SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

Content Area	**Eighth Grade Math**

First Nine Weeks

Standards/Indicators Addressed:

Standard: 8-2: The student will demonstrate through the mathematical processes an understanding of operations with integers, the effects of multiplying and dividing with rational numbers, the comparative magnitude of rational and irrational numbers, the approximation of cube and square roots, and the application of proportional reasoning.

8-2.1* Apply an algorithm to add, subtracts, multiply and divide integers. (C3)
8-2.2* Understand the effect of multiplying and dividing a rational number by another rational number. (B2)
8-2.3* Represent the approximate location of irrational numbers on a number line. (B2)
8-2.4* Compare rational and irrational numbers by using the symbols ≤, ≥, <, >, and =. (B2)
8-2.5* Apply the concept of absolute value. (C3)
8-2.6* Apply strategies and procedures to approximate between two whole numbers the square roots and cube roots of numbers less than 1000. (C3)
8-2.7* Apply ratios, rates, and proportions. (C3)

Standard: 8-4: The student will demonstrate through the mathematical processes an understanding of the Pythagorean theorem; the use of ordered pairs, equations, intercepts, and intersections to locate points and lines in A coordinate plane; and the effect of a dilation in a coordinate plane. the effect of a dilation in a coordinate plane.

8-4.1* Apply the Pythagorean theorem. (B3)
8-4.2* Use ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate plane. (C3)

Standard: 8-5: The student will demonstrate through the mathematical processes an understanding of the proportionality of similar figures; the necessary levels of accuracy and precision in measurement; the use of formulas to determine circumference, perimeter, area, and volume; and the use of conversions within and between the U.S. Customary System and the metric system.

8-5.7* Use multi-step unit analysis to convert between and within U.S. customary system and the metric system. (C3)

*** These indicators are covered in the following 4 Modules for this Nine Weeks Period.**

Module 1-1 Number Structure and Relatinships – Rational and Irrational Numbers

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines
Module 1-1 Lesson A: Square Roots and Cube Roots **8-2.6** Apply strategies and procedures to approximate between two whole numbers the square roots and cube roots of numbers less than 1000. (C3)	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – see Appendix A	See Instructional Planning Guide Module 1-1 "Introductory Lesson A"	See Instructional Planning Guide Module 1-1 "Lesson A 'Assessing the Lesson'"
Module 1-1 Lesson B: Rational and Irrational Numbers **8-2.3** Represent the approximate location of irrational numbers on a number line.B2 (B2) **8-2.4** Compare rational and irrational numbers by using the symbols ≤, ≥, <, >, and =. (B2)		See Instructional Planning Guide Module 1-1 "Introductory Lesson B	See Instructional Planning Guide Module 1-1 "Lesson B 'Assessing the Lesson'"

Module 1-1 Lesson C: **Absolute Value** **8-2.5** Apply the concept of absolute value. (C3)		See Instructional Planning Guide Module 1-1 "Introductory Lesson C	See Instructional Planning Guide Module 1-1 "Lesson C 'Assessing the Lesson'
Module 1-2 Operations and Proportional Reasoning			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 1-2 Lesson A: Operations with Integers **8-2.1** Apply an algorithm to add, subtract, multiply and divide integers. (C3)	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – see Appendix A	See Instructional Planning Guide Module 1-2 "Introductory Lesson A" See Instructional Planning Guide Module 1-2, Lesson A "Additional Instructional Strategies"	See Instructional Planning Guide Module 1-2 "Lesson A 'Assessing the Lesson'"
Module 1-2 Lesson B: Effects of Multiplying and Dividing a Rational Number by Another Rational Number **8-2.2** Understand the effect of multiplying and dividing a rational number by another		See Instructional Planning Guide Module 1-2, "Introductory Lesson B"	See Instructional Planning Guide Module 1-2 "Lesson B 'Assessing the Lesson'"

rational number. (B2)			
Module 1-2 Lesson C: Ratios, Rates, and Proportions **8-2.7** Apply ratios, rates, and proportions. (C3)		See Instructional Planning Guide Module 1-2 "Introductory Lesson C" See Instructional Planning Guide Module 1-2, Lesson C "Additional Instructional Strategies"	See Instructional Planning Guide Module 1-2 "Lesson C 'Assessing the Lesson'"
Module 1-3 Conversions			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 1-3 Lesson A: Multi-step Unit Analysis **8-5.7** Use multi-step unit analysis to convert between and within U.S. customary system and the metric system. (C3)	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM)	See Instructional Planning Guide Module 1-3 "Introductory Lesson A"	See Instructional Planning Guide Module 1-3 "Lesson A 'Assessing the Lesson'"

	Textbook Correlations – see App A		
Module 1-4 Plane and Proportional Reasoning			
Module 1-4 Lesson A: Exploring Linear Functions **8-4.2** Use ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate plane. (C3)	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – see Appendix A	See Instructional Planning Guide Module 1-4 "Introductory Lesson A"	See Instructional Planning Guide Module 1-4 "Lesson A 'Assessing the Lesson'"
Module 1-4 Lesson B: Pythagorean Theorem **8-4.1** Apply the Pythagorean Theorem. (B3)		See Instructional Planning Guide Module 1-4, "Introductory Lesson B" See Instructional Planning Guide Module 1-4, Lesson B "Additional Instructional Strategies"	See Instructional Planning Guide Module 1-4 "Lesson B 'Assessing the Lesson'"

MODULE

1-1

Number Structure and Relationships – Rational and Irrational Numbers

This module addresses the following indicators:

8-2.3 - Represent the approximate location of irrational numbers on a number line.

8-2.4 - Compare rational and irrational numbers by using the symbols ≤, ≥, <, >, and =.

8-2.5 - Apply the concept of absolute value.

8-2.6 - Apply strategies and procedures to approximate between two whole numbers the square roots and cube roots of numbers less than 1000.

This module contains 3 lessons. These lessons are INTRODUCTORY ONLY. Lessons in S3 begin to build the conceptual foundation student need. ADDITIONAL LESSONS will be required to fully develop the concepts.

I. *Planning the Module*

Continuum of Knowledge

Indicator 8-2.3 Represent the approximate location of irrational numbers on a number line.

In seventh grade, students work involved approximating the location of perfect squares on the number line (7-2.2).
In eighth grade, students represent the approximate location of irrational numbers on a number line. This is the first time students are introduced to the concept of irrational numbers and their approximate location on a number line.

Indicator 8-2.4 Compare rational and irrational numbers by using the symbols $\leq$, $\geq$, < , >, and =.

In sixth grade, students compare rational numbers and whole number percentages through 100 by using symbols $\leq$, $\geq$, < , >, and = (6-2.3).
In seventh grade, students focused on comparing rational numbers and square roots of perfect squares (7-2.3).
In eighth grade, students compare rational and irrational numbers by using the symbols $\leq$, $\geq$, < , >, and =.

Indicator 8-2.5 Apply the concept of absolute value.

Seventh grade was the first experience with absolute value as a distance away from zero (7-2.4).
In eighth grade, students apply the concept of absolute value (8-2.5).

Indicator 8-2.6 Apply strategies and procedures to approximate between two whole numbers the square roots or cube roots of numbers less than 1,000.

In seventh grade, students gained an understanding of the inverse relationship between squaring and finding the square roots of perfect squares (7-2.10).
In eighth grade, students apply strategies and procedures to approximate between two whole numbers the square roots or cube roots of numbers less than 1,000 (8-2.6). They also represent the approximate location of irrational numbers on a number line.

Key Concepts/Key Terms

*These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

*Real numbers
*Rational Numbers
*Irrational Numbers
*Square Roots
*Cube Roots
*Absolute Value
*Absolute value Symbol | |
*Terminating Decimal
*Non-terminating Decimal
*(Square/Cube) root ($\sqrt{\ }$)
*Pi (π)
* $\sqrt[3]{\ }$ (cube root)
*Approximation

II. Teaching the Lesson(s)

8-2.3 → Represent the approximate location of irrational numbers on a number line.

For this indicator, it is **essential** for students to:
- Understand that irrational numbers can be approximated as a decimal
- Recall the value of pi (It is used a benchmark irrational number by National Association for Educational Progress (NAEP).
- Understand that every number on a number line is either rational or irrational.
- Understand how to find the cube root of a number
- Determine the value between which an irrational number occurs using perfect squares or cubes
- Recall the perfect squares and their values
- Approximate for square roots and cubes roots less than 1000 (8-2.6)

For this indicator, it is **not essential** for students to:
- Compute the exact value of irrational numbers

8-2.4 → Compare rational and irrational numbers by using the symbols ≤, ≥, <, >, and =.

For this indicator, it is **essential** for students to:
- Understand the meaning of rational numbers
- Find the square root and cube root
- Understand the difference between ≤ and ≥
- Translate numbers to same form, where appropriate before comparing numbers
- Translate between the fraction and percents

- Recall the benchmark fractions and common fraction – decimal equivalents.

For this indicator, it is **not essential** for students to:

- Find the exact value of an irrational number

8-2.5 → Apply the concept of absolute value. (C3)

For this indicator, it is **essential** for students to:

- Understand that absolute value is a distance from zero, not a direction.
- Understand that distance is always a positive value; therefore, the absolute value is always positive

For this indicator, it is **not essential** for students to:

- None noted

8-2.6 → Apply strategies and procedures to approximate between two whole numbers the square roots and cube roots of numbers less than 1000.

For this indicator, it is **essential** for students to:

- Understand the meaning of square roots
- Understand the meaning of cube roots
- Understand the relationship between area and square root and volume and cube roots.

For this indicator, it is **not essential** for students to:
None noted

1. Teaching Lesson A: *Square Roots and Cube Roots*

Students are fine tuning their estimation skills and rounding abilities when they are choosing the location for an irrational number on a number line. Students should be aware that every number on the number line is either rational or irrational. Discussion should include why numbers are rational vs. irrational. A rational number can be represented as a ratio of two integers where the denominator does not equal zero and an irrational number can not be written as a ratio of two integers. Irrational numbers when written as decimals do not terminate or have a repeating pattern the way rational numbers do.

With an understanding of what absolute value means, students can work examples that involve finding the absolute value. Examples could include the difference between below and above sea level, freezing temperatures which are below and above zero, being in the red as a deficit balance and then in the black for having a positive bank balance, deposits and withdrawals, cars that are going in different directions, yards gained or lost in a football game, etc.

Students will need to develop conceptual understanding and be expected to move to fluency in regard to square roots and cube roots approximations. Cube roots will be a new topic for students but one that understanding can build from prior knowledge of square roots. As examples: the square root of 50 is between 7 and 8, the cube root of 25 is between 2 and 3. Looking ahead, once students can approximate square roots, they will be able to find missing sides of a right triangle using the Pythagorean Theorem.

a. Indicators with Taxonomy

8-2.6 → Apply strategies and procedures to approximate between two whole numbers the square roots and cube roots of numbers less than 1000. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson A: Square Roots and Cube Roots

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. *Teaching Student Centered Mathematics: Grades 5-8*.

Squares and Cubes

Teacher Note: When asked to find roots, answers should be given as whole numbers and/or decimals rounded to the hundredths place.

This activity is intended to provide a good introduction to square and cube roots. From this point, the concept of roots of any degree is easily developed.

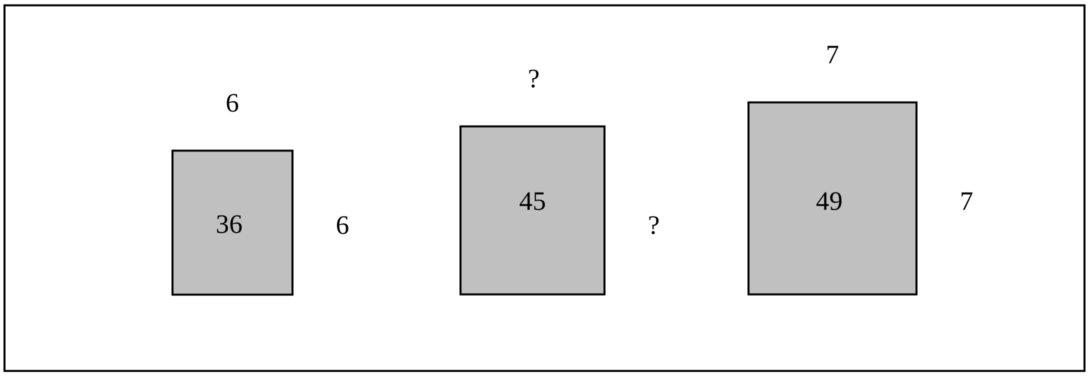
SQUARES

MATERIALS:

- "Square Strips" – 2 for each pair of students
- Teacher example strip to use at overhead
- one calculator for each pair of students

PROCEDURES

Begin by sharing the example strip on the overhead or at the board. The edges of the first and last figures are consecutive whole numbers. The areas of all three figures are given. The students' task is to use a calculator to find the edge of the figure in the center. Students may not use the square root key.



Questions to activate students' thinking:

- Look at the square with edges labeled 6. What is the relationship between the edges and the area of the square?
- What about the square with edges labeled 7?
- Since we know that 45 is between 36 and 49, what can we infer about the edges of the middle square?
- Explain how to use the calculator to find the mystery number without using the square root key.

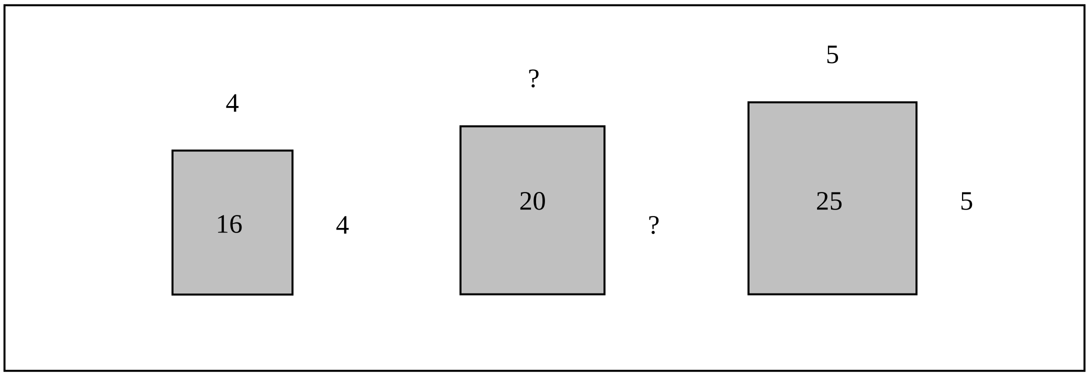
When students have a clear understanding of how to proceed, give each pair of students an envelope with Student Strips A and B enclosed. The pairs of students work together to find the mystery number for the middle squares. Students should record their "guesses" as they work. As pairs finish, have them form quads to discuss their work.

Have Student Strips C and D in separate envelopes if students want to practice further.

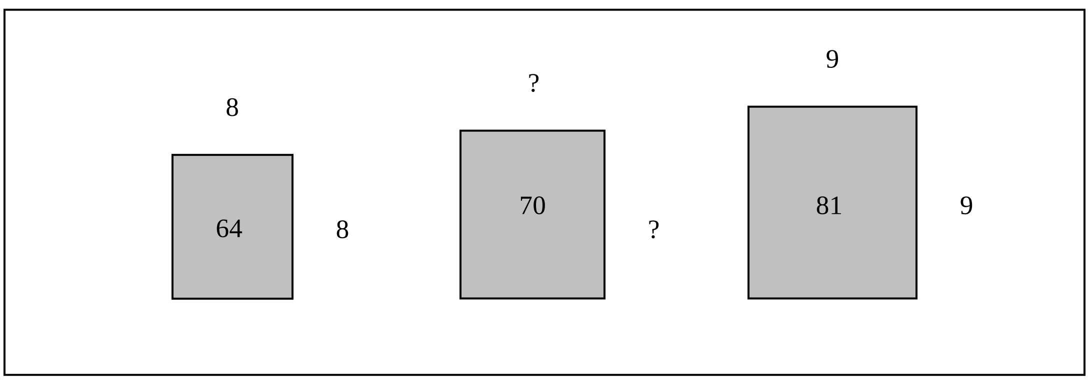
You may also have students who work more quickly create their own "strips" and exchange them with other pairs.

STUDENT STRIPS

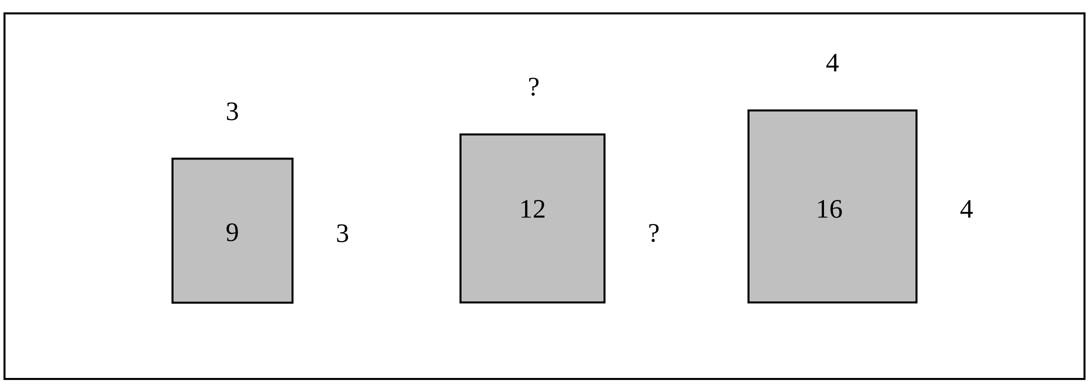
STRIP A



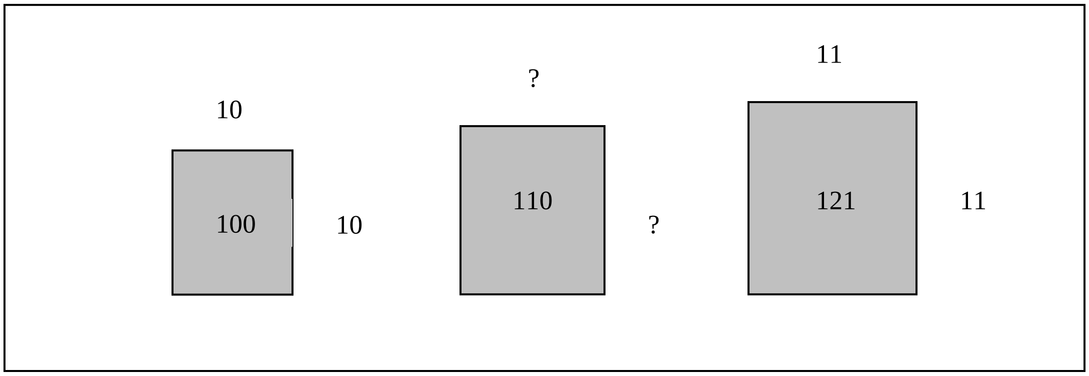
STRIP B



STRIP C



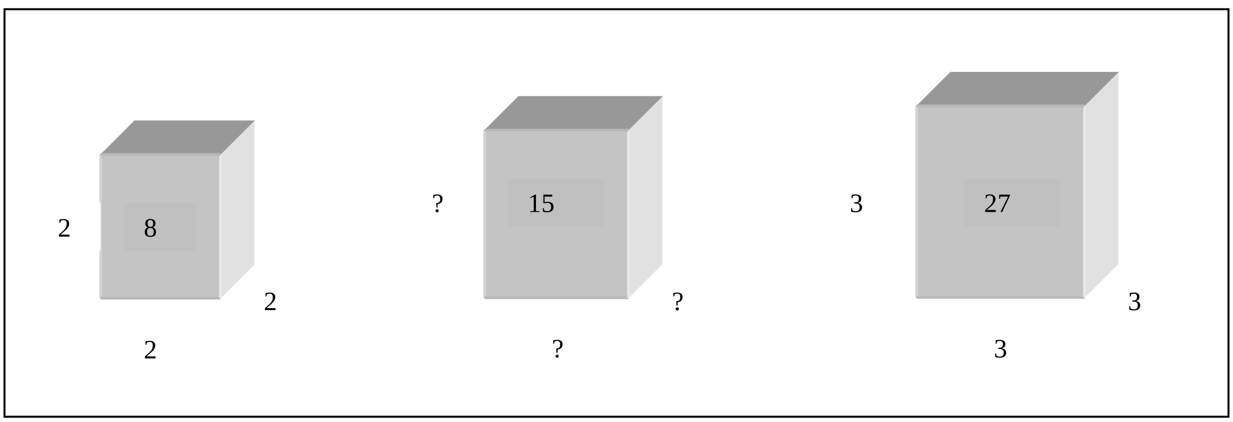
STRIP D



CUBES

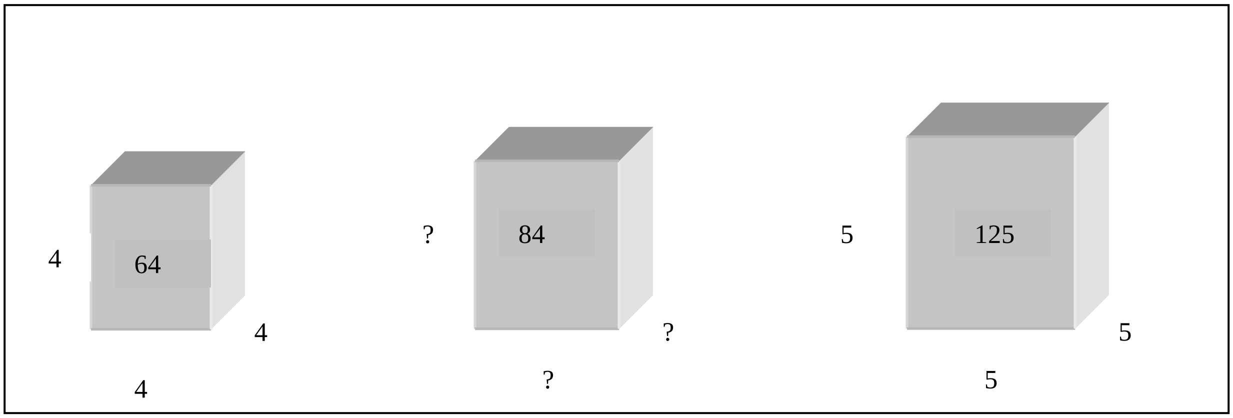
Follow the same procedures as for **SQUARES.** In this case, students may not use the carrot key to find the cube.

