similar in appearance to a Punnett square, a tool students will use in the genetics and heredity lessons related to this math lesson.

- Repeat the process using a spinner and a coin.



- Set a timer for 60 seconds. When time is called, display the following slide*:*



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   - How do the theoretical probabilities relate to the experimental probabilities in the trials you conducted?

- SPIN AND FLIP representations

  tree diagram

list

RH	RT
YH	YT
GH	GT
BH	BT

table

	(spin) Red	**(spin) Yellow**	**(spin) Green**	**(spin) Blue**
Quarter heads	**HR**	**HY**	**HG**	**HB**
Quarter Tails	**TR**	**TY**	**TG**	**TB**

NOTE: This table is very similar in appearance to a Punnett square, a tool students will use in the genetics and heredity lessons related to this math lesson.

- Teacher notes for further discussion:
  - In each of these discussions, the outcomes for event 1 and event 2 are independent of each other, meaning one coin's outcome does not affect the other.
  - In the "Flip Two Coins" discussion, Coin 1 (Quarter) has 2 outcomes, while Coin 2 (Penny) also has 2 outcomes. Since there are 2 outcomes for coin 1 and 2 outcomes for coin 2, there are 4 possible outcomes for flipping the two coins.

  - In the “Spin and Flip” discussion, the spinner has 4 outcomes, while the coin has 2 outcomes. There are a total of 8 outcomes when spinning the spinner, then flipping the coin.
  - If we were to roll a die numbered 1-6, then spin a spinner with 4 equal sections with different colors, what would be a way to calculate the total number of outcomes WITHOUT creating a tree diagram, table or list? We just need the number of total outcomes….

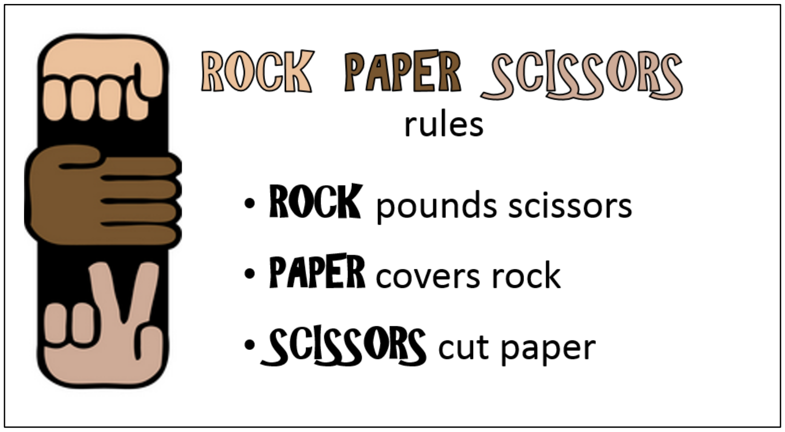
NOTE: Students should see that the product of the number of outcomes for each event will be the total number of outcomes for both events combined.

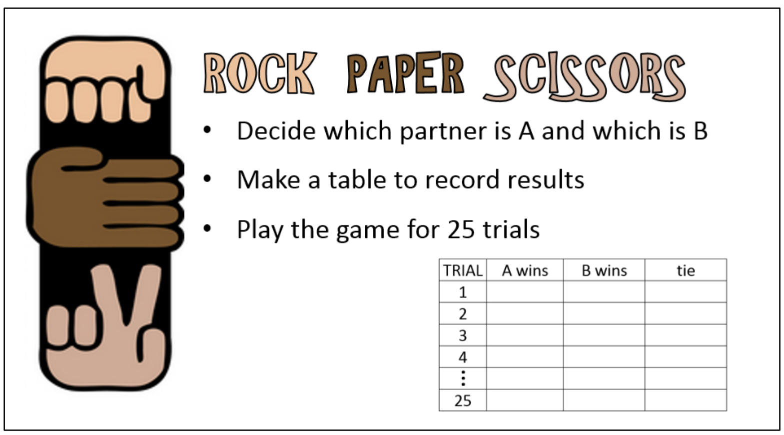
  - “Flip Two Coins”: 2 outcomes coin one x 2 outcomes coin two = 4 total outcomes for flipping two coins
  - “Spin and Flip”: 4 outcomes on the spinner x 2 outcomes on the coin = 8 total outcomes for spinning and flipping
  - “Die and Spinner”: 6 outcomes on the die x 4 outcomes on the spinner = 24 total outcomes for rolling and spinning

