SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

Content Area	**Eighth Grade Math**

Third Nine Weeks

Standards/Indicators Addressed:

Standard 8-4: The student will demonstrate through the mathematical processes an understanding of the Pythagorean theorem; the use of ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate plane; and the effect of a dilation in a coordinate plane.

8-4.3* Apply a dilation to a square, rectangle, or right triangle in a coordinate plane. (C3)
8-4.4* Analyze the effect of a dilation on a square, rectangle, or right triangle in a coordinate plane. (B4)

Standard 8-5: The student will demonstrate through the mathematical processes an understanding of the proportionality of similar figures; the necessary levels of accuracy and precision in measurement; the use of formulas to determine circumference, perimeter, area, and volume; and the use of conversions within and between the U.S. Customary System and the metric system.

8-5-1* Use proportional reasoning and the properties of similar shapes to determine the length of a missing side. (C3)
8-5-2* Explain the effect on the area of two-dimensional shapes and on the volume of three-dimensional shapes when one or more of the dimensions are changed. (B2)
8-5.3* Apply strategies and formulas to determine the volume of the three-dimensional shapes cone and sphere. (C3)

8-5.4* Apply formulas to determine the exact (*pi)* circumference and area of a circle. (C3)

8-5.5* Apply formulas to determine the perimeters and areas of trapezoids. (C3)

8-5-6* Analyze a variety of measurement situations to determine the necessary level of accuracy and precision. (B4)

Standard 8-6: The student will demonstrate through the mathematical processes an understanding of the relationships between two variables within one population or sample.

8-6.1* Generalize the relationship between two sets of data by using scatterplots and lines of best fit. (B2)
8-6.2* Organize data in matrices or scatterplots as appropriate. (B4)
8-6.8* Interpret graphic and tabular data representations by using range and the measures of central tendency (mean, median, and mode). (B2)

*** These indicators are covered in the following 5 Modules for this Nine Weeks Period.**

Module 3-1 Plane and Proportional Reasoning

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines
Module 3-1 Lesson A: Dilations **8-4.3** Apply a dilation to a square, rectangle, or right triangle in a coordinate plane. (C3) **8-4.4** Analyze the effect of a dilation on a square, rectangle, or right triangle in a coordinate plane. (B4)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) Textbook Correlations – see Appendix A	See Instructional Planning Guide Module 3-1 "Introductory Lesson A"	See Instructional Planning Guide Module 3-1 "Lesson A 'Assessing the Lesson'"

Module 3-2 Perimeter, Circumference, Area and Volume			
Module 3-2 Lesson A: Changing Dimensions **8-5.2** Explain the effect on the area of two-dimensional shapes and on the volume of three-dimensional shapes when one or more of the dimensions are changed. (B2)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) **Textbook Correlations - See** Appendix A	See Instructional Planning Guide Module 3-2 "Introductory Lesson A"	See Instructional Planning Guide Module 3-2 "Lesson A 'Assessing the Lesson'"
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 3-2 Lesson B: Volume of Cones and Spheres **8-5.3** Apply strategies and formulas to determine the volume of the three-dimensional	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching	See Instructional Planning Guide Module 3-2, "Introductory Lesson B"	See Instructional Planning Guide Module 3-2 "Lesson B 'Assessing the Lesson'"

shapes cone and sphere. (C3)	Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) **Textbook Correlations Appendix A**		
Module 3-2 Lesson C: Circumference and Area of Circles **8-5.4** Apply formulas to determine the exact (*pi)* circumference and area of a circle. (C3)		See Instructional Planning Guide Module 3-2 "Introductory Lesson C"	See Instructional Planning Guide Module 3-2 "Lesson C 'Assessing the Lesson'"
Module 3-2 Lesson D: Trapezoids **8-5.5** Apply formulas to determine the perimeters and areas of trapezoids. (C3)		See Instructional Planning Guide Module 3-2 "Introductory Lesson D"	See Instructional Planning Guide Module 3-2 "Lesson D 'Assessing the Lesson'"
Module 3-2 Lesson E: Accuracy and Precision 8-5.6 Analyze a variety of measurement situations to determine the necessary level of accuracy and precision. (C4)		See Instructional Planning Guide Module 3-2 "Introductory Lesson E"	See Instructional Planning Guide Module 3-2 "Lesson E 'Assessing the Lesson'"

Module 3-3 Proportional Reasoning			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 3-3 Lesson A: Similar Figures **8-5.1** Use proportional reasoning and the properties of similar shapes to determine the length of a missing side. (C3)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) **Textbook Correlations Appendix A**	See Instructional Planning Guide Module 3-3 "Introductory Lesson A"	See Instructional Planning Guide Module 3-3 "Lesson A 'Assessing the Lesson'"
Module 3-4 Collection and Representation			
Module 3-4 Lesson A:	NCTM's Online Illuminations http://illuminations.nctm.org/	See Instructional Planning Guide Module 3-4 "Introductory Lesson A"	See Instructional Planning Guide

Matrices **8-6.2** Organize data in matrices or scatterplots as appropriate. (B4) **Module 3-4 Lesson B: Scatterplots** **8-6.2** Organize data in matrices or scatterplots as appropriate. (B4)	NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) **Textbook Correlations** See Appendix A		Module 3-4 "Lesson A 'Assessing the Lesson'"
Module 3-5 Data Analysis			
Indicator	**Recommended Resources**	**Suggested Instructional Strategies**	**Assessment Guidelines**
Module 3-5 Lesson A: Generalizing Relationships Using Scatterplots and Lines of Fit **8-6.1** Generalize the relationship between two sets of data by using scatterplots and lines of	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th	See Instructional Planning Guide Module 3-5 "Introductory Lesson A"	See Instructional Planning Guide Module 3-5 "Lesson A 'Assessing the Lesson'"

best fit. (B2)	Edition, John Van de Walle		
Module 3-5 Lesson B: Interpreting Data Using Range and the measures of central tendency (mean, median, and mode). **8-6.8** Interpret graphic and tabular data representations by using range and the measures of central tendency (mean, median, and mode). (B2)	www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) **Textbook Correlations Appendix A**	See Instructional Planning Guide Module 3-5 "Introductory Lesson B"	See Instructional Planning Guide Module 3-5 "Lesson B 'Assessing the Lesson'"

MODULE

3-1

Plane and Proportional Reasoning – *Part II*

This module addresses the following indicators:

8-4.3 Apply a dilation to a square, rectangle, or right triangle in a coordinate plane.
8-4.4 Analyze the effect of a dilation on a square, rectangle, or right triangle in a coordinate plane.

This module contains 1 lesson. These lessons are INTRODUCTORY ONLY. Lessons in S3 begin to build the conceptual foundation student need. ADDITIONAL LESSONS will be required to fully develop the concepts.

I. Planning the Module

Continuum of Knowledge

Indicator 8-4.3 Apply a dilation to a square, rectangle, or a right triangle in a coordinate plane.

In sixth grade the students applied strategies to find the missing vertices of various polygons (6-4.2).
In eighth grade, students apply a dilation to a square, rectangle, or a right triangle in a coordinate plane (8-4.3)

Indicator 8-4.4 Analyze the effect of a dilation on a square, rectangle, or right triangle in a coordinate plane.

In sixth grade the students applied strategies to find the missing vertices of various polygons (6-4.2).
In eighth grade, students apply a dilation to a square, rectangle, or a right triangle in a coordinate plane (8-4.3)

Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

*Dilation
*Coordinate Plane
*Scale Factor
*Multiple
*Ordered Pairs
*Image
*Center of dilation
*Proportional Reasoning
*Notation
A'- A prime and denotes the image of the original or pre-image.

II. Teaching the Lesson(s)

A dilation is defined as a transformation that copies, enlarges, or reduces a figure or image. Students should be given practical opportunities to investigate the process of applying dilation to various polygons. One strategy would be to have students construct a square, rectangle or a right triangle in a coordinate plane and record the original coordinate points next to the vertices of the shape. Students should then multiply these coordinate pairs by a number (this number is called the scale factor) to create a similar figure. The new shape should then be constructed using the new coordinate pairs and the new coordinates recorded next to the vertices of the new shape. By doing so, students have performed a dilation on the figure. This provides the opportunity to discuss similarities and differences between the old and new ordered pairs and the shapes they produced to assist in the development of mental models. Please note that if the scale factor of the dilation is greater than 1 the image will be bigger than the original. If the scale factor of the dilation is less than 1 but greater than 0, the image will be smaller than the original. Once students have a deeper understanding of the effect of a dilation, the teacher can then move to more abstract examples by giving them only the coordinates of the vertices of a polygon and having them find the coordinates of a dilation of the given polygon, given the scale factor.

8-4.3 → Apply a dilation to a square, rectangle, or right triangle in a coordinate plane.