Teacher Strip

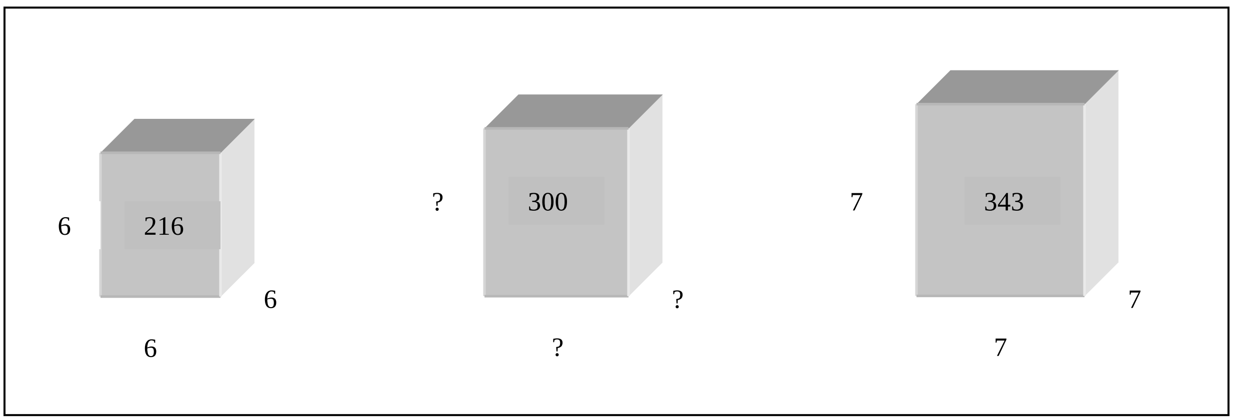


STUDENTS STRIPS

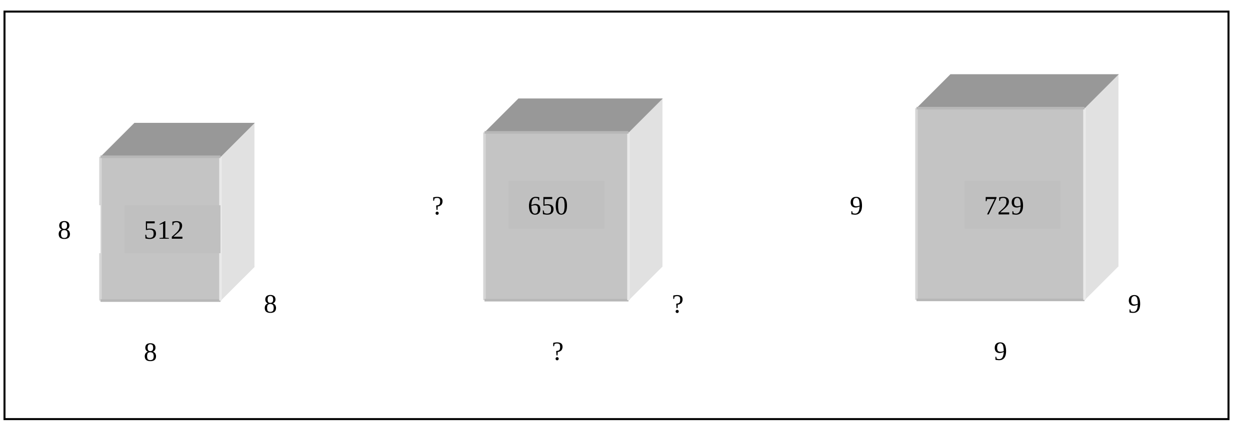
STRIP A



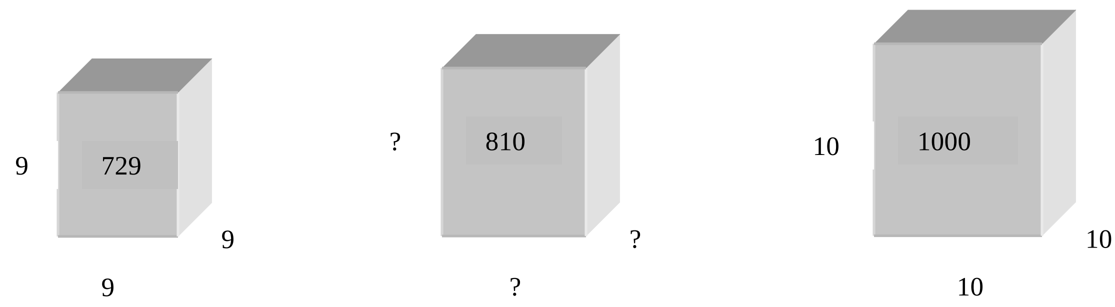
STRIP B



STRIP C

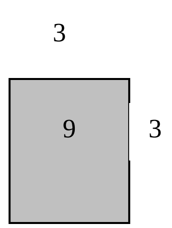
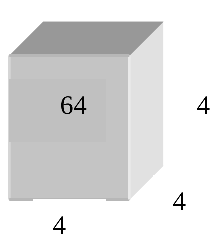


STRIP D



Note: By using the visual representation of squares and cubes, a common mistake that students make may be avoided. Students often mistakenly compute 3^2 as if the exponent were a factor, so they come up with 6 instead of 9. They do the same when the exponent is 3. If students gain an understanding of the relationships between multiplication, exponent, and radical, they'll be much less likely to make this common error. It will also lay a good foundation for working with other roots.

Relationships between squares and cubes

Visual	Multiplication expression	Exponent expression	Radical expression
	3 x 3 = 9	$3^2 = 9$	$\sqrt{9} = 3$
	4 x 4 x 4 = 64	$4^3 = 64$	$\sqrt[3]{64} = 4$

c. Misconceptions/Common Errors

- Students often mistakenly compute 3^2 as if the exponent were a factor, so they come up with 6 instead of 9. They do the same when the exponent is 3. If students gain an understanding of the relationships between multiplication, exponent, and radical, they'll be much less likely to make this common error. It will also lay a good foundation for working with other roots.

- Students may think that the square root is dividing by 2 and the cube root is dividing by 3

- Whenever students see a radical sign, they immediately think the number is irrational. A possible approach is to consider roots from the opposite direction. Rather than ask, "What is the square root of 64 or the cube root of 27"?, we might suggest that every number is the square (second) root, the third root, the fourth root and so on, of some number. For example, 3 is the second root of 9 and the third root of 27, etc. From this vantage point students can see that "square root" is just a way of indicating a relationship between two numbers. That the cube root of 27 is 3 indicates special relationship between 3 and 27. *(Lesson B will repeat this information as it will help address this misconception as well)*

d. Additional Instructional Strategies/Differentiation

- When asked to find roots, answers should be given as whole numbers and/or decimals rounded to the hundredths place.

- The focus of the indicator is to approximate values; therefore, students should use their conceptual understanding of square roots and cube roots to approximate these values.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- illuminations.nctm.org (Lessons, Activities and Related WebLinks)
- nlvm.usu.edu (National Library of Virtual Manipulatives)
- Oneplacesc.org (ETV Streamline and more!)
- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

Calculators should be allowed during this lesson since the lesson is about the connections and patterns, not the computation UNLESS they are approximating square and cube roots between 2 whole numbers. For this, they should not use a calculator.

f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS

- What is the relationship between the edges of the square and the area of the square?
- How might you find the square root of a number that is not a perfect square?
- How do you get as close as possible to the square root of a number that is not a perfect square?
- What is the relationship between the edges of the cube and the volume of the cube?
- How might you find the cube root of a number that is not a perfect cube?
- How do you get as close as possible to the cube root of a number that is not a perfect cube?

2. *Teaching Lesson B: Rational and Irrational Numbers*

a. Indicators with Taxonomy

8-2.3 → Represent the approximate location of irrational numbers on a number line. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

8-2.4 → Compare rational and irrational numbers by using the symbols ≤, ≥, <, >, and =. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson B: Rational and Irrational Numbers

Parts 1 and 2 are scaffolding lessons to gain teacher insight and prepare students for mastery of the above indicators.
Parts 3 and 4 are introductory lessons building on parts 1 and 2 allowing the indicators to be explored more fully.

Introductory Lesson B – Part 1

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. *Teaching Student Centered Mathematics: Grades 5-8*.

Repeater or Terminator

Students must create a table of the first 20 unit fractions. For example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, etc. The table should include the prime factorization of each denominator and include the decimal form of the fraction. (Calculators should be used here).

For example:

Unit Fraction	Prime Factorization of Denominator	Decimal Form
$\frac{1}{2}$	2 (prime)	0.5
$\frac{1}{3}$	3 (prime)	0.3333…
$\frac{1}{4}$	2 x 2 = 4	0.25
Continue the chart until the first 20 unit fractions have been entered….		

Discussion should center around what types of rational numbers have terminating decimal forms and which have repeating decimal forms? Once conjectures have been made, then they should be tested with other unit fractions – keeping denominators between 21 and 100.

FORMATIVE ASSESSMENT QUESTIONS:
Ask for patterns and interpretation of patterns. Which patterns result in decimal forms of rational numbers that terminate? Which repeat? Why do these patterns occur?

Teacher note: the only fractions with terminating decimal equivalents factor into all 2s and/or all 5s. Because, every terminating decimal can be written as a fraction with a denominator that is a power of 10 and 2 and 5 are the only factors of 10. If any other prime is a factor in the denominator, no multiple of it will be a power of 10.

Introductory Lesson B – Part 2

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. *Teaching Student Centered Mathematics: Grades 5-8*.

How Close is Close

Start with students selecting two fractions and two decimals they think are "really close." (It doesn't matter what fractions or decimals they choose – or how close they are.) Challenge them to find 10 fractions between the two they chose and then two decimals between the two decimals they chose. Students could "check" their work by using a calculator. This will allow them to self assess and then look for patterns and begin to discuss why they are correct or incorrect. This is a way to determine student understanding of fractions and decimals. Students must generate their own ideas for ordering fractions and decimals. This activity should serve as FORMATIVE ASSESSMENT. Discussions and discoveries should give students a chance to build their own understanding of ordering fractions and decimals.

Introductory Lesson B – Part 3

Post It Note Numberline

As students walk into the classroom, they will be given a post-it note with a rational or irrational number recorded on it. A number line will be on the board. The number line should include both a negative and positive side. The students will assess the meaning of the numbers they were given. Then each student will post his or her number in the correct approximate location. Once all numbers have been posted, the class will check for correctness and discuss what the reasoning was for each location.

Introductory Lesson B – Part 4

What's On My Back?

Another lesson to compare rational and irrational numbers is to play the "Who Am I?" game. As the students enter the room, each one will have a label (post-it note) on his back. The label will have a rational or irrational number on it. The students will circulate among each other asking questions of their classmates to discern their identity. As the students find out "who" they are, they will assemble themselves in order from least to greatest. Ordering the students according to the numbers that were on their backs, simulates locating rational and irrational numbers on a number line.

c. Misconceptions/Common Errors

Indicator 8-2.3
Represent the approximate location of irrational numbers on a number line.

- Students using calculators need to realize that the display may only have 8 digits visible. This could be a very long repeating decimal and not be easily identified.
- When students see a radical sign, they mistakenly think that the number is irrational

Indicator 8-2.4
Compare rational and irrational numbers by using the symbols $\leq$, $\geq$, < , >, and =.

- Students using calculators need to realize that the display may only have 8 digits visible. This could be a very long repeating decimal and not be easily identified.
- When students see a radical sign, they mistakenly think that the number is irrational

d. Additional Instructional Strategies/Differentiation

Indicator 8-2.3
Represent the approximate location of irrational numbers on a number line.

- Discuss with students why a number is labeled as *rational* or *irrational*.
  Provide practice with rounding and estimation for students to perfect their skills. Knowledge that every number on a number line is either rational or irrational and solid connections with rounding will better enable students to place numbers in the appropriate place on a number line.
- Although students do not have to compute the exact value of irrational numbers without a calculator, they should use a strategy to determine between which two numbers the irrational number lies. For example, to locate square root of 79, students should reason that 79 is between 64 and 81; therefore, the square root of 79 is between 8 and 9 and probably closer to nine because 79 is closer to 81. The answer is ≈ 8.9.

Indicator 8-2.4

Compare rational and irrational numbers by using the symbols ≤, ≥, < , >, and =.

- In indicator 8-2.3, students approximate the value of irrational numbers; therefore, when students compare numbers the value should not be close. For example, compare the square root of 3 and 1.752. Students only know that the square root of 3 is between 1 and 2 not that it equals 1.732. So this would be a difficult comparison. A more appropriate comparison may be square root of 3 and 2.5. They know for sure what the relationship is between the two numbers.

- To alleviate the misconception that a number in a radical sign is irrational, consider roots from the opposite direction. Imagine that every number is a square root, a cube root, a fourth root, etc so that students realize that a root is just a way of expressing a relationship between two numbers. For example, rather than ask, "What is the square root of 64 or the cube root of 27"?, we might suggest that every number is the square (second) root, the third root, the fourth root and so on, of some number. Three is the second root of 9 and the third root of 27, etc. From this vantage point students can see that "square root" is just a way of indicating a relationship between two numbers. That the cube root of 27 is 3 indicates special relationship between 3 and 27.
- Calculators should be allowed unless students are estimating between square roots and cube roots.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

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- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

Calculators SHOULD be allowed…. The lesson is about seeing patterns in finding rational and irrational equivalencies.

f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS:

Part 1 → Formative Assessment Questions are included within the

notes for the introductory lesson

Part 2 → The entire introductory lesson is about formative assessment – it is meant for the teacher to gather information about where students are in terms of understanding

Part 3 → Why did you place yourself between ___ and ___?; What were you thinking as you looked for your place in line?

Part 4 → Listen to questions students ask each other to determine what their number is on their back. Make notes on a clipboard about the questions you hear. Use the questions as some you can pose to students for discussion purposes.

3. *Teaching Lesson C: Absolute Value*

a. Indicators with Taxonomy

8-2.5 → Apply the concept of absolute value. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson C: Absolute Value

The next lesson will begin with a discussion involving opposites. Use the following questions to engage students in discussion:

- If four red chips represent a negative four, what would its opposite look like?
- What changed? (The direction to move from zero)
- Where would that be on the number line?
- How far are both numbers from zero?
- Do you see a pattern with regard to opposite integers and their distance from zero?

Write the following on the board, "The absolute value of an integer is its relative distance from zero". Show and explain the notation for absolute value. (Examples might include | 4 | = 4 and | $^-$4 | = 4) Ask students what do they notice about the absolute value of each? (They are both the same distance from zero.) Ask students to explain in writing what this means to them in light of the work they have done so far. Collect explanations to assess for understanding. (Note: This is just an introduction to Absolute Value. It will be discussed further in the subtraction of integers.)

After students have mastered finding absolute value, they should also be able to evaluate algebraic expressions involving absolute value. For example, find $|a|$ if a = -4. More examples include, find:

$|-a|$ if a = 4 → answer: $|-4|$ = 4

$|-a|$ if a = -4 → answer: $|-(-4)| = |4|$ = 4

-$|a|$ if a = 7 → answer: -$|7|$ = -7

-$|-a|$ if a = 7 → answer: -$|-7|$ = -7

Etc.
This helps prepare students to solve equations and graph absolute value functions later in Algebra.

c. Misconceptions/Common Errors

- Students may think that distance from zero can be a negative number.
- When students only evaluate numerical expressions they learn to do the computation and then ignore or "remove" the minus sign. This causes difficulty in later grades when values are unknown or variables are involved.
- Students often make the mistake of thinking that absolute value is always "taking the opposite sign." That is why stressing the distance definition is so crucial. When students are evaluating expressions such as - $|-5|$, they should treat the $|\ |$ as parentheses and solve that part first, then take the outside sign. It is the same as -1 x (the absolute value of -5.)

d. Additional Instructional Strategies/Differentiation

- To deepen conceptual understanding, students need opportunities to compare absolute values using order symbols. For example, |-5 | □ -|4 |
- Students can work examples that involve finding the absolute value such as finding the difference between below and above sea level, freezing temperatures which are above and below

zero, being in the red as a deficit balance and then in the black for having a positive bank balance, deposits and withdrawals, cars that are going different directions, yards gained or lost in a football game, etc.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- illuminations.nctm.org (Lessons, Activities and Related WebLinks)
- nlvm.usu.edu (National Library of Virtual Manipulatives)
- Oneplacesc.org (ETV Streamline and more!)
- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS:

Refer back to the questions posed at the start of the introductory lesson. Pose these same questions at the close of the day on an exit slip for students. When reviewing the exit slips, focus on the last question. The questions from the start of the lesson are below:

- If four red chips represent a negative four, what would its opposite look like?
- What changed? (The direction to move from zero)
- Where would that be on the number line?
- How far are both numbers from zero?
- Do you see a pattern with regard to opposite integers and their distance from zero?

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

Indicator 8-2.6 Apply strategies and procedures to approximate between two whole numbers the square roots and cube roots of numbers less than 1000.

The objective of this indicator is to apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is on computational fluency with strategies for approximating cube and square roots values, the essence of the strategy is rooted in conceptual understanding. The learning progression to **apply** requires students to recall and understand the meaning of square roots and cube roots. Students explore concrete and/or pictorial models of square roots and cube roots using area and volume. Students generalize connection among these representation and finding the approximate value of cube and square roots. They generalize mathematical statements (8-1.5) about the connection between these representations to develop a strategy for determining approximations. As students explore these representations, they use correct and clearly written or spoken word to communicate their reasoning (8-1.6).

Indicator 8-2.3 Represent the approximate location of irrational numbers on a number line.

The objective of this indicator is to represent which is in the "understand conceptual" knowledge of Bloom's Taxonomy. To represent is to translate from one form to another; therefore, students develop an understanding of irrational number by translate them from numerical form (number) to graphical form (number line). The learning progression to **represent** requires students to recall the concept of rational number. Students understand the characteristics of irrational numbers and make the connection to their prior knowledge of rational numbers. Students explore how to represent these types of numbers in the correct location on a number line as their relate to rational numbers and justify their placement of a rational number on a number line using general mathematical statements (8-1.5) based on inductive and deductive reasoning (8-1.3).

