SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

Content Area	**Eighth Grade Math**

Second Nine Weeks

Standard/Indicators Addressed:
Standard: 8-3: The student will demonstrate through the mathematical processes an understanding of equations, inequalities, and linear functions.

- **8-3.1*** Translate among verbal, graphic, tabular, and algebraic representations of linear functions. (B2)
- **8-3.2*** Represent algebraic relationships with equations and inequalities. (B2)
- **8-3.3*** Use commutative, associative, and distributive properties to examine the equivalence of a variety of algebraic expressions. (C3)
- **8-3.4*** Apply procedures to solve multi-step equations. (C3)
- **8-3.5*** Classify relationships between two variables in graphs, tables and/or equations as either linear or nonlinear. (B2)
- **8-3.6*** Identify the coordinates of the *x*- and *y*-intercepts of a linear equation from a graph, equation, and/or table. (C1)
- **8-3.7*** Identify the slope of a linear equation from a graph, equation, and/or table. (C1)

*** These indicators are covered in the following 3 Modules for this Nine Weeks Period.**

Module 2-1 Representations, Properties and Proportional Reasoning

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines
Module 2-1 Lesson A: Algebraic Relationships **8-3.2** Represent algebraic relationships with equations and	NCTM's Online Illuminations http://illuminations.nctm.org NCTM's Navigations Series SC Mathematics Support Document	See Instructional Planning Guide Module 2-1 "Introductory Lesson A"	See Instructional Planning Guide Module 2-1 "Lesson A 'Assessing the Lesson'"

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inequalities. (B2)	Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations –see Appendix A		
Module 2-1 Lesson B: Properties and Algebraic Expressions **8-3.3** Use commutative, associative, and distributive properties to examine the equivalence of a variety of algebraic expressions. (C3)		See Instructional Planning Guide Module 2-1, "Introductory Lesson B"	See Instructional Planning Guide Module 2-1 "Lesson B 'Assessing the Lesson'"
Module 2-2 Solve Mathematical Situations			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-2 Lesson A: Solving Multi-Step Equations **8-3.4** Apply procedures to solve multi-step equations. (C3)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John	See Instructional Planning Guide Module 2-2 "Introductory Lesson A"	See Instructional Planning Guide Module 2-2 "Lesson A 'Assessing the Lesson'"
Module 2-2 Lesson B: Identifying x and y intercepts of Linear		See Instructional Planning Guide Module 2-2, "Introductory Lesson B"	See Instructional Planning Guide Module 2-2 "Lesson B

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Equations **8-3.6** Identify the coordinates of the *x*- and *y*-intercepts of a linear equation from a graph, equation, and/or table. (C1)	Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – see Appendix A		'Assessing the Lesson'"
Module 2-2 Lesson C: Linear Functions **8-3.1** Translate among verbal, graphic, tabular, and algebraic representations of linear functions. (B2) **8-3.7** Identify the slope of a linear equation from a graph, equation, and/or table. (C1)		See Instructional Planning Guide Module 2-2 "Introductory Lesson C"	See Instructional Planning Guide Module 2-2 "Lesson C 'Assessing the Lesson'"
Module 2-3 Pattern, Relationships and Functions			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 2-3 Lesson A: Linear or Nonlinear	NCTM's Online Illuminations http://illuminations.nctm.org/	See Instructional Planning Guide Module 2-3 "Introductory Lesson A"	See Instructional Planning Guide Module 2-3 "Lesson A

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8-3.5 Classify relationships between two variables in graphs, tables and/or equations as either linear or nonlinear.	NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – see Appendix A		'Assessing the Lesson'"

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MODULE

2-1

Representations, Properties and Proportional Reasoning

This module addresses the following indicators:

8-3.2 Represent algebraic relationships with equations and inequalities.
8-3.3 Use commutative, associative and distributive properties to examine the equivalence of a variety of algebraic expressions.

This module contains 2 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation student need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

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I. Planning the Module

Continuum of Knowledge

8-3.2 Represent algebraic relationships with equations and inequalities.

In sixth grade, students represent algebraic relationships with variables in expressions and simple equations and inequalities (6-3-3). In the seventh grade, the learning extends to include representing the solution of a two-step inequality on a number line (7-3.5).

In eighth grade, represent algebraic relationships with equations and inequalities (8.3-2).

In Elementary Algebra, students analyze given information to write a linear function that models a given problem situation (EA-5.9) and analyze given information to write a linear inequality in one variable that models a given problem situation (EA-5.12).

8-3.3 Use commutative, associative and distributive properties to examine the equivalence of a variety of algebraic expressions.

In fifth grade, students identified applications of commutative, associative and distributive properties (5-3.4). In sixth grade, students learned to show that two expressions (numbers only) are equivalent by applying the commutative, associative, and distributive properties (6-3.4) and to apply inverse operations to solve simple one step equations (6-3.5).

In eighth grade, students use commutative, associative, and distributive properties to examine the equivalence of a variety of algebraic expressions.

In Algebra I, students will carry out a procedure using the real properties of numbers (including commutative, associative and distributive) to simplify expressions (EA-2.5)

Key Concepts/Key Terms

*These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

*Equations
*Inequalities
*Commutative Property
*Associative Property
*Distributive Property

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*Less than <
*Less than or equal to ≤
*Greater than >
*Greater than or equal to ≥
*Multiple representations
*Numerical expressions
*Algebraic expressions
*Variable
*Constant
*Coefficient
*Equivalency
Linear patterns
Nonlinear patterns
Like terms
Unlike terms
Substitute
Terms

II. Teaching the Lesson(s)

1.Teaching Lesson A: Algebraic Relationships

Although students have worked with these properties in previous grades, they may still need to support their understanding through the use of concrete and pictorial models. Connecting what the students know about finding the area of rectangles to the distributive property is one strategy. Students will need practice with several examples in order to understand this concept. For example, prove the following by using an area model: 6 x 53 = 6(40) + 6(13)

Teachers should connect numeric patterns in tables to equations and inequalities to support conceptual understanding.

Students may give examples of numbers that would satisfy the inequality and develop a rule or a verbal description. For example,

- Describe the solutions of the inequality $x < 2$ or $x > 1$.
  Answer: All real numbers between 1 and 2 but not including 1 or 2

Once students are able to write inequalities, have them apply what they know by posing contextual situations. For example,
- Water can take on 3 forms: solid, liquid or gas. Under ordinary conditions, water is a solid at temperatures of

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32°F or lower and a gas at temperatures of 212°F or higher.

- What inequality describes when water is NOT a liquid? Answer: $t \leq 32$ or $t \geq 212$
- What inequality describes when water IS a liquid? Answer: $32 < t < 212$

For this indicator, it is **essential** for students to:

- Understand pattern generalizations (example: Less than is the same as <)
- Identify patterns as linear and nonlinear
- Translate from one representation to another (graph, table, word)
- Write an equation/inequality that represents a situation (words, graph, table)
- Read inequalities and equations using appropriate terminology. For example,
    - $-23 < x$ (-23 is less than x)
    - $-23 < x < 100$ (-23 is less than x which is less than or equal to 100) OR (x is between -23 and 100 and equal to 100)
- Understand that a closed circle is inclusive of the number it represents and therefore is associated with ≤ and ≥.
- Understand that an open circle is **not** inclusive of the number it represents and therefore is associated with < and >.

For this indicator, it is **not essential** for students to:

- Solve the equation or inequality
- Graph the equation or inequality

a. Indicators with Taxonomy

8-3.2 → Represent algebraic relationships with equations and inequalities. (B2)

Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson A: Algebraic Relationships

Part 1: Read the inequality:

a) $-23 < x$ (-23 is less than x)
b) $x < 100$ (x is less than 100)
c) $-23 < x < 100$ (-23 is less than x which is less than 100 OR x is between -23 and 100)

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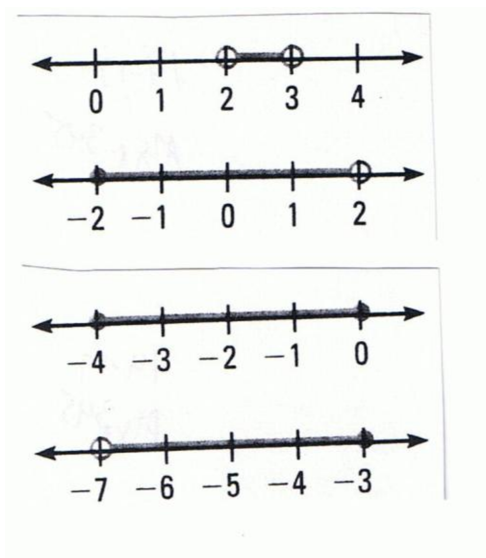
d) $0 \leq x < 18$ (0 is less than or equal to x which is less than 18 OR x is x is greater than or equal to 0 but less than 18)

<u>Part 2</u>: Write the inequality based on the number line: (Students should discuss open and closed circles on the number line graphs – the closed circle is inclusive of the number it represents therefore are associated with the symbols $\leq$ and $\geq$. The open circle is not inclusive of the number it represents, therefore is associated with the symbols $<$ and $>$.)



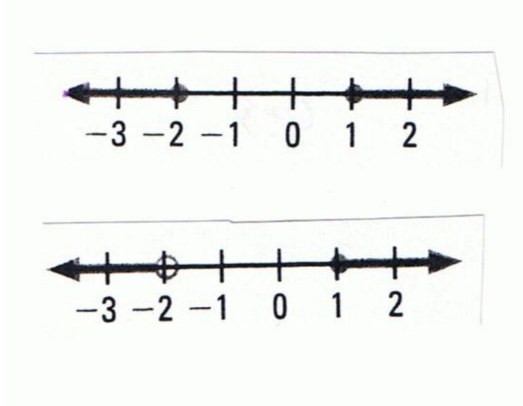
<u>Part 3</u>: Write an inequality that represents the statements:
a) x is greater than -6 and less than -1 ($-6 < x < -1$)
b) x is less than or equal to -2 and greater than -4. ($-4 < x \leq -2$)

<u>Part 4</u>: Read the following inequality: $x \leq -3$ OR $x > 0$ (x is less than or equal to -3 OR greater than 0)

Can you show the possible solutions on a number line? What is different from the inequalities listed above in Part 3? *Answer: There is an "OR" component – the possible solutions are not between the two numbers listed.*

Draw the following possible solutions on the number line.
$x \leq -2$ or $x \geq 1$
$x < -2$ or $1 \leq x$

Answers below:



Once students are able to write inequalities, have them apply what they know by posing contextual situations.

For example:
Water can take on 3 forms: solid, liquid or gas. Under ordinary conditions, water is a solid at temperatures of 32°F or lower and a gas at temperatures of 212 °F or higher. What inequality describes when water is NOT a liquid? *($t \le 32$ or $t \ge 212$)*
What inequality describes when water IS a liquid? *($32 < t < 212$)*

c. Misconceptions/Common Errors

Students may think that the inequality symbols indicate the direction of the shading of the graph.

d. Additional Instructional Strategies

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

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f. Assessing the __Lesson__

Formative assessment strategies are embedded throughout the introductory lesson.

2 . Teaching Lesson B: Properties and Algebraic Expressions

For this indicator, it is **__essential__** for students to:

- Recall the commutative, associative and distributive properties
- Simplify an expression by collecting like terms
- Verify equivalency by substituting values

For this indicator, it is **__not essential__** for students to:

- Distribute variables when simplifying expressions

a. Indicators with Taxonomy

8-3.3 → Use commutative, associative, and distributive properties to examine the equivalence of a variety of algebraic expressions. (C3)

Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson B: Properties and Algebraic Expressions

Materials Needed: Combining Like Terms Handout (1 for each student), Highlighters

FIRST, review the commutative, associative and distributive properties in isolation. This lesson will combine the properties and use them to determine equivalencies of algebraic expressions.

The handout for this lesson is given first, followed by the teacher notes for instructing the lesson.

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collecting like terms

name______________________

Date______________________

class______________________

Like terms	Unlike terms
1 & 13	4 & 13x
2y & 8y	$2y$ & $8y^2$
3rs & 4rs	3r & 4rs
$\frac{1}{4}m^2$ & $7m^2$	$\frac{1}{4}m^2$ & $7b^2$

Examine the table at the left. Look carefully to see what the ***terms*** have in common — and what they don't. Then answer the following questions.

a. Explain why the terms in the left column are ***like terms*** and terms in the right column are ***unlike terms***.

b. Describe what it means for terms to be ***like terms***.

Like terms can be collected – or ***combined*** – by adding or subtracting the ***coefficients***.

What is a ***coefficient***?

For example…

$2x + x = 3x$ ***BUT*** $2x + 1 \neq 3x$ Why?

$rs + rs = 2rs$ ***BUT*** $rs + r + s \neq 2rs$ Why?

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And what about...?

$3y + 2y^2$

Does that equal $\mathbf{5y^2}$?   Why or why not?

When you collect or combine like terms, you ***simplify*** the expression.