For this indicator, it is **essential** for students to:

- Understand the meaning of dilation
- Recall the characteristics of a square, rectangle and right triangle
- Apply a dilation factor to enlarge or reduce squares, rectangles, and right triangles.
- Multiply the ordered pairs by the dilation factor.
- Use the new coordinates to graph the image of the pre-image.
- Understand that if the scale factor of the dilation is greater than 1, the image will be bigger than the original.
- Understand if the scale factor of the dilation is less than 1 but greater than 0, the image will be smaller than the original.
- Understand that the image and pre-mage may overlap or one may be inside the other.
- Use appropriate notation to denote the image and pre-image

For this indicator, it is **not essential** for students to:

- Apply a dilation factor to enlarge or reduce irregular shapes.

8-4.4 → Analyze the effect of a dilation on a square, rectangle, or right triangle in a coordinate plane.

For this indicator, it is **essential** for students to:

- Name the new coordinates of a polygon when given the coordinates and the scale factor.
- Understand that a fractional scale factor will reduce the pre-image and a scale factor greater that 1 will enlarge the pre-image.
- Know that a scale factor of 1 will copy the image.
- Analyze the impact the scale factor has on the area of the polygon.
- Understand that the image and pre-image are similar shapes.
- Use proportional reasoning to analyze relationships

For this indicator, it is **not essential** for students to:

- Apply a dilation to irregular shapes.

1. Teaching Lesson A: *Dilations*

a. Indicators with Taxonomy

See the following Sample:

8-4.3 → Apply a dilation to a square, rectangle, or right triangle in a coordinate plane. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

8-4.4 → Analyze the effect of a dilation on a square, rectangle, or right triangle in a coordinate plane. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

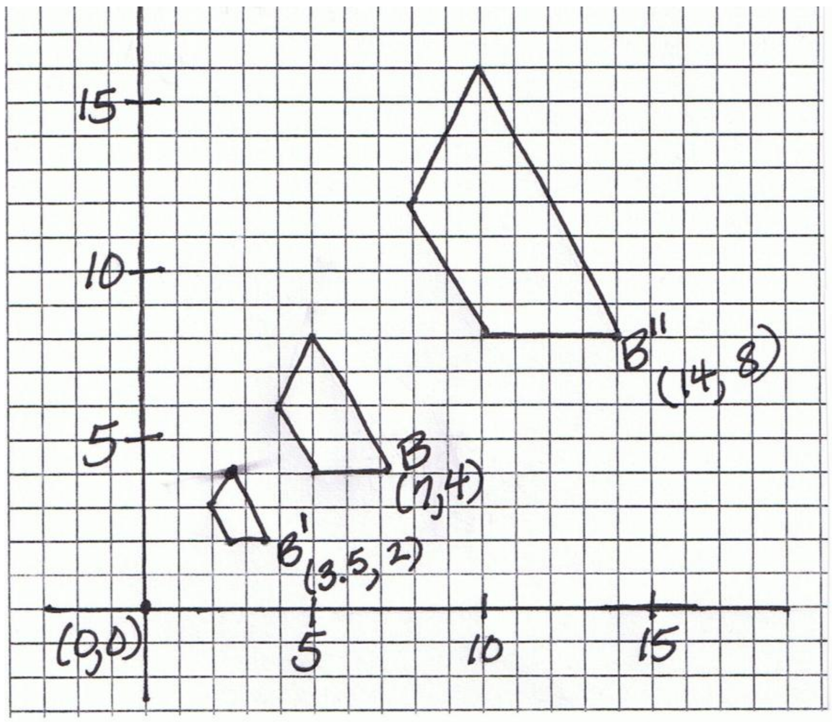
b. Introductory Lesson A: *Dilations*

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. *Teaching Student Centered Mathematics: Grades 5-8*.

Students begin with a four-sided figure in the first quadrant. They then make a list of the coordinates and make a new set of coordinates by multiplying each of the original coordinates by 2. They plot the resulting shape. What is the result?
Now students multiply the original coordinates by ½ and plot that shape. What is the result?

Next, students draw a line from the origin to a vertex of the largest shape on their paper. Repeat for one or two additional vertices and ask for observations.

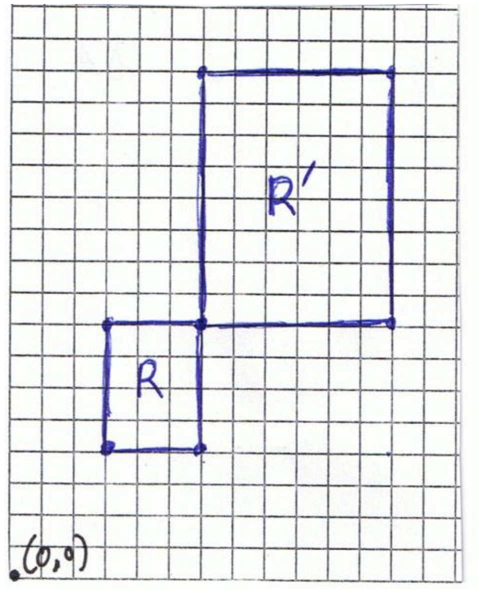
What might it look like?



<u>Another consideration</u>:

What would have to be done to the rectangle R formed by (3,4), (6,4), (3,8) and (6,8) to increase it's area by 4 times? (Answer: Increase each dimension (length and width) by 2 times.)

How would that be represented by a dilation (what would the coordinates of the new figure? *(Answer: The x coordinate or the length would be increased by 2 times and the y coordinate or the width would be increased by 2 times resulting in Figure R' which has vertices (6,8), (12,8), (6,16), and (12,16)). See Figure provided:*



What dilation would give me an area increased by 9 times the original figure R? *(Answer: the coordinates would be (9,12), (18,12), (9,24) and (18,24). Found by multiplying each coordinate by 3, therefore 3 x 3 = 9.)*

c. Misconceptions/Common Errors

Indicator 8-4.3 Apply a dilation to a square, rectangle, or a right triangle in a coordinate plane.

- Students sometimes have difficulty plotting ordered pairs, by wanting to reverse the x and y axis "moves." If this is the case, location activities should precede or be used in a differentiated lesson for those students having difficulty. Going back to the basics may be needed – creating pictures based on a series of ordered pairs (a connect the dots of sorts) or having students design a picture, generate the list of ordered pairs that connect to produce the picture, trade with another student and create their picture from the list of ordered pairs is a fun way to practice.
- Students often think that all dilation factors will always enlarge the pre- image. They do not connect the idea that multiplying by a fraction or decimal less than one but greater than zero reduces the size and that multiplying by a whole number 1 and greater enlarges the image.

Indicator 8-4.4 Analyze the effect of a dilation on a square, rectangle, or right triangle in a coordinate plane.

- Students often think that dilations will always enlarge the pre-image. They do not connect the idea that multiplying by a fraction or decimal less than one but greater than zero reduces the size and that multiplying by a whole number 1 and greater enlarges the image.
- Student may not understand that if the anchor point or the center of dilation is a point within the pre-image the dilation factor would not be applied to that point.

d. Additional Instructional Strategies/Differentiation –

Indicator 8-4.3 Apply a dilation to a square, rectangle, or a right triangle in a coordinate plane.

- Students should be given several opportunities to investigate the process of applying dilation to various polygons. A dilation is defined as a transformation that copies, enlarges, or reduces an image or polygon. One strategy would be to have students construct a square, rectangle or a right triangle in a coordinate plane and record the original coordinate points next to the vertices of the shape. Students should then multiply these coordinate pairs by a number (this number is called the scale factor) to create a similar figure. The new shape should then be constructed using the new coordinate pairs and the new coordinates recorded next to the vertices of the new shape. By doing so, students have performed a dilation on the figure. This provides the opportunity to discuss similarities and differences between the old and new ordered pairs and the shapes they produced to assist in the development of mental models. Once students have a deeper understanding of the effect of a dilation, the teacher may want to move to more abstract examples by giving them only the coordinates of the vertices of a polygon and having them find the coordinates of a dilation of the given polygon, given the scale factor.
- Geoboards could be used when first create dilations

Indicator 8-4.4 Analyze the effect of a dilation on a square, rectangle, or right triangle in a coordinate plane.

- This is an extension of 8-4.3. When students are provided with examples of dilations, they should be able to identify them as either being a dilation or as not being a dilation. As part of this indicator, students also explore the effect of a dilation on the area of the figures.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- Specific Resource:
  - http://www.mathopenref.com/dilate.html- interactive dilation and describes another technique that can be used to create dilations.
- illuminations.nctm.org (Lessons, Activities and Related WebLinks)
- nlvm.usu.edu (National Library of Virtual Manipulatives)
- Oneplacesc.org (ETV Streamline and more!)
- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS

How do the lengths of the sides and the areas of the figure compare when the coordinates are multiplied by 2? Or ½?

What if they are multiplied by 3?

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

Indicator 8-4.3 Apply a dilation to a square, rectangle, or a right triangle in a coordinate plane.