Indicator 8-2.4 Compare rational and irrational numbers by using the symbols $\leq$, $\geq$, $<$, $>$, and $=$.

The objective of this indicator is to understand which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge should include numerous examples. The learning progression to **compare** requires students to recall common fraction/decimal equivalents and identify perfect square roots. They understand how to find the approximate value of irrational numbers. Students understand equivalent symbolic expressions (8-1.4) and translate numbers to a common form, if necessary. Students use their conceptual understanding to determine the location of irrational numbers. They compare without dependence on a traditional algorithm and use concrete models to support understanding where appropriate. Students recognize mathematical symbols $<$, $>$, $\geq$, $\leq$ and = and their meanings. As students analyze the relationships to compare percentages and rational numbers, they evaluate conjectures and explain and justify their answer to classmates and their teacher. Students should use correct and clearly written or spoken words, variables and notation to communicate their reasoning (8-1.6).

Indicator 8-2.5 Apply the concept of absolute value.

The objective of this indicator is to understand, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning; therefore, the student's conceptual knowledge of absolute value should include numerous real world examples and non-examples. The learning progression to **understand** requires students to recall the concept of integers and their position in relation to zero. When directed to determine the absolute value of a number, students should understand that absolute value is a distance away from zero and does not include which direction (positive or negative) away from zero. They illustrate this understanding by representing absolute value relationship on the number line. Students use their understanding to generate and solve complex problems to deepen conceptual knowledge. They use correct and clearly spoken words, variables and notation to communicate their understanding (8-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Between what two whole numbers is the square root of 79?

    Answer: 8 and 9

2. Describe the relationship between 4 and its square of 16.

    Answer: it can be written as 4 x 4 or 4^2 because a square has equal length and width, resulting in an area of 16 for a square.

3. Describe the relationship between 6 and its cube of 216.

Answer: it can be written as 6 x 6 x 6 or 6^3 because a cube has equal length, width and height, resulting in a volume of 216 for a cube.

4. Find the square root of 572. Describe the steps you took to do so.

Answer:

I knew that $20^2 = 400$ and $25^2 = 625$, so the answer had to be between 20 and 25. I started with 24^2 because 572 is closer to 625.

$24^2 = 576$ (too big)
$23^2 = 529$ (too small)
$23.5^2 \approx= 552.25$ (too small)
$23.8^2 \approx= 566.44$ (too small)
$23.9^2 \approx= 572.21$ (a little too big)
$23.85^2 \approx 568.82$
$23.87^2 \approx 569.8$
$23.89^2 \approx 570.73$

The square root of 572 is between 23.89 and 23.9.

5. Find the cube root of 950. Describe the steps you took to do so.

Answer:

I knew that $9^3 = 729$ and $10^3 = 1000$, so the answer had to be between 9 and 10. I started with 9.5^3 because 950 is closer to 1000.

$9.5^3 \approx 857.38$
$9.7^3 \approx 912.68$
$9.9^3 \approx 970.3$
$9.8^3 \approx 941.2$
$9.85^3 \approx 955.67$
$9.84^3 \approx 952.76$
$9.83^3 \approx 949.86$

The cube root of 950 is between 9.83 and 9.84.

6. Place the following numbers on a number line.

a) $\frac{2}{9}$ b) 0.23 c) $\frac{1}{5}$ d) $\frac{2}{11}$

Answer: d, c, a, b

7. Compare the following three numbers using $\leq$, $\geq$, <, >, *or =.*

a) $\frac{6}{11}$, $\frac{10}{13}$, 0.55

Answer: $\frac{10}{13} > 0.55 > \frac{6}{11}$

8. Place the following numbers on a number line.

a) $\frac{3}{4}$ b) 0.45 c) $\frac{2}{7}$ d) $\frac{5}{11}$

Answer: c, d, b, a

9. Compare the following three numbers using $\leq$, $\geq$, <, >, or =.

a) $\frac{5}{6}$, $\frac{2}{3}$, 0.74

Answer: $\frac{5}{6} > 0.74 > \frac{2}{3}$

10. Determine if the statement is true or false... justify or prove your answer.

a) The opposite of a number is always negative.
(false… The opposite of -5 is 5 which is positive)
b) The absolute value of a number is never negative.
(true by definition of distance)

11. What is the value of - $|-2|$? (*answer (b))*

a) 2 b) -2 c) $|2|$ d) $|-2|$

12. Evaluate the following if x = -6.

a) $|x|$ b) $|-x|$ c) $-|x|$ d) $-|-x|$

13. True or false? Justify or prove your answer.

a. The expression, -a is never positive.
Answer: (false) → if a = -4, then –a = -(-4) = 4, which is positive.

b. The expression $|a|$ is always greater than or equal to a.
Answer: (true) → if a is negative, then the absolute value of a will be positive, therefore greater. If a is 0 or greater, then the absolute value will be that number, which is equal.

c. The absolute value of a negative number is always negative.

Answer: (false) → the absolute value of -3 is 3.

14. What is/are the solution(s) of $|x| = 18$? *Answer: (18 and -18)*

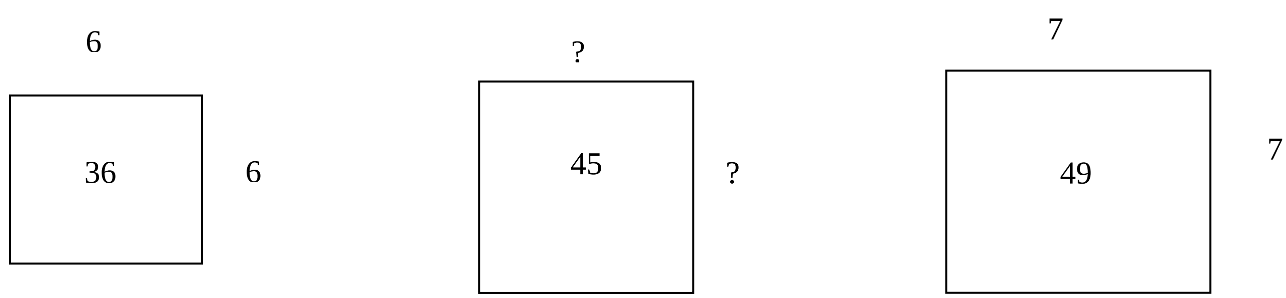
15. Evaluate the following if x = -9.

a) $|x|$ b) $|-x|$ c) $-|x|$ d) $-|-x|$

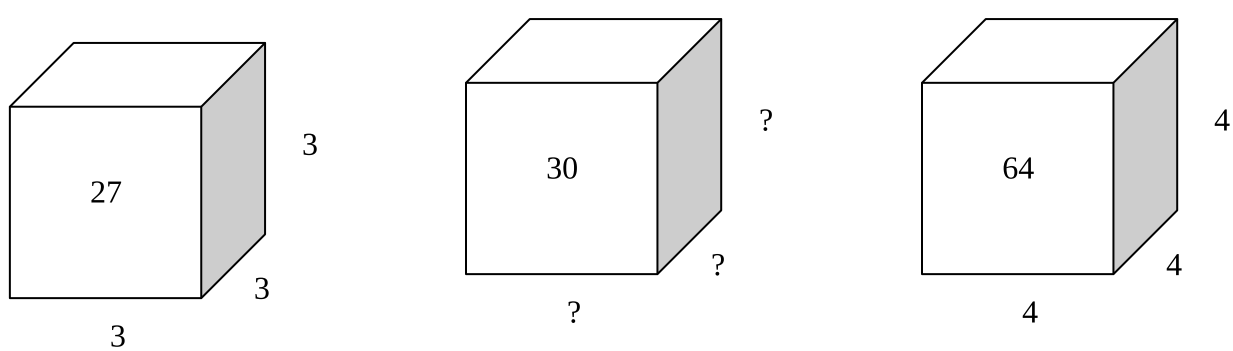
Answers: 9, 9, -9, -9

16. Find the edge of the figures in the center of each set A and B:

SET A



SET B



MODULE

1-2

Operations and Proportional Reasoning

This module addresses the following indicators:

8-2.1 **Apply an algorithm to add, subtract, multiply and divide integers.**
8-2.2 **Understand the effect of multiplying a rational number by another rational number.**
8-2.7 **Apply ratios, rates and proportions.**

This module contains 3 lessons. These lessons are INTRODUCTORY ONLY. Lessons in S3 begin to build the conceptual foundation student need. ADDITIONAL LESSONS will be required to fully develop the concepts.

I. Planning the Module

Continuum of Knowledge

Indicator 8-2.1 Apply an algorithm to add, subtract, multiply, and divide integers.

> In seventh grade, students generated their own strategies to add, subtract, multiply, and divide integers (7-2.8).
> In eighth grade, students apply an algorithm to add, subtract, multiply and divide integers (8-2.1).

Indicator 8-2.2 Understand the effect of multiplying and dividing a rational number by another rational number.

> In seventh grade, students learned to apply an algorithm to multiply and divide fractions and decimals (7-2.9).
> In eighth grade, students gain an understanding of the effect of multiplying and dividing a rational number by another rational number.

Indicator 8-2.7 Apply ratios, rate and proportions

> In sixth grade, students understand the relationship between ratio/rate and multiplication/division (6-2.6). In seventh grade, students apply ratios, rates, and proportions to discounts, taxes, tips, interest, unit costs and similar shapes (7-2.5).
>
> In eighth grade, students apply ratios, rates and proportions (8-2.7).

Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students**.**

*Integers
*Rational numbers
*Ratios
*Rates
*Proportions
*Additive inverse
*Multiplicative inverse
*Zero pairs
*Sum
*Product
*Quotient
*Effect

II. Teaching the Lesson(s)

Eighth grade expands the application of computational skills to all operations involving integers. Students in seventh grade generated strategies to add, subtract, multiply, and divide integers. As a result of sharing those generated strategies, students developed a conceptual understanding of integer operations. In other words student work with integers was limited to concrete and pictorial models. Therefore, it is beneficial to discuss strategies students developed with pictorial representation before moving into the algorithm. The emphasis for eighth grade is to apply an algorithm to add, subtract, multiply and divide. As a result, by the end of eighth grade student should exhibit fluency when solving a wide range of addition, subtraction, multiplication, and division problems involving integers.

Eighth grade students should be able to use all operations to solve a variety of problems involving integers and explain the meaning and effects of those operations when working with integers. Operations should also be introduced in context so that students develop an understanding of the algorithm instead of being taught arbitrary procedures.

Parts Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. *Teaching Student Centered Mathematics: Grades 5-8*.

Before integers, the "+" and "—" were only used for addition and subtraction. Models are essential in both the realization of the concept of integers and the application of an algorithm to add, subtract, multiply and divide integers. The two models introduced in the following lessons appear quite different, but are mathematically alike involving two concepts – quantity and opposite. Students should experience both models and then discuss how the two are alike.

Teachers should refer to the additive and multiplicative inverses as instructing. Teachers will need to provide a solid understanding of addition prior to subtraction and multiplication prior to division.

Proportional reasoning is developed over time through reasoning. It is the "ability to think about and compare multiplicative relationships between quantities. These relationships are represented symbolically as ratios." (Van de Walle, page 154).

When using cross-multiplying as a method for solving problems, teachers should provide students with meaningful experiences to develop a deep understanding. Cross multiplication is a powerful technique but needs to be taught with understanding. One way to develop cross multiplying is to create equivalent ratios by multiplying each term by 1:

$$\frac{2}{7} = \frac{a}{3} \qquad \frac{3 \times 2}{3 \times 7} = \frac{7 \times a}{7 \times 3} \qquad 3 \times 2 = 7 \times a$$

8-2.1 → Apply and algorithm to add, subtract, multiply and divide integers.

For this indicator, it is **essential** for students to:
- Connect concrete and/or pictorial models to an algorithm (numbers only)
- Understand the meaning of additive inverse
- Understand that addition means combining. Subtracting an integer is the same as adding the opposite, a "double negative."
- Gain computational fluency

For this indicator, it is **not essential** for students to:
- None noted

8-2.2 → Understand the effect of multiplying and dividing a rational number by another rational number.

For this indicator, it is **essential** for students to:
- Multiply and divide rational numbers
- Understand that division does not always result in a smaller answer
- Understand that multiplication does not always result in a larger answer
- Recognize fractional forms of one
- Understand the concept of equivalent fractions (same value; different form)
- Understand that zero is a rational number
- Understand why division by zero is not possible

For this indicator, it is **not essential** for students to:
- None noted

8-2.7 → Apply ratios, rates, and proportions.

For this indicator, it is **essential** for students to:
- Understand the meaning of ratio, rate and proportions
- Develop strategies for determining discounts
- Connect the similar ideas computing discounts, taxes, tips and interest in order to develop strategies
- Understand that with discounts, they pay less and with taxes, tips and interest they pay more
- Understand the concept of percents
- Multiply by a decimal

- Understand the relationship between unit cost and ratio and rate
- Understand the proportionality of similar shapes
- Solve proportions using an appropriate strategy
- Explore real world examples that goes beyond discounts, taxes, tips,etc.. that were explored in 7th grade

For this indicator, it is **not essential** for students to:

- None noted

1. Teaching Lesson A: Operations with Integers

a. Indicators with Taxonomy

8-2.1 → Apply and algorithm to add, subtract, multiply and divide integers. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson A: Operations with Integers

PART A: Integer Operations – Addition and Subtraction

Notes: Positive and negative numbers are measured distances to the right and left of 0. It is important to remember that signed values are directed distances and not points on a line. The points on the number line are not models of integers; the directed distances are. To emphasize this for students, represent all integers with arrows, and avoid referring to the number line coordinates as "numbers." Poster board arrows of different whole-number lengths can be made in two colors, yellow pointing to the right for positive quantities and red to the left for negative quantities. The arrows help students think of integers as directed distances. A positive arrow never points left; a negative arrow never points right. Furthermore, each arrow is a quantity with both length (magnitude or absolute value) and direction (sign). These properties remain for each arrow regardless of its position on the number line. Small versions of the arrows can easily be cut from poster board for individual students to work with. Students may also use red and yellow colored pencils as they progress from the concrete to the pictorial on their way to using algorithms.

SUGGESTED LESSON PROGRESSION
1A.) Combining positive quantities
1B.) Combining negative quantities
2.) Zero Pairs
3.) Combining mixed quantities (include "zero groups" in this lesson)
4A.) Subtracting Quantities
4B.) "Double" Negatives

Part 1: COMBINING POSITIVE QUANTITIES/COMBINING NEGATIVE QUANTITIES

***Teacher Note*: There are at least two big take-aways for students from this lesson. The first is that if groups of the same sign are combined, the result is a bigger group of the same sign. So, combining groups of positive quantities results in a bigger positive group; and combining groups of negatives results in a bigger negative group. Secondly, they should be able to communicate that + and – are more than operational signs; their job description has been expanded to include identifying integers as either positive or negative.**

Part 1A: COMBINING POSITIVE QUANTITIES

MATERIALS:

- two-sided red/yellow counters
- red and yellow colored pencils (at least one of each per pair of students)
- students need paper and/or math notebooks to sketch/record examples

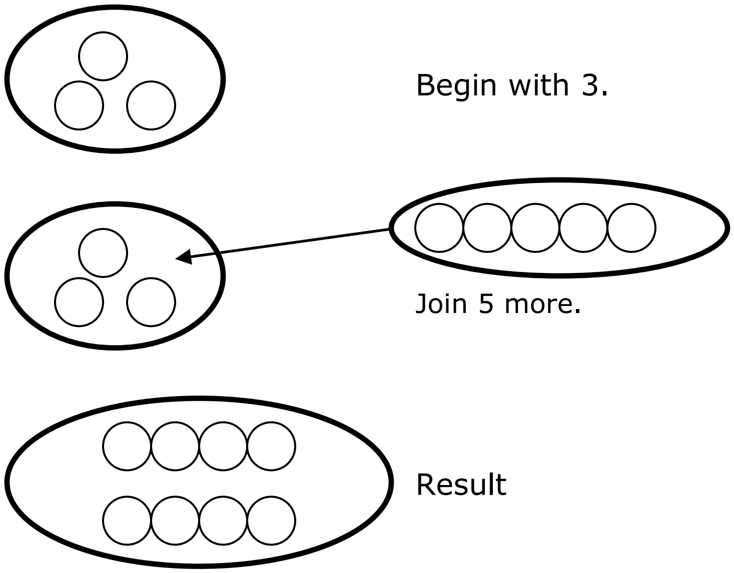
Begin by adding whole numbers. Students are familiar with the process, so using the yellow counters should help cement the concept of combining positive groups of integers. (white = yellow)

The examples shown/given in the lesson are not sufficient for students to connect the concept to an algorithm or procedure. You must either supply further examples or solicit them from the students.

All examples should be modeled for the students as they use their own counters at their desks.

All examples should also be sketched to provide a pictorial representation of the concrete model. Students should sketch their work on their own paper or in their math notebook using the colored pencils.

EX: 3 + 5



EX: 2 + 7

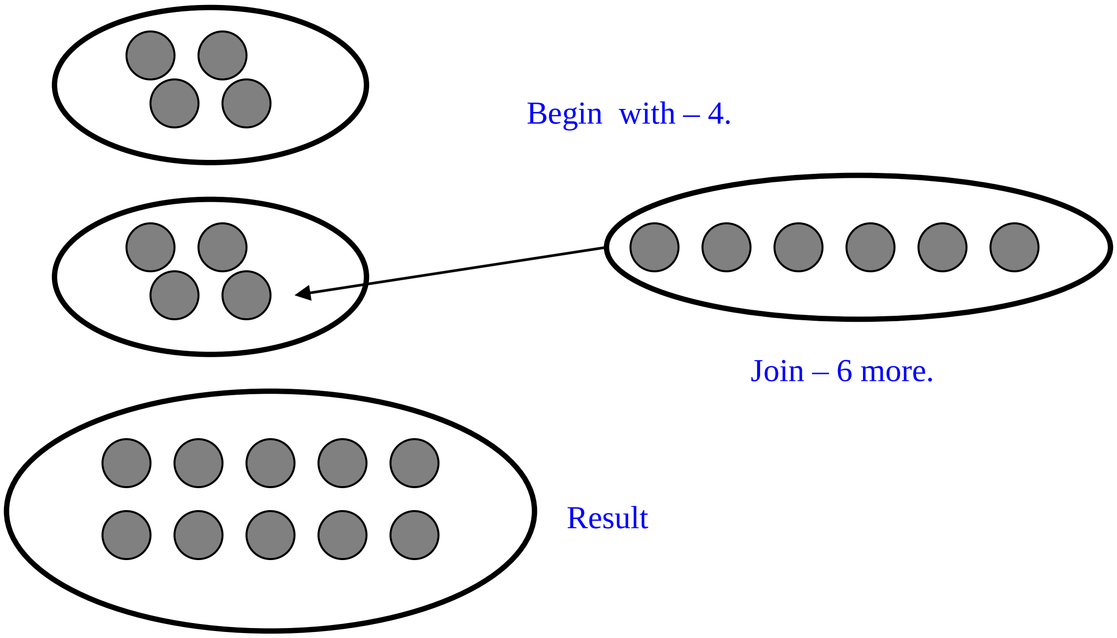
EX: 4 + 9

Model these and other examples as above.

Ask students to create their own examples. Even though it is “only adding,” make sure they use the counters to build the sentences and sketch their work on their own paper or in their math notebook using yellow colored pencils.

Part 1B: COMBINING NEGATIVE QUANTITIES (grey = red)

EX: - 4 + - 6



EX: - 8 + - 2

EX: - 1 + - 5

Model these and other examples as above.

Ask students to create their own examples. Make sure they use the counters to build the sentences and sketch their work on their own paper or in their math notebook using yellow colored pencils.

Part 2: ZERO PAIRS

MATERIALS:
- two-sided red/yellow counters [white = yellow and grey = red]
- red and yellow colored pencils (at least one of each for each pair of students)
- students need paper and/or math notebooks to sketch/record examples

Ask: What happens if you combine equal groups of positive and negative integers? Let's start with +1 and – 1.

Use the counters to model 1 – 1.

How else can it be expressed (commutative property)?

$1 - 1$; $-1 + 1$; $1 + {}^{-}1$

It may be useful to quickly model each expression to emphasize that they're all equal.