- Probability Notation:
  - With compound events, we state probabilities as “the Probability of event 2 given the probability of event 1) or P(event1|event 2).
  - With the “flip two coins” above, find the P(H|H), P(H|T), P(T|H) and the P(T|T) - all possible outcomes….
  - With the “spin and flip” above, find the P(R|H), P(G|T), P(B|H)

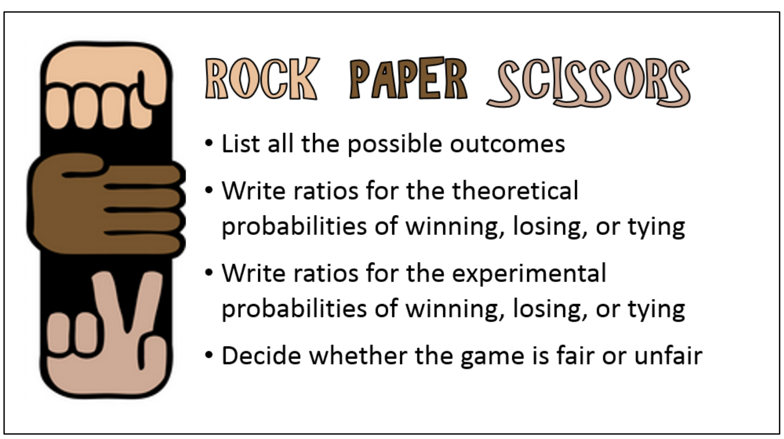
Explore

- Students work with Elbow Partners.
- Students will play two versions of the game Rock, Paper, Scissors.
- Distribute Rock, Paper, Scissors rules and directions handouts (Handout 1). You may choose to give a copy to each student, make class posters, put a class set in sheet protectors, or display electronically.





TRIAL	A wins	B wins	tie
1			
2			
3			
4			
⋮			
25			



- Pairs Square: Have pairs form groups of four to discuss their answers.
- Circulate as students work to ask guiding questions and check student progress. Take note of students who make lists, tables, or tree diagrams.
- Bring students together to discuss whole group. Ask for volunteers or use popsicle sticks to choose students to share.
    - All possible outcomes (sample space). Try to get an example of each of these: list, table, or tree diagram. If one of those is missing, offer it as another possibility.
    - Theoretical probability of winning: 1:3
    - Theoretical probability of losing: 1:3
    - Theoretical probability of tying: 1:3
    - Where do these fit on a number line between 0 and 1?
    - Experimental probability: will vary – ask students to share some of theirs
    - Why do different pairs have different experimental probabilities?
    - Is this game fair or unfair?

Matrix:

	Player 2		
Player 1	Paper	Scissors	Rock
Paper	PP	PS	PR
Scissors	SP	SS	SR
Rock	RP	RS	RR

Tie

Tie: 3/9 or 1/3 Player 1: 3/9 or 1/3 Player 2: 3/9 or 1/3

Demonstrate or describe how to calculate the total number of outcomes for each round of the game.

Answer: Player 1 has 3 independent outcomes (sample space of P,S,R) and Player 2 has 3 independent outcomes (P,S,R). The matrix constructed is a 3 x 3 for 9 outcomes in the compound probabilities.