"We-Do's" (That means we do them together.)

1. $2w + 2w =$
2. $4x - x =$
3. $3y + 7y - 6 =$
4. $2b^2 + b^2 =$
5. $2m^2 + m^2 - 5m =$
6. $6x + 3 - x =$
7. $5pq + 6q =$

"U-Do's" (That means 'u' do them on your own.)

Simplify the following expressions

1. $17x + 4 - 3 =$
2. $8rs - 6r =$
3. $16w + 3 - w =$
4. $3t - 2t + 9t =$
5. $19n - 19n^2 =$
6. $4x - 4xy - 4x =$
7. $12y^2 + 6x + 3y^2 + x =$
8. $f^2 + 3f + 4f - 6 =$

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Some expressions for warm-ups…

1. $1y + 6y - 2y =$
2. $5 + 11rz - 3z =$
3. $t + 9t - 4 =$
4. $5p + 4\ p - 18pr =$
5. $18r - 18rd + 18r =$
6. $2h^3 + 4h =$
7. $10v + 5k^3 - v + 20k^3 =$
8. $\frac{1}{2}xy + \frac{1}{4}x =$
9. $7 + 5w - 4w + w^2 =$
10. $5a + 6ab + a^2 =$

END OF HANDOUT

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TEACHER NOTES: Collecting Like Terms

MATERIALS:

- Collecting Like Terms handout (1 for each student)

PROCEDURES:

Project the Collecting Like Terms handout for the students. They should work on their own copies while you work with them on the overhead/board.

Talk through the handout with them. Solicit answers from the students using a random method (popsicle sticks, playing cards, etc) to be sure everyone is with you. Highlight like and unlike terms and encourage students to do the same on their own copies.

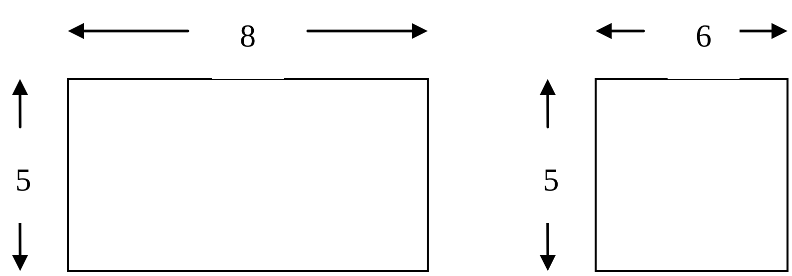
Work through the "We-Do's" together. Allow students to work with a partner to complete the "U-Do's."

Transition to Distributive Property

Collecting like terms allows us to simplify expressions so that substituting values in or solving them is more easily accomplished. Sometimes, though, an expression can't be simplified by collecting like terms. That's where the distributive property comes in.

Teacher note:

You need to draw and label these figures as you go. Students should also draw and label. The questions are intended to connect what they know about finding the area of rectangles to the distributive property. You may write the questions or simply ask them.



What's the combined area of these two figures? How do you know? Describe how you found it.
Possibilities:

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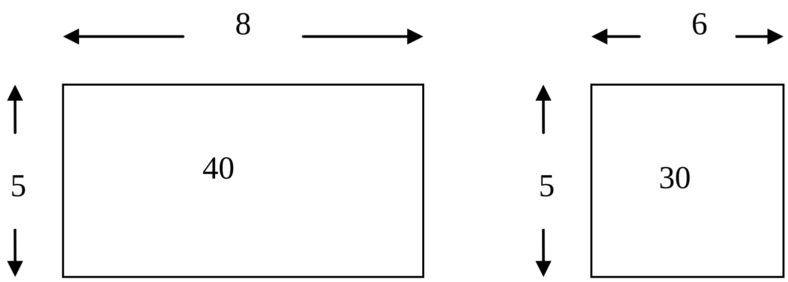
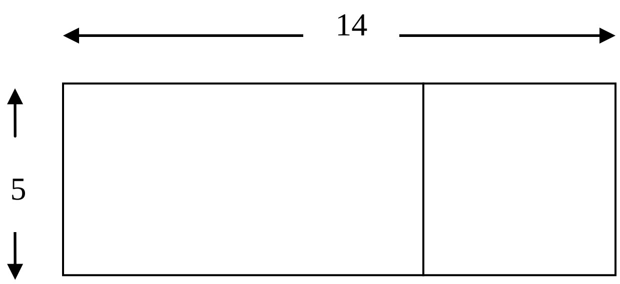
8 x 5 = 40; 5 x 6 = 30; add 40 and 30 to get 70

Add 8 and 6 to get 14; multiply by 5 to get 70

GO VISUAL!!

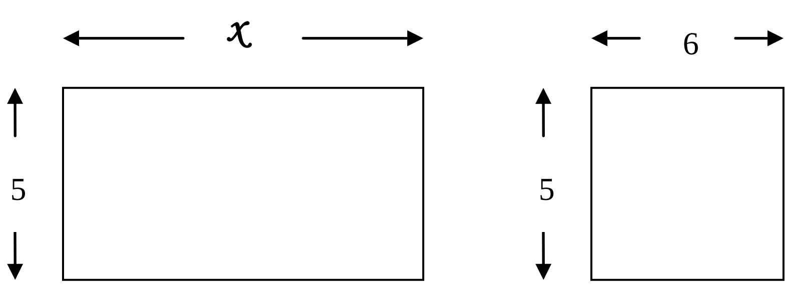


40 + 30 = 70



5 X 14 = 70

But what if…



Now, how do you find an expression for the combined areas of the two figures?

What can you find? The smaller rectangle is 30.

What about the larger? What does "5 x **x** " mean? And what does it equal?

5 x **x** means 5 **x**'s or 5 groups of 1 **x**. So… 5 x **x** = 5**x**

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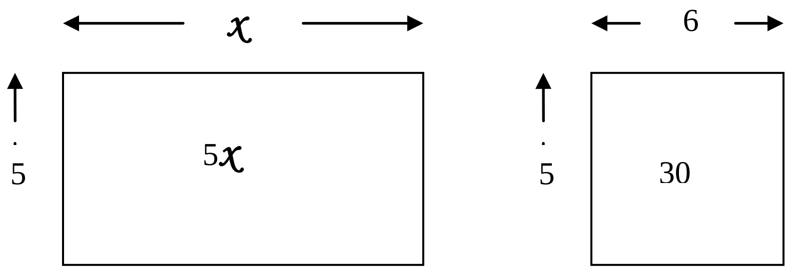
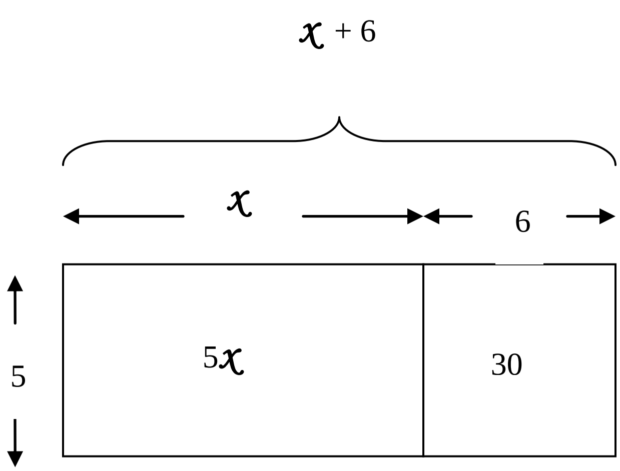
Which leads us to:



$5x + 30$

Considering what we know about collecting like terms, can $5\mathbf{x} + 30$ be simplified?

Look at it this way:



$5x + 30$

So…

$5\,(x + 6) = 5x + 30$

TEACHER NOTE: IT WILL TAKE MORE THAN ONE EXAMPLE LIKE THIS FOR STUDENTS TO ALL GRASP THE CONCEPT.

c. Misconceptions/Common Errors

Students may collect like terms that are not alike. By building the concept properly, students are less likely to make this error. Hands-on activities using manipulatives to represent variables are helpful in support their understanding of like terms. For example, use Popsicle sticks to represent "a" and straws to represent "b". Have students represent the expression 5a+ 2b + a + 3b using the manipulatives. Then tell them to organize them into like groups. Ask them what did they get using manipulatives? 6 Popsicle sticks and 5 straws. Then they write an expression: 6a + 5b. Ask them can they add the two groups? No because they can't add sticks and straws; they are not alike. So 6a + 5b ≠ 11ab.

d. Additional Instructional Strategies

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

Possible technology resource:
http://www.studystack.com/menu-47007

f. Assessing the Lesson

Samples of effective questions are posed throughout the lesson. Pay attention to student responses during the lesson.

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III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

8-3.2 Represent algebraic relationships with equations and inequalities.

The objective of this indicator is to represent which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To understand means to construct meaning; therefore, the students' focus is on building conceptual knowledge of the relationships between the forms. The learning progression to **represent** requires students to understand the concepts of equivalency and inequalities. Students analyze algebraic relationships (words, tables and graphs) to determine known and unknown values and the operations involved. They generate descriptions of the observed relationship and generalize the connection (8-1.7) between their description and structure of algebraic equations or inequalities. Students explain and justify their ideas with their classmates and teachers using correct and clearly written or spoken words, variables and notation to communicate their ideas (8-1.6). Students then compare the relationships (words, tables and graphs) to their equation, inequality or expression to verify that each form conveys the same meaning.

8-3.3 Use commutative, associative and distributive properties to examine the equivalence of a variety of algebraic expressions.

The objective of this indicator is to use which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Although the focus of the indicator is to use which is knowledge of specific steps and details, learning experiences should integrate both memorization and concept building strategies to support retention. The learning progression to **use** requires student to explore a variety of examples of these number properties using a various types of numbers. They analyze these examples and generalize connections (8-1.7) about what they observe using correct and clearly written or spoken language (8-1.6) to communicate their understanding. Students translate these connections into mathematical statements using variables. Students connect these statements to the terms commutative, associative and distributive. Students then develop meaningful and personal strategies that enable them to recall these relationships.

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The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Write an inequality that represents the set of numbers:
    a) All real numbers less than -6 or greater than 2.
    Answer: (x < -6 or x > 2)
    b) All real numbers greater than 7 or less than 0.
    Answer: (x > 7 or x < 0)
    c) All real numbers greater than or equal to -3 and less than 17.
    Answer: (-3 ≤ x < 17)

2. Describe the solutions of the inequality x < 2 or x > 1.
    Answer: (All real numbers between 1 and 2 but not including 1 or 2.)

3. Write the inequality that matches the following situation. Then answer the questions below.

    A public transit system charges fares based on age. Children under 4 ride free. Those who are 4 or older but less than 12 pay half fare. People who are at least 12, but younger than 70, pay full fare. Those 70 and older pay a reduced fare.

    a. What can you say about those who pay half fare?
    Answer: 4≤ x < 12
    b. What about those who have reduced rates or free fares?
    Answer: Y ≥ 70 or y < 12

4. a. Prove 8 x 43 = 8(30) + 8(13) by using an area model.

    b. What is another way to solve 8 x 43 using an area model?
    Possible Answer: 8(40) + 8(3) but this would be drawn out….(There are more answers than this one given)

5. Use a model to multiply 3(2x + 5). How can the commutative property of addition be used to prove the distributive property?
Answer: 3 groups of "2x + 5" → commutative property allows you to group the 3 groups of 2x and the 3 groups of 5. Resulting in 6x + 15.

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6. Use the commutative, associative, and/or distributive property to get from expression A to expression B.

Expression A	Expression B
5(7x – 8) – 6(7x – 8)	-7x + 8

Answer: Expression A: 5(7x – 8) – 6(7x – 8) = (5-6)(7x – 8) = (-1)(7x – 8) = -7x + 8
OR *5(7x – 8) – 6(7x – 8) = 35x – 40 – 42x + 48 = 35x – 42x – 40 + 48 = -7x + 8 (There may be other methods to arrive at the correct simplification.)*

7. You have a big Science test coming up and you are concerned about how it will effect your grade. It will have 60 multiple choice questions and you would like to know how many questions you will have to answer correctly in order to receive each grade.
a. How many questions correct will yield each letter grade.

	A	B	C	D	F
	90%-100%	80%-89%	70%-79%	60%-69%	0%-59%
# Correct answers needed	?	?	?	?	?

b. Write inequalities for the number of questions that you need to answer correctly in order to receive each grade. Let x represent the number of questions answered correctly.

	A	B	C	D	F
	90%-100%	80%-89%	70%-79%	60%-69%	0%-59%
# Correct answers needed	? $x \geq 54$	? $48 \leq x < 53$	? $42 \leq x < 48$	? $36 \leq x < 42$	? $X < 36$

c. Have students draw number lines as another representation for these solutions.