The objective of this indicator is to apply which is in the "apply procedural knowledge" cell of the Revised Taxonomy. To apply is to carry out or use a procedure in various situations. The learning progression to **apply** requires students to recall and understand the meaning of dilation. They also recall the characteristics of squares, rectangles and right triangles. Students explore examples where they multiply by several different scale factors. They generalize connections (8-1.7) among examples where the factors are between zero and one or greater than one. They generalize mathematical statements (8-1.5) about these relationships using correct and clearly written or spoken words (8-1.6). They use their understanding of the relationships to apply dilations to other examples and explain and justify their answers to their classmates and teacher.

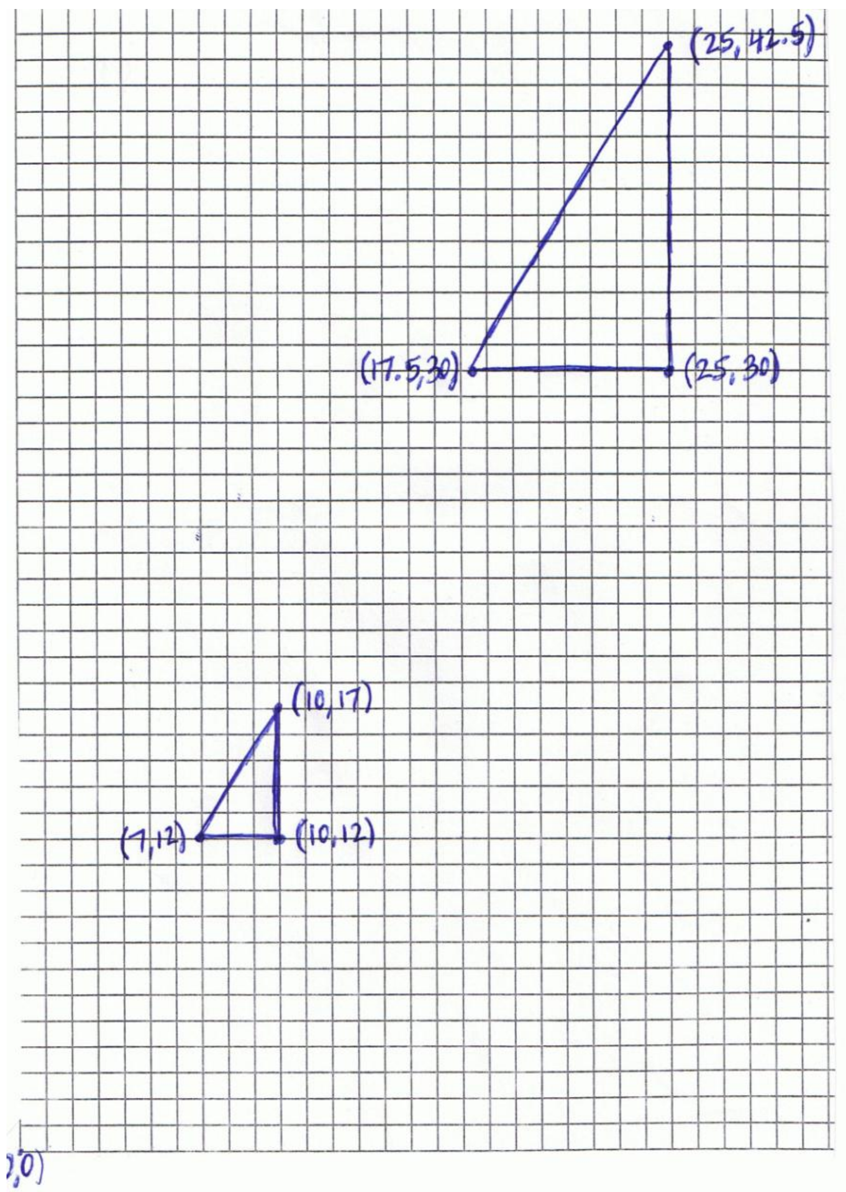
Indicator 8-4.4 Analyze the effect of a dilation on a square, rectangle, or right triangle in a coordinate plane.

The objective of this indicator is to analyze which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To analyze means to break material down into its constituent parts and determine how the parts relate to each other and the overall purpose; therefore, students examine the coordinates and area of figures (parts) and determine how their relate to dilations (purpose). The learning progression to **analyze** requires students to recall the meaning of dilations and understand how to perform dilations. They recognize the relationships among scale factor and proportional relationship between the pre-image and image. They explore a variety of examples and generalize connections (8-1.7) among those examples in order to generalize mathematical statements (8-1.5) related to the effect of dilations. Students use deductive reasoning (8-1.3) to move from generalized statements to specific relationships in order to describe the effects.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Consider the closed figure with the following vertices: (10,17), (10,12), and (7,12). Label this figure T. What scale would allow this figure to dilate to Figure T' with the following vertices: (25, 42.5), (25,30), and (17.5,30)?

(Answer: Scale Factor of 2.5). See Drawing Provided:



2. Explain your answer to the following: Can the area of a figure be increased by 2 times (doubled) by using whole number scale factors? *You may want to create a figure on a coordinate grid.*

(Answer: No, if the x coordinate was increased by 2 times and the y coordinate was increased by 2 times, the area was quadrupled -meaning the length x width change was 2 x 2 or 4 times. When the area is doubled, then the scale factor of the x coordinate and the y coordinate must have a product of 2 and only $\sqrt{2}$ x $\sqrt{2}$ will give you a product of 2. And for a dilation, the two scale factors must be the same number. Remember by changing one scale factor and not the other, the resulting figure will be a translation of the other or some other transformation.

MODULE

3-2

Perimeter, Circumference, Area and Volume

This module addresses the following indicators:

8-5.2 Explain the effect on the area of two-dimensional shapes and on the volume of three-dimensional shapes when one or more of the dimensions are changed.
8-5.3 Apply strategies and formulas to determine the volume of the three-dimensional shapes cone and sphere.
8-5.4 Apply formulas to determine the exact (*pi)* circumference and area of a circle.
8-5.5 Apply formulas to determine the perimeters and areas of trapezoids.
8-5.6 Analyze a variety of measurement situations to determine the necessary level of accuracy and precision.

This module contains 5 lessons. These lessons are INTRODUCTORY ONLY. Lessons in S3 begin to build the conceptual foundation student need. ADDITIONAL LESSONS will be required to fully develop the concepts.

I. Planning the Module

Continuum of Knowledge

Indicator 8-5.2 Explain the effect on the area of two-dimensional shapes and on the volume of three dimensional shapes when one or more of the dimension are changed.

> In fourth grade, students generated strategies to determine the area of rectangles and triangles (4-5.5). They extend this knowledge to apply formulas to determine area of rectangles, triangles and parallelograms in fifth grade (5-5.4). Also in fifth grade, students apply strategies and formulas to determine volume of rectangular prisms (5-5.5). In seventh grade, the students apply strategies and formulas to determine volume of prism, pyramid, and cylinder (7-5.2).
>
> In eighth grade, the students will explain the effect on the area of two-dimensional shapes and on the volume of three dimensional shapes when one or more of the dimension are changed (8-5.2).
>
> In Geometry, the students will analyze how changes in dimensions affect the perimeter and extend the list of geometric figures to include quadrilaterals and other regular polygons (G-4.4). Spheres are added when analyzing the changes in dimensions and how it affects the volume of objects (G-7.3).

Indicator 8-5.3 Apply strategies and formulas to determine the volume of the three-dimensional shapes cone and sphere.

> In fifth grade, students applied strategies and formulas to determine the volume of a rectangular prism (5-5.5). In seventh grade, students applied formulas to find the volume of prisms, pyramids, and cylinders (7-5.2).
>
> In 8th grade, students apply strategies and formulas to determine the volume of the three-dimensional shapes cone and sphere (8-5.3).
>
> In Geometry, students extend their knowledge of volume to include hemispheres and composite objects (G-7.2). Students will also analyze how changes in dimensions affect the volume of a geometric object (G-7.3).

Indicator 8-5.4 Apply formulas to determine the exact (pi) circumference and area of a circle.

> In sixth grade, students applied strategies to find the circumference and area of a circle using an approximation of pi (3.14 and 22/7) (6-5.2). They also learned the relationship among circumference,

diameter, and radius of a circle (6-5.1). In seventh grade, the students applied strategies and formulas to determine the surface area and volume of the three dimensional shapes prisms, pyramid and cylinder (7-5.2).

In eighth grade, students should apply formulas to determine the exact (pi) circumference and area of a circle.

In Geometry, students will carry out a procedure to compute the circumference and area of circles (G-5.1 and G-5.2). The students will also analyze how a change in the radius affects the circumference or area of a circle (G-5.3).

Indicator 8-5.5 Apply formulas to determine the perimeters and areas of trapezoids.

In fifth grade, students applied formulas to determine the perimeters and areas of triangles, rectangles and parallelograms (5-5.4). In sixth grade, students found the perimeter and area of irregular shapes (6-5.5). In seventh grade, students generate strategies to determine the perimeters and areas of trapezoids (7-5.3).

In eighth grade, students apply formulas to determine the perimeters and areas of trapezoids (8-5.5).