Ask students to give other examples of zero pairs. Model them using the counters. Sketch your work on the overhead or board. Students should be working with their own counters and sketching their work. They need to come to the conclusion that equal groups of positives and negatives form zero pairs. Other ways to consider it: they "zero out" or "cancel each other out."

Part 3: COMBINING MIXED QUANTITIES

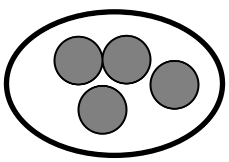
MATERIALS:
- two-sided red/yellow counters [white = yellow and grey = red]
- red and yellow colored pencils (at least one of each for each pair of students)
- students need paper and/or math notebooks to sketch/record examples

The examples shown/given in the lesson are not sufficient for students to connect the concept to an algorithm or procedure. You must either supply further examples or solicit them from the students.

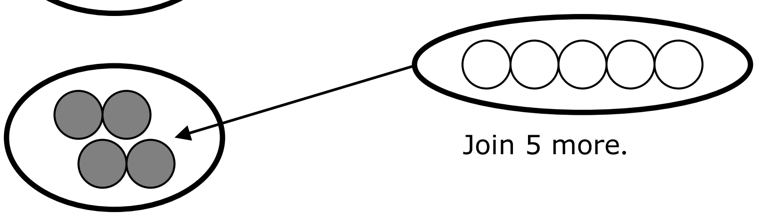
All examples should be modeled for the students as they use their own counters at their desks.

All examples should also be sketched to provide a pictorial representation of the concrete model. Students should sketch their work on their own paper or in their math notebook using the colored pencils.

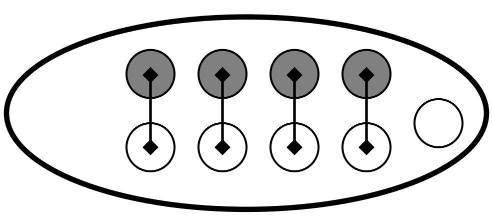
EX: - 4 + 5



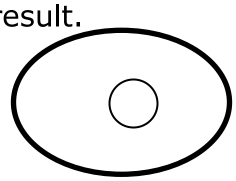
Begin with – 4.



Join 5 more.



Cancel pairs of + and – ("zero pairs") to get result.



EX: 7 + - 4
EX: - 3 + 5

Model these and other examples as above.

Ask students to create their own examples. Make sure they use the counters to build the sentences and sketch their work on their own paper or in their math notebook using yellow colored pencils.

<u>Part 4A: SUBTRACTING QUANTITIES</u>

MATERIALS:

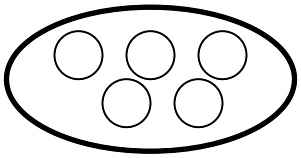
- two-sided red/yellow counters [white = yellow and grey = red]
- red and yellow colored pencils (at least one of each for each pair of students)
- students need paper and/or math notebooks to sketch/record examples

The examples shown/given in the lesson are not sufficient for students to connect the concept to an algorithm or procedure. You must either supply further examples or solicit them from the students.

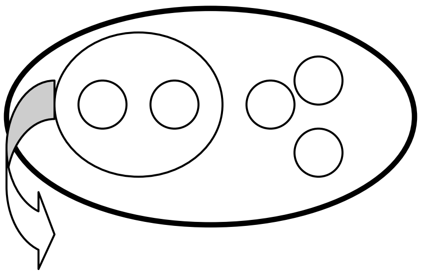
All examples should be modeled for the students as they use their own counters at their desks.

All examples should also be sketched to provide a pictorial representation of the concrete model. Students should sketch their work on their own paper or in their math notebook using the colored pencils.

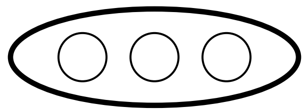
EX: 5 – 2



Begin with 5.



Remove 2 for the result.



Result

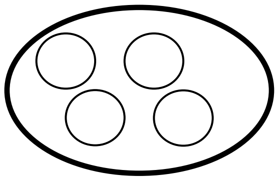
EX: 7 - 4

EX: 2 - 1

Model these and other examples as above.

Ask students to create their own examples. Make sure they use the counters to build the sentences and sketch their work on their own paper or in their math notebook using yellow colored pencils. Ask students to create their own examples. Make sure they use the counters to build the sentences and sketch their work on their own paper or in their math notebook using yellow colored pencils.

EX: 4 – 5

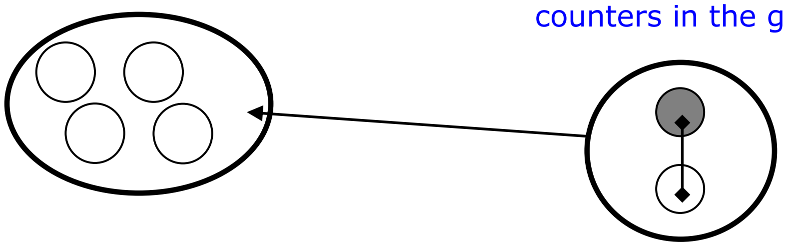


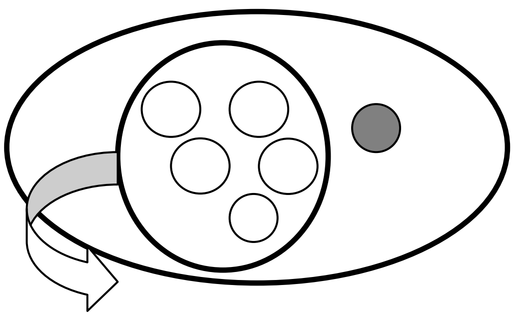
Begin with 4.

positive

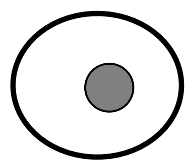
You want to remove 5, but there are only 4

counters in group.
Add 1 zero pair so that there are 5 positive counters in the group.





Remove 5 for the result.



2 - 7

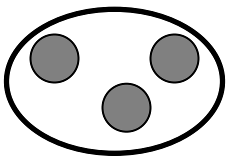
1 - 4

Model these and other examples as above.

Ask the students to give examples (one at a time) and model those. Students should put the examples in the empty boxes on the recording sheet, use the counters, and sketch their work. When sufficient examples have been modeled and solved (use your professional judgment), ask the students to generalize what happens when combining mixed groups of integers. You should hear something like this: Find the difference between the two groups; the larger group determines the sign of the solution. Again, more mathematically, find the difference in the absolute values and the answer takes the sign of the greater value.

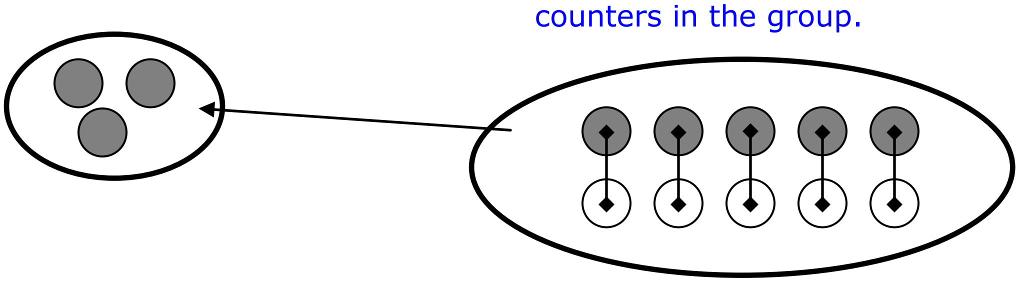
EX:

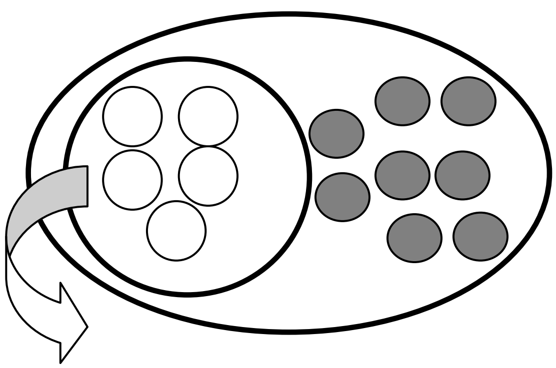
- 3 – 5 =



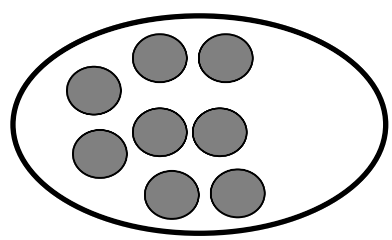
Begin with – 3.

You want to remove 5, but there are only negative counters in group.
Add 5 zero pairs so that there are 5 positive counters in the group.





Remove 5 for the result.



- 2 – 7
- 4 – 9

Model these and other examples as above.

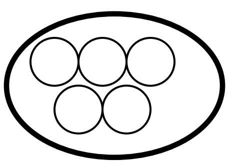
Ask students to create their own examples. Again, insist that students use the counters to build the sentences and sketch their work on their own paper or in their math notebook using red colored pencils.

When students have a collection of examples of combining both positive quantities and negative quantities, ask them to generalize a "rule" for combining these quantities. They should be able to see that they can add the groups and keep the sign. More mathematically speaking, add the absolute values and keep the sign of the two groups. This leads to the idea that an integer takes the sign immediately preceding it. Thus – 4 + - 9 has the same value as – 4 – 9.

<u>Part 4B: DOUBLE NEGATIVES</u>

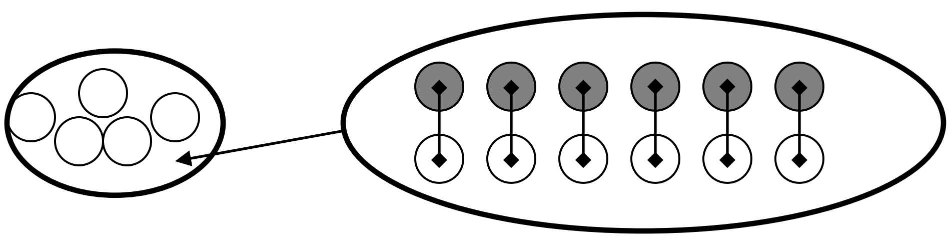
<u>Suggestion</u>: Begin by subtracting from positive integers.

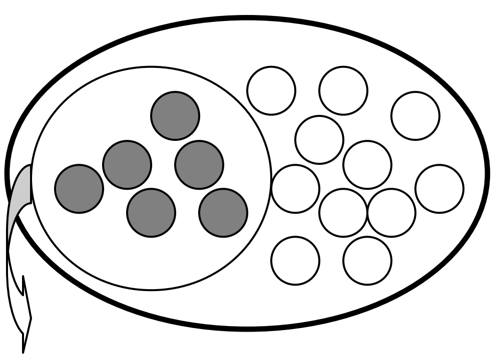
5 – $^{-}6$



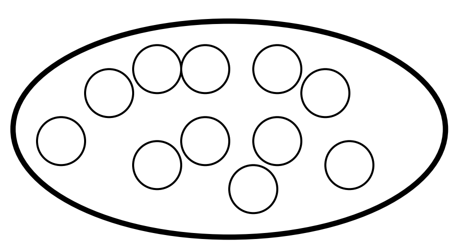
Begin with 5.

You want to remove $^{-}6$, but there are no negative counters in group. *Add 6 zero groups so that there are 6 negative counters.*





Remove $^{-}6$ for the result.



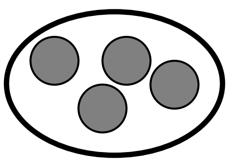
EX: $3 - {}^{-}4$
EX: $8 - {}^{-}5$

Model these and other examples as above.

At this point, ask students to make a generalization about what to expect when subtracting a negative from a positive. They should recognize that subtracting the negative gives the same result as adding the opposite. Subtracting ${}^{-}6$ from 5 has the same result as adding ${}^{+}6$ to 5. This shows clearly in the models and sketches.

Before modeling the next example, ask students if they think their generalization about adding the opposite will hold true when subtracting a negative from a negative.

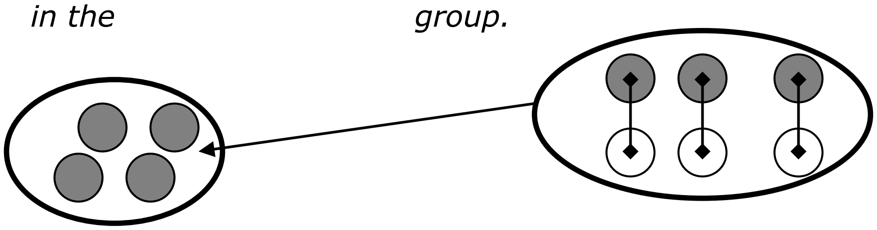
EX: $-4 - -7$

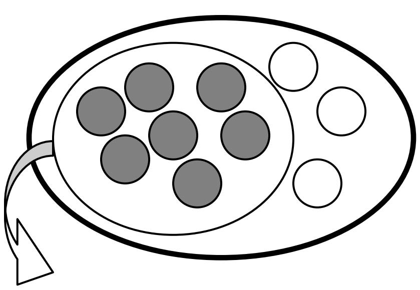


Begin with ${}^{-}4$.

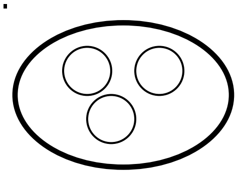
You want to remove ${}^{-}7$, but there are only 4 negative counters available.

Add 3 zero groups so that there are 7 negative counters in the group.





Remove ${}^{-}7$ for the result.



So…is $^{-}4 - {}^{-}7$ the same as $^{-}4 + 7$? It is INDEED! Let's see if it works with other examples.

EX: $^{-}9 - {}^{-}3$

EX: $^{-}10 - {}^{-}5$

This is usually the hardest process for students to generalize. They need to recognize that subtracting the negative gives the result of adding the opposite. It may take modeling and sketching many expressions.

<u>TEACHER NOTE</u>: Context problems need to be done here for students to relate the concept to.

PART B: MULTIPLYING AND DIVIDING INTEGERS

Students will work with the teacher to complete a multiplication chart below and study it for patterns in order to make a rule for multiplying integers. Because division is the inverse of multiplication, the same rules apply for dividing integers as apply for multiplying them. The inverse could be shown with numbers they already know by using fact families.

TEACHER COPY (A blank student copy is on the following page.)

	$^-3$	**$^-2$**	**$^-1$**	**0**	**1**	**2**	**3**	
3	$^-9$	$^-6$	$^-3$	0	3	6	9	**3**
2	$^-6$	$^-4$	$^-2$	0	2	4	6	**2**
1	$^-3$	$^-2$	$^-1$	0	1	2	3	**1**
0	0	0	0	0	0	0	0	**0**
$^-1$	$^-3$	$^-2$	$^-1$	0	1	2	3	**$^-1$**
$^-2$	$^-6$	$^-4$	$^-2$	0	2	4	6	**$^-2$**
$^-3$	$^-9$	$^-6$	$^-3$	0	3	6	9	**- 3**

STUDENT PAGE (Each student will need a copy of the blank handout.)

	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	
3				0				3
2				0				2
1				0				1
0	0	0	0	0	0	0	0	0
$^{-}1$				0				$^{-}1$
$^{-}2$				0				$^{-}2$
$^{-}3$				0				- 3

Fold the bottom half up under the top half.

	-3	-2	-1	0	1	2	3	
3				0				3
2				0				2
1				0				1
0	0	0	0	0	0	0	0	0
-1				0				-1
-2				0				-2
-3				0				-3

Now fold the left half behind the right.

	-3	-2	-1	0	1	2	3	
3				0				3
2				0				2
1				0				1
0	0	0	0	0	0	0	0	0

0	1	2	3	
0				3
0				2
0				1
0	0	0	0	0

0	1	2	3	
0	3	6	9	3
0	2	4	6	2
0	1	2	3	1
0	0	0	0	0

Ask students to fill in the first quadrant.

Box factors so they stand out.

Key questions:

What sign did the factors have in quadrant 1?
What sign did the products have?

Fold the left half out so it can be seen.

	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	
3	$^{-}9$	$^{-}6$	$^{-}3$	0	3	6	9	3
2	$^{-}6$	$^{-}4$	$^{-}2$	0	2	4	6	2
1	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	1
0	0	0	0	0	0	0	0	0

Ask students to continue that pattern across zero into the second quadrant.

Box factors so they stand out.

Key questions:

What sign did the factors have in quadrant 2? What sign did the products have?

Unfold the bottom half.

	-3	-2	-1	0	1	2	3	
3	-9	-6	-3	0	3	6	9	3
2	-6	-4	-2	0	2	4	6	2
1	-3	-2	-1	0	1	2	3	1
0	0	0	0	0	0	0	0	0
-1	3	2	1	0	-1	-2	-3	-1
-2	6	4	2	0	-2	-4	-6	-2
-3	9	6	3	0	-3	-6	-9	-3

Now quadrants 1 and 2 are completed.

Ask students to complete quadrants 3 and 4.

Box quadrant 3 factors.

Key questions:

In quadrant 3, what sign did the factors have? What sign did the products have?

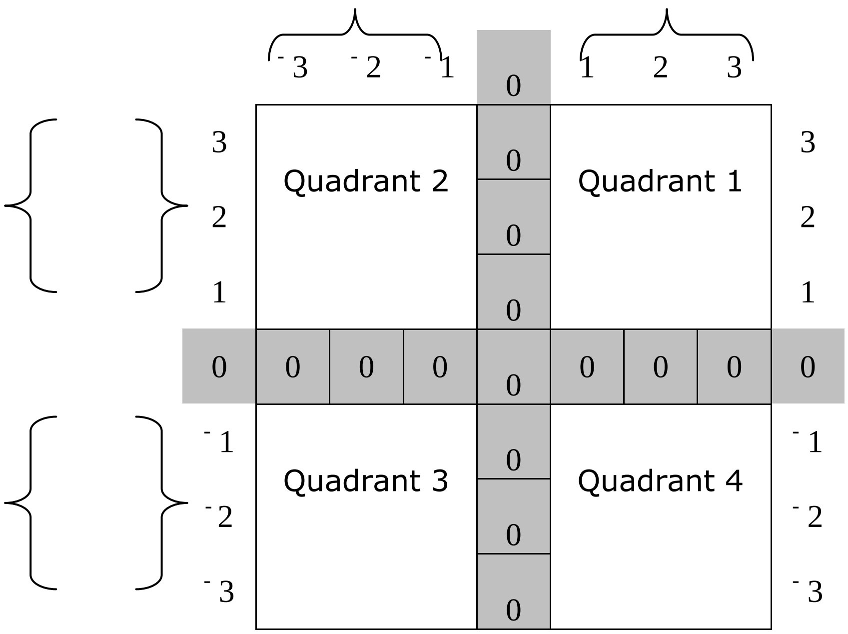
Box quadrant 4 factors.