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8. You must stack books with a thickness of 6 centimeters on a shelf that has a
height of 74 centimeters

a. Use a model or picture to show the number of books you can stack.
b. Write and solve an inequality to find the possible number of books that you can stack.
c. Draw a graph and write an equation that gives the height y of stacked books as a function of the number x of books. Explain your solution using the graph and equation(s) you drew/wrote.
d. Which method would you use if the shelf had an height of 120 centimeters.
(model, solving an inequality, or drawing a graph) Explain.

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MODULE

2-2

Solve Mathematical Situations

This module addresses the following indicators:

8-3.1 Translate among verbal, graphic, tabular, and algebraic representations of linear functions.
8-3.4 Apply procedures to solve multi-step equations.
8-3.6 Identify the coordinates of the x- and y-intercepts of a linear equation from a graph, equation, and/or table.
8-3.7 Identify the slope of a linear equation from a graph, equation, and/or table.

This module contains 3 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S3 begin to build the conceptual foundation student need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

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I. Planning the Module

Continuum of Knowledge

8-3.1 Translate among verbal, graphic, tabular, and algebraic representations of linear functions.

In fifth grade, students match tables, graphs, expressions, equations, and verbal descriptions of the same problem situation (5-3.3). In seventh grade, students represent proportional relationships with graphs, tables and two-step equations (7-3.6)

In eighth grade, students translate among verbal, graphic, tabular and algebraic representations of linear functions (8-3.1) and classify relationships between two variables in graphs, tables and/or equations as either linear or nonlinear (8-3.5) and applying abstract reasoning (8-1.1) in regards to the various representations.

In Elementary Algebra, students will carry out a procedure to write an equation of a line (EA-4.1, EA-4.2, EA-4.3), carry out a procedure to graph a line (EA-5.1), and carry out a procedure to determine the slope of a line from data given tabularly, graphically, symbolically and verbally (EA-5.6).

8-3.4 Apply procedures to solve multi-step equations.

In sixth grade, students begin to build the foundation by using inverse operations to solve simple one-step equations with whole numbers solutions and variables with whole number coefficients (6-3.5). In 7th grade, students use inverse operations to solve two-step equations (with rational numbers) and two-step inequalities (7-3.4).

In 8th grade, the process continues as students apply procedures to solve multi-step equations (8-3.4).

In Elementary Algebra, students will demonstrate through the mathematical processes an understanding of the real number system and operations involving exponents, matrices and algebraic expressions (EA2).

8-3.6 Identify the coordinates of the x- and y-intercepts of a linear equation from a graph, equation, and/or table.

Eighth grade is the first time that students are first introduced to slope-intercept form (y=mx + b) of a linear equation (linear function).

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In Elementary Algebra, students analyze the effects of changes in the slope, *m*, and the y-intercept, *b,* on the graph of *y=mx +b* (EA-5.2). Students also carry out a procedure to determine the x-intercept and y-intercept of lines from data given tabularly, graphically, symbolically, and verbally (EA-5.5)

8-3.7 Identify the slope of a linear equation from a graph, equation, and/or table.

In seventh grade, students analyze tables and graphs to describe the rate of change between and among quantities (7-3.2). Students gain an understanding slope as a constant rate of change (7-3.3)

In eighth grade, students identify the slope of a linear equation from a graph, equation, and/or table (8-3.7). Students did not relate slope to linear equations in seventh grade.

In Elementary Algebra, students carry out a procedure to determine the slope of a line from data given tabularly, graphically, symbolically, and verbally (EA-5.6).

Key Concepts/Key Terms

*These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

*Nonlinear
*Constant
*Coefficient
*Multiple representations
*Functions
∗Multi-step equations
∗Properties of Equalities
∗Properties of Real Numbers
∗X-intercept
∗Y-intercept
∗Multiple representations
∗Rise
∗Run
∗Slope
∗Coefficient
∗Rate of change
Solving equations

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II. Teaching the Lesson(s)

1. Teaching Lesson A: Solving Multi-step Equations

a. Indicators with Taxonomy

A connection can be made here to order of operations in that when solving equations or inequalities (particularly two-step). We proceed in isolating the variable by doing the order of operations in reverse order. (See 6th grade Algebra Indicator 6-3.2 (Apply order of operations to simplify whole-number expressions) for information on prior knowledge for order of operations.)

For this indicator, it is **essential** for students to:

- Recall and understand the slope intercept form of a linear function
- Construct a table of values given a linear equation
- Analyze verbal descriptions for key words that relate to linear concepts such as slope, rate of change, initial value, y-intercept, etc..
- Produce the other two representations when given one of them (produce a table given the graph or the equation)
- Identify the slope and y-intercept when given a linear equation in slope intercept form.
- Identify increasing and decreasing linear patterns from the graph, table and equation
- Connect numeric patterns in tables and graphs in coordinate planes.

For this indicator, it is **not essential** for students to:
None noted

8-3.4 → Apply procedures to solve multi-step equations. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson A: Solving Multi-step Equations

Materials Needed:

- Algebra tiles (1 set for each pair of students)
- Overhead Algebra tiles

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Simplifying by collecting like terms

Begin by reviewing the process for solving two-step equations by using inverse operations. It may be beneficial to have students use the Algebra tiles to build and solve a couple of the equations to strengthen the review, before solving them without the tiles.

Example 1:

Write the following equation on the overhead/board. Have students build the equation using the Algebra tiles.

$2\mathbf{x} + 7 + \mathbf{x} = 16$

Once students have had the chance to build the equation at their desks, have a student build it on the overhead or using some kind of technology. (Promethean board, Smartboard, etc.)

Ask: How is this equation different from the ones we solved earlier? Answer: There are more than two terms set equal to the answer.

Ask: How might we begin to solve this equation?
Answer: To begin solving this equation, we have to simplify the part that has more than two terms. We do this by collecting like terms.

How do you collect like terms? [This should be a review for the students.]

Ask: How might we model collecting like terms? Have student(s) model how to do this using the Algebra tiles at their desks.

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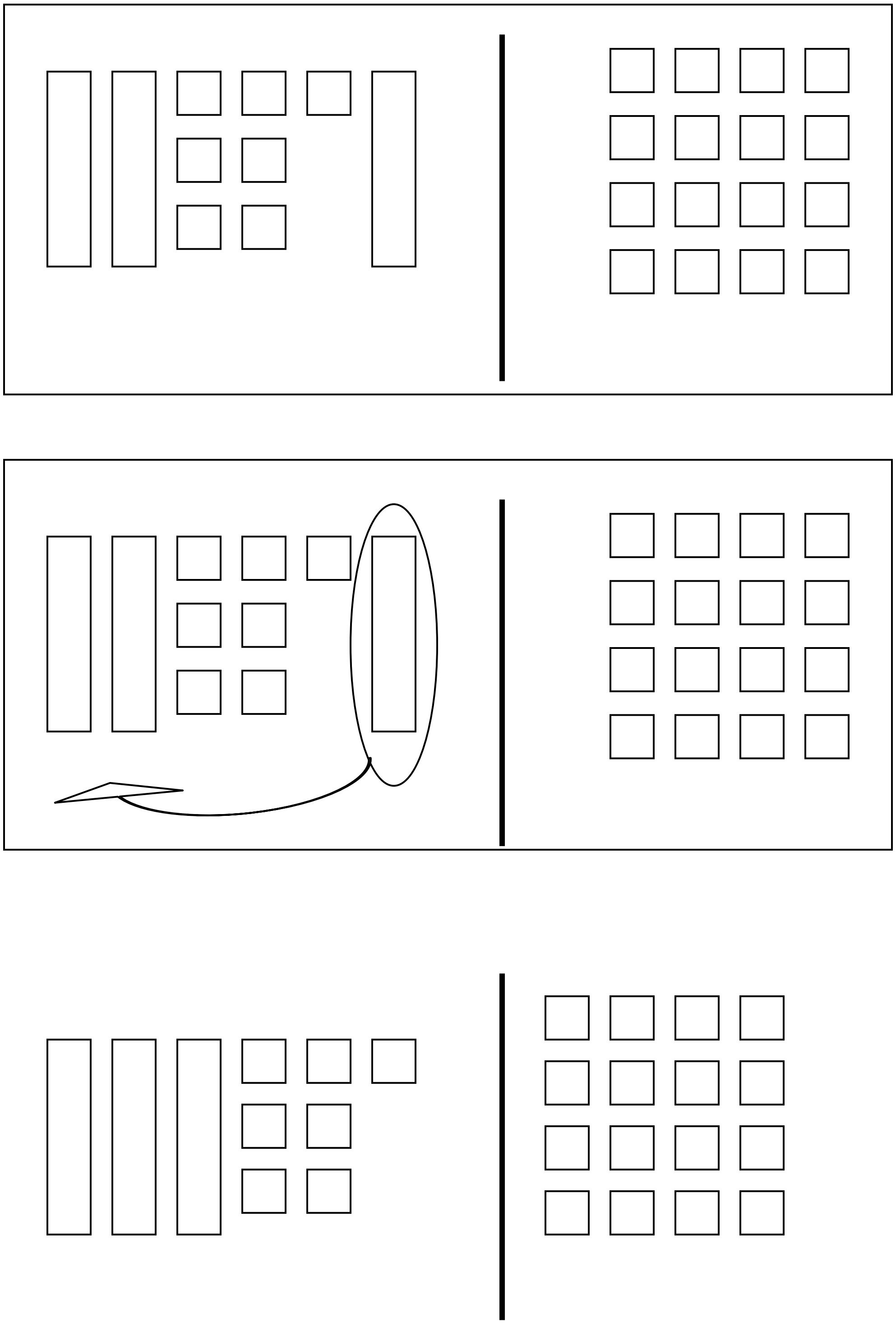
$2x + 7 + x = 16.$



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Ask: If we collect like terms and simplify 2**x** + 7 + **x** = 16, what do we get?
Answer: 3x + 7 = 16

Students should be able to look at their work and tell you the simplified expression is 3**x** + 7 = 16. They should also be able to use prior knowledge to find the solution. You may use the tiles to work through to the solution.

Example 2:

Another example using Algebra tiles may be necessary.

m + 2m – 4 = 14

Example 3:

One football team beat another by 12 points. The total number of points scored by both teams was 38 points. Write an equation to find out what each team scored. Let ***p*** equal the winning team's score.

p = the winning team's score
p – 12 = the losing team's score

So…
p + ***p*** – 12 = 38

Simplify. 2p – 12 = 38

Solve it.

2p = 50

p = 25

The winning team scored 25 points, and the losing team scored 13 points.

Example 4:

The sum of three consecutive integers is 96. Write an equation and find out what the three integers are. Let ***n*** equal the least integer.

n = the least integer
n + 1 = the second integer
n + 2 = the third integer

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So…

n + ***n*** + 1 + ***n*** + 2 = 96

Simplify. $3n + 3 = 96$

Solve it.

$3n = 93$

$n = 31$

The three consecutive integers are 31, 32, and 33.

Example 5:

Bill and his younger sister Jan collect marbles. Together, they have 94 marbles. Bill counted that he has 4 more than twice as many marbles as Jan. Let m stand for the number of marbles Jan has. Write and solve an equation to find out how many marbles each of them has.

Answer:
m = the number of marbles Jan has
2m + 4 = number of marbles Bill has

$2m + 4 + m = 94$
$3m + 4 = 94$
$3m = 90$
$m = 30$

Jan has 30 marbles.
Bill has 64 marbles.

<u>Simplifying by using the distributive property</u>

Example 1:
$3(n + 2) = 32$

Ask: How would we go about simplifying this equation?

Collecting like terms isn't always the first step.

How do you find the combined area of the two rectangles?

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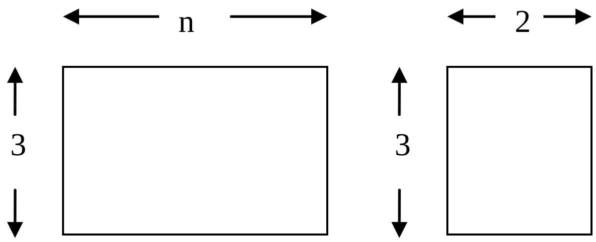
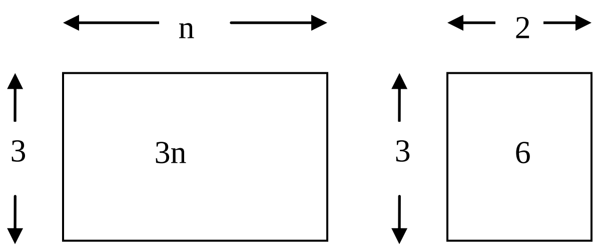


(This is a review of the area model used to introduce the Distributive Property. Students should remember that they can multiply 3 x n to get the area of one rectangle and 3 x 2 to get the area of the second rectangle. This leads them to the expression 3n + 6.)

Say: So in addition to being able to combine like terms to simplify equations, the distributive property is also an available and useful tool.



So if we simplify 3 (n + 2) = 32 using the distributive property, what do we get? ***3n + 6 = 32.***

From there, we're able to solve the equation to find out what n is.