In Geometry, students will carry out a procedure to compute both the perimeter and area of quadrilaterals, regular polygons, and composite figures (G-4.1 and G-4.2). They will also use their knowledge of quadrilaterals to analyze how changes in dimensions affect the perimeter and area (G-4.4).

Indicator 8-5.6 Analyze a variety of measurement situations to determine the necessary level of accuracy and precision.

Students have been taught to use units of measurement since the 1st grade. Levels of precision and accuracy are new skills to be introduced to 8th grade students for the first time.

Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

*Area
*Volume
*Base
*Side

*Pi
*Circumference
*Perimeter
*Accuracy
*Precision
*Review of the different units of measurement (Metric and U.S. Customary)
*Perimeter
*Height
*Diameter
*Radius
*Volume
*Three-Dimensional Figures
*Geometric figures (rectangles, triangles, parallelograms, rectangular prism and pyramid, triangular prism and pyramid, cylinder)

*Sphere and Cone (review characteristics)

*Area of Base

II. Teaching the Lessons

In order for students to explain how the area of a two-dimensional shape is affected when dimensions change, they need experiences with models and representations drawn on a grid. Computer software and the use of a SmartBoard would be beneficial in demonstrating the change in the lengths of sides of the shape and help with the students' understanding of the affects of the changes.

For students to apply strategies and formulas to determine volume of cones and spheres, students need to have a conceptual understanding of volume. Students should also use their prior knowledge of finding a volume of a cylinder (MS 7-5.2) when applying a formula to find the volume of a cone. To apply strategies to determine the volume of a sphere is a little more complicated. Students could use the formula for a pyramid to determine the formula for volume of a sphere. Another comparison used for finding the volume of a sphere can be made with the volume of a cylinder. Students should apply these formulas in problem solving situations to determine the volumes of cones and spheres.
In eighth grade, students are extending the learning and applying the formulas to find the exact circumference and area of a circle. Students need to be able to work with both diameter and radius when finding area of a circle. Students should be able to rely on their prior knowledge of circumference and area of a circle to now apply the formulas.

In eighth grade, students are using that understanding to apply the perimeter and area formulas for a trapezoid. Students should discuss the formulas and relate the formula components to a representation of a trapezoid.

Students should develop an understanding of precision and accuracy. Students should discuss and examine how objects are measured and how the results are expressed. Students need to understand that a measurement is precise only to one-half of the smallest unit used in a measurement. When measuring, students need to determine to what degree of accuracy is needed.

8-5.2 Explain the effect on the area of two-dimensional shapes and on the volume of three-dimensional shapes when one or more of the dimensions are changed.

For this indicator, it is **essential** for students to:

- Recall area and volume formulas for geometric figures
- Calculate area and volume of geometric figures
- Understand the concept of volume and cubic units as well as area and square units.
- Examine patterns and draw conclusions based on the patterns that exist

For this indicator, it is **not essential** for students to:

- Use spheres when changing the dimensions and analyzing the effect on volume
- Analyze how changes in dimensions effect perimeter

8-5.3 Apply strategies and formulas to determine the volume of the three-dimensional shapes cone and sphere.

For this indicator, it is **essential** for students to:

- Understand the concept of height of a solid
- Be able to fluently calculate with fractions, decimals, and exponents
- Know the estimated equivalent of pi as 3.14 and $\frac{22}{7}$
- Recall the formula to find area of a circle
- Understand the concept of volume and cubic units
- Be able calculate volume of spheres and cones
- Identify characteristics of cones and sphere from verbal and pictorial representations
- Estimate answers by rounding

For this indicator, it is **not essential** for students to:

- Use Pythagorean Theorem to find the height of a cone
- Find exact solutions (in terms of pi)
- Calculate with measurements that are irrational

8-5.4 Apply formulas to determine the exact (*pi)* circumference and area of a circle.

For this indicator, it is **essential** for students to:

- Recall the formula for area and circumference
- Find the diameter when given the radius and vice-versa
- Understand the relationship between an estimate of pi and the exact value
- Calculate the exact area and circumference when the diameter and radius is a rational number.
  For example, find the exact area of a circle with a radius of 4 in. The area is 16π.
- Leave answers in π in the answer
- Find the area or circumference of a fractional part of a circle (half, three-fourths, etc..

For this indicator, it is **not essential** for students to:

- Calculate the exact area and circumference when the diameter or radius is irrational.
- Give an approximation for the area or circumference of a circle
- Find the length of the diameter or radius when given the circumference or area in a real-world situation.

8-5.5 Apply formulas to determine the perimeters and areas of trapezoids.

For this indicator, it is **essential** for students to:

- Recall the formula for area and perimeter of trapezoids
- Recall the attributes of a trapezoid
- Understand how to locate the height and bases of a trapezoid given in pictorial and verbal form
- Substitute values correctly into the formula
- Simplify

For this indicator, it is **not essential** for students to:

- Calculate the area and perimeter when the height or bases are irrational.

8-5.6 Analyze a variety of measurement situations to determine the necessary level of accuracy and precision.

For this indicator, it is **essential** for students to:

- Recall the different types of measurement in both U.S. Customary and Metric units
- Determine what type of accuracy is needed for a given real-life situation
- Distinguish between the concepts of accuracy and precision
- Understand that a measurement is precise only to one-half of the smallest unit used in a measurement
- Identify situations where accuracy or precision is more appropriate

- Understand that there is really no limit to how precisely an object can be measured. It depends on the measurement tool being used

For this indicator, it is **not essential** for students to:

- Measure attributes to a certain accuracy or precision

1. Teaching Lesson A: Changing Dimensions

a. Indicators with Taxonomy

8-5.2 → Explain the effect on the area of two-dimensional shapes and on the volume of three-dimensional shapes when one or more of the dimensions are changed. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson A: Changing Dimensions

In order for students to explain how the area of a two-dimensional shape is affected when dimensions change, they need experiences with models and representations drawn on a grid. Computer software and the use of a SmartBoard would be beneficial in demonstrating the change in the lengths of sides of the shape and help with the students' understanding of the affects of the changes.

Give students a rectangle with a width of 2 and a length of 5 drawn on a grid. Have the students determine the area of the given shape. Then have them draw a rectangle whose length is twice as long and determine the area. Have them complete a table or chart to record the measurements (width, length, and area). With a different or the "new" drawn rectangle, change both dimensions and have the students determine the effect on the area. Students should be given many opportunities to explore the effects on the area of two-dimensional shapes.

An example of a table you can use is:

Sketch of Original Shape	Original Dimensions of the Edges (units)	Original Area (square units)	New Dimensions of the Edges (units)	New Area (square units)

This same type of teaching strategy can be used when explaining the effects on the volume of a three-dimensional shape. Start with a cube then use other three-dimensional shapes. A table similar to the one above can be used to record information being sure to replace area with volume and square units with appropriate cubic units .

More Practice

Using another two-dimensional shape, begin altering the lengths of the edges and recording the new lengths and areas in a table or chart. Group the students and have them do the same with other two-dimensional shapes.
Share the data in a class discussion. Lead the students in formulating ideas about how the lengths of the edges affect areas of these two-dimensional shapes.

For three-dimensional shapes, have students apply the same strategy for explaining the effects on the volume when one or more dimensions are changed.

c. Misconceptions/Common Errors

Indicator 8-5.2 Explain the effect on the area of two-dimensional shapes and on the volume of three dimensional shapes when one or more of the dimension are changed.

- Students may forget that volume is measured in cubic units and area is measured in square units.

- Students may not understand the meaning of doubling or tripling a dimension.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

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f. Assessing the <u>Lesson</u>

<u>FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS</u>

What happens to the area of a 2x3 rectangle when;

- Double the length of the width?
- Double the length of each side?

What happens to the volume of rectangular prism with dimensions of 3x5x8 when all the dimensions are tripled? Find the new volume and compare it with the volume of the original prism.

What happens to the area of parallelogram when you;

- Double the length of the base?
- Double the length of base and the height?

What happens to the volume of rectangular prism with when all the dimensions are tripled?

2. Teaching Lesson B: *Volume of Cones and Spheres*

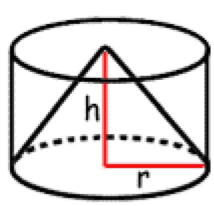
a. Indicators with Taxonomy

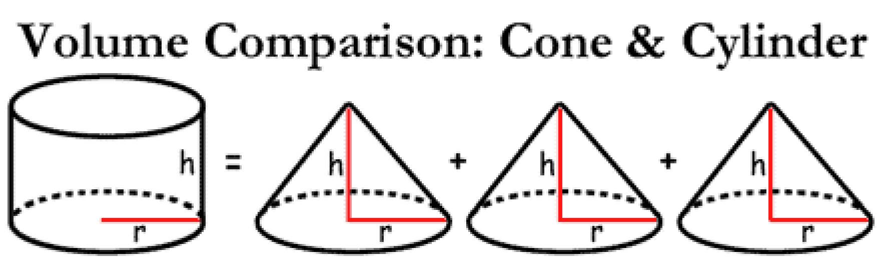
8-5.3 → Apply strategies and formulas to determine the volume of the three-dimensional shapes cone and sphere. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson B: *Volume of Cones and Spheres*

Students should also use their prior knowledge of finding volume of a cylinder (MS 7-5.2) when applying a formula to find the volume of a cone.
Have students decompose cylinders by slicing them in order to develop a conceptual understanding of volume formula (Volume = Area of base x Height). Students should be given the opportunity to explore the relationship between the volumes of cylinders and cones.