END DAY ONE

BEGIN DAY TWO

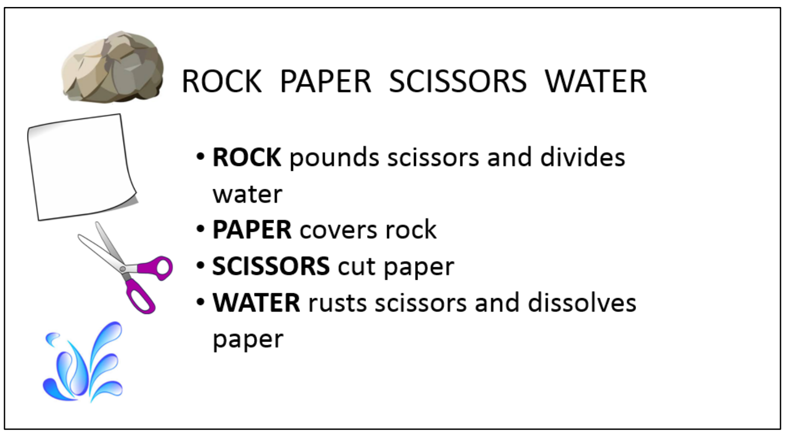
Say, "Yesterday, we looked at the game ROCK PAPER SCISSORS and what happens with three independent events. Today, we're going to add an element to the game and see what happens when there are four independent events."

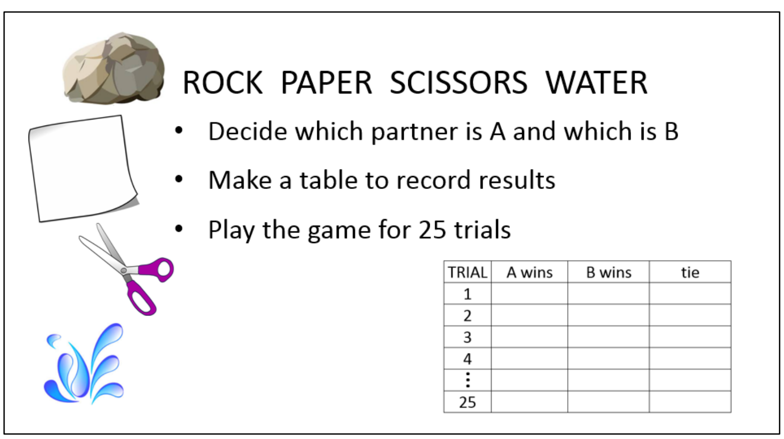
Students work with Elbow Partners.

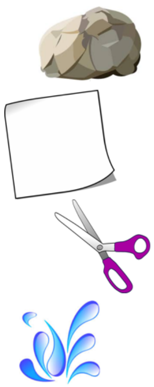
Rock, Paper, Scissor, Water

Before they start, have them make a prediction about how many different outcomes they think there will be.

- Distribute Rock, Paper, Scissors, Water rules and directions cards (Handout 2). You may choose to give a copy to each student, make class posters, put a class set in sheet protectors, or display electronically.







ROCK PAPER SCISSORS WATER

- List all the possible outcomes
- Write ratios for the theoretical probabilities of winning, losing, or tying
- Write ratios for the experimental probabilities of winning, losing, or tying
- Decide whether the game is fair or unfair

- Pairs Square: Have pairs form groups of four to discuss their answers.
- Circulate as students work to ask guiding questions and check student progress. Take note of students who make lists, tables, or tree diagrams.
- Bring students together to discuss whole group. Ask for volunteers or use popsicle sticks to choose students to share.
  - All possible outcomes (sample space). Try to get an example of each of these: list, table, or tree diagram. If one of those is missing, offer it as another possibility.
  - Theoretical probability of winning:
  - Theoretical probability of losing:
  - Theoretical probability of tying:
  - Where do these fit on a number line between 0 and 1?
  - Experimental probability: will vary – ask students to share some of theirs
  - Why do different pairs have different experimental probabilities?
  - Is this game fair or unfair?

Matrix:

	Player 2			
Player 1	Paper	Scissors	Rock	Water
Paper	PP	PS	PR	PW
Scissors	SP	SS	SR	SW
Rock	RP	RS	RR	RW
Water	WP	WS	WR	WW

Tie

Tie: 1/4 Player 1: 6/16 or 3/8 Player 2: 6/16 or 3/8

Demonstrate or describe how to calculate the total number of outcomes for each round of the game.

Answer: Player 1 has 4 independent outcomes (sample space of P,S,R,W) and Player 2 has 4 independent outcomes (P,S,R,W). The matrix constructed is a 4 x 4 which shows 16 outcomes in the compound probabilities.