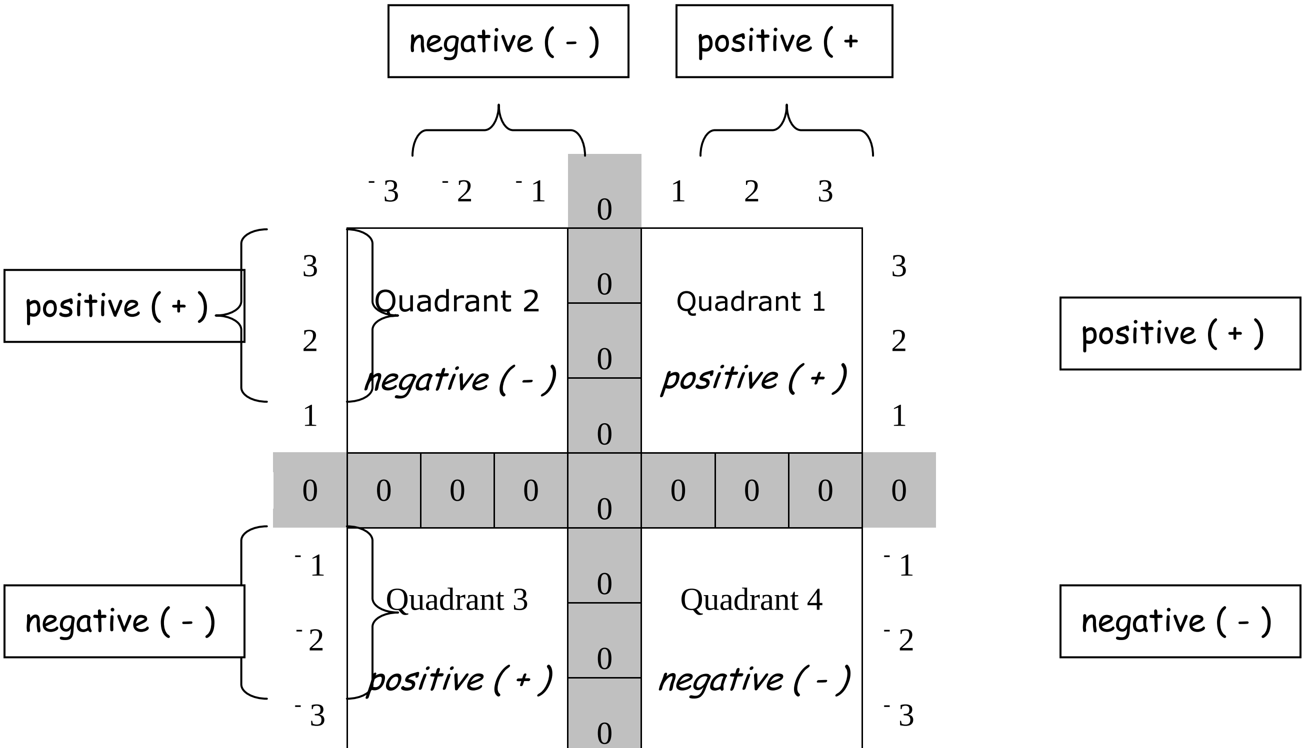
Key questions:

In quadrant 4, what sign did the factors have? What sign did the products have?

Put this on the back of the student copy of the empty chart.

Use it to review the patterns.





In quadrant one, what were the signs of the factors? *both positive*	What was the sign of the products? *positive*
In quadrant two, what were the signs of the factors? *one was positive, one was negative*	What was the sign of the products? *negative*
In quadrant three, what were the signs of the factors? *both negative*	What was the sign of the products? *positive*
In quadrant four, what were the signs of the factors? *one was positive, one was negative*	What was the sign of the products? *negative*

c. Misconceptions/Common Errors

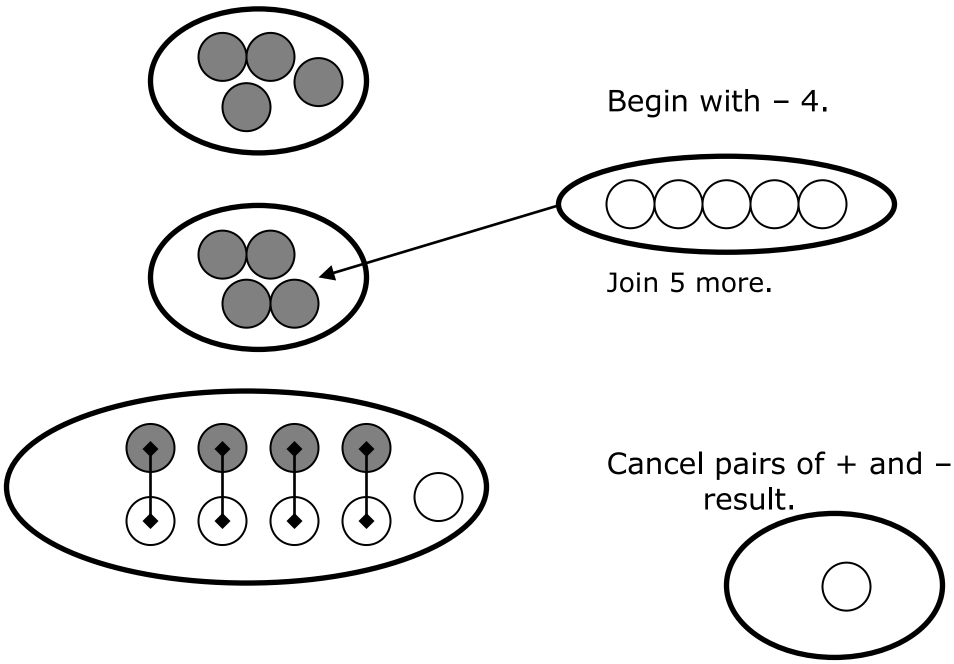
Although rules are fun saying and are easy for students to remember, many students still struggle with operations with integers because they lack a conceptual understanding of these operations. Rules such as "like signs give positive products" and "unlike signs give negative products" are deceptively simple. Correct answers and meaningful justification of these answers are equally important.

d. Additional Instructional Strategies/Differentiation

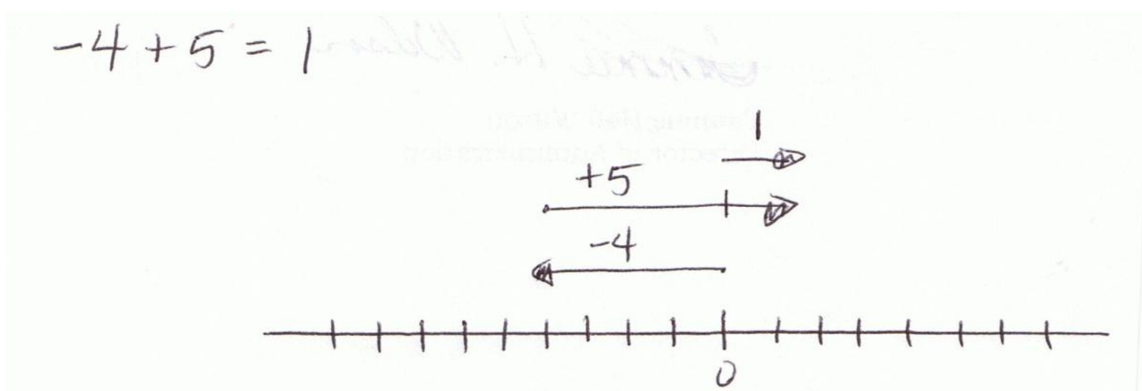
- Use arrows to represent all integers and don't refer to number line coordinates as "numbers." The arrows help students think of the integers as directed distances.
- Allow students to use colored pencils (yellow for positive and red for negative) to connect the concept of the models they learned in previous grade levels.
- It is important to use appropriate mathematical terminology when discussing strategies. Using words like flip may be easier for students to understand but they decrease the student's conceptual understanding.
- A strategy for moving students through operations with integers is to ensure that students have a thorough understanding of addition and subtraction prior to instructing multiplication and division. Progress from combining (addition) of positive integers, to combining negative integers, and finally to combining unlike signs, a mixture of positive and negative integers that include zero pairs.
- Although students have generated strategies in 7^{th} grade and explored problems in context, the use of these problems is still essential to helpful students deepen their understanding of procedures.
- When using colored counters it may be helpful to relate them to the process of using the number line to help students make additional understanding.

- Relate the counters to the number line. See the following example:

EX: - 4 + 5



Number Line Representation:



e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- illuminations.nctm.org (Lessons, Activities and Related WebLinks)
- nlvm.usu.edu (National Library of Virtual Manipulatives)
- Oneplacesc.org (ETV Streamline and more!)
- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS are posed throughout the lesson. Pay close attention to student responses during the lesson. Student responses and student questions will guide instruction. Further formative assessment questions may be needed based on student understanding.

3. Teaching Lesson B: Effects of Multiplying and Dividing a Rational Number by Another Rational Number

a. Indicators with Taxonomy

8-2.2 → Understand the effect of multiplying and dividing a rational number by another rational number. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

***b. Introductory Lesson B:* Effects of Multiplying and Dividing a Rational Number by Another Rational Number**

PART I

Use a calculator to complete the following table using the rule given.
Look for patterns and answer the questions below:

Use the rule: $a \div b = c$	a	b	c
	16	2	
	16	4	
	16	0.5	
	0	16	
	0	4	
	16	0	
	4	0	
	0.5	0.5	
	0.5	0.25	

What pattern do you see in the division?
What pattern do you see in the inverse of division (multiplication) related to the chart?

Consider the following pattern:

$\frac{16}{2} = 8$ because $2 \times 8 = 16$.

$\frac{0}{16} = 0$ because $16 \times 0 = 0$.

So, what happens with $\frac{16}{0}$?

(answer: undefined, therefore resulting in an error message because $\frac{16}{0} \neq 0$ because $0 \times 0 \neq 16$ and $\frac{16}{0} \neq 16$ because $0 \times 16 \neq 16$. Apply the pattern from above.)

PART 2

Use a calculator to complete the following table using the rule given.
Look for patterns and answer the questions below:

Use the rule: a X b = c	a	b	c
	16	2	
	16	0.5	
	16	0	
	0	16	
	32	0.5	
	2	0.5	
	32	0.25	
	0.5	0.5	
	0.5	0.2	

What pattern do you see in the multiplication?
What pattern do you see in the inverse of multiplication (division) related to the chart?

c. Misconceptions/Common Errors

- Students may experience difficulty understanding why division by zero is impossible. Discussion should focus on the resulting effect on the product and quotient when the operation is performed.

d. Additional Instructional Strategies/Differentiation

- The focus of the indicator is an emphasis on understanding relationships not on computational fluency. Understanding these relationships will also be beneficial to students as they perform operations with rational numbers.
- Provide contextual opportunities for students to focus on the effects to the products and quotients of operations with rational numbers.
- One strategy for helping students understand why division by zero is not possible is to examine the relationship between rational numbers (fractions) and their decimal equivalent. The denominator x quotient = the numerator. For example, ½ = 0.5; therefore, 2 x 0.5 must equal 1. Students sometimes state the #/0 = 0 so by their logic 4/0 =0 but if that were true then 0 x 0 should equal 4.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

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- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS:

The questions posed during the lesson should serve as formative assessment for the introductory lesson. Pay close attention to student responses during the lesson. Student responses and student questions will guide instruction. Further formative assessment questions may be needed based on student understanding.

4. Teaching Lesson C: Ratios, Rates, and Proportions

a. Indicators with Taxonomy

8-2.7 → Apply ratios, rates, and proportions. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson C: Ratios, Rates, and Proportions

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. *Teaching Student Centered Mathematics: Grades 5-8*.

Rectangle Ratios

MATERIALS:

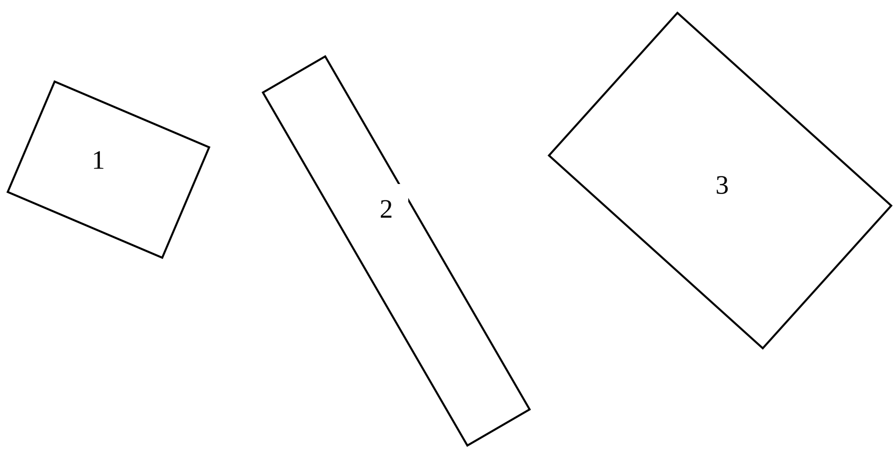
- rectangle handout (included – one per pair of students)
- recording sheet (included – one for each student)
- scissors (one per pair of students)

Rectangle Handout info

Rectangles A, I, and D have sides in the ratio of 3 to 4. Rectangles C, F, and H have sides in the ratio of 5 to 8. Rectangles J, E, and G have sides in the ratio of 1 to 3. Rectangle B is a square, so the sides are in the ratio of 1 to 1.

PROCEDURES:

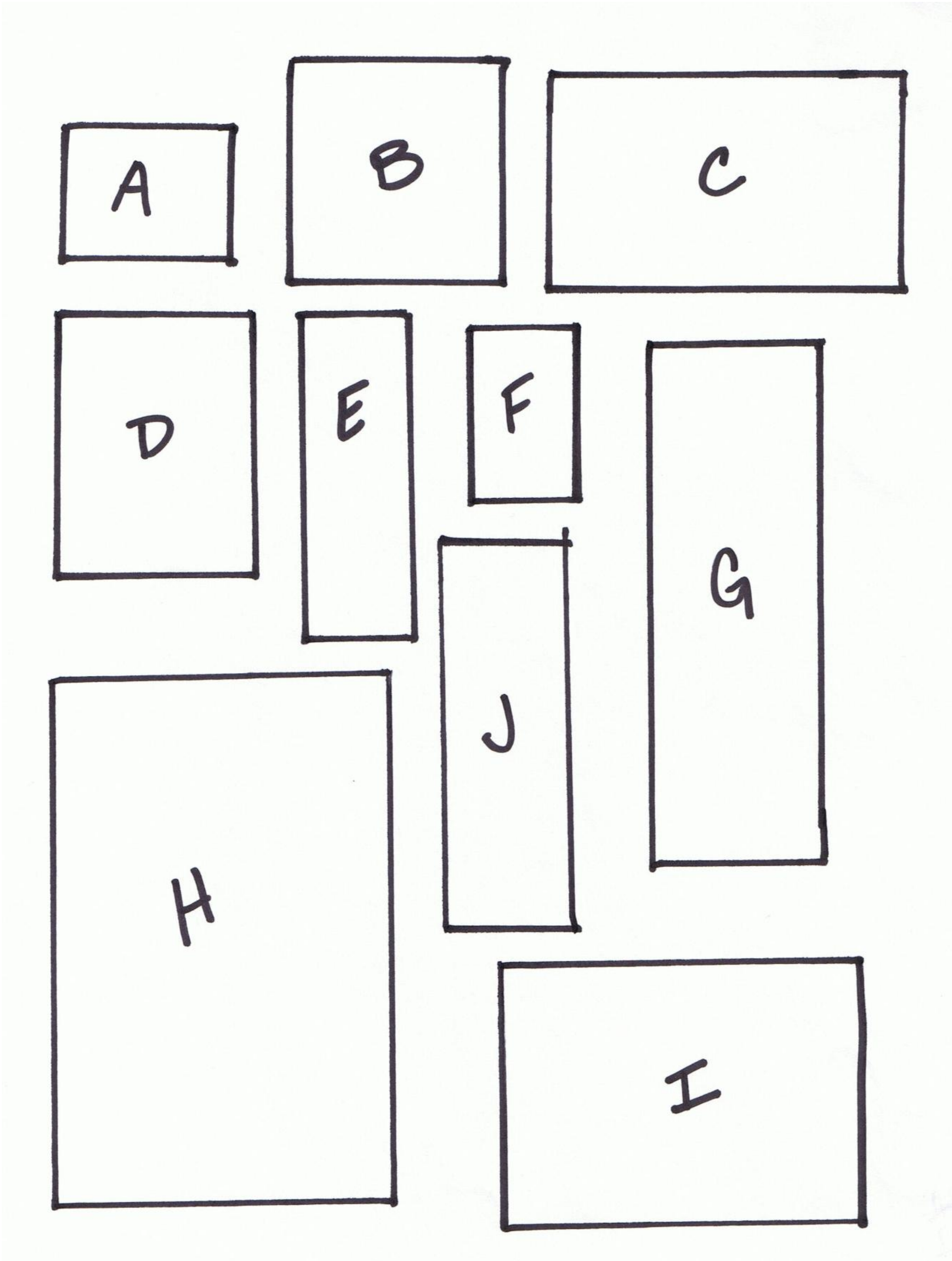
To get students thinking about their task, draw three rectangles on the board or overhead with two that are similar and one that is clearly dissimilar to the other two (as in the example below). Ask students to explain which rectangle doesn't belong with the other two. This should give them a chance to refresh their memories in regard to the definition of "similar."



Distribute the rectangle handouts and have students cut out the rectangles. Their task is to group the rectangles into three sets of three that are similar and one leftover that doesn't fit.

When pairs have decided on their groupings, pull the class back together and ask students to explain why they grouped their rectangles as they did. Don't evaluate their answers.

Next, have them measure and record the sides of each rectangle to the nearest half-centimeter. A sheet for them to use is included. Each student should have his own recording sheet to refer to later. When they've done their measuring, have them determine the ratio of the short to long sides for each rectangle. Again, they can record their results on the data sheet. Discuss the results and ask students to offer explanations. If the groups are made of similar rectangles, the ratios within the group will be the same.


A
B
C
D
E
F
G
J
H
I

Rectangle Ratios

Name:
Partner:
Class:
Date:

Use the tables below to organize your sets. Then measure the short and long side of each rectangle to the nearest half-centimeter. Record your findings.

Set 1

Rectangle (letter	Measure in cm		Ratio of sides
	short side	long side	short/long

Set 2

Rectangle (letter	Measure in cm		Ratio of sides
	short side	long side	short/long

Set 3

Rectangle (letter	Measure in cm		Ratio of sides
	short side	long side	short/long

Leftover

Rectangle (letter	Measure in cm		Ratio of sides
	short side	long side	short/long

<u>Trucks and Boxes</u>

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. *Teaching Student Centered Mathematics: Grades 5-8*.

MATERIALS:

- trucks and boxes cards (samples included – you will need more for this lesson to be effective)

Key questions to address student understanding: THESE SHOULD BE ASKED THROUGHOUT THE LESSON.

1. How did you decide on your matches?
2. What do the pictures mean? What information do they provide?
3. Create another ratio to match the 1 truck to 3 boxes card.
4. Create another ratio to match the 1 truck to 4 boxes card.
5. How are these ratios different from "plain old fractions?"
6. What makes these ratios rates?
7. What other rates can you think of?

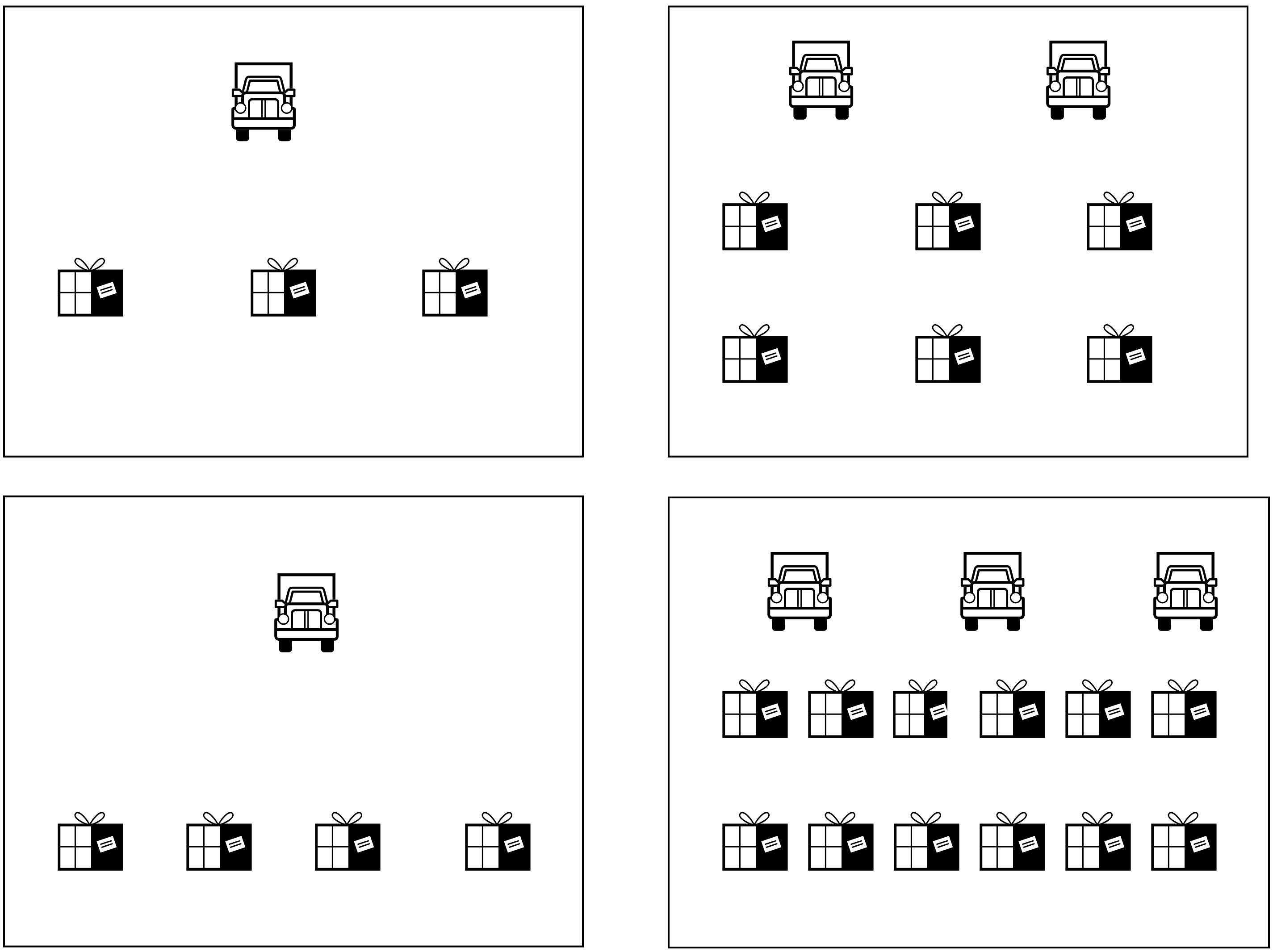
PROCEDURES:

- Groups students in pairs.
- Give each pair a set of cards.