Divide your class into groups. Give each group a different size cylinder and cone (each have the same size base and height) and use either a container of water or sand. Have each group predict the relationship between the volume of the cylinder to the volume of the cone. Then have them fill the cone with water or sand and pour it into the cylinder until the cylinder is filled. Students will see that the cone needs to be filled three times in order to fill the cylinder. Therefore, the volume of the cylinder is 3 time the volume of the cone (3:1) or the volume of the cone is 1/3 the volume of the cylinder with the same base and height regardless of the shape of the base or the position of the vertex ($V = \frac{1}{3}(A \times h$, where A is area of the base).



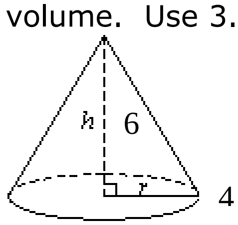


To help clarify their understanding of this relationship, ask your student questions such as:

- What would be the volume of a cone with the same height and base as a cylinder whose volume is 600 cubic inches?
- What would be the volume of a cylinder with the same height and base as a cone whose volume is 50 cubic centimeters?

The volume formula for the cylinder is $V = \pi r^2 h$ and the volume formula for the cone is $V = \frac{1}{3}\pi r^2 h$. The use of visual displays focusing on the entire procedures when using the formulas would be helpful to the students. Give the students several problems in which they must use the formula to find the volume of a cone. You may want to model the process for using the formula for finding the volume for the first problem.

- Given a cone with radius of 4cm and a height of 6cm, find its volume. Use 3.14 for pi.

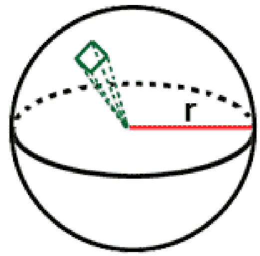


$V = \frac{1}{3}\pi r^2 h$
$V = \frac{1}{3}(3.14)(4)^2(6)$
$V = \frac{1}{3}(3.14)(16)(6)$ Using order of operations square first
$V = \frac{1}{3}(301.44)$ Multiply and then divide by 3
$V = 100.48\ cm^3$

- Find the volume:

Cone	Cone	Cone
r = 12 m	r = 3 in.	r = 3.5 cm
h = 10 m	h = 4 in.	h = 2 cm

To apply strategies to determine the volume of a sphere is a little more complicated. But, students could use the formula for a pyramid ($V = \frac{1}{3}Bh$, where B is area of the base) to determine the formula for volume of a sphere ($V = 4/3\ \pi r^3$).



The volume of the sphere would then be the sum of the volumes of all the pyramids. To calculate this, we would use the formula for volume of a pyramid, namely V = Bh/3. We would take the sum of all the pyramid bases, multiply by their height, and divide by 3.

First, the sum of the pyramid bases would be the surface area of a sphere, SA = $4\pi r^2$.

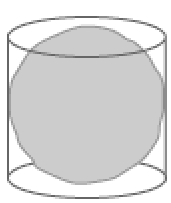
Second, the height of each of the pyramids is the radius of the sphere, r.

Third, we divide by three. The result of these three actions is SA = $(4\pi r^2)(r)/3 = 4\pi r^3/3$.

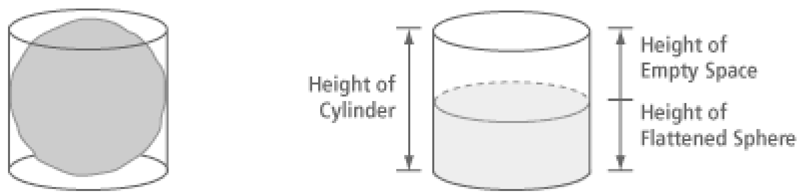
Example 1: If r = 300 mi (a sun), then the volume would be
$V = 4\pi r^3/3 = 4(3.14)(300 \text{ mi})^3/3 = 4(3.14)(27{,}000{,}000 \text{ mi}^3)/3 = 113{,}040{,}000 \text{ mi}^3$.

Example 2: If r = 4 mm (a small ball), then the volume would be
$V = 4\pi r^3/3 = 4(3.14)(4 \text{ mm})^3/3 = 4(3.14)(64 \text{ mm}^3)/3 = 267.9 \text{ mm}^3$.

Another comparison used for finding the volume of a sphere can be made with the volume of a cylinder, $V = \pi r^2 h$ or the area of its base times its height which is $(\pi r^2)(2r)$ or $2\pi r^3$. The sphere does not fill the entire cylinder; but its volume is 2/3 of the volume of a cylinder. So, $2/3(2\pi r^2) = 4/3\pi r^3$. For example, use a sphere and cylinder that have the same height and diameter.



Either fill the sphere with water or sand and pour into cylinder or make the sphere out of playdough or clay and flatten the sphere so that it tightly fits the bottom of the cylinder. Be sure that all the air and space is out of the dough.



Using water or sand is easier to see the relationship. Students should observe that the volume of the sphere is 2/3 the volume of the cylinder.

More Practice - Find the volume of the sphere with radius of 3 cm.

c. Misconceptions/Common Errors

Indicator 8-5.3 Apply strategies and formulas to determine the volume of the three-dimensional shapes cone and sphere.

- It is difficult for students to understand conceptually that volume of the cone is 1/3 the volume of a cylinder.
- Students may not understand that multiplying by 1/3 is the same as dividing by 3.
- Students confuse the height and lateral height of a pyramid.
- Students may not use order of operations when simplifying the formula.
- Students have difficulty remembering volume is cubic units.
- Students may confuse the diameter and the radius.

d. Additional Instructional Strategies/Differentiation

- **Connections to Literature:** *Counting on Frank* by Rod Clement is a great picture book that offers scenarios for students to explore and apply their knowledge and understanding of volume. http://illuminations.nctm.org/LessonDetail.aspx?ID=L203 **(***Counting on Frank* lesson plan)

e. Technology

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f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS

What is the relationship of the volume of the cone to the volume of a cylinder with the same base and height?

Find the volume of the cone with a radius of 4 cm and a height of 9 cm.

What is the relationship of the volume of a sphere to the volume of a cylinder with the same height and diameter?

Find the volume of a sphere with a radius of 5 inches.

What is the relationship of the volume of the cylinder to the volume of a cone with the same base and height?

Find the volume of the cone with a radius of 6 cm and a height of 9 cm.

3. Teaching Lesson C: Circumference and Area of Circles

a. Indicators with Taxonomy

8-5.4 → Apply formulas to determine the exact (pi) circumference and area of a circle. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson C: Circumference and Area of Circles

This lesson is very straight forward. In order to get the exact circumference or area of a circle, the symbol for pi must be in the answer. For example, find the area of a circle with a radius of 4. The formula is $A = \pi r^2$. The answer would be 16π.
For circumference, $C = \pi d$ or $2\pi r$.

Give the students several circles with various radii or diameters and ask them to find the exact circumference and/or area.

Example: In track and field, a men's discus has a radius of 11 cm. Find the circumference and area of the discus. Use 3.14 for pi.

c. *Misconceptions/Common Errors*

- Students may not understand that leaving pi in the answer is the exact value of circumference and area.
- Students may use the diameter instead of the radius to find area of a circle.
- Students may interchange the area and circumference formula.
- Students may forget that area is measured in square units and circumference is a linear measure.

d. Additional Instructional Strategies/Differentiation

- For this indicator, students are asked to leave pi in the answer to determine the exact value of the circumference and area. A discussion about the difference between leaving pi and using 3.14 or (22/7) may be needed.

e. Technology

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f. *Assessing the Lesson*

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS

- Give the two formulas used to determine the circumference of a circle.
- What is the relationship between the diameter and the radius of a circle?
- Why is the symbol for pi used in the answers when finding the exact circumference and area of a circle?
- Why is the symbol for pi used in the answers when finding the exact circumference and area of a circle?

4. *Teaching Lesson D: Trapezoids*

a. *Indicators with Taxonomy*

8-5.5 → Apply formulas to determine the perimeters and areas of trapezoids. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. *Introductory Lesson D: Trapezoids*

Remind the students that they developed strategies for finding the perimeter and area of trapezoids in grade seven.

Given figure 1 below, ask the students to tell you the formula for area of a trapezoid. Help them along by discussing the attributes of the trapezoid. Ask them if the formula for area of a trapezoid is similar to another area formula that they have already developed. Be sure they understand each component of the formula.