Explain

NOTE: Begin this section of the lesson during Day Two and complete it by the end of Day Three.

- Tell students they will use what they've learned about games of chance to design games of their own.
- Distribute a Design a Game task card (Handout 3) to each student.
- Make materials available.
    - blank spinners
    - 6-sided dice
    - 8-sided dice
    - 10-sided dice
    - cards
    - coins
    - markers or colored pencils
    - straight edges
    - paper clips (for spinners)
- Give students a set amount of time to work on their games. It may be helpful to divide the period into timed segments.
- If possible, take pictures of the students' games.
- Students return any non-consumable materials.

Exit ticket: Set aside 10 minutes at the end of class for this

Choose a game designed by another pair of students and analyze it. Decide whether it meets the criteria of being a fair game. Explain why you think the game is fair.

Websites that have virtual simulations:

http://www.transum.org/software/SW/Dice/

http://www.shodor.org/interactivate/activities/Marbles/

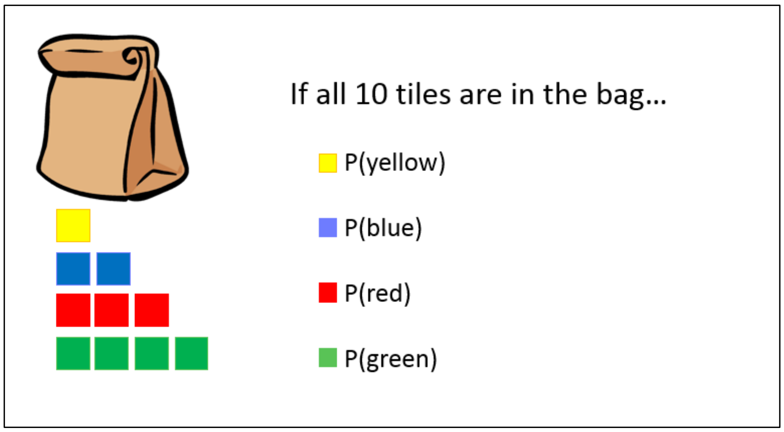
https://www.random.org/dice/?num=5

https://illuminations.nctm.org/adjustablespinner/

BEGIN DAY FOUR

Quick review of previous concepts. Students use the You PROBABLY Remember handout (Handout 4) to take notes.

- Display the paper bag and color tiles.
- Remind students that the P(event) is: $\dfrac{\text{number of ways the event can occur}}{\text{total number of possible outcomes}}$
- Display the following slide:



- Lead students through finding the probability of randomly pulling one of each of the tiles from the bag.
    - P(yellow)
    - P(blue)
    - P(red)
    - P(green)
    - P(white)

- Remember to reinforce that probabilities fall between 0 and 1.
- Say, "What if I were to randomly pull one tile, *then replace it* before I draw again? What is the likelihood of randomly pulling first a yellow then a blue?" Demonstrate how to write that using proper notation: P(Y|B).
- Remind students that for independent events, this looks like:

# of ways given outcome can occur for event 1 X # of ways given outcome can occur for event 2
P(Y) $\frac{1}{10}$ X P(B) $\frac{2}{10}$

So… P(Y|B) is $\frac{1}{10}$ x $\frac{2}{10}$ = $\frac{2}{100}$ simplified to $\frac{1}{50}$. The probability of randomly drawing yellow then blue with replacement is $\frac{1}{50}$.

- Have students find P(R|Y) and check with their Elbow Partner when done.

P(R|Y) is $\frac{3}{10}$ x $\frac{1}{10}$ = $\frac{3}{100}$ The probability of randomly drawing red then yellow with replacement is $\frac{3}{100}$.

- Have students add the words "with replacement – independent" to the section of their notes that is first labeled COMPOUND PROBABILITY – it's directly under the graphic showing the bag and tiles.