$3n = 26$

$n = 8\frac{2}{3}$

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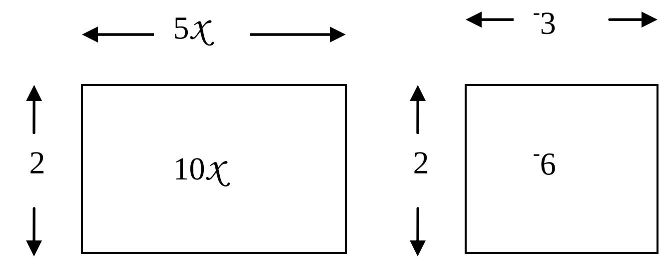
Example 2:

$2 (5x - 3) = 14$



$10x – 6 = 14$

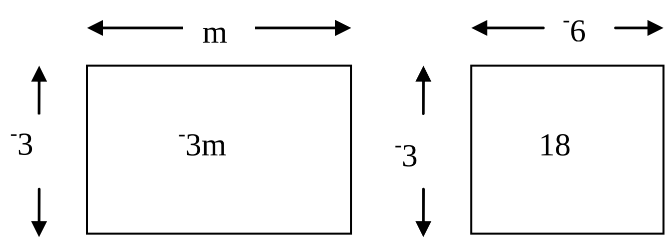
Solve it.

$10x = 20$

$x = 2$

Example 3:

$^{-}3 (m – 6) = 4$



$^{-}3m + 18 = 4$

Solve it.

$\mathbf{^{-}3m = {}^{-}14}$

$\mathbf{m =} 4\frac{2}{3}$

Using the distributive property and combining like terms to simplify equations

Example 1:

$2(a - 1) - a = 16$

Ask: What do you see in this equation?
Answer: A situation where I need to use the distributive property to simplify the parentheses.

Ask: What equation is the result of using the distributive property to simplify?
Answer: $2a - 2 - a = 16$

Ask: Is the equation completely simplified?
Answer: No, like terms need to be combined

What's the result? $\mathbf{a - 2 = 16}$

Solve it. $\mathbf{a = 18}$

Example 2

$9(2c + 5) + 3c = {}^{-}60$

Ask: What is the first step?
Answer: Use the distributive property to simplify the parentheses.

Ask: What is the result?
Answer: $18c + 45 + 3c = {}^{-}60$

Ask: What is the next step?
Answer: Combine like terms.
Ask: What is the result?
Answer: $21c + 45 = {}^{-}60$

Ask: Is the equation simplified?
Answer: Yes.

Say: Solve it.

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$21c + 45 = {}^{-}60$

$21c = {}^{-}105$

$c = {}^{-}5$

c. Misconceptions/Common Errors

Students may forget to reverse the order of operations when solving.

d. Additional Instructional Strategies

- Although students have solved equations in the past, proficiency with procedures is support by understanding of concepts. Students may still use concrete and pictorial models to support understanding.
- The types of problems students encounter should move beyond basic procedural knowledge. Sample problems:
- Explain how to solve 3 (9 + 4a) – 19 = 32
- Together, Donald, Yolanda, and Iris made 27 birdhouses for a school fair. Yolanda made the fewest number of birdhouses. Donald made one more than Yolanda, and Iris made one more than Donald. Write and solve and equation to find out how many birdhouses each of them made. Let n = the number of birdhouses Yolanda made
- Comparing multiple strategies for solving the same problems builds both conceptual and procedural knowledge.
- Making a connection to using the order of operations in reverse order may be helpful to students when solving equations or inequalities (particularly two-step)

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

This is a suggestion for an Algebra tile resource: www.nlvm.usu.edu

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f. Assessing the Lesson

Formative assessment

By discussing the examples and listening carefully to student responses, you should be able to discern the level of your students' understanding at various points during the lesson.

2. Teaching Lesson B: Identifying x and y intercepts of a linear equation

When presented with an equation in this form (y= mx + b), 8th grade students should be able to translate among the graphic, tabular, and algebraic representation of the linear function. Students should also progress to having the ability to identify the slope and the x- and y-intercepts when given a linear equation in slope-intercept form. In the slope-intercept form, y=mx+b, the slope (m) and the y-intercept (b) is easily recognized, although it is important that students have the opportunity through graphing activities to discover what the "m" and "b" in slope-intercept form are.

The x-intercept is not as easily seen when given the slope-intercept form and time and emphasis should be placed on the students understanding that any point on the x-axis (an x-intercept) has a y value of zero, and likewise any point on the y-axis (a y-intercept) has an x value of zero. An in depth understanding of intercepts is essential for students to have the foundation needed for Algebra I and will enable the student to not only recognize the x- and y-intercepts on a graph, but also know that to find the x-intercept in an equation all they need to do is plug in zero for y and to find the y-intercept plug in zero for x. Knowing this also makes it easy to look for and recognize these values in a table, the x-intercept being when the y=0 in the table and the y-intercept being when x=0 in the table.

For this indicator, it is **essential** for students to:
· Understand that the x-intercept is in the form (x,0)
· Understand that the y-intercept is in the form (0,y)
· Identify slope (m) and y-intercept (b) of a linear equation
· Understand that the x-intercept is where the graph crosses the x axis
· Understand that the y-intercept is where the graph crosses the y axis

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For this indicator, it is **not essential** for students to:

· Identify x- and y-intercepts from a graph that are not integers

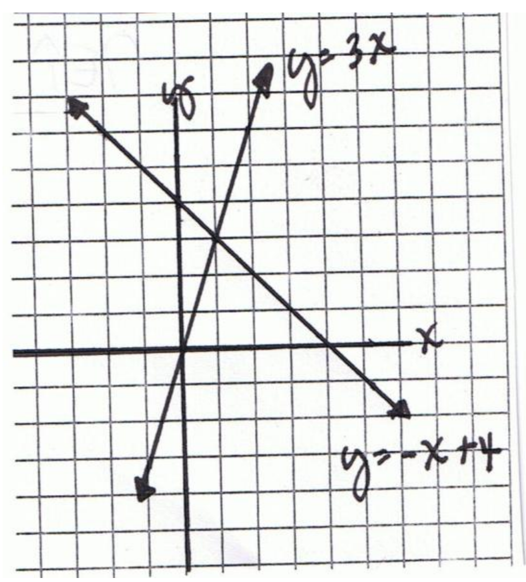
a. Indicators with Taxonomy

8-3.6 → Identify the coordinates of the x- and y-intercepts of a linear equation from a graph, equation, and/or table. (C1)
Cognitive Process Dimension: Remember
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson B: Identifying x and y intercepts of a linear equation

Materials Needed:

Display the following graph for the students.



What is the x-intercept of the line y = 3x?
What is the y-intercept of the line y = 3x?
What is the x-intercept of the line y = -x + 4?
What is the y-intercept of the line y = -x + 4?

Consider the format of a line y = mx + b for each line:

a) y = 3x + 0
   What is the y-intercept of this line?
b) Y = -x + 4
   What is the y-intercept of this line?

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c) What do you notice about y = mx + b and the y-intercept? *Answer: the constant, what's added to the x term is the y-intercept. (This will be addressed more as the lesson progresses).*

The y-intercept is the point in which a line crosses the y-axis (review from module 4) In the equation, y = mx + b, it is the constant or the "b" term.

What do you notice about the value of y when it is on the x-axis? *It is always 0.*
Therefore, whenever x = 0, you can find a y-intercept. So, how does that help find the y-intercept of the following equations?

a) $y = -3x + 5$

b) $y = \frac{1}{4}x - 9$

c) $y = 7x + 2$

d) $y = 8x - \frac{1}{2}$

Remember back to module 4 when you graphed equations of lines. You could choose any x-value, substitute it into the equation and find the corresponding y-value. So, what should you do here if you know x = 0 will help you find the y-intercept? *Answer: substitute in 0 for x and solve for y.*

e) $y = -3x + 5$ → if $x = 0$, then $y = -3(0) + 5$ or $y = 5$

f) $y = \frac{1}{4}x - 9$ → if $x = 0$, then $y = \frac{1}{4}(0) - 9$ or $y = -9$

g) $y = 7x + 2$ → if $x = 0$, then $y = 7(0) + 2$ or $y = 2$

h) $y = 8x - \frac{1}{2}$ → if $x = 0$, then $y = 8(0) - \frac{1}{2}$ or $y = -\frac{1}{2}$

What do you notice about the equation's parts and the y-intercepts calculated? *Answer: That the b constant is the y-intercept.*

What do you notice about the value of x when it is on the y-axis? *It is always 0.*
Therefore, whenever y = 0 you can find the x-intercept. So, how does that help you find the x-intercept of the same equations?

a) $y = -3x + 5$ → substitute 0 for y → $0 = -3x + 5$ and solve the equation for x. →

$$0 = -3x + 5$$
$$-5 = -3x$$

$X = \frac{5}{3}$, therefore the line crosses the x-axis at $\frac{5}{3}$.

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b) $y = \frac{1}{4}x - 9 \rightarrow$ substitute 0 for y $\rightarrow 0 = \frac{1}{4}x - 9$ and solve the equation for x. $\rightarrow$

$0 = \frac{1}{4}x - 9$

$9 = \frac{1}{4}x$

36 = x or x = 36, therefore the line crosses the x-axis at 36.

c) $y = 7x + 2 \rightarrow$ substitute 0 for y $\rightarrow$ $0 = 7x + 2$ and solve the equation for x. $\rightarrow$

$0 = 7x + 2$

$-2 = 7x$

$-\frac{2}{7} = x$, therefore the line crosses the x-axis at $-\frac{2}{7}$.

d) $y = 8x - \frac{1}{2} \rightarrow$ substitute 0 for y $\rightarrow$ $0 = 8x - \frac{1}{2}$ and solve the equation for x. $\rightarrow$

$0 = 8x - \frac{1}{2}$

$\frac{1}{2} = 8x$

$x = \frac{1}{2} \div 8$ or $\frac{1}{16}$, therefore the line crosses the x-axis at $\frac{1}{16}$.

<u>Special Case to Explore:</u>

Graph the line, <u>y = 0.</u> *Remember back to Module 4.*
Set up a t-table. Does it matter what x is? *It didn't in the graphs in module 4….*
What if x = 9? *Y will be 0.*
What if x = -3? *Y will be 0.*
Is there any value of x where y won't be 0? *No.*
So graph the ordered pairs found: (9, 0) and (-3, 0), then construct your line.

What does the line look like? *It is a horizontal line passing through the origin OR the equation for the x-axis.*

Now let's look at the line <u>x = 0</u>. Set up a t-table.
What values of x can you use? *Only 0 because x is always 0 in this case.*
So your t-table will yield ordered pairs such as (0, 4), (0, -2) or (0, 5).
Graph these points and construct the line.
What does the line look like? *It is a vertical line passing through the origin OR the equation of the y-axis.*

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c. Misconceptions/Common Errors

Students confuse which variable as a value of zero for each intercepts.

d. Additional Instructional Strategies

Not only should students identify x and y intercepts from each form but they should also make connections among the forms; therefore, each time students examine one representation, they should examine how it looks in the other forms. For example, once the y-intercept is identified from slope-intercept form, illustrate what it looks like on the graph and in a table.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:
http://www.mathexpression.com/y-intercept.html

f. Assessing the Lesson

Formative assessment occurs throughout the lesson during the questioning. Student responses should guide the instruction.

3. Teaching Lesson C: Linear Functions

8-3.1 → Translate among verbal, graphic, tabular, and algebraic representations of linear functions. (B2)

For this indicator, it is **essential** for students to:
- Recall and understand the slope intercept form of a linear function
- Construct a table of values given a linear equation
- Analyze verbal descriptions for key words that relate to linear concepts such as slope, rate of change, initial value, y-intercept, etc..
- Produce the other two representation when given one of them (produce a table given the graph or the equation)

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- Identify the slope and y-intercept when given a linear equation in slope intercept form.
- Identify increasing and decreasing linear patterns from the graph, table and equation
- Connect numeric patterns in tables and graphs in coordinate planes.

For this indicator, it is **not essential** for students to:
None noted

8-3.7 → Identify the slope of a linear equation from a graph, equation, and/or table. (C1)

For this indicator, it is **essential** for students to:
- Have a conceptual understanding of slope (rate of change)
- Use the process of rise over run to find the slope from graph
- Understand that slope is the steepness and direction of the line.
- Recall that the slope is the coefficient of x in y=mx + b and y = b + mx
- Understand the effect the m has on the equation and the graph
- Review the vocabulary related to the coordinate plane.

For this indicator, it is **not essential** for students to:
- Identify the slope for horizontal and vertical lines

a. Indicators with Taxonomy

8-3.1 → Translate among verbal, graphic, tabular, and algebraic representations of linear functions. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

8-3.7 → Identify the slope of a linear equation from a graph, equation, and/or table. (C1)
Cognitive Process Dimension: Remember
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson C : Linear Functions

Materials Needed:
- y=mx handouts (These will serve as the students' notes for the lesson.)
- colored pencils: black, red, blue, green, orange
- index cards for the students to use as straight edges
- Graphing Calculator

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PROCEDURES:

PART 1A

- □ Begin by asking the students to relate the equation ***y = mx*** to the coordinate grid. Review vocabulary such as: ordered pair, coordinate, x-axis, and y-axis. They should notice that the equation contains both an x and a y.