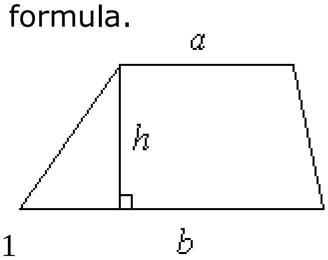

Fig.1

The formula is A = ½ ($base_1$ + $base_2$) height. Using the figure above A = ½ (a + b)h.

Give the students dot paper or geoboards and have them find the area of several trapezoids. Have them record their work on a separate sheet of paper and explain the procedure used to find the area if each trapezoid.

Ask the students how they would find the perimeter of any quadrilateral. Then ask the students how they would find the perimeter of a trapezoid in figure 2 if a = 4, b = 7, c = 5, d = 3, and h = 4.

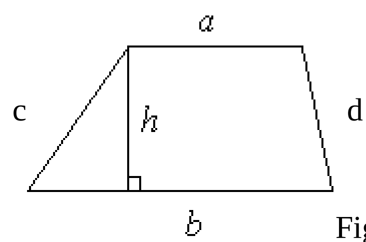

Fig.2

c. *Misconceptions/Common Errors*

- Students may think that the base of a trapezoid always means the bottom.
- Students may forget to add the lengths of the bases together first.
- Students may select the length of one of the legs of the trapezoids instead of the height.
- Students may interchange the area and perimeter formula.
- Students may forget that area is measured in square units and perimeter is a linear measure.

d. *Additional Instructional Strategies/Differentiation*

- In seventh grade, students generated strategies to find area and perimeter of trapezoid. Students will need opportunities to connect their generated strategies with the formulas.

e. Technology

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f. Assessing the Lesson

Formative Assessment Notes: Observe the students finding the area of various trapezoids in the "Introductory Lesson" section.

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS

Explain the procedure for finding the area of a trapezoid as well as the procedure for finding its perimeter.

5. Teaching Lesson E: Accuracy and Precision

a. Indicators with Taxonomy

8-5.6 → Analyze a variety of measurement situations to determine the necessary level of accuracy and precision. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson E: Accuracy and Precision

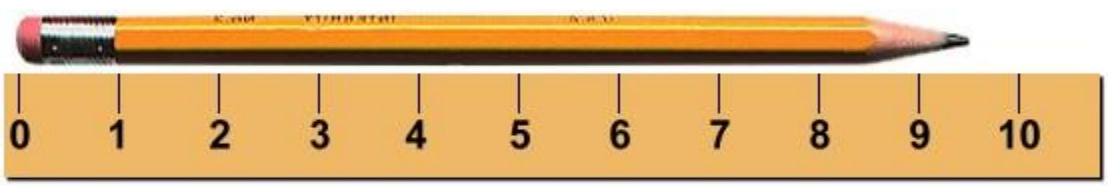
Students should discuss and examine how objects are measured and how the results are expressed. Students need to understand that a measurement is precise only to one-half of the smallest unit used in a measurement. When measuring, students need to determine to what degree of accuracy is needed.

For example: when measuring a wall for molding to fit, students need to understand that the measurement taken needs to be very accurate so that the molding will fit and not be short (no gaps) and the smallest unit would be an inch or a centimeter as well as using small fractional parts. On the other hand, determining the number of 6-foot sections of molding needed for the job, the nearest foot would probably be sufficient.
We have to distinguish between "accuracy," which means the closeness of a measurement to the exact value, and "precision," which means the claimed or implied closeness. For example, if I said my desk was 2 meters wide, and you said it was 2.345 meters wide, your answer would be more precise (claiming that you know it down to the millimeter); but if the desk is really 2.123 meters wide, then my answer is more accurate!

The precision of a measurement describes the <u>units</u> you used to measure something. For example, you might describe your height as *'about 6 feet'*. That wouldn't be very precise. If however you said that you were *'74 inches tall'*, that would be *more* precise.

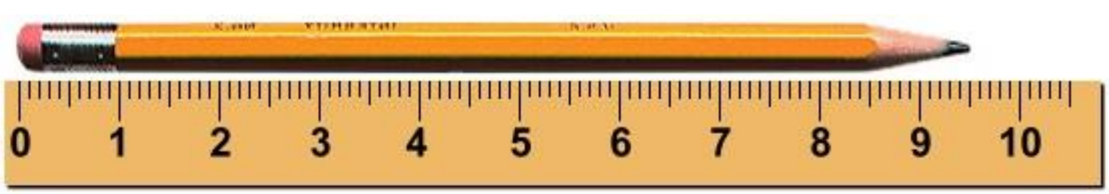
The smaller the unit you use to measure with, the more precise the measurement is.

In mathematics, it is often necessary to make measurements that are as precise as you can make them. This requires that you use measuring instruments with smaller units. For example, measure the pencil below.



How long is the pencil? Students might say it's about 9 centimeters. Some might guess and say 'about 9.5 centimeters, but the decimal place is just a guess. Because the smallest unit on the ruler you are using is one centimeter, the precision of your measurement is to the nearest centimeter.

Now look at the picture below, where we are using a different ruler to measure the pencil.



How long is the pencil? Students might say is 'about 9.5 centimeters. Again, some might guess and say 'about 9.51 centimeters, but the second decimal place is just a guess. Because the smallest unit on the ruler you are now using is one millimeter(one tenth of a centimeter), the precision of your measurement is to the nearest millimeter, or tenth of a centimeter. This second measurement is <u>more precise</u>, because you used a <u>smaller unit</u> with which to measure. Discuss with the students this statement; the smaller the unit, the more precise the measurement.

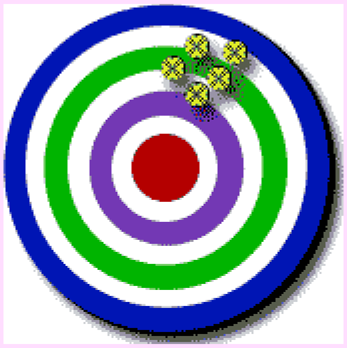
Discuss with the students why it is impossible to make a perfectly precise measurement. (There is really no limit to how precisely an object can be measured. It depends on the tool being used.)

The accuracy of a measurement describes how close it is to the 'real' value. This real value need not be very precise; it just needs to be the 'accepted correct value'. Accuracy is the difference between our own calculation and the accepted real value. For example, suppose your math textbook tells you that the value of Pi is 3.14. You do a careful measurement by drawing a circle and measuring the circumference and diameter, and then you divide the circumference by the diameter to get a value for Pi of 3.16. The accuracy of your answer is how much it differs from the accepted value. In this case, the accuracy is 3.16 - 3.14 = 0.02. The precision with which the accepted value has been measured is not important. All that matters is *how different* your measurement is from that value; the bigger the difference, the less accurate your measurement. Also,

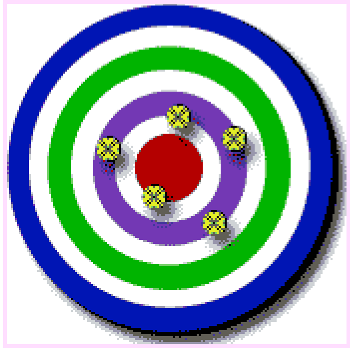
Accuracy depends on the instrument you are measuring with. But as a general rule: The degree of accuracy is half a unit each side of the unit of measure.

A dart board is a common example of showing the difference between accuracy and precision. Discuss the three boards below with your students.

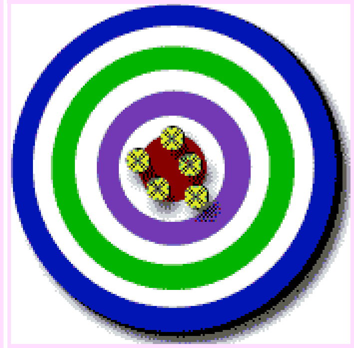
Examples of Precision and Accuracy:



Low Accuracy
High Precision



High Accuracy
Low Precision



High Accuracy
High Precision

Have the student s describe how the darts would appear on the dart board if there is no precision or no accuracy. (Ans. – darts would be far apart and unevenly spaced)

Have students discuss and examine various occupations that deal with precision and accuracy (nurse, carpenter, architect, and etc.).

c. *Misconceptions/Common Errors*

- Students may confuse the concepts of precision with accuracy.

d. *Additional Instructional Strategies/Differentiation*

- While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

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f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS

- What is meant by precision with regards to measurement?
- What determines how accurate you are when measuring ?
- What is meant by precision with regards to measurement?

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

Indicator 8-5.2 Explain the effect on the area of two-dimensional shapes and on the volume of three dimensional shapes when one or more of the dimension are changed.