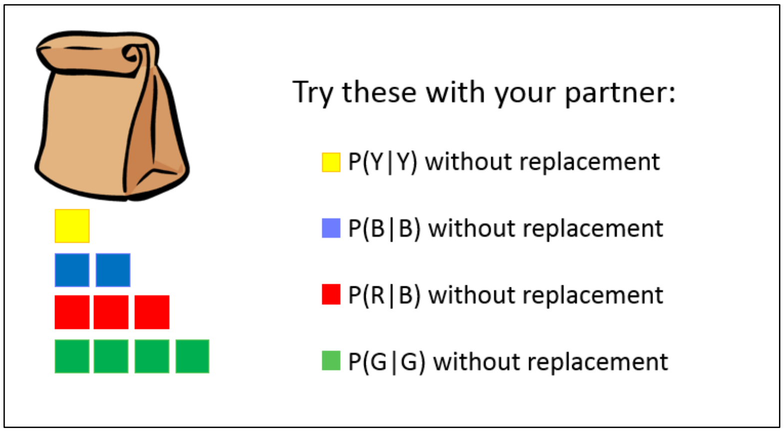
Shift gears to COMPOUND PROBABILITY WITHOUT REPLACEMENT

- Say, "Now, what if I change the rule so that I DO NOT REPLACE the first tile before pulling the second tile? What would the P(Y|B) become? Remember, the probability for independent events is:

# of ways given outcome can occur for event 1 X # of ways given outcome can occur for event 2

- Students discuss the question with their Elbow Partner.
- Possible guiding questions:
    - How many total tiles are in the bag before the first tile is pulled?
    - How many total tiles are in the bag before the second tile is pulled?
    - How many yellow tiles are there?
    - How many blue tiles are there?
- Use popsicle sticks to randomly call on students to share their thinking.
- Share the solution to the question.
  In the first draw, there are 10 tiles in the bag and only one is yellow. Therefore, the probability of drawing yellow is $\frac{1}{10}$. If the tile is not replaced, for the second draw there are 9 tiles in the bag and two are blue. Therefore, the probability of drawing blue from the remaining tiles is $\frac{2}{9}$.
  So…P(Y|B) without replacing the tile after the first draw is $\frac{1}{10}$ x $\frac{2}{9}$ which is $\frac{2}{90}$ which may be simplified to $\frac{1}{45}$.
- Have students add the words "without replacement – dependent" to the line at the top of page 2.

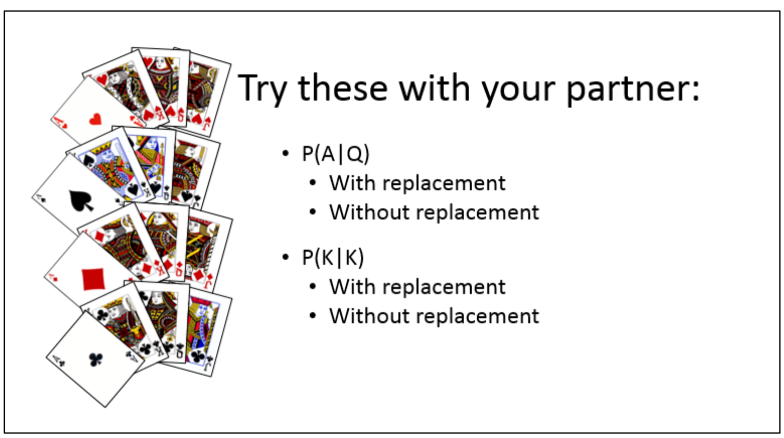
- Display the following slide:



- Use popsicle sticks to randomly call on students to share their thinking.
- Share the solutions with students.

  - P(Y|Y) without replacement: $\frac{1}{10}$ x $\frac{0}{9}$ = $\frac{0}{90}$.
  - P(B|B) without replacement: $\frac{2}{10}$ x $\frac{1}{9}$ = $\frac{2}{90}$ which may be simplified to $\frac{1}{45}$.
  - P(R|B) without replacement: $\frac{3}{10}$ x $\frac{2}{9}$ = $\frac{6}{90}$ which may be simplified to $\frac{1}{15}$.
  - P(G|G) without replacement: $\frac{4}{10}$ x $\frac{3}{9}$ = $\frac{12}{90}$ which may be simplified to $\frac{2}{15}$.