- □ Tell them you're going to be exploring this equation using the graphing calculators. Their "job" is to graph and figure out what "m" tells them about each equation. This is a good time to revisit the word coefficient. Point out that in each equation, x has a coefficient and that the m represents the coefficient in y = mx. If no one mentions it, ask them what the coefficient is in $\mathbf{y_1}$. They may need to be reminded that a variable standing alone always has a coefficient of 1.

- □ Show them how to set their calculators up. They need to clear any equations from the "y=" screen, set the window, and turn off any stat plots. So that all students are viewing the same part of the coordinate plane, tell them to set the window as follows: X_{min} and Y_{min} are – 10; X_{max} and Y_{max} are 10; and X_{scl} and Y_{scl} are 1.

- □ Show them how to enter $\mathbf{y_1 = x}$ and graph that first line. They should use the black colored pencil to sketch the line on their papers. They will leave y=x graphed on their calculators; it will serve as a reference point.

  NOTE: Have the students graph the subsequent lines at y_2=. This will allow them to compare each of the other lines to y=x. Also, have them label the lines on their paper as they graph them.

- □ Move on to y_2. After they've got that one sketched, discuss:

*What happened when the coefficient changed from 1 to 2?
*Take a look at y_3. Don't call out! Think about what the coefficient may do to that line.

- □ Graph and sketch y_3. Discuss:

*What happened to the line? Did you make a good guess?
*How are the 3 lines oriented in relation to the axes?
*Take a look at y_4. Don't call out! Think about what the coefficient may do to that line.

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- Graph and sketch y_4. Discuss:

*What happened to the line? Did you make a good guess? How does this line compare to the first three we graphed? Why do you think it looks like that?”
*Finally, take a look at y_5. Don’t call out! Think about what the coefficient may do to that line.

- Graph and sketch y_5. Discuss:

*What happened to the line? Did you make a good guess? How does this line compare to the others?

- Ask a couple of students to suggest coefficients for the class to graph. The idea is to get a couple more whole numbers to compare to the first 3 lines. Graph y = x first, and then graph the lines the students suggested on the extra grids and discuss them. THEN get a couple more fraction coefficients. Again, graph y = x , then graph the “fraction” lines. Lead students to notice/discover how the coefficients affect the slope. Just don’t introduce that word yet. Talk about steepness and direction.

<u>PART 1B</u>

The PROCEDURES for this section are the same as for PART 1A. Students really need to pay attention to the signs of the coefficients this time. NOTE: The operation key for subtraction IS NOT used to indicate negative numbers on a graphing calculator. The key for entering a negative sign has parentheses around it. (-)

When you get to the part where you asked students to suggest coefficients for extra graphs, just use the opposites of the ones they gave for PART 1A. That way, this part of the lesson parallels what you’ve already done.

<u>“Plug It In”</u>

What you’re aiming for:

- Students will use the substitution principle and their prior knowledge of solving one-step equations to find points on the lines they’ve already graphed using the ***y=*** function on the graphing calculators.
- Through discussion, students will solidify their understanding of how ***m*** affects lines.
- Students will “see” the relationship between slope and the ordered pairs that are on the line by examining t-tables to identify how x

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and y change, as well as showing how the points on the line "move."

Procedures:

Use the equations from the y=mx handouts to demonstrate how to create t-tables to find actual values for x and y.

- Give out copies of the "Plug It In" handouts.

- Discuss the first equation: $y_1 = x$.

*Finding some ordered pairs for our equations is very simple. You can use a tool called a t-table. Find the grid that's labeled "y = x."

*So that we'll all be able to work together, we'll use the values for x that are listed in the table.

*We're going to use the substitution principle to plug the values for x in. Use what you know about solving equations to find the values for y.

*Have students complete the t-table to get the y values.

*Now have the students graph the ordered pairs and compare the line that goes through those points to the line they graphed using the calculator. They should notice that the lines are the same.

- Do this with each of the equations.

When all of the tables and graphs are complete, show the students how to use the t-tables to help them find slope.

y = x

x	y
-2	-2
-\-1	-1
0	0
1	1
2	2

y = 2x

x	y
-2	-4
-1	-2
0	0
1	2
2	4

y = 4x

x	y
-2	-8
-1	-4
0	0
1	4
2	8

y = $\frac{1}{2}x$

x	y
-4	-2
-2	-1
0	0
2	1
4	2

y = $\frac{1}{4}x$

x	y
-8	-2
-4	-1
0	0
4	1
8	2

<u>y = mx + b Lesson</u>

- Students will continue to explore slope.
- Students will explore how b affects the position of the line. By the end of the lesson, students will be able to identify b as the y-intercept.

<u>PART 2A</u>

- Have the students use the graphing calculators to graph each line. They shouldn't require too much guidance given that they've already completed Parts 1A and 1B.
- Each of the grids is labeled with an equation. The screen beans are asking questions the students need to answer.
- There are extra grids for the students to use if they need to. Have them write the equations themselves. Use the list below.

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<u>EXTRA</u> y=mx + b equations:

y = 3x + 4
y = 4x +3
y = $\frac{1}{8}x$
y = $\frac{3}{10}x+5$

Swap the signs around to let students graph different lines.

They should be able to recognize that m is slope and determines the steepness and direction of the line. They should also recognize that b shows where the line crosses the y-axis.

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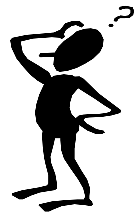
y=mx



Name: ________________________________

Group members: ________________________________

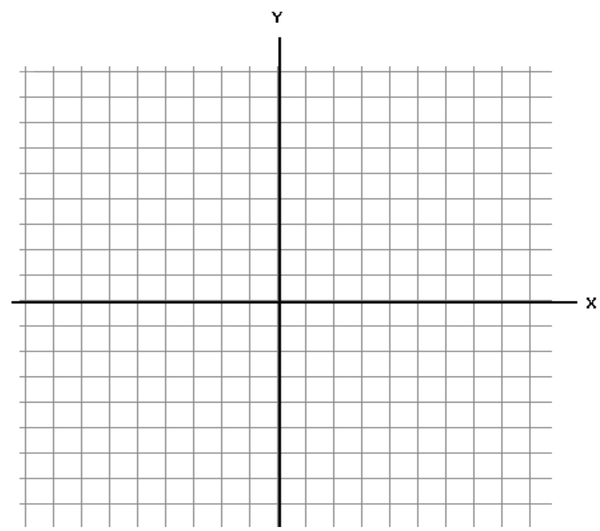
____

Date: ____________ Class pd: ________________

PART 1A

Graph each of the equations in the table below. Use the coordinate grid to the right of the table. ***PAY ATTENTION*** to the color indicated for each line!!

EQUATION (y=mx)	COLOR
$y_1 = x$	black
$y_2 = 2x$	red
$y_3 = 4x$	blue
$y_4 = \frac{1}{2}x$	green
$y_5 = \frac{1}{4}x$	orange



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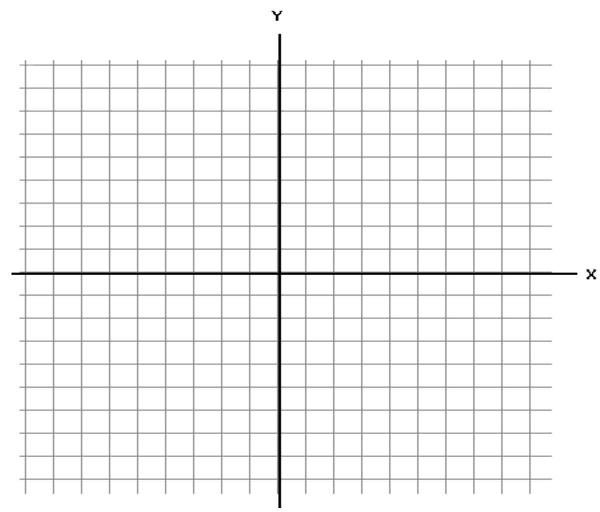
PART 1B

Graph AGAIN! This time, watch both the ***sign*** and the ***color*** !

EQUATION (y=mx)	COLOR
$y_1 = -x$	black
$y_2 = -2x$	red
$y_3 = -4x$	blue
$y_4 = -\frac{1}{2}x$	green
$y_5 = -\frac{1}{4}x$	orange



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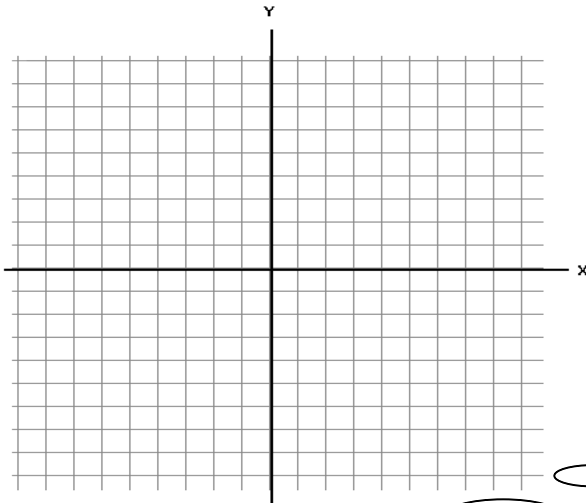
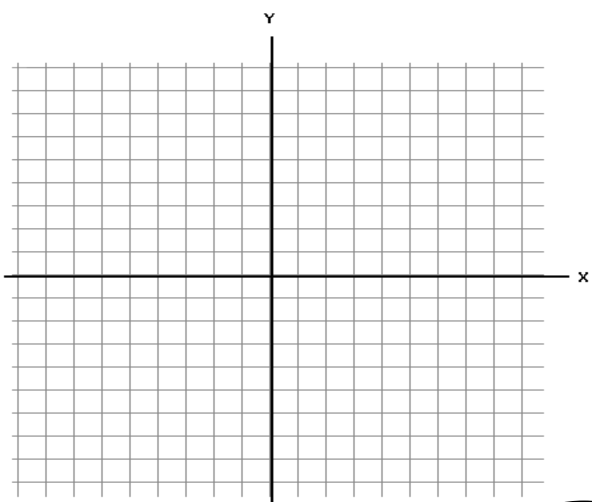
PART 1A : Extra Grids



EQUATION (y=mx)	COLOR
$y_1 = x$	black
$y_2 =$	red
$y_3 =$	blue
$y_4 =$	green
$y_5 =$	orange

Each coefficient is an integer…What does this do to the line?



EQUATION (y=mx)	COLOR
$y_1 = x$	black
$y_2 =$	red
$y_3 =$	blue
$y_4 =$	green
$y_5 =$	orange

Each coefficient is a fraction…What does this do to the line?

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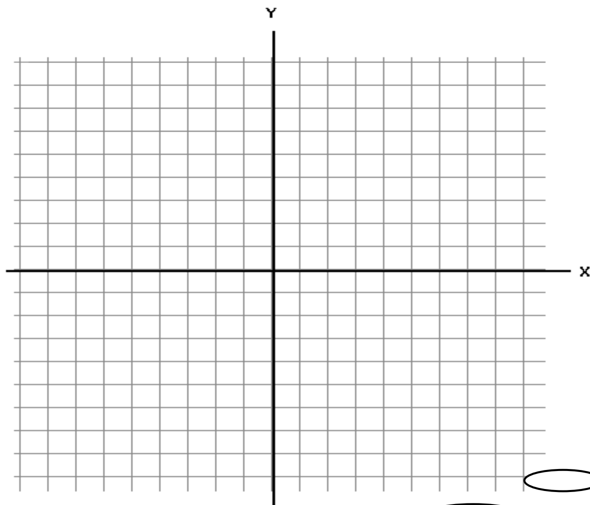
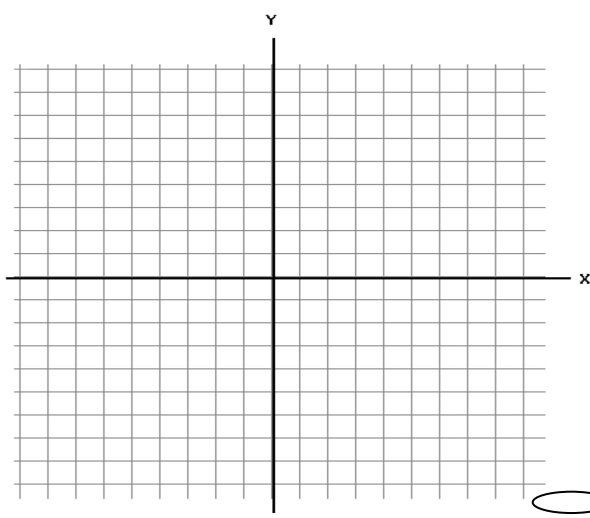
PART 1B : Extra Grids



Each coefficient is an integer…What does this do to the line?