The objective of this indicator is to explain, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples. For this reason, the student's conceptual knowledge of how the area of two-dimensional shapes and the volume of three dimensional shapes when one or more of the dimension are changed should be evaluated using a variety of examples. To explain is to construct a cause and effect model; therefore, explanation should include the cause (doubling, tripling, etc..) and the effect (area doubles, etc..). The learning progression to **explain** requires that students recall the formulas for area and volume as well as calculate these answers with fluency. Students should use their previous knowledge of area and volume and apply it to a variety of examples in order to examine the patterns that exist. Students draw conclusions based on these patterns and use questioning techniques to either prove or disprove their conclusions

(8-1.2). This process should be completed separately for two-dimensional and three-dimensional figures. Throughout this process students should use correct terminology to communicate their thoughts clearly through either speaking or writing (8-1.6).

Indicator 8-5.3 Apply strategies and formulas to determine the volume of the three-dimensional shapes cone and sphere.

The objective of this indicator is to apply, which is in the "apply procedural" knowledge cell of the Revised Taxonomy. Procedural knowledge is knowledge of specific steps or strategies to solve a problem or problem situation. Although the focus is to gain computational fluency with calculating volume of spheres and cones, the learning progression should integrate strategies to enhance both conceptual and procedural knowledge. The learning progression to **apply** requires students to recall the formula used to find the area of a circle. They understand the concept of volume being the number of cubic units to fill a space. It is essential for students to model these geometric figures to discover the relationships among the solids and their connections to the formulas used (8-1.1). Students also need to generalize and communicate the formula to see relationships among the solids and connections to the symbols in the formulas (8-1.4). Students should also use correct and clearly written or spoken words to explain their reasoning for their answers (8-1.6). Students apply these procedures in context as opposed to only rote computational exercises and generalize connections among representational forms and real world situations (8-1.7). They engage in meaningful practice to support retention.

Indicator 8-5.4 Apply formulas to determine the exact (pi) circumference and area of a circle.

The objective of this indicator is to apply, which is in the "apply procedural" knowledge cell of the Revised Taxonomy table. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem. Procedural knowledge is also tied to knowledge of criteria for determining when to use appropriate procedures. The learning progression to **apply** requires students to recall the formulas for both circumference and area. The students must be able to determine which formula would be used with respect to the context of the problem presented. A variety of real-world situations should be presented so that the students are comfortable differentiating between the concepts of circumference and area (8-1.7). Once the correct formula is selected, students will be required to simplify the equation and also examine the reasonableness of their answer. Since an approximation for area and circumference were found in a previous grade, students will understand the distinct symbolic forms that represent the same relationship (8-1.4) and will use correct and clearly written or spoken words to communicate their answer (8-1.6).

Indicator 8-5.5 Apply formulas to determine the perimeters and areas of trapezoids.

The objective of this indicator is to apply, which is in the "apply procedural" knowledge cell of the Revised Taxonomy table. Procedural knowledge is knowledge of specific steps or strategies that can be used to solve a problem. Procedural knowledge is also tied to knowledge of criteria for determining when to use appropriate procedures. The learning progression to **apply** requires students to recall the formulas for both perimeter and area. The students determine which formula would be used with respect to the context of the problem presented. A variety of real-world situations should be presented so that the students are comfortable differentiating between the concepts of perimeter and area (8-1.7). Once the correct formula is selected, students are required to simplify the equation using correct mathematical procedures. Students should examine the reasonableness of their answer and will use correct and clearly written or spoken words to communicate their answer (8-1.6).

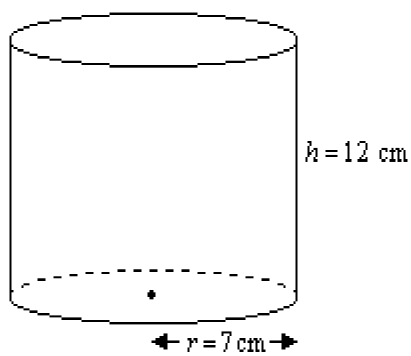
Indicator 8-5.6 Analyze a variety of measurement situations to determine the necessary level of accuracy and precision.

The objective of this indicator is to analyze, which is in "analyze conceptual" knowledge cell of the Revised Taxonomy table. Analyze requires students to break material (measurement) into its constituent parts (accuracy and precision) and to determine how the parts relate to one another and to an overall structure or purpose. Conceptual knowledge is not bound by specific examples; therefore, the student's conceptual knowledge of precision and accuracy should be assessed using a variety of real-life examples (8-1.7). The learning progression to **analyze** requires students to recall basic units of measurements and to apply them in real-world situations. When given these authentic situations, students should be able to differentiate between the concepts of precision and accuracy when asked specific questions. They also generate problems (8-1.1) that relate to accuracy and precision. Students should be able to explain there answers using correct and clearly written or spoken words to communicate their answers to both teachers and their classmates (8-1.6).

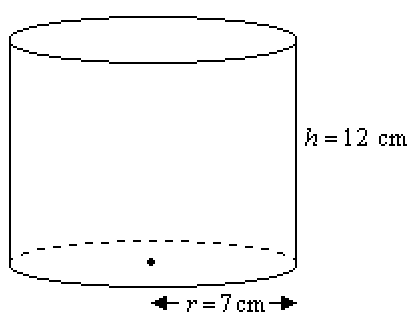
The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Explain the effect on the area of a triangle whose height is 4 meters and base is 3 meters when:
    a. Height is doubled.
    b. Both dimensions are quadrupled.

2. Explain the effect on the volume of this cylinder when:
    a. the radius is doubled.
    b. the height is doubled.



3. Explain the effect on the volume of this cylinder when:
    a. the radius is doubled and the height is halved.



4. Explain the effect on the area of a triangle whose height is 6 meters and base is 4 meters when:
   a. Height is doubled.
   b. Both dimensions are tripled.

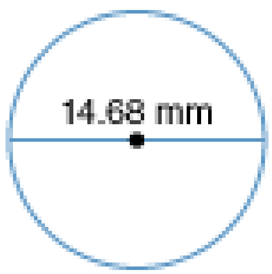
5. Find the volume of a cone with a diameter of 10 cm and height of 12 cm.

6. If the volume of a cylinder is 45 cubic inches, find the volume of a sphere that has the same height and diameter as the cylinder.

7. Find the volume of a cone with a diameter of 10 cm and height of 12 cm.

8. If the volume of a cylinder is 63 cubic inches, find the volume of a sphere that has the same height and diameter as the cylinder.

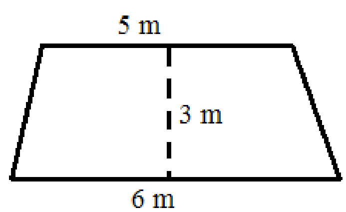
9. Find the exact circumference and area of this circle.



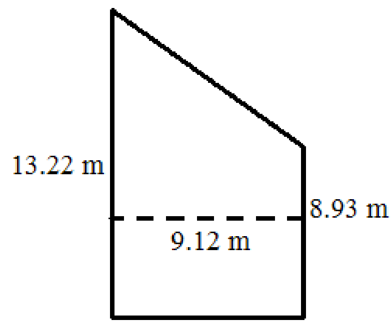
10. Sam has a circular pool in his backyard. The pond has a diameter of 28 feet. Calculate the exact circumference and exact area of Sam's pond.

11. There is a circular fountain in the front yard of Smith Middle School. The fountain has a diameter of 17 feet. Calculate the exact circumference and exact area of the fountain.

12. Mr. Henry's pet chickens live in a shed with a trapezoid floor. He wants to use brown paper to cover the floor. How many square meters does Mr. Henry need to cover the shed floor? (Show all your work)

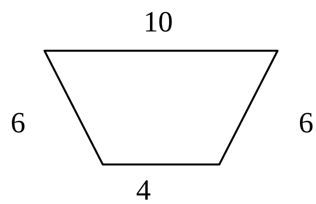


13. Use the formula to find the area of this trapezoid.



3.

14. Find the perimeter of this trapezoid.



15. Find the area of a trapezoid with following dimensions:
$b_1 = 4$, $b_2 = 8$, and $h = 5$.

16. Using the information in the problem above for a trapezoid, can you find the perimeter of the figure? Why or why not?

17. Which situation would require the most precision?
   a. weighing a dog
   b. weighing the amount of sugar in a cookie recipe
   c. weighing the ingredients in a prescription drug pill
   d. weighing vegetables at the market

MODULE

3-3

Proportional Reasoning

This module addresses the following indicators:

8-5.1 Use proportional reasoning and the properties of similar shapes to determine the length of a missing side.

This module contains 1 lesson. These lessons are INTRODUCTORY ONLY. Lessons in S3 begin to build the conceptual foundation student need. ADDITIONAL LESSONS will be required to fully develop the concepts.

I. Planning the Module

Continuum of Knowledge

Indicator 8-5.1 Use proportional reasoning and the properties of similar shapes to determine the length of a missing side.

In sixth grade, students use proportional reasoning to determine unit rates (6-5.6). They also use scale to determine distance (6-5.7). They gained an understanding of the relationship between ratio/rate and multiplication/division (6-2.6). They also classify geometric shapes as being similar (6-4.8). In seventh grade, students must apply proportional reasoning to find missing attributes of similar shapes (7-4.8).