- Display the following slide:



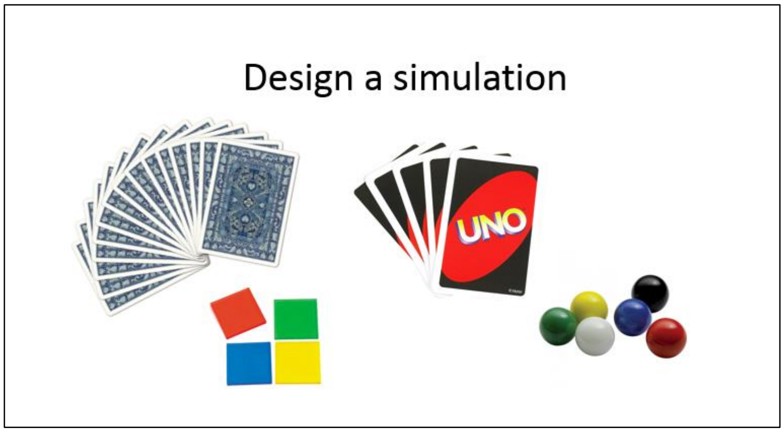
- Use popsicle sticks to randomly call on students to share their thinking.
- Share the solutions with students.

  - P(A|Q) with replacement: $\frac{4}{16}$ x $\frac{4}{16}$ = $\frac{16}{256}$ which may be simplified to $\frac{1}{16}$.

- P(A|Q) without replacement: $\frac{4}{16}$ x $\frac{4}{15}$ = $\frac{16}{240}$ which may be simplified to $\frac{1}{15}$.
- P(K|K) with replacement: $\frac{4}{16}$ x $\frac{4}{16}$ = $\frac{16}{240}$ which may be simplified to $\frac{1}{15}$.
- P(K|K) without replacement: $\frac{4}{16}$ x $\frac{3}{15}$ = $\frac{12}{240}$ which may be simplified to $\frac{1}{20}$.

Note: This would be a great place to review simplifying factors before multiplying so that the product is either already in simplest form or at least closer to it.

- Display the following slide and distribute the Design a Simulation task sheet (Handout 5) to students:



- Give partners a set amount of time to work.
- When time is up, Pairs Square to form small groups of four to analyze each other's simulations and complete the Small Group Task described at the bottom of the Design a Simulation task sheet.
- Circulate as students work to ask guiding questions and check the progress of student work.
- When the small groups are close to completing the Small Group Task, visit each group briefly and have one member draw a card or slip of paper labeled either INDEPENDENT or DEPENDENT. The group should decide which simulation they want to share with the class that involves this type of probability.
- Have each small group share their simulation and thinking with the class. Give each group approximately 3 – 5 minutes.

Rules:

ROCK PAPER SCISSORS

rules

- **ROCK** pounds scissors
- **PAPER** covers rock
- **SCISSORS** cut paper

Work directions:

ROCK PAPER SCISSORS

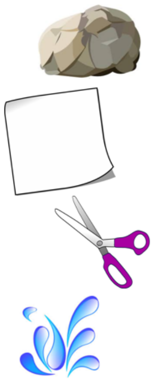
- Decide which partner is A and which is B
- Make a table to record results
- Play the game for 25 trials

TRIAL	A wins	B wins	tie
1			
2			
3			
4			
⋮			
25			

ROCK PAPER SCISSORS

- List all the possible outcomes
- Write ratios for the theoretical probabilities of winning, losing, or tying
- Write ratios for the experimental probabilities of winning, losing, or tying
- Decide whether the game is fair or unfair

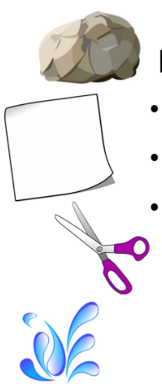
Rules:



ROCK PAPER SCISSORS WATER

- **ROCK** pounds scissors and divides water
- **PAPER** covers rock
- **SCISSORS** cut paper
- **WATER** rusts scissors and dissolves paper

Work directions:



ROCK PAPER SCISSORS WATER

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TRIAL	A wins	B wins	tie
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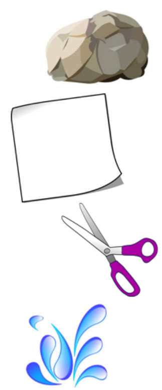
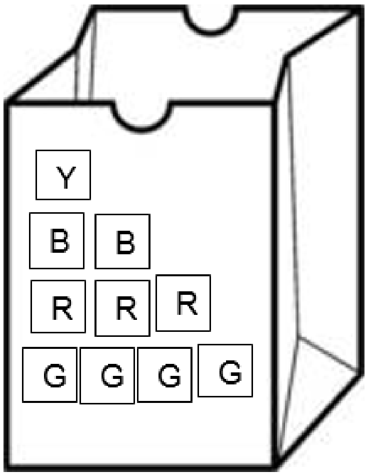
25			



ROCK PAPER SCISSORS WATER

- List all the possible outcomes
- Write ratios for the theoretical probabilities of winning, losing, or tying
- Write ratios for the experimental probabilities of winning, losing, or tying
- Decide whether the game is fair or unfair

You **PROBABLY** Remember…



- P(event) is: ________________________________

SIMPLE PROBABILITY

If all 10 tiles are in the bag, find:

- ☐ P(yellow)
- ☐ P(blue)
- ☐ P(red)
- ☐ P(green)

COMPOUND PROBABILITY ____

What if you were to randomly draw one tile, *then replace it* **before** you draw again? What is the likelihood of randomly pulling first a yellow tile, them a blue? P(Y|B)

# of ways given outcome can occur for event 1 x # of ways given outcome can occur for event 2

P(Y) x P(B)

So… P(Y|B) = ________

Now find P(R|Y) (with replacement) and check with your Elbow Partner when you're finished.

P(R|Y) = ________

over

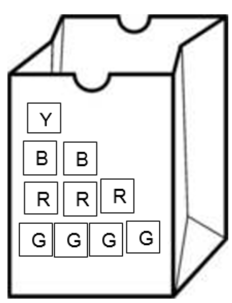
COMPOUND PROBABILITY ____

Think about the same bag of tiles. Work with your Elbow Partner and decide what you think P(Y|B) is if you draw a tile and *do not replace it* before you draw the second tile.

This is still true…

# of ways given outcome can occur for event 1 x # of ways given outcome can occur for event 2

P(Y|B) without replacement = ☐



Try these with your partner:

- ❑ P(Y|Y) without replacement
- ❑ P(B|B) without replacement
- ❑ P(R|B) without replacement
- ❑ P(G|G) without replacement

Try these with your partner:

- P(A|Q)
  - With replacement
  - Without replacement
- P(K|K)
  - With replacement
  - Without replacement

In your own words, explain:

- simple probability
- independent compound probability
- dependent compound probability

Design a Simulation

Partner Task:

With your partner, describe a simulation which would involve compound, independent events.

Possible materials:

- Full deck of regular playing cards with Jokers removed (52 cards total)

- Partial deck of regular playing cards.
  - Only face cards
  - Only face cards and aces
  - Only number cards 2 – 10
  - Only aces and number cards 2 – 10

- Partial deck of Uno cards
  - 1 color of number cards 0 – 9
  - 2 colors of number cards 0 – 9
  - 3 colors of number cards 0 – 9
  - 4 colors of number cards 0 – 9

- Some number of color tiles
  - specify which colors and how many of each

- Some number of solid colored marbles
  - Specify which colors and how many of each

When you and your partner have decided which materials you would use, describe at least 5 of the compound, independent events that would be possible. Remember: compound, independent means with replacement.

- Use proper notation: P(event 1|event 2)
- Write the probability for each of the events.
- Each person should record their work on their own paper or in their own notebook.

Small Group Task:

Work with your small group to analyze your original simulations.

Then, rewrite the events in your simulations such that compound, dependent events are used. Remember: compound, dependent means without replacement.

- Use proper notation: P(event 1|event 2)
- Write the probability for each of the events.
- Each person should record their work on their own paper or in their own notebook.