EQUATION (y=mx)	COLOR
$y_1 = x$	black
$y_2 =$	red
$y_3 =$	blue
$y_4 =$	green
$y_5 =$	orange



Each coefficient is a fraction…What does this do to the line?

EQUATION (y=mx)	COLOR
$y_1 = x$	black
$y_2 =$	red
$y_3 =$	blue
$y_4 =$	green
$y_5 =$	orange

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Study the lines you graphed in **PART 1A** to answer the following questions.

1. Describe what you observe about the direction and steepness when you compare the original line, y_1, to the lines y_2, y_3, y_4, and y_5.

2. The placement of a line is where it crosses the y-axis. Describe the placement(s) of the lines you graphed.

Study the lines you graphed in **PART 1B** to answer the following questions. (Yes, the questions are the same, but the answers *may not* be. ☺)

1. Describe what you observe about the direction and steepness when you compare the original line, y_1, to the lines y_2, y_3, y_4, and y_5.

2. The placement of a line is where it crosses the y-axis. Describe the placement(s) of the lines you graphed.

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3. What is the *same* about the equations in PART 1A and PART 1B? What is *different*?

VOCABULARY BREAK!!

slope:

CONCLUSIONS thus far…

Describe what you notice if m is an integer (that is, not a fraction).

Describe what you notice if m is a proper fraction.

Describe what you notice if m is positive.

Describe what you notice if m is negative.

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Grade 8

Plug It In!

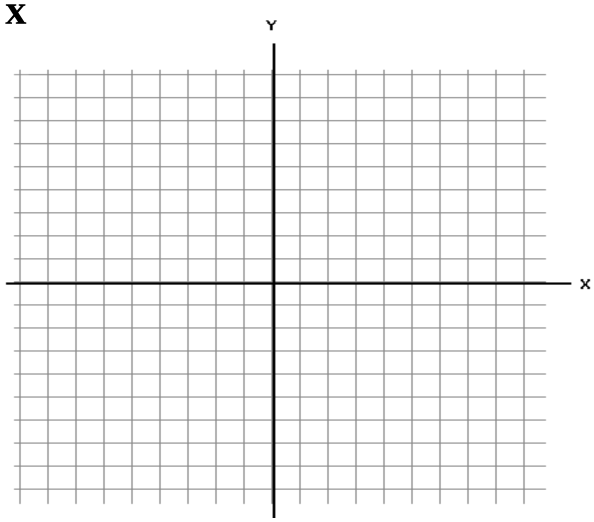
Name: ____________________________________

Group members: ____________________________________

____

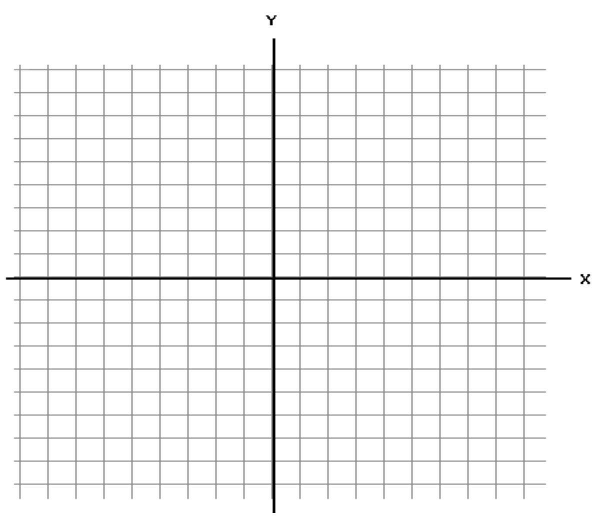
Date: ______________Class pd: ____________________

y = x



x	y
-2	
-1	
0	
1	
2	

y = 2x



x	y
-2	
-1	
0	
1	
2	

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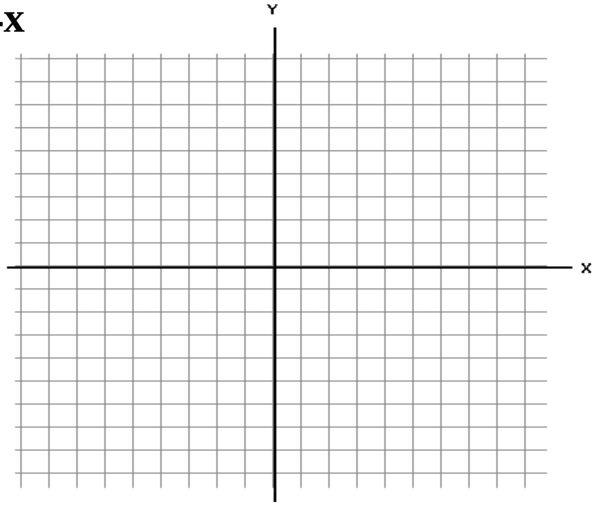
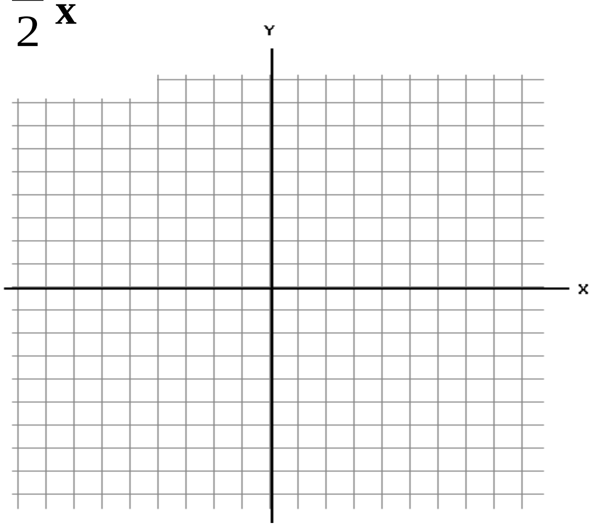
y = 4x



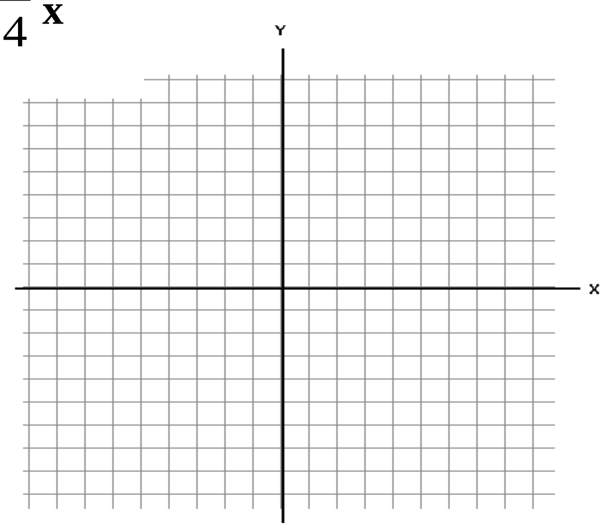
x	y
-2	
-1	
0	
1	
2	

y = $\frac{1}{2}$ **x**



x	y
-4	
-2	
0	
2	
4	

y = $\frac{1}{4}$ **x**



x	y
-8	
-4	
0	
4	
8	

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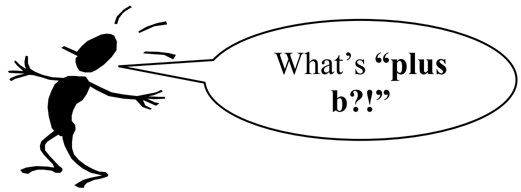
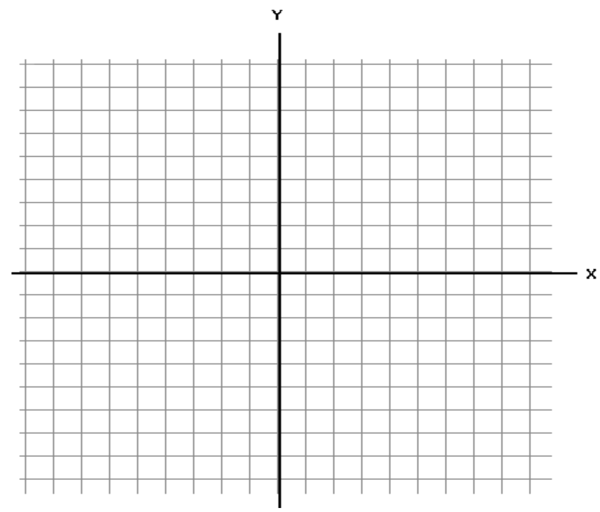
y=mx+b



PART 2A

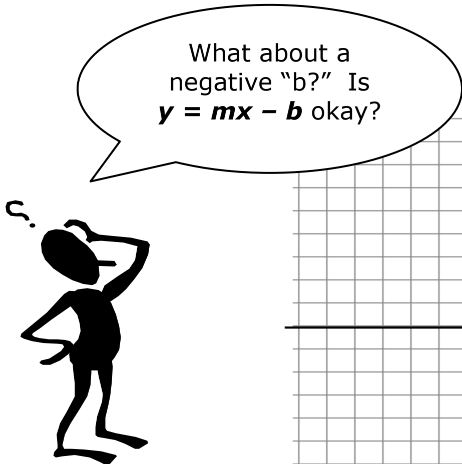
The "+ b" will affect the lines. Graph to find out how *b* affects the lines.

EQUATION (y=mx + b)	COLOR
$y_1 = x + \frac{3}{2}$	black
$y_2 = 2x + 1$	red
$y_3 = 4x + 2$	blue
$y_4 = \frac{1}{2}x + 3$	green
$y_5 = \frac{2}{3}x + 4$	orange

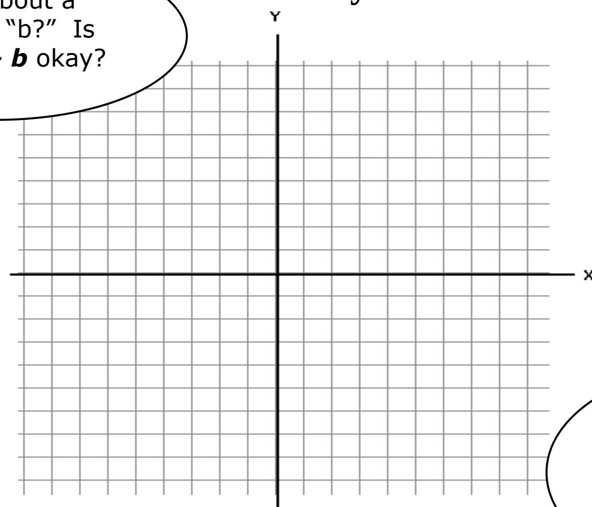


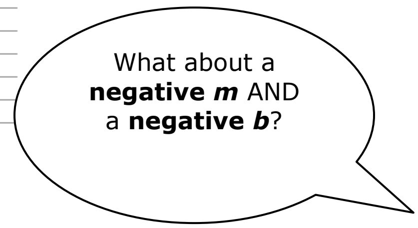
SO…What happened?

(Consider *placement* and *slope* in your answer. ☺)

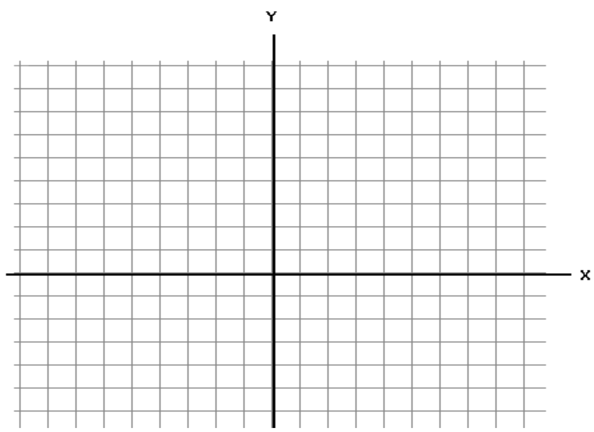


$y = 2x - 1$

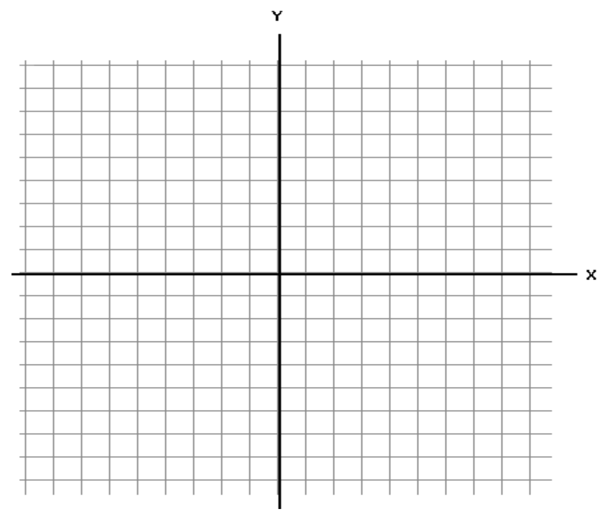




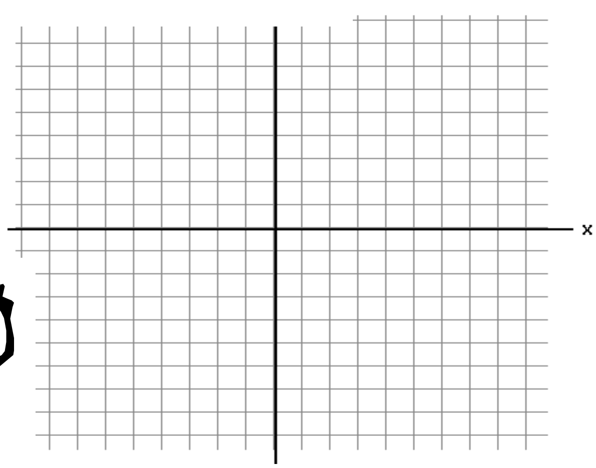
$y = 4x - 2$

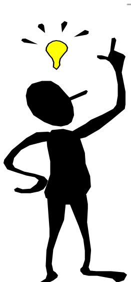


$y = -2x - 1$



$y = -x - \frac{3}{2}$





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Both $y = mx$ and $y = mx + b$ (OR $y = mx - b$) are called equations of a line.