Students are to match cards with equivalent ratios. This moves them from the visual representation to the numerical. It also introduces the concept of ratios as rates (number of boxes per truck or vice versa). The unit rate cards depict exactly one truck. The idea of unit rate should be discussed with the students.

SAMPLE CARDS

NOTE: These cards can be easily created with stickers and index cards. Check the teacher section at an office supply store. The cards also don't have to be trucks and boxes; they could be any combination that makes sense: children and balls; people and pizzas; coins and bills, etc.



<u>A Table of Ratios</u>

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. *Teaching Student Centered Mathematics: Grades 5-8*.

Organizing information in a table is a good way to show how two sets of data are related to each other.
It is easy to use addition when looking for proportional relationships, but encourage students to use multiplicative relationships when working with proportions.

Given the following situations, build a ratio table and use it to answer the questions given.

a) A person who weighs 160 pounds on Earth will weigh 416 pounds on Jupiter. How much will a person who weighs 120 pounds on Earth weigh on Jupiter?
b) 5 out of 8 seniors live in apartments at the local college. How many of the 30 senior math majors are likely to live in an apartment?
c) The tax on a purchase of $20.00 is $1.12. How much tax will there be on a purchase of $45.50?

Three Sample Solutions for Scenario A:

1)

÷ 2 ÷ 2 x 3

Earth Weight	160	80	40	120
Jupiter Weight	416	208	104	312

2)

÷ 2 add add

Earth Weight	160	80	40	120
Jupiter Weight	416	208	104	312

3)

÷ 8 add x 5

Earth Weight	160	20	100	120
Jupiter Weight	416	52	260	312

Scale Drawings

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. Teaching Student Centered Mathematics: Grades 5-8.

Provide students with a simple geometric figure, including the dimensions. Have students create a new drawing that is either larger or smaller than the given one. Provide students with one dimension of the new drawing. Have students determine the other dimensions by solving proportions.

Discounts, Taxes, and Tips
Pose several problems in involving discounts, taxes, and tips for students to solve.
For example,

(1) If a jacket, marked $58, is on sale for 20% off, how much will you pay?
(2) A computer costs $1,500, not including sales tax. What is the total cost if the sales tax is 7%.
(3) A restaurant bill comes to $28.50, including sales tax. You plan to leave a 20% tip. How much did you spend at the restaurant?

c. Misconceptions/Common Errors

- Students may still struggle with converting decimals.

- Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. *Teaching Student Centered Mathematics: Grades 5-8*.

  Watch for students who are using a mechanical method (such as the cross-product algorithm). These methods do not develop proportional reasoning and should not be encouraged ***or introduced*** until students have had many experiences with intuitive and conceptual methods. If students are using such methods, insist that they also come up with another way to find a solution.

  Notice how students approach problems. Do they use a variety of means or do they always use the same approach? Students tend to use unit rates to compare ratios, even when another approach may be easier. If students always use the unit rate approach, encourage them to consider different approaches. A nonflexible or algorithmic approach, even if correct, may signal that a student is simply following a rule.

d. Additional Instructional Strategies/Differentiation

- Although students are not required to compute the final cost, it may be helpful for them to understand that discounts (subtracting from original amount) and taxes, tips and interest (add to the original amount).

- Representing these concepts as pictorial or concrete models may help students deepen their understanding of these procedures. For example, using a hundreds charts to represent $100 dollars. Shade appropriate blocks to represent discounts like 20% off mean subtracting 20 blocks or interest of 30% means adding 30 more blocks.

- The cross multiplying algorithm for solving proportions is a valid strategy that should be taught with understanding. Using proportional reasoning is another powerful strategy that focuses more on a conceptual understanding of proportional relationships as opposed to a traditional algorithm. Proportional reasoning always allows students make estimates.

- Suggested Literature Connection:
  "If you Hopped Like A Frog" by David M. Schwartz
  In this book, the author uses proportional reasoning to determine what it would be like if we had the powers/dimensions of animals. The proportions are based on factual data.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- illuminations.nctm.org (Lessons, Activities and Related WebLinks)
- nlvm.usu.edu (National Library of Virtual Manipulatives)
- Oneplacesc.org (ETV Streamline and more!)
- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

Spreadsheets can be used to create tables and compute ratios.

f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS:

These were the key questions asked during the lesson.

How did you decide on your matches?
What do the pictures mean? What information do they provide?
Create another ratio to match the 1 truck to 3 boxes card.
Create another ratio to match the 1 truck to 4 boxes card.
How are these ratios different from "plain old fractions?"
What makes these ratios rates?
What other rates can you think of?

As you work with students in solving proportional reasoning tasks, continue to think about the type of reasoning students are using.

(a) Do they distinguish between situations that are proportional and nonproportional?
(b) Are they flexible in the way they attempt to solve proportions?
(c) Are there differences in thinking about different types of proportional situations?
(d) Do students seem to understand rates as ratios?

How students deal with these ideas reflects the development of their proportional thinking.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

Indicator 8-2.1 Apply and algorithm to add, subtract, multiply and divide integers.

The objective of this indicator is to apply which is in the "apply procedural" cell of Revised Taxonomy. Although the focus is to gain computational fluency with operations with integers, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **apply** requires students to recall and understand the concept of integers. Students explore these operations in context as opposed to rote memorization of rules. They apply their conceptual knowledge of integers to transfer their understanding of concrete and/or pictorial representations to symbolic representations (numbers only) by generalizing connections among these representational forms and real world situations (8-1.7). Students use correct and clearly written or spoken words to communicate their understanding (8-1.6). Students engage in repeated practice using pictorial models, if needed, to support learning. Lastly, students should evaluate the reasonableness of their answers using appropriate strategies.

Indicator 8-2.2 Understand the effect of multiplying and dividing a rational number by another rational number. (B2)

The objective of this indicator is to understand which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand is to construct meaning; therefore, students develop a conceptual understanding of these effects. The learning progression to **understand** requires students to recall the meaning of rational numbers. Students understand how to multiply and divide rational numbers using an appropriate algorithm. Given examples, students generate and evaluate conjectures (8-1.2) about the effect on the product or quotient. They use inductive reasoning (specific to general) to generalize mathematical statements about the effect using correct and clearly written or spoken words and notation (8-1.6). They then generate and solve complex problems such as division by zero.

Indicator 8-2.7 Apply ratios, rates, and proportions.

The objective of this indicator is to apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is procedural, students also use their conceptual understanding of ratio, rate and proportion to solve problems. The learning progression to **apply** requires students to recall and understand the meaning of the concepts rate, ratio and proportions. . Students explore a variety of problems (beyond discounts, taxes, tips, interest, unit cost and similar shapes) in context to generalize connections (8-1.7). They analyze pictorial and/or concrete models, where appropriate, to support conceptual understanding of these procedures. They generate mathematical statements (8-1.5) related to how to use ratios, rates and proportions to solve problems and use correct and clearly written and spoken words, variable and notation to communicate their understanding (8-1.6). Students should use their understanding to generate and solve more complex problems using ratio, rate and proportion (8-1.1).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Represent the following with manipulatives and sketch your steps and result.

a) -8 + 3
b) -4 – 2
c) 5 – (-4)

Answers will be based on representations. The end results are: a) -5 b) -6 c) 9

2. Your high school football team needs 9 yards to score a touchdown. The last four plays result in a 5 yard gain, a 2 yard gain, a 12 yard loss and a 15 yard gain. Does your team score a touchdown? If not, how many yards do they still need?

Represent your answer using either manipulatives or an expression.

Answer: yes, but should be accompanied by drawings or algebraic representation.

3. Predict whether or not the product of A x B will be larger or smaller than A. AFTER PREDICTING, find the product and explain the results.

A	B	Product Prediction (>A or <A)	Actual Product
0.5	0.75		
0.5	4.25		
0.35	$\frac{20}{7}$		

Answers Below:

A	*B*	*Product Prediction (>A or <A)*	*Actual Product*
0.5	*0.75*	*Less than A*	*0.375*
0.5	*4.25*	*Greater than A*	*2.125*
0.35	$\frac{20}{7}$	*Greater than A*	*.999999999*

4. When you multiply a number smaller than 1 by another number, the product will always be less than or equal to one or both of the numbers. True or False? *Answer: True*

5. Prove or justify the following statement:
   Dividing by 0 yields an undefined result. *Answer: see notes in lesson for the reasoning…*

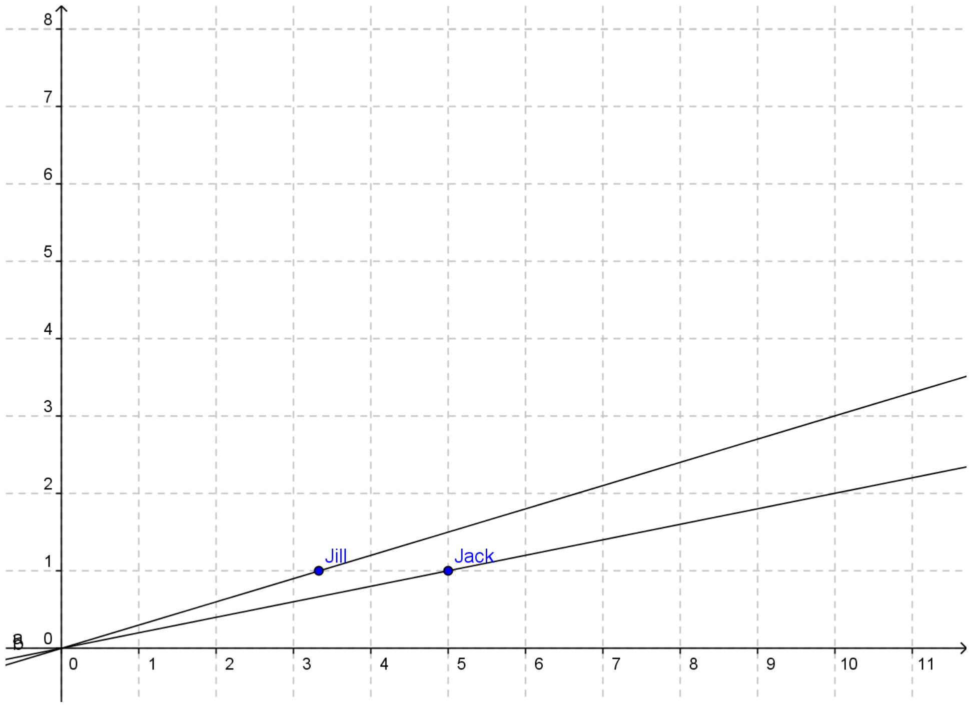
6. Jack and Jill were out picking strawberries at the local Pick Your Own Berries field. Jack snacked on 5 berries every 25 minutes. Jill ate 3 berries every 10 minutes. If they both picked at about the same speed for the same length of time, who will bring home more berries? Or will they bring home the same amount of berries? (You must be able to explain why your answer makes sense using a table, graph, or a picture)

Possible Answers

Jack → 5 berries every 25 minutes → 1 berry every 5 minutes
Jill → 3 berries every 10 minutes → 1 berry every 3 1/3 minutes

Jill was eating berries faster than Jack, so Jack will have more berries to bring home.

In 50 minutes (I picked 50 because it's a multiple of both 25 and 10), Jack will eat 10 berries, and Jill will eat 15 berries. Jack will bring home more berries than Jill.



7. Which rectangle is closer to a square... a 3 x 5 rectangle or an 8 x 10 rectangle? *Answer: 8 x 10 because it's the same as 4 x 5 which is closer to a square than 3 x 5. Other rationales may be discussed.*

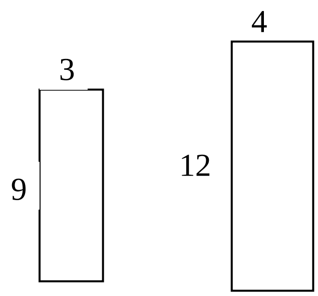
8. Debt to Income ratios are found by comparing the total expenses to the total income.

a. Use the information to determine the debt to income ratio for Roberta. .

Type of Debt	Expenses	Income
Mortgage	650	3950
Credit Cards	325	
Car Payment	415	
Utilities	120	

b. If Roberta pays off/does not have her credit card bill to buy a house what would be her new debt to income ratio?

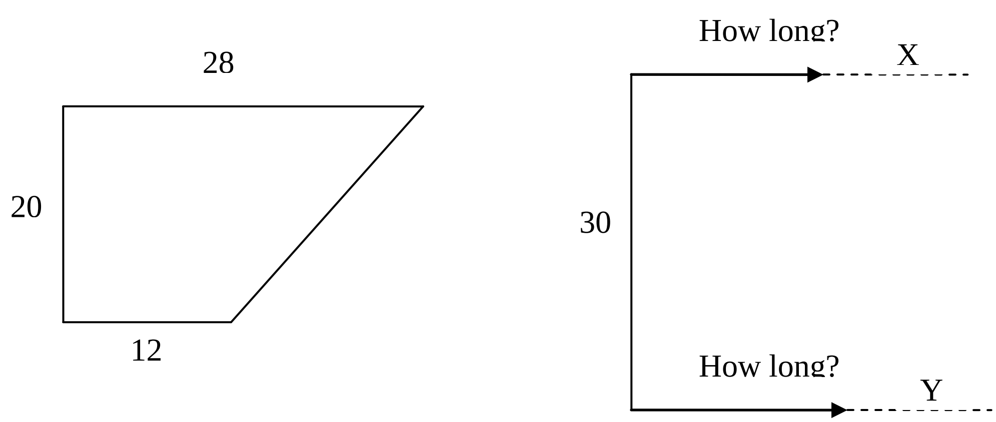
9. a. What is the ratio of the sides of the two similar rectangles?



b. Complete the ratio table for the similar rectangles above.

Short side	3	4	5	6	?	n
Longer side	9	12	?	?	30	?

10. Use the figure below and determine what the length of segment X and segment Y will be.



11. When in Australia you can exchange $4.50 in the U.S. dollars for $6 Australian. How much is $17.50 Australian in U.S. dollars?

12. At the local college, 5 out of every 8 seniors live in apartments. How many of the 30 senior math majors are likely to live in an apartment?

13. Talknow phone company charges $.70 for every 15 minutes. Horizon phone company charges $1.00 for 20 minutes. Which company is offering the cheaper rate?

MODULE

1-3

Conversions

This module addresses the following indicators:

8-5.7 Use multi-step unit analysis to convert within U.S. customary system and the metric system.

This module contains 1 lesson. These lessons are INTRODUCTORY ONLY. Lessons in S3 begin to build the conceptual foundation student need. ADDITIONAL LESSONS will be required to fully develop the concepts.

I. *Planning the Module*

Continuum of Knowledge

In fourth grade, students recalled basic conversion facts within the U.S. Customary System for length, time, weight and capacity (4-5.8). They used equivalencies to convert units of length, weight, and capacity within the Customary System (4-5.3). In fifth grade, students recalled equivalencies associated with length, liquid volume and mass in the metric system (5-5.8) and converted within the metric system (5-5.3). In seventh grade, students recall equivalencies (7-5.4) and used one step unit analysis to convert within and between the U.S. Customary System and metric system (7-5.5)

In eighth grade, students use multi-step unit analysis problems to convert within the U.S. Customary System and the metric system, as well as, between the systems (8-5.7).

In Elementary Algebra, students will use dimensional analysis to convert units of measure within a system (EA-2.4)

Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

*Conversions
*Unit Analysis (dimensional analysis)
*System (Customary or Metric)
*Unit cancellation

II. Teaching the Lesson(s)

Students should know that 1 yd = 3 ft and 1 ft = 12 inches. The set up might look like this:

$$\frac{10\ \cancel{yd}}{1} \times \frac{3\ \cancel{ft}}{1\ \cancel{yd}} \times \frac{12\ in}{1\ \cancel{ft}} = \frac{10 \times 3 \times 12\ in}{1 \times 1 \times 1} = 360 \text{ inches.}$$

Students should be required to show each step in the process in order to help them develop and understanding. Several examples should be provided for students to work and all work should be done in context.

8-5.7 → Use multi-step unit analysis to convert between and within U.S. customary system and the metric system.

For this indicator, it is **essential** for students to:

- Understand proportional reasoning
- Multiplying fractions
- Simplify expression

- Understand that each equivalency when written as a fraction is a form of one
- Use equivalencies to convert between systems
- Set up ratios to convert using unit analysis:

  For example, how many inches are in 10 yards? Students should know that 1 yd = 3 ft and 1 ft = 12 inches. The set up might look like this:

$$\frac{10\ \cancel{\text{yd}}}{1} \times \frac{3\ \text{ft}}{1\ \cancel{\text{yd}}} \times \frac{12\ \text{in}}{1\ \text{ft}} = \frac{10 \times 3 \times 12\ \text{in}}{1 \times 1 \times 1} = 360\ \text{in.}$$

For this indicator, it is **not essential** for student to:

- Use unit analysis for conversions within the metric system (although it works if they choose to use it). Students can rely on their knowledge of computing metric conversions (place value equivalencies or multiplying/dividing by powers of 10).

1.Teaching Lesson A: Multi-step Unit Analysis

a.Indicators with Taxonomy

8-5.7 → Use multi-step unit analysis to convert between and within U.S. customary system and the metric system. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson A: Multi-step Unit Analysis

* *Review fractions or ratios that simplify to 1. For example:*

$\frac{2}{2}$ *or* $\frac{60\ \text{min}}{1 hour}$ *or* $\frac{1\ foot}{12 inches}$.

Discuss why these simplify to 1. The last two are conversion factors – There are 60 minutes in an hour; in 1 foot there are 12 inches.

* *Also discuss cross canceling when you multiply ratios. For example:*

Compare $\frac{2}{3} x \frac{3}{5} = \frac{2x3}{3x5} =$

$\frac{2}{5} x \frac{3}{3}$ *(by the commutative property in the denominator)* = $\frac{2}{5} x$ *1 (by simplifying the ratio)* = $\frac{2}{5}$

TO: $\frac{2}{\cancel{3}_{1}} x \frac{\cancel{3}^{1}}{5} = \frac{2}{3} x \frac{3}{5} = \frac{2}{5}$ *because the 3's can cancel each other out.*

* *Next, introduce Unit Analysis in CONTEXT!*
For example: You kept track of the number of miles you drove in your car and the amount of gasoline used for two months.

Number of Miles	*290*	*242*	*196*	*237*	*184*
Number of Gallons	*12.1*	*9.8*	*8.2*	*9.5*	*7.8*

a) What was the average mileage for a gallon of gasoline? (Round to the nearest tenth.)
b) Estimate the distance you can drive on 18 gallons.

Answers:

a) Rate is in $\frac{miles}{gallon}$.

The total miles is 1149. The total gallons is 47.4 gallons.