1. Compare the two forms. Tell what is the same and different about them.

2. Describe:

    a. the effect of a positive m.

    b. the effect of a negative m.

3. Describe how you can tell how steep a line will be.

4. Describe:

    a. the effect of a positive b.

    b. the effect of a negative b.

    c. the effect of not having a b at all.

c. Misconceptions/Common Errors

No typical student misconceptions noted at this time.

d. Additional Instructional Strategies

While additional learning opportunities are needed, no suggestions are included at this time.

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e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

<u>Specific resources</u>

illuminations.nctm.org
click "lesson"
- ✓ 6-8
- ✓ algebra

in search, type "constant dimensions"

illuminations.nctm.org
click "weblink"
click "algebra"
scroll to "Rent-a-Car"

<u>GRAPHING CALCULATORS</u> ARE NECESSITIES.

f. Assessing the <u>Lesson</u>

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS are posed throughout the lesson. Pay close attention to student responses during the lesson.

III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

8-3.1 Translate among verbal, graphic, tabular, and algebraic representations of linear functions.

The objective of this indicator is to *translate,* which is in the "understand conceptual" knowledge cell of the Revised Bloom's Taxonomy. To understand is to construct meaning. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge of translating among verbal, graphic, tabular,

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and algebraic representations of linear functions should include a variety of examples. The learning progression to translate requires students to recall and understand the multiple methods of expressing real world functional relationships (words, graphs, equations and tables). Students analyze these representations simultaneously to generalize connections among these forms (8-1.7) such as the location of the y-intercept on the graph, in the table and in the equation. They use correct and clearly written and spoken words, variables and notation to communicate their understanding of these generalizations. Students then use these generalizations to translate among representations. They explain and justify their understanding to their classmates and teacher.

8-3.4 Apply procedures to solve multi-step equations.

The objective of this indicator is use which is in the “apply procedural” of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus is to gain computational fluency with solving multi-step equations, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to use requires students to explore and understand the concepts of equivalency, inequality and variables using concrete models such as balance scales. Student use this understanding of balance to analyze a variety of examples of simple multi-step equations. Students use inductive reasoning (8-1.3) to generalize connections (8-1.7) among types of equations/inequalities and generate mathematical statements (8-1.5) related to how these equations can be solved. They use concrete manipulatives and pictorial to support their conceptual understanding. Student engage in meaningful practice to support retention of these processed and check their answers.

8-3.6 Identify the coordinates of the x- and y-intercepts of a linear equation from a graph, equation, and/or table.

The objective of this indicator is to *remember*, which is in the “remember procedural” knowledge cell of the Revised Bloom’s Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus is on remembering procedures to identify the coordinates of the x- and y-intercepts of a linear equation from a graph, equation and/or table,

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the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **remember** requires students to recall that the x intercept is where the graph crosses the x axis and the y-intercept is where the graph crosses the y axis. By generalizing connections (8-1.7) among forms, students use this understanding to create a coordinate representation of intercepts using zero appropriately. They graph these points to examine the graphical form. They analyze the equation to determine how these coordinates connect i.e. the y-intercept is the value of b. They generate other values in the table and observe how the intercept fit into the table. Students generalize mathematical statements (8-1.5) about the relationships between and among the representations using correct and clearly written or spoken words, variables, and notation (8-1.6).

8-3.7 Identify the slope of a linear equation from a graph, equation, and/or table.

The objective of this indicator is to remember which is in the "remember procedural" knowledge cell of the Revised Bloom's Taxonomy. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem or problem situation. Although the focus is on remembering procedures to identify, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **remember** requires students to recall and understand the connection between the rate of change and slope of a line. Students use their understanding of slope from tables and graphs to explore slope with equations. Students analyze a variety of linear equations and their graphs and generalize connections (8-1.7) between the coefficient of x, the direction of the line and the steepness of the line. Students generalize mathematical statements (8-1.5) about these connections using correct and clearly written or spoken words, variables, and notation (8-1.6). After having sufficient experiences with slope from equations, students use this understanding to identify slope from a graph, table and equation.

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The following example of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Simplify and solve.

a. $36 = y - 5y - 12$

36 = - 4y – 12
48 = - 4y
- 12 = y

b. $5 - 2 (y - 5) = 7$

5 – 2y + 10 = 7
- 2y + 15 = 7
- 2y = - 8
y = 4

2. Explain how to solve 3 (9 + 4a) – 19 = 32

Answer: Begin by using the distributive property to simplify the parentheses. The result is 27 + 12a – 19 = 32.

Next, combine 27 and – 19. The result is 8 + 12a = 32.

Subtract 8 (add – 8) to both sides. The result is 12a = 24

Divide both sides by 12. a = 2

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3. Together, Donald, Yolanda, and Iris made 27 birdhouses for a school fair. Yolanda made the fewest number of birdhouses. Donald made one more than Yolanda, and Iris made one more than Donald. Write and solve and equation to find out how many birdhouses each of them made. Let n = the number of birdhouses Yolanda made.

Answer:
n = Yolanda
$n + 1$ = Donald
$n + 2$ = Iris

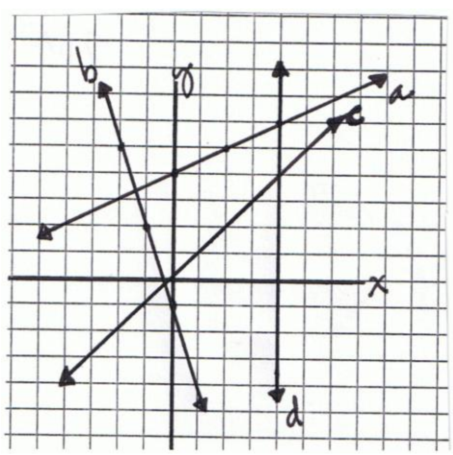
$n + n + 1 + n + 2 = 27$

$3n + 3 = 27$

$3n = 24$

$n = 8$
Yolanda made 8 birdhouses, Donal made 9, and Iris made 10.

4. Consider the following graph to answer the questions below:



a. What is the x-intercept of line d?
b. What is the x-intercept of line c?
c. What is the y-intercept of line a?

5. How do you find the y-intercept of any equation? Explain your reasoning.

6. How do you find the x-intercept of any equation? Explain your reasoning.

7. Graph the following or use a table or algebraic method to find the x and y-intercepts.

   a) $y = 3x + 4$

   b) $y = -\frac{1}{2}x - 1$

   c) $y = -5x + 2$

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8. Match the graphs to the corresponding equations.

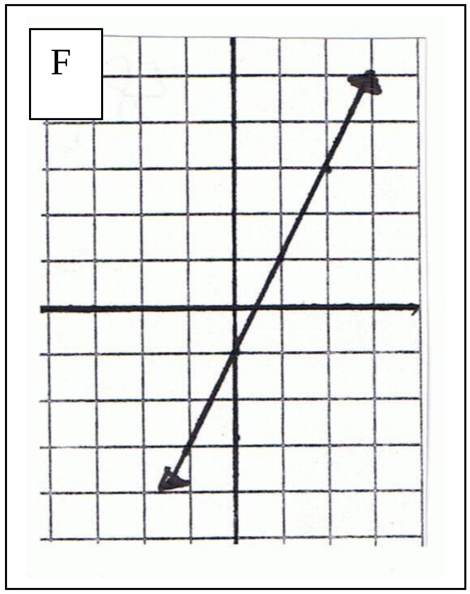
a) $y = x - 1$

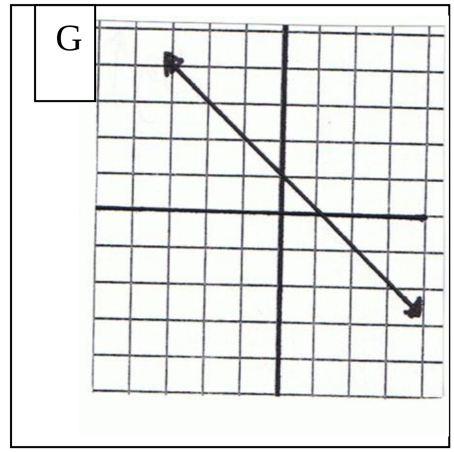
b) $y = x + 3$

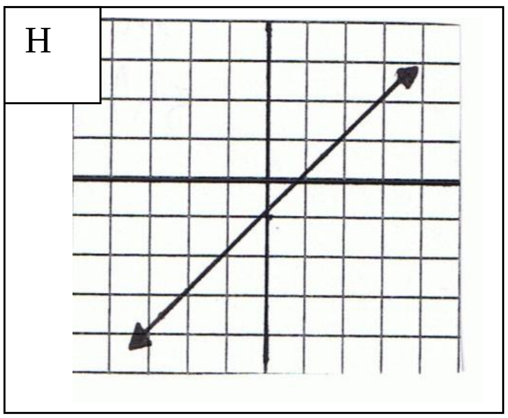
c) $y = 2x - 1$

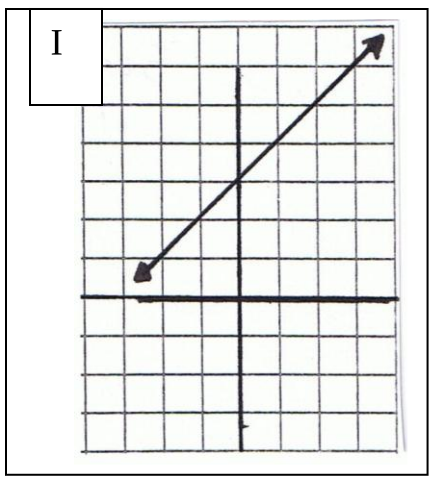
d) $y = -x + 1$

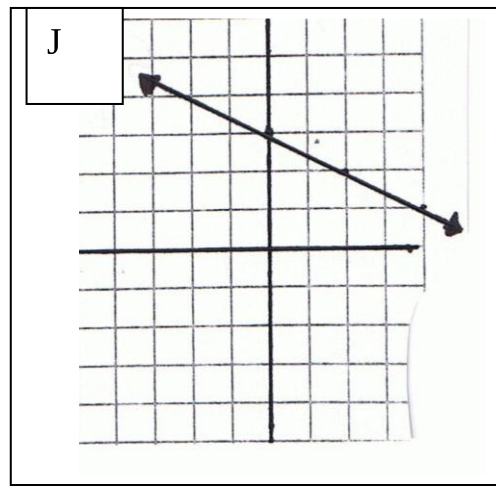
e) $y = -\frac{1}{2}x + 3$











9. a) Graph the line y = 3.
   b) What does the line look like?
   Answer: *It is a horizontal line passing through 3 on the y-axis.*

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10. For the chart below, make three checks.

    a. Determine if the line is above the origin, below the origin, or through the origin. What variable determines the position on the y-axis?

    b. Determine if the line is uphill or downhill. What variable or what about the variable determines this?

    c. Determine if the line is steeper or flatter in reference to the line y = x. What variable or what about the variable determines this?

y = mx + b	**above the origin**	**below the origin**	**through the origin**	**uphill**	**downhill**	**steep**	**flat**
$y = 3x + 1$							
$y = -2x - 4$							
$y = \frac{1}{4}x + 7$							
$y = \frac{3}{5}x - 2$							
$y = 5x$							
$y = -4x - 9$							
$y = 7x - 1$							
$y = -\frac{1}{3}x$							
$y = -5x + \frac{3}{2}$							

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Answers:

$y = mx + b$	above the origin	below the origin	through the origin	uphill	downhill	steep	flat
$y = 3x + 1$	X			X		X	
$y = -2x - 4$		X			X	X	
$y = \frac{1}{4}x + 7$	X			X			X
$y = \frac{3}{5}x - 2$		X		X			X
$y = 5x$			X	X		X	
$y = -4x - 9$		X			X	x	
$y = 7x - 1$		X		X		X	
$y = -\frac{1}{3}x$			X		X		X
$y = -5x + \frac{3}{2}$	X				X	X	

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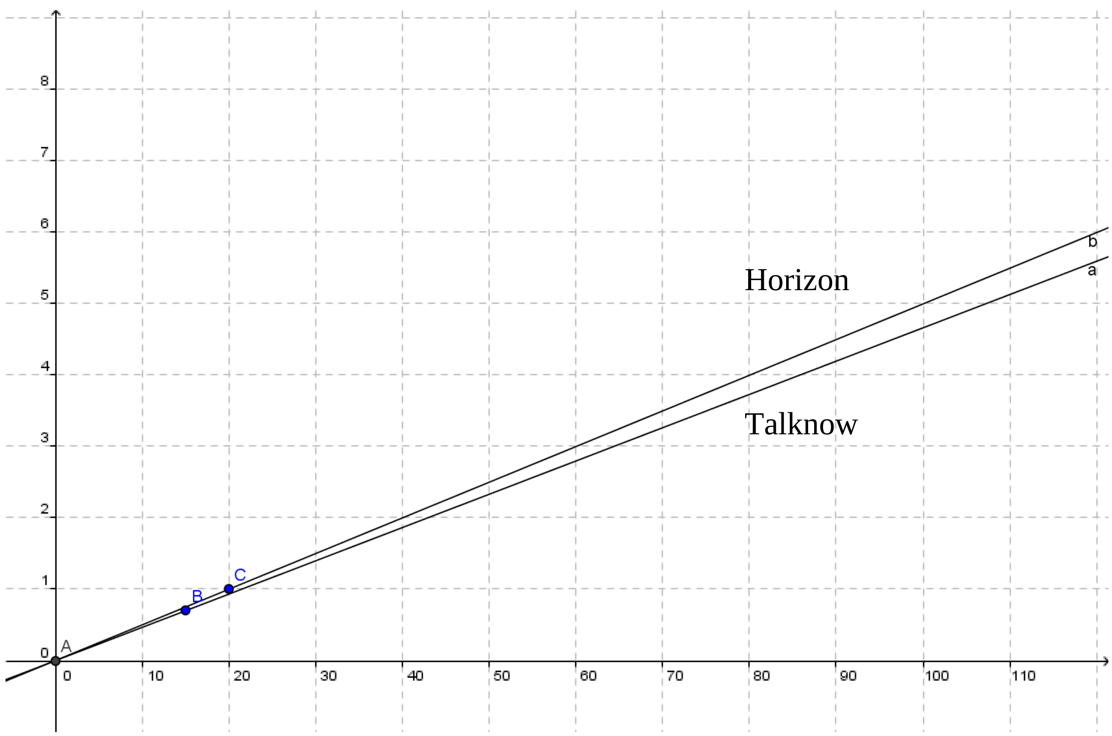
11. Talknow phone company charges $.70 for every 15 minutes. Horizon phone company charges $1.00 for 20 minutes. Which company is offering the cheaper rate?

Answer: Table of values.

Talknow	x	15	30	45	60	
Plan	y	.70	1.4	2.1	2.8	

Horizon	x	20	40	60	80	
Plan	y	1	2	3	4	



MODULE

2-3

Patterns, Relationships and Functions

This module addresses the following indicators:

8-3.5 Classify relationships between two variables in graphs, tables, and/or equations as either linear or nonlinear.

This module contains 1lesson. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation student need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

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I. Planning the Module

Continuum of Knowledge

In 7th grade, students analyze tables and graphs to describe the rate of change among quantities and to determine if the rate of change is constant (7-3.2). They gained an understanding of slope as constant rate of change (7-3.3).

In 8th grade, students gain the ability to classify relationships between two variables in graphs, tables, and/or equations as linear or nonlinear (8-3.5).

In Elementary Algebra, students use this understanding to classify a variation as direct (linear) or inverse (EA-3.6).

Key Concepts/Key Terms

*These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

∗Linear
∗Nonlinear
∗Constant
∗Coefficient
∗Variable
*Multiple representations

II. Teaching the Lesson(s)

1. Teaching Lesson A: Linear or Non-Linear

While the introduction of slope takes place in 7th grade, it is in 8th grade that students gain the ability to identify and classify relationships between two variables in graphs, tables, and/or equations as linear or nonlinear and to compare and contrast the properties of each. Students in the 8th grade should have the opportunity to discover that a linear relationship will have a constant rate of change. In regards to equations, students must learn that a linear equation has no operations other than addition, subtraction, and multiplication of a variable by a constant, that the variables cannot be multiplied or appear in the denominator, and that a linear equation cannot contain variables with exponents other than one. If a relationship does not fit these criteria, students should recognize the relationship as nonlinear.

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It is in the 8th grade that students are first exposed to the slope-intercept form of a linear equation ($y=mx+b$), with m being the slope and b the y-intercept. They learn that a linear function is a function whose ordered pairs satisfy a linear equation and that any linear function can be written in slope-intercept form. Because equations are commonly written in this form, students should be able to recognize anything that can be written in slope-intercept form as linear.

Whether using a two variable table, a coordinate plane graph, or an equation, the goal for 8th graders is to be able to classify data as linear or nonlinear. Each model should be seen as a tool to support the conclusions drawn from another model. The relationship between variables determined by students with one format should be confirmed easily by using the other two methods for representing the same data.

Making connections between the numeric, pictorial, and algebraic models should be the center of instruction designed to develop this concept and is imperative for a strong foundation as students progress to Algebra I. The ability to recognize patterns in any form and to classify them will increase students' understanding of linear functions, a major topic of Algebra I.

For this indicator, it is **essential** for students to:

- Understand the structure of a linear equation i.e. constant multiplied by a variable (+ or –) a constant or constant (+ or –) a constant multiplied by a variable
- Understand that linear equations do not have variables in the denominator
- Understand that a linear equation cannot contain variables with exponents other than one.
- Recognize that if a relationship does not fit these criteria, the relationship is nonlinear.
- Determine if a given set of data or equation is linear.
- Make connections between numeric, pictorial, and algebraic models.

For this indicator, it is **not essential** for students to:
None noted

a. Indicators with Taxonomy

8-3.5 → Classify relationships between two variables in graphs, tables and/or equations as either linear or nonlinear. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

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b. Introductory Lesson A: Linear or Non-Linear

Materials Needed: Graphing Calculator

Students should be given the following sets of equations to produce graphs and tables. *You will need to help them organize their data/equations/tables and graphs.* They should use a graphing calculator to do this. *If one is not available, see the technology section for a link to one online.*

We are not testing to see if they can graph "by hand," we are classifying relationships and discovering what yields linear and non-linear relationships. DO ONE SET AT A TIME, FOLLOWING EACH SET WITH THE SERIES OF QUESTIONS BELOW IT.

Set A	Set B	Set C	Set D
$y = x$	$y = x^2$	$y = x^3$	$y = \frac{1}{x}$
$y = x + 4$	$y = 2x^2 + 3$	$y = 2x^3 - 1$	$y = \frac{1}{x} - 1$
$y = 2x$	$y = \frac{1}{2} x^2$		$y = \frac{2}{x} + 1$
$y = 2x + 2$	$y = \frac{1}{4} x^2 - 1$		
$y = \frac{1}{2} x$			
$y = \frac{1}{2} x - 3$			
QUESTIONS TO ADDRESS CONCEPT OR PATTERNS			
Set A	Set B	Set C	Set D
What do you notice about exponents and the "x" variable?	What do you notice about exponents and the "x" variable?	What do you notice about exponents and the "x" variable?	What do you notice about the location of the "x" variable?
What is the shape of the graph?	What is the shape of the graph?	What is the shape of the graph?	What is the shape of the graph?
How does the coefficient of x affect the shape of the graph?	How does the coefficient of x affect the shape of the graph?	How does the coefficient of x affect the shape of the graph?	
How does the constant (such as 2 in the equation $y = 2x + 2$) affect the shape of the graph?	How does the constant (such as 3 in the equation $y = 2x^2 + 3$) affect the shape of the graph?	How does the constant (such as -1 in the equation $y = 2x^3 - 1$) affect the shape of the graph?	How does the constant (such as -1 in the equation $y = \frac{1}{x} - 1$) affect the shape of the graph?

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Based on the findings in each set, which set(s) would you classify as linear? Explain your reasoning and complete the characteristics chart below. **Teacher note**: In regards to equations, students must learn that a linear equation has no operations other than addition, subtraction, and multiplication of a variable by a constant, that the variables cannot be multiplied or appear in the denominator, and that a linear equation cannot contain variables with exponents other than one. If a relationship does not fit these criteria, students should recognize the relationship as nonlinear.

Linear Characteristics	***Non-Linear Characteristics***
See above note for possible responses	***See above note for possible responses***

c. Misconceptions/Common Errors

Students may struggles with the exponent of the variable being one because it is not visible.

d. Additional Instructional Strategies

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

Graphing calculators should be used.

- http://www.shodor.org/interactivate/activities/Graphit/
- http://www.coolmath.com/graphit/

f. Assessing the Lesson

Some sample questions:

What patterns are you noticing?
What connections are you making?

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III. Assessing the __Module__

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

1. Make matches of three – one from each section (one graph, one equation, one table)

SECTION 1

f) $y = x^2 + 2$

g) $y = \frac{1}{x} + 2$

h) $y = \frac{1}{2}x^2 + 2$

i) $y = 2x + 2$

j) $y = \frac{1}{2}x + 2$

SECTION 2

k)

X	Y
-2	1
-1	1.5
0	2
1	2.5
2	3

l)

X	Y
-2	6
-1	3
0	2
1	3
2	6

m)

X	Y
-2	4
-1	2.5
0	2
1	2.5
2	4

n)

X	Y
-2	1.5
-1	1
0	Und
1	3
2	2.5

o)

X	Y
-2	-2
-1	0
0	2
1	4
2	6

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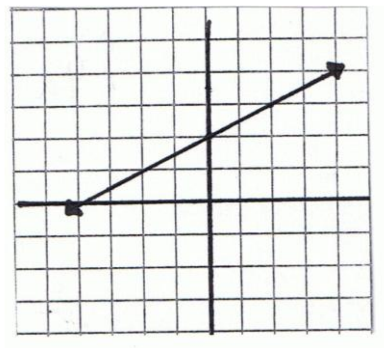
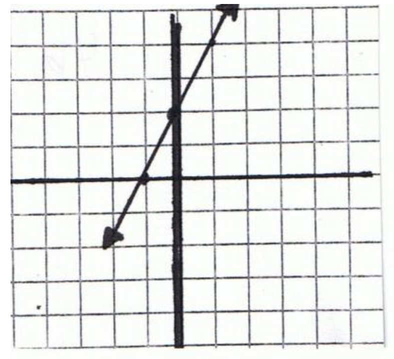
SECTION 3

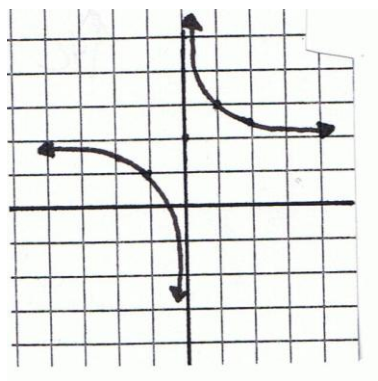
a)



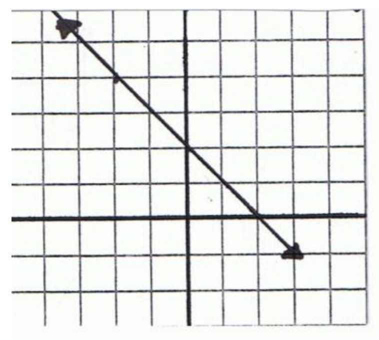
b)



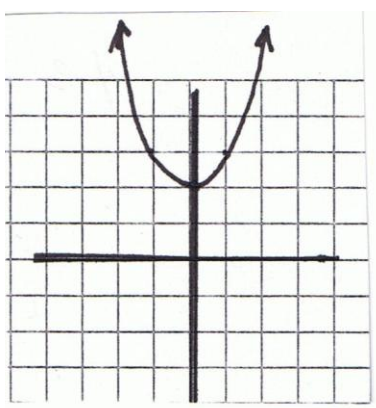
c)



d)



e)



ANSWERS: e,f,l c,g,n d,h,m b,i,o a,j,k

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2. Refer to Number 1 for the following:
    a. Which of the graphs, equations and tables represent a linear relationship?
    b. Which are nonlinear?
    c. What are you basing your decisions on?
    d. How do you know by looking at the graph?
    e. How do you know by looking at the equation?

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