The rate is $\frac{1149}{47.4}$ *or* $24.2\frac{miles}{gallon}$.

b) You should be able to multiply the number of miles per gallon by the number of gallons which is 18, but let's look at WHY unit analysis would work here.

$24.2\frac{miles}{gallon}$ *x 18 gallons* $= \frac{24.2 miles}{1\cancel{gallon}} \times \frac{18\cancel{gallons}}{1}$ *(the gallons unit cancels out) and leaves you with…*

$= \frac{24.2 miles x 18}{1}$ *= (24.2)(18)miles = 435.6 miles*

<u>More Examples</u>:

a) You are cooking and need to convert 4 gallons into pints. How could unit analysis help you do this?

Start with your unit rate: $\frac{4 gallons}{1} = \frac{?\,pints}{1}$.

What are your conversion factors?
1 gallon = 4 quarts; 1 quart = 2 pints

So…don't jump too far… think, if you can convert to quarts, then to pints it would be simpler….

Therefore:

$\frac{4 gallons}{1}$ *x* $\frac{4 quarts}{1 gallon}$

Conversion factor – remember it is set up with gallons in the denominator because gallons was the numerator in the starting rate and the gallons units need to cancel out….

Leaving you with...

$\frac{4}{1}$ *x* $\frac{4 quarts}{1}$. *NOW → The next conversion factor needed would convert quarts to pints. It is the factor of* $\frac{2 pints}{1 quart}$*. So multiply that factor by the rates you currently have…*

$\frac{4}{1}$ *x* $\frac{4 quarts}{1}$ *x* $\frac{2 pints}{1 quart}$.

The quarts units now cancel out, leaving you with $\frac{4}{1}$ *x* $\frac{4}{1}$ *x* $\frac{2 pints}{1}$ =

$\frac{4x4x2 pints}{1x1x1}$ = $\frac{32 pints}{1}$ *= 32 pints.*

b) a car uses fuel at a rate of 19 miles per gallon. Estimate how many miles the car can travel on 13 gallons of fuel. (Answer: 247 miles)

SO, HOW DOES THIS APPLY TO THE METRIC SYSTEM?

Let's take 18 meters and convert to millimeters….

$\frac{18 meters}{1}$ x $\frac{10 decimeters}{1 meter}$ x $\frac{10 centimeters}{1 decimeter}$ x $\frac{10 millimeters}{1 centimeter}$ =

$\frac{18x10x10x10 millimeters}{1}$ = 18,000 millimeters

Now, let's take 1860 centimeters and convert to meters…

$\frac{1860 centimeters}{1}$ x $\frac{1 decimeter}{10 centimeters}$ x $\frac{1 meter}{10 decimeters}$ = $\frac{1860x1x1 meter}{1x10x10}$ =

$\frac{1860 meters}{100}$ = 18.6 meters

c. Misconceptions/Common Errors

- Students may have trouble with placing their units in the correct position of the fraction to get their units to "cancel."
- Students need to know ALL conversion factors. Without knowing conversion factors, they cannot and will not accurately use unit analysis even if they know the correct procedure.

For example: When converting 720 seconds to minutes, they do this… $\frac{720\sec onds}{1}$ x $\frac{60\sec onds}{1\min ute}$, then the units will not cancel out.. INSTEAD, it should be: $\frac{720\sec onds}{1}$ x $\frac{1\min ute}{60\sec onds}$.

d. Additional Instructional Strategies/Differentiation

- A review of fractions or ratios that simplify to 1 and the process of cross simplification (cancelling) may need to be reviewed.
- Although the focus of the indicator is procedural, exploring unit analysis in context builds conceptual understanding and supports retention. For example,

You kept track of the number of miles you drove in your car and the amount of gasoline used for two months.

Number of Miles	290	242	196	237	184
Number of Gallons	12.1	9.8	8.2	9.5	7.8

a. What was the average mileage for a gallon of gasoline? (Round the answer to the nearest tenth)
b. Estimate the distance you can drive on 18 gallons.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- illuminations.nctm.org (Lessons, Activities and Related WebLinks)
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- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS:

Ask students what the ultimate goal is in their conversion? What are the units in the end result? What conversion factor will get you started?

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

Indicator 8-5.7 Use multi-step unit analysis to convert between and within U.S. customary system and the metric system.

The objective of this indicator is use, which is the "apply procedural" knowledge cell of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem. The focus is to gain computational fluency with conversions within and between the US Customary and metric system. The learning progression to **use** requires students to recall the conversion equivalencies. They understand the relationship between the equivalencies and a form of one. Students review teacher generated examples of one step unit analysis problems and generalize mathematical statements (8-1.5) summarizing how that strategy may be applied to two step problems. They explain and justify their strategy using correct and clearly written or spoken words (8-1.6). They apply a strategy to other examples and real world situations (8-1.7) and use their understanding of proportional reasoning to justify their answers.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. A bald eagle can fly at a rate of 30 miles per hour.

    a) use unit analysis to find a bald eagle's flying rate in miles per minute. (*Answer: 0.5* $\frac{miles}{\min ute}$ *)*

    b) how many minutes would it take a bald eagle to fly 6 miles? (Answer: 12 minutes)

2. A Cheetah can run at a rate of 70 miles per hour. Use unit analysis to find the speed of the cheetah in miles per minute. Round to the nearest tenth.

    (Answer: 1.2 $\frac{miles}{\min ute}$)

3. *Convert 720 seconds to hours.*

    (Answer: $\frac{1}{5}$ *of an hour or 0.20 hours)*

4. What conversion factors are needed to convert from kilograms to dekagrams?

    Now, Convert 110 kilograms to dekagrams.
    (Answer: 11,000 dekagrams)

MODULE 1-4

Plane and Proportional Reasoning – Part I

This module addresses the following indicators:

8-4.1 Apply the Pythagorean theorem.
8-4.2 Use ordered pairs, equations, intercepts, and intersections to locate points and lines on a coordinate plane.

This module contains 2 lessons. These lessons are INTRODUCTORY ONLY. Lessons in S3 begin to build the conceptual foundation student need. ADDITIONAL LESSONS will be required to fully develop the concepts.

I. Planning the Module

Continuum of Knowledge

Indicator 8-4.1 Apply the Pythagorean Theorem.

In seventh grade the students learned about the inverse relationship between perfect squares and square roots (7-2.10)
In eighth grade, students apply the Pythagorean Theorem (8-4.1).
This topic is also addressed in high school Geometry.

Indicator 8-4.2 Use ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate plane.

In the sixth grade students represented with ordered pairs of integers the location of a point in the coordinate grid (6-4.1). In seventh grade, students analyzed tables and graphs to determine the rate of change between and among quantities (7-3.2). Students gained an understanding of slope as a constant rate of change (7-3.3).
In eighth grade, students use ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate system (8-4.2). They also identified the coordinates of the *x*- and *y*- intercepts of a linear equation from a graph, equation, and/or table (8-3.6)

Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

*Pythagorean Theorem
*Hypotenuse
*Legs
*Right triangle
*Square root
*Radical
*Coordinate Plane
*Ordered Pairs
*Equations
*Intercepts (x-intercept and y-intercept)
*Intersections
*Slope
*Function table
*Function rule
*y-axis

*x-axis
*Linear function
*Mathematical Formula
$a^2+b^2=c^2$

II. Teaching the Lesson(s)

Eighth grade is the first time that students are introduced to the Pythagorean Theorem, a theorem they will use in all mathematics classes throughout high school, and even into college. Because of its frequent use in mathematics, it is **extremely** important that students understand (not just memorize) the Pythagorean Theorem and its applications. The Pythagorean relationship states that if a square is constructed on each side of a right triangle, the sum of the areas of the two smaller squares equals the area of the largest square. The Pythagorean Theorem is most frequently represented as $a^2 + b^2 = c^2$, with "*a*" and "*b*" being the legs (the two sides that create the right angle) and "*c*" being the hypotenuse (the side directly across from the right angle) of a right triangle.

Because of the importance of this theorem and its frequent use, students should have an opportunity to ***discover*** the Pythagorean relationship through a hands-on activity. Students should have the opportunity to actually form squares on each side of a right triangle (it is important to remind students that this theorem can **only** be used with a **right** triangle), then actually cut out the squares kinesthetically and visually see that the two smaller squares will fit into the largest square. This will help the students to better understand the equation representation, $a^2 + b^2 = c^2$ of this theorem. After completing this activity, students should transition into using the Pythagorean Theorem to solve for a missing side of a right triangle. Because this involves solving an equation, two-step equation solving is a prerequisite for the application of the Pythagorean Theorem. When solving for a missing side, students should give the solution as a whole number (in the case of a perfect square) and as a radical expression (example, $\sqrt{5}$, in the case of a non-perfect square).

Eighth grade students should also be able to determine if a triangle is a right triangle when given the three side measures. They can determine this by plugging these numbers into the Pythagorean Theorem, $a^2 + b^2 = c^2$. Students should understand that if the result is a true equation, then the triangle **is** a right triangle, and vice versa, if the result is a false equation, then the triangle is **not** a right triangle. When doing this, it is important for students to remember that "*c*" will always be the longest length.

Students also need extensive practice in graphing linear functions using a variety of strategies. It is also important for students to see the connection or relationship between the various strategies, and that each strategy has a situation where it is best used. This indicator covers a broad range of knowledge required by the students. Teachers should incorporate into their lessons plotting ordered pairs and graphing an equation by making a table of ordered pairs that fit that equation. Students should understand the terminology *intercepts* and *intersections* and use them to locate lines and points in the coordinate plane.

Another aspect of this indicator is that eighth grade students should be able to use the coordinate plane to locate vertices, given the coordinates of a vertex and length of adjacent sides of a polygon. The foundation for this was provided in 6th grade where shapes were oriented horizontally. Shapes in 8th grade should be oriented both horizontally and vertically, and in all quadrants of the coordinate plane.

8-4.1 → Apply the Pythagorean Theorem.

For this indicator, it is **essential** for students to:

- Recall the formula
- Understand the relationship among the sides beyond memorization of the formula
- Know that the hypotenuse is always the longest side and that the legs are connected to the right angle.
- Know how to use the Pythagorean Theorem in situations involving both perfect and non-perfect squares.
- Recognize when the Pythagorean Theorem is being described in a story problem

For this indicator, it is **not essential** for students to:

- None noted

8-4.2 → Use ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate plane.

For this indicator, it is **essential** for students to:

- Use an equation to locate points by creating a function table
- Use points from a table to graph a line
- Use (plot) intercepts to graph a line
- Determine the point of intersection after graphing two equations or from a given graph.
- Understand that the point of intersection is where both equations have the same value for x and y.

- Understand that a line consists of infinitely many points
- Understand that the x-intercept is the result of the intersection between a line and the x-axis line. It is in the form (x, 0)
- Understand that the y-intercept is the result of the intersection between a line and the y-axis line. It is in the form (0, y).

For this indicator, it is **not essential** for students to:

- Know that the point of intersection is the solution to a system of equations.
- Solve for the *x*- and *y*- intercept without the graph.

1. Teaching Lesson A: Exploring Linear Functions

a. Indicators with Taxonomy

See the following Sample:

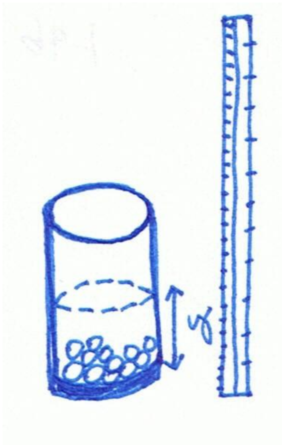
8-4.2 → Use ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate plane. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson A: Exploring Linear Functions

Adapted from: Winter, Mary Jean & Carlson, Ronald J., 1993. *Algebra Experiments I: Exploring Linear Functions*.

Set up stations around the room. Students follow instructions on the handouts to complete the stations. Set up is also included in the handouts. The purpose is to graph ordered pairs that result in linear patterns due to a linear relationship. There are 3 experiments: The Raven and the Jug, Walking the Plank and Stretching Springs.

The Raven and the Jug set up followed by handout instruction and recording sheet.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"THE RAVEN AND THE JUG" **Linear Data Experiment**

Materials Needed: Jar, Clear Centimeter Ruler, Marbles

Instructions:

x = number of marbles in the jar
y = height of the water

THESE ARE THE STEPS....CHECK OFF EACH AS YOU COMPLETE IT.

_____ **(1)** **Fill the jar with ONLY about 2 inches of water.**

_____ **(2)** **Add some marbles (count how many you add – this is your first x value), and measure the height of the water level from the bottom of the jar – this is your first y value.) Fill in the first row of your t-table.**

_____ **(3)** **Continue adding several marbles at a time, each time measuring the new height. EACH TIME, the x is always the total marbles in the jar, and y is the new water height. MAKE SURE YOU COMPLETE YOUR T-TABLE.**

_____ **(4)** **ONCE YOUR T-TABLE IS COMPLETE, graph the scatterplot of your ordered pairs (points from your t-table). SET YOUR SCALE TO COUNT BY 1, 2, 3, 4, 5… (WHATEVER WORKS BEST) – Remember the homework assignment from yesterday?**

_____ **(5)** **LABEL EACH AXIS – the x and y axes should be labeled by what they represent, what they are (number of marbles, height of water).**

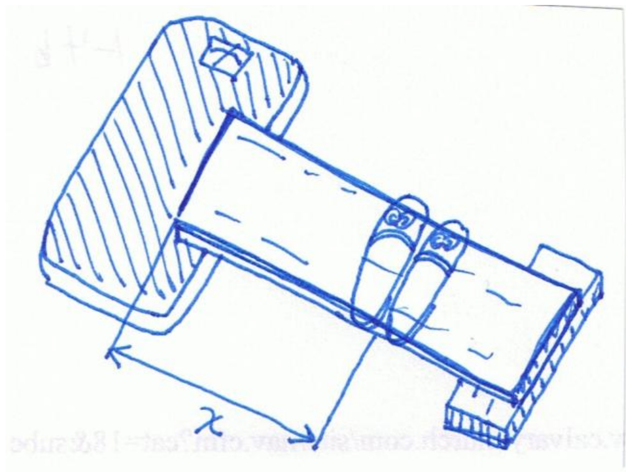
_____ **(6)** **DRAW THE LINE OF BEST FIT through your data points on your graph.**

T-TABLE **SCATTERPLOT**

X	Y
number of marbles	height of water

QUESTIONS: What is the relationship between x and y? WHY are they related? What does one have to do with the other? ________________________________
____
____

Walking the Plank set up followed by handout instruction and recording sheet.



"WALKING THE PLANK" **Linear Data Experiment**

Materials Needed: Board, Bathroom Scale, Book (Textbook), Yardstick

Instructions: Set the experiment up by placing one end of the board on the bathroom scale and one end on the textbook.

x = distance the person "walking" is from the end of the board
y = weight showing on the scale

THESE ARE THE STEPS….CHECK OFF EACH AS YOU COMPLETE IT.

_____ (1) Have your "walker" stand at the end of the board (farthest from the scale)

_____ (2) Measure the distance he/she is standing from the end. Record this distance in the x column of the t-table. Record the weight showing on the scale in the y column of the t-table.

_____ (3) The walker should continue to take one step at a time toward the scale. AFTER EACH STEP, you should record the distance from the end of the board and the person's weight. MAKE SURE YOU COMPLETE YOUR T-TABLE.

_____ (4) ONCE YOUR T-TABLE IS COMPLETE, graph the scatterplot of your ordered pairs (points from your t-table). SET YOUR SCALE TO COUNT BY 1, 2, 3, 4, 5… (WHATEVER WORKS BEST) – Remember the homework assignment from yesterday?

_____ (5) LABEL EACH AXIS – the x and y axes should be labeled by what they represent, what they are (distance from end of board, weight).

_____ (6) DRAW THE LINE OF BEST FIT through your data points on your graph.

T-TABLE **SCATTERPLOT**

X distance from end of board	Y weight shown on scales

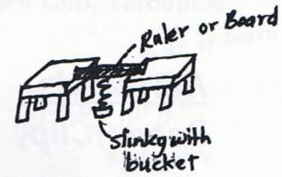
QUESTIONS: What is the relationship between x and y? WHY are they related? What does one have to do with the other? ______________________________
____
____

Stretching Springs Set up and Instruction Sheet

"STRETCHING SPRINGS" **Linear Data Experiment**

Materials Needed: Slinky, Small Plastic Cup "Bucket" with Paper Clip, Yardstick, Ruler, Masking Tape, Marbles

Instructions: Set up the experiment across two desks as demonstrated to the right → → → →



x = number of marbles in the bucket
y = distance the bottom of the bucket is from the floor

THESE ARE THE STEPS….CHECK OFF EACH AS YOU COMPLETE IT.

_____ (1) Start with x = 2 marbles, and measure for the y distance. Fill in the first part of the t-table.

_____ (2) Add 2 or 3 marbles, and measure for the y distance. (x will now be the total of marbles in the bucket.)

_____ (3) Continue adding several marbles at a time, each time measuring the new distance. MAKE SURE YOU COMPLETE YOUR T-TABLE.

_____ (4) ONCE YOUR T-TABLE IS COMPLETE, graph the scatterplot of your ordered pairs (points from your t-table). SET YOUR SCALE TO COUNT BY 1, 2, 3, 4, 5… (WHATEVER WORKS BEST) – Remember the homework assignment from yesterday?

_____ (5) LABEL EACH AXIS – the x and y axes should be labeled by what they represent, what they are (number of marbles, or distance to floor).

_____ (6) DRAW THE LINE OF BEST FIT through your data points on your graph.

T-TABLE **SCATTERPLOT**

X number of marbles	Y distance from floor

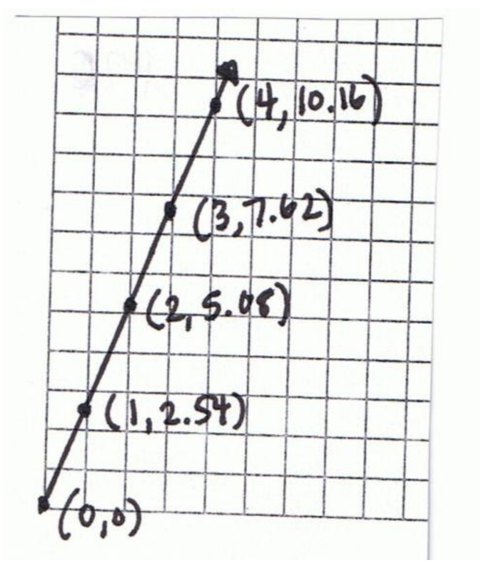
QUESTIONS: What is the relationship between x and y? WHY are they related? What does one have to do with the other? ______________________________
____
____

Teacher Note: Part 2 is a teacher questioning and facilitating approach to the lesson. Therefore, it is written in question and *possible answer* format. Generate answers from students and don't get stuck on a "script." The questions listed are all important points or concepts for students to grasp however.

Part 2: Transition: All of the data experiments yielded a linear or almost linear relationship. Consider the following table as a comparison of inches (x) to centimeters (y). The comparisons are given as ordered pairs. Graph the ordered pairs using inches on the x axis and centimeters on the y axis.

X (inches)	Y (centimeters)
0	0
1	2.54
2	5.08
3	7.62
4	10.16

Answer:



Why is this relationship a "perfectly straight" line? *Answer you're looking for is along the lines of… "The relationship between x and y is constant. Every inch is another 2.54 centimeters."*

What are all lines made up of? *Ordered pairs*

What are ordered pairs made up of? *X and Y coordinates*
Therefore all lines are made up of x's and y's.
So, how does that help us graph a line? *(continue…)*

Consider the following equation of a line: $y = 2x + 1$.
What are some ordered pairs that make up that line?
Think: Since all lines are made up of x's and y's, if I know an x, I can find y.

So, what's x? Does it matter? Choose a value. For example, $x = 1$.
In the equation $y = 2x + 1$, what is y when $x = 1$?

Use substitution: $y = 2(1) + 1 = 2 + 1 = 3$
Therefore the resulting ordered pair is (1,3) because $x = 1$ & $y = 3$.
Is one ordered pair enough to graph a line? *No, because it's only one point and you wouldn't know what direction to move toward.*

So, we need more ordered pairs. Find some more y's for any values of x. Complete the table below:

X	Y = 2x + 1	Y
1	Y = 2(1) + 1	3
2	Y = 2(2) + 1	5
3	Y = 2(3) + 1	7
4	Y = 2(4) + 1	9
5	Y = 2(5) + 1	11

The resulting ordered pairs are (1,3), (2,5), (3,7), (4,9) and (5,11)
How many more points (ordered pairs) are on the line $y = 2x + 1$?
Infinite number, because it consists of many pairs of x's and y's.

Graph the ordered pairs, then graph the linear equation.

Now graph the following equations:

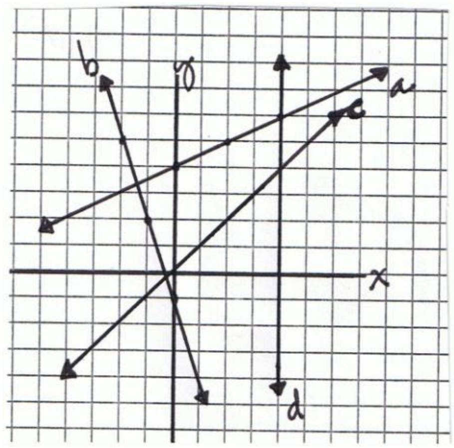
$Y = 3x + 3$
$Y = -2x + 2$
$Y = \frac{1}{2}x - 4$ (Think on this one… what values used for x would make the equation easier to solve once substituted? – *possibly even numbers…. Numbers divisible by 2 to yield a whole number)*

<u>Notes to Share with Students</u>:

Intercepts are points where a line crosses the x axis or the y axis. The x intercept is where the line crosses the x axis and the y intercept is where the line crosses the y axis. Intercepts can be written as ordered pairs.

Intersections are ordered pairs where two lines meet. These could be in any quadrant or on any axis.

Consider the following graph of several lines.

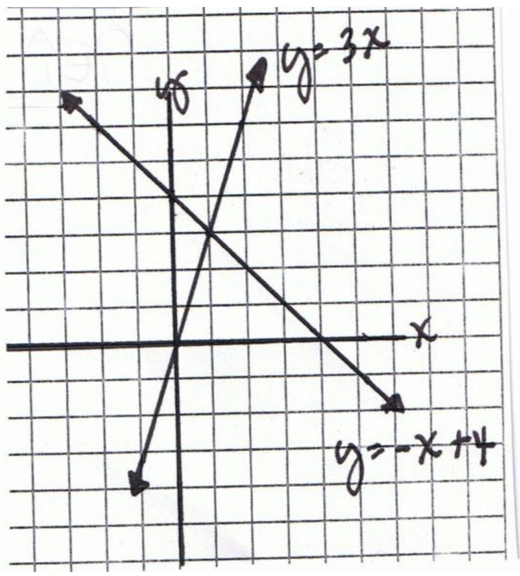


- Which line contains the following points (4,4) and (0,0)? *Answer: c*
- What two lines intersect at (4,4)? *Answer: c & d*
- What is the line that has an x-intercept of 4? *Answer: d*
- What line has a y-intercept of 4? *Answer: a*
- What line has the point (0,-1) on it? Is there another way to name this point? *Answer b, yes it's the y intercept of -1*

- Graph the following two lines and find their point of intersection. How could you prove this intersection is the point they have in common?

  $Y = 3x$ and $Y = -x + 4$

  Answer:



Prove by looking at the tables for each equation. When the tables are constructed, the common point is (1,3).

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies/Differentiation

- To clarify the intent of this indicator, replace the word locate with graph. For example,
  - Students use ordered pairs to graph points and lines.
  - Students use equations to graph points and line.
  - Students use intercepts to graph points and line.
  - Students use intersections to graph points and line.

- The intent of the indicator as it relates to intersections of lines is not to solve a system of linear equations. Real world story problems can be used to explore the concept of intersection. For example, at Kira's Video, they charge $12 a month and $0.50 for each video. At Arika's video there is no fee to join but charges $1 for each video. For how many movies will the cost at each video store by the same?

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

ETV Streamline
Discovering Algebra with Graphing Calculators: Graphing a Line

These are suggestions for resources:

- illuminations.nctm.org (Lessons, Activities and Related WebLinks)
- nlvm.usu.edu (National Library of Virtual Manipulatives)
- Oneplacesc.org (ETV Streamline and more!)
- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS
Formative assessment occurs throughout the lesson during the questioning. Student responses will guide the instruction. Further formative assessment questions may be needed based on student understanding.

2. Teaching Lesson B: Pythagorean Theorem

a. Indicators with Taxonomy

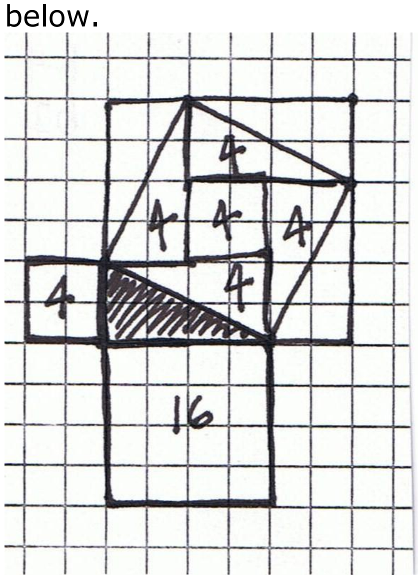
8-4.1 → Apply the Pythagorean Theorem. (B3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson B: Pythagorean Theorem

Suggested Literature Connection:
- Children's book: "What's Your Angle Pythagoras" By: Julie Ellis

PART I
Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. Teaching Student Centered Mathematics: Grades 5-8.

Have students draw a right triangle on half-centimeter grid paper. Assign each student a different triangle by specifying the lengths of the two legs. Students are to draw a square on each leg and the hypotenuse and find the area of all three squares. (NOTE: For the square on the hypotenuse, the exact area can be found by making each of the sides the diagonal of a rectangle as shown in the figure below.



Make a table of the area data. For example:

	Area of Square on Leg 1	Area of Square on Leg 2	Area of Square on Hypotenuse
Based on the example shown in figure 7.18	4	16	20
Based on the triangle with legs lengths of 3 and 4 and an hypotenuse of 5	9	16	25
Based on the triangle with legs lengths of 5 and 12 and an hypotenuse of 13	25	144	169

Ask students to look for relationships between the squares.

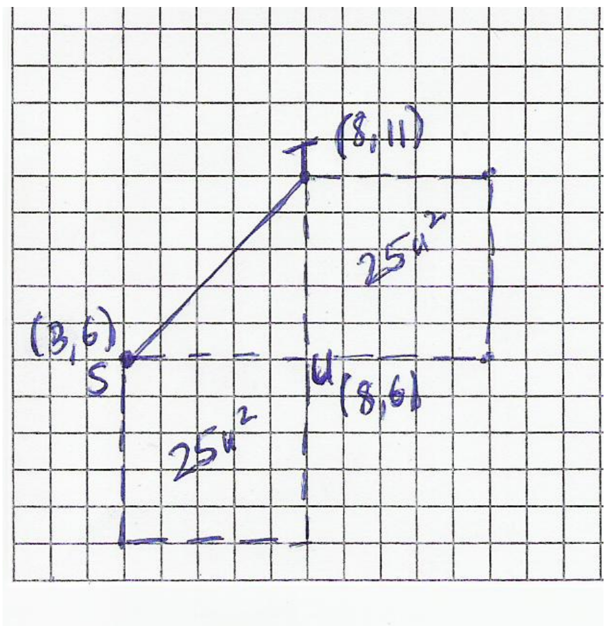
PART II

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. *Teaching Student Centered Mathematics: Grades 5-8*.

<u>The Distance Formula</u>

The geometric version of the Pythagorean relationship is about area. The following activity has students use the coordinate grid and the Pythagorean relationship to develop a formula for the distance between points.

Example:



Have students draw a line between two points in the first quadrant that are not on the same horizontal or vertical line. *(For example in Figure A, segment $\overline{ST}$.* The task is to use only the coordinates of the endpoints to calculate the distance between them in terms of the units on the grid. To this end, suggest that they draw a right triangle using the line as the hypotenuse△*(For example in Figure A, △ STU.)* The vertex at the right angle will share one coordinate from each endpoint. Students can compute the areas of the squares on the legs and add to find the area of the square on the hypotenuse. *(In figure A, the area of each square off the legs is 25 units^2. The area of the square on the hypotenuse therefore is 50 units^2.)* The length of the original line segment (the distance between the points) is the number whose square is the area of the square on the hypotenuse. *(Therefore in figure A, the $\sqrt{50}$).* (This last statement is a geometric interpretation of square root). By computing the areas of these squares students can compute the length of the original line.

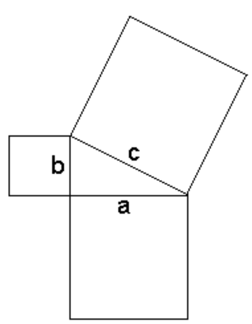
Have students look through all their calculations and see how the coordinates of the two endpoints were used. Challenge students to use the same type of calculations to get the distance between two new points without drawing any pictures. *(It's the distance formula:* $\sqrt{((x-x)^2+(y-y)^2)}$ *for points S and T in example A)*

c. Misconceptions/Common Errors

- The students often forget to take the square root of the sum to find the length of the missing side.
- Students will often misuse the formula thinking all values go in the place of *a* and *b* because they do not have a sound understanding of the difference between the legs and the hypotenuse. One strategy to help students remember the difference is to have them think of the right angle symbol (the square that verifies an angle is right) as a table and that a table has legs, therefore the lines extending from the right angle are the legs.

d. Additional Instructional Strategies/Differentiation –

- Although the focus of the indicator is procedural, students need to have an understanding of the "concept" of the Pythagorean Theorem. Conceptual knowledge is based on relationships; therefore, students should gain an understanding of how these values connect beyond reciting the formula.
- The Pythagorean relationship states that if a square is constructed on each side of a right triangle, the sum of the areas of the two smaller squares equals the area of the largest square. The Pythagorean Theorem is most frequently represented as $a^2 + b^2 = c^2$, with "*a*" and "*b*" being the legs (the two sides that create the right angle) and "*c*" being the hypotenuse (the side directly across from the right angle) of a right triangle.

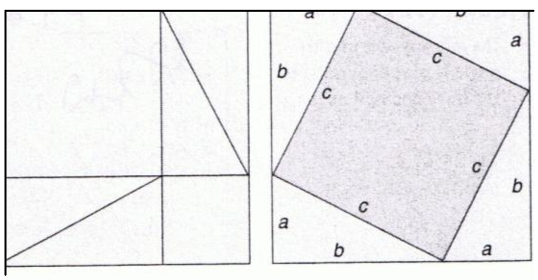


It is extremely important that the students

understand (not just memorize) the different applications of the Pythagorean Theorem. The students should be given a variety of story problems that can be solved by using the Pythagorean Theorem. An example of this: A diver dove off of the dock and swam 50 feet to the where the buoy was attached to the bottom of the lake. The buoy is 25 feet above his head; how far is it from the dock to the buoy? This question requires the students to draw a picture and to realize that they are looking for a missing leg; the information that was given is the length of the hypotenuse and one leg.

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. *Teaching Student Centered Mathematics: Grades 5-8*.

Use the two drawings in the figure below to create a proof of the Pythagorean relationship.



The area of the large square is $(a + b)^2 = a^2 + 2ab + b^2$

The same area is also c^2 plus 4 times the area of one triangle.

$c^2 + 4\left(\frac{1}{2}ab\right) = c^2 + 2ab$

So, $c^2 + 2ab = a^2 + 2ab + b^2$

And, $c^2 = a^2 + b^2$

The above are two proofs of the Pythagorean relationship. The two squares together are proof without words. Can you supply the words? The second proof is the algebraic proof based on the right-hand figure.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- Specific Resource:
  - illuminations.nctm.org
    click "Activities"
    ✓ 6-8
    search "Pythagorean Theorem"
    choose: "Proof without Words: Pythagorean Theorem"

- illuminations.nctm.org (Lessons, Activities and Related WebLinks)
- nlvm.usu.edu (National Library of Virtual Manipulatives)
- Oneplacesc.org (ETV Streamline and more!)
- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS are posed throughout the lesson. Pay close attention to student responses during the lesson. Further formative assessment questions may be needed based on student understanding.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

Indicator 8-4.2 Use ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate plane.

The objective of this indicator is to use which is in the "apply procedural" cell of the Revised Taxonomy. Procedural knowledge is not only knowledge of steps and techniques but also knowing when to use appropriate those steps. The learning progression to **use** requires students to recall how to plot points and how to write points given the graph. Students use their understanding of ordered pairs, equations, intercepts and intersections to create point and lines on the coordinate plane. They generalize connections (7-1.7) among these concepts and understand that each is a distinct symbolic form that represents the same linear relationship (7-1.4). Students translate from equation to ordered pairs to graph, from intercepts to graph and explore real world problems to gain a conceptual understanding of intersection.

Indicator 8-4.1 Apply the Pythagorean Theorem.

The objective of this indicator is to apply which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is to gain computational fluency with using the Pythagorean Theorem, the learning progression should also build the student's conceptual knowledge in order to support retention. The learning progression to **apply** requires students to recall explore the Pythagorean relationship using a variety of examples. Students use their observations to generalize connections (8-1.7) among the areas of the squares. They then generalize a mathematical statement (8-1.5) summarizing this connection using correct and clearly written or spoken words (8.1.6). Students translate this verbal description to mathematical notation understanding that both are equivalent symbolic expressions of the same relationship (8-1.4). They use their conceptual understanding of the Pythagorean Theorem as they engage in problem solving and repeated practice to gain computational fluency.

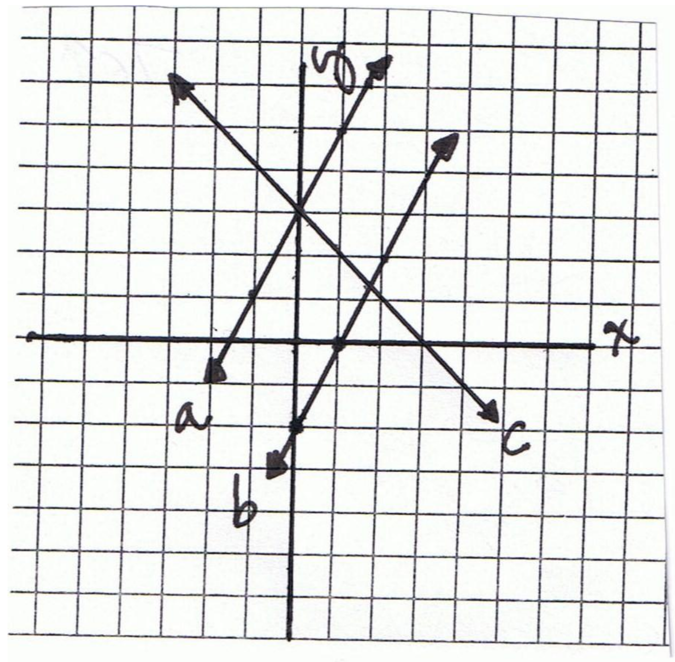
The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. a. Graph the following two lines and find their point of intersection. How could you show that this intersection is the point they have in common?

$$Y = -2x - 2 \text{ and } Y = \frac{2}{3}x - 2$$

   b. Is there anything special about their point of intersection? *Answer: It's the y intercept of -2 or the ordered pair (0, -2).*

2. Consider the following graph of several lines.



a. Which line contains the following points (0,3) and (2,1)?
   Answer: c
b. What two lines intersect at (0,3)?
   Answer: c & a
c. Is there another name for the point (0,3) on each of the two lines that cross it?
   Answer: Yes, the y intercept is 3.
d. What is the line that has an x-intercept of 3?
   Answer: c
e. Which two lines have no intersection?
   Answer: a & b
f. What type of lines are these?
   Answer: Parallel

3. You have a rope with 24 equally spaced knots tied in it. Form a right triangle with the rope and give the length of each side. How can you use this rope to tell that the triangle is a right triangle?

> *Answer: 6 knots, 8 knots and 10 knots. This is the same ratio as the 3,4,5 triangle – just a multiple of 2.*

4. Let students generate strategies to determine Pythagorean triplets:

Multiples of 3,4,5				*Multiples of 5,12,13*		
3	*4*	*5*		*5*	*12*	*13*
6	*8*	*10*		*10*	*24*	*26*
9	*12*	*15*				
12	*16*	*20*				

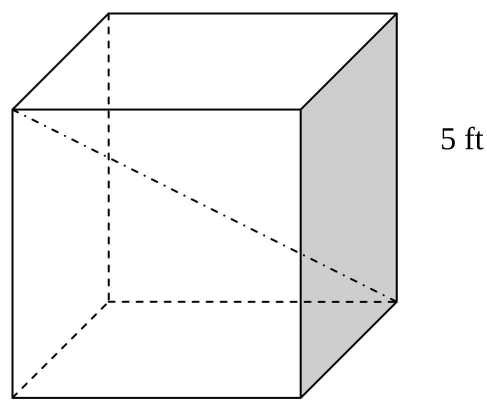
Consider 7,24,25 and 8,15,17 for enrichment.

5. The National Safety Council recommends placing the base of a ladder one foot from the wall for every three feet of the ladders length. How high can a 15-foot ladder safely reach?

Answer: This is a right triangle with the hypotenuse length 15ft (the ladder, and the leg 5ft (the distance from the wall to base of the ladder). Pathagorean Theorem can be used to determine the height using:

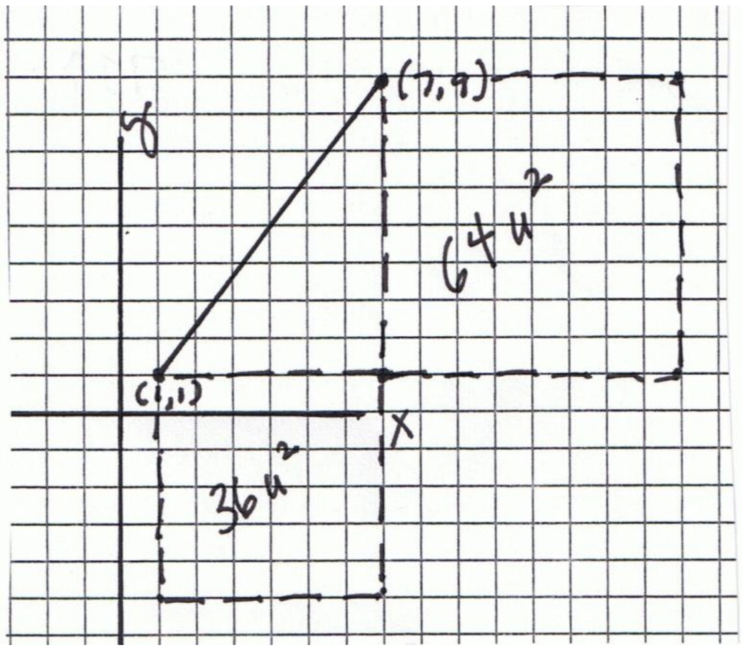
$a^2 + 5^2 = 15^2$; $a^2=200$; $a=\sqrt{200}$; $a=10\sqrt{2} = 14.14$

6. Find the length of the diagonal of the cube.



Answer: The triangle has height 5, length $\sqrt{50}$ (5^2+5^2=length of diagonal of the base), therefore; the length of the diagonal of the cube is $5^2+\sqrt{50}^2=\sqrt{75}$

5. Find the distance between the following points (1,1) and (7,9). Use the Pythagorean theorem to justify your answer. Show your answer algebraically as well.



Algebraically: The distance from x = 1 to x = 7 is 6 units. The distance from y = 1 to y = 9 is 8 units. Therefore the legs are 6 units and 8 units long. Therefore a = 6 and b = 8. Using the Pythagorean theorem, a^2 + b^2 = c^2, then 36 + 64 = c^2. If 100 = c^2, then c by way of square root is 10. The distance is 10.

The distance formula yields the same result:

$\sqrt{((x-x)^\wedge 2+(y-y)^\wedge 2)}$.

$\sqrt{((7-1)^\wedge 2+(9-1)^\wedge 2)}$ **=** $\sqrt{36+64}$ **=** $\sqrt{100}$ **=** 10.