In eighth grade, students use proportional reasoning and the properties of similar shapes to determine the length of a missing side (8-5.1).

The concepts of proportional reasoning and similar figures will be used throughout high school Geometry.

Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students**.**

*Ratio
*Proportional
*Dilation
*Magnify
*Shrink
*Enlarge
*Reduce
*Congruent
*Corresponding
*Similar
*Similar Figures
*Ratio of similitude - The ratio of the corresponding sides of similar figures is called the ratio of similitude. This ratio of similitude is also the ratio of the corresponding diagonals, altitudes, medians, angle bisectors, apothems and perimeters.

II. Teaching the Lesson

Proportional reasoning is a crucial component of middle school work and is connected to many topics in algebra (study of linear functions, y = kx), geometry (dilations), measurement (finding length of missing side), and statistics (reasoning about data from a relative-frequency histogram). It is also a difficult concept for students, and they need to work with activities that represent real world contexts so that they can sense the reasonableness of solutions to problems.

Students will apply and analyze these methods in problem solving situations and additionally will use the cross-product or cross-multiplication method to solve a proportion. Students at this level will deepen their understanding of equivalent ratios and think flexibly about rational numbers through application of concepts introduced in 6th grade and developed in 7th grade.

Students should develop a deep understanding of the method of cross-multiplying through meaningful experiences. Cross multiplication is a powerful technique but needs to be taught with understanding. Expressing the terms as equivalent ratios is one way of developing cross multiplication. They need to be able to analyze and explain how the method works and know when it is reasonable to make use of this method. One way to develop cross multiplying is to create equivalent ratios by multiplying each term by 1:

$$\frac{2}{7} = \frac{a}{3} \qquad \frac{3 \times 2}{3 \times 7} = \frac{7 \times a}{7 \times 3} \qquad 3 \times 2 = 7 \times a$$

It is important for students to extend their understanding of ratio and proportion as they explore similarity. They combine the knowledge of attributes of similar figures with knowledge of proportionality to find measures of missing sides, beginning with explorations of various types of similar figures. Students must understand that pairs of similar shapes have proportional perimeters and side lengths. Problems that involve creating and analyzing scale drawings give students good experience with similarity and proportionality.

Similarity as well as proportionality can be understood at a physical, numeric, graphic, or symbolic level. By seeing the connections among each of the representations, students gain a more in-depth understanding than if these levels were viewed independently.

8-5.1Use proportional reasoning and the properties of similar shapes to determine the length of a missing side.

For this indicator, it is ***essential*** for students to:

- Recognize similar polygons as shapes that have the same shape, but different sizes

- Recognize that corresponding angles are congruent in similar shapes
- Recognize corresponding sides as proportional
- Find corresponding sides of any pair of similar polygons (by observation and by angle measurement)
- Solve proportions
- Find the missing side length going from the smaller shape to the larger and from the larger to the smaller shape
- Apply proportional reasoning to similar shapes in different geometric "families"

For this indicator, it is **<u>not essential</u>** for students to:

- Find the missing angle measures in similar figures
- Use the ratio of similitude to calculate measurements related to the diagonals, altitudes, medians, angle bisectors, apothems and perimeters of similar figures.

1. Teaching Lesson A: Similar Figures

a. Indicators with Taxonomy

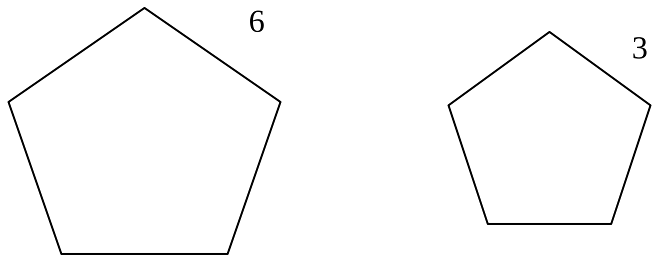
8-5.1 → Use proportional reasoning and the properties of similar shapes to determine the length of a missing side. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson A: Similar Figures

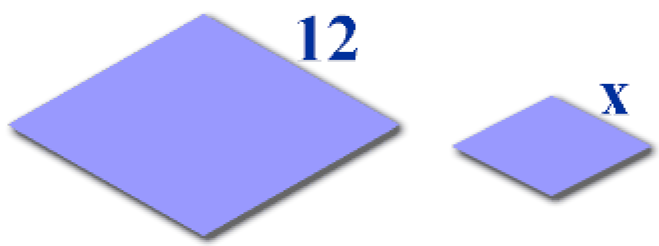
To use proportional reasoning and properties of similar shape, have students start with rectangles. They are easily sorted into "families" or groups by examining ratios of corresponding sides. When a family of rectangles is nested (rectangles share the lower left corner) on top of each other, the diagonals will line up and corresponding vertices will form a straight line.

The ratio of the corresponding sides of similar figures is called the ratio of similitude. This ratio of similitude is also the ratio of the corresponding diagonals, altitudes, medians, angle bisectors, apothems and perimeters.

Draw these 2 pentagons on the board or SmartBoard. Have students determine if these two pentagons are similar. Their response should be yes and the ratio of similitude is 6: 3 (reduced 2 : 1).



These two rhombi are similar and their ratio of similitude is 3 : 1. Have the students find x.



Given that rectangle ABCD is similar to rectangle A'B'C'D' where A corresponds to A' and so on. Have the students draw the rectangles and label the vertices for each rectangle
If AB = 12, A'B' = 4, AD = 9, find A'D'.

*The examples shown are not sufficient for students to master the concept. You must supply further examples.

c. Misconceptions/Common Errors

- Students may have difficulty determining which sides are corresponding sides.

d. Additional Instructional Strategies/Differentiation

- Students should begin with explorations of drawings for various types of geometric figures. This knowledge should be extended to problems in different contexts.

- Example: Given that rectangle ABCD is similar to rectangle A'B'C'D' where A corresponds to A' and so on, if AB = 12, A'B' = 4, AD = 9, find A'D'.

- Problems that involve creating and analyzing scale drawings also give students good experience with similarity and proportionality.
- To use proportional reasoning and properties of similar shapes, have students start with rectangles. They are easily sorted into "families" or groups by examining ratios of corresponding sides. When a family of rectangles is nested (rectangles share the lower left corner) on top of each other, the diagonals will line up and corresponding vertices will form a straight line.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- Specific Resources:
  - Lesson 6: In Your Shadow http://illuminations.nctm.org/LessonDetail.aspx?ID=L515 lesson 6
- illuminations.nctm.org (Lessons, Activities and Related WebLinks)
- nlvm.usu.edu (National Library of Virtual Manipulatives)
- Oneplacesc.org (ETV Streamline and more!)
- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS

In triangle ABC, angle A = 90º and angle B = 35º. In triangle DEF, angle E = 35º and angle F = 55º. Are the triangles similar? Justify your reasoning using properties of similar figures.

III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

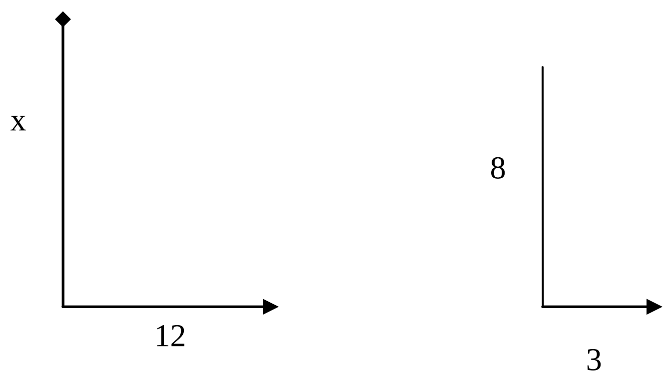
Indicator 8-5.1 Use proportional reasoning and the properties of similar shapes to determine the length of a missing side.

The objective of this indicator is to <u>use</u> proportional reasoning, which is in the "apply procedural" knowledge cell of the Revised Taxonomy table. Procedural knowledge is tied to knowledge of criteria for determining when to use appropriate procedures. The learning progression to **use** proportional reasoning requires students to <u>recall</u> the characteristics of similar polygons. After students <u>determine</u> that given polygons are similar figures, they must <u>compare</u> the characteristics of the polygons in order to <u>locate</u> the corresponding parts. Students will <u>use</u> proportions fluently in order to find a missing side length. Students should <u>understand</u> similarity as well as proportionality at a physical, numeric, graphic, and symbolic level. By <u>generalizing</u> the connections (8-1.7) among each of the representations, the students will gain a more in-depth understanding than if these representations were viewed independently. Students <u>use</u> correct and clearly written or spoken words in order to communicate their ideas about the proportionality of these similar figures (8-1.6). They <u>use</u> proportional reasoning to determine if their answer is reasonable.

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. A vertical flagpole casts a shadow 12 feet long at the same time that a nearby vertical post 8 feet casts a shadow 3 feet long. Find the height of the flagpole. Explain your answer.

Answer or possible solution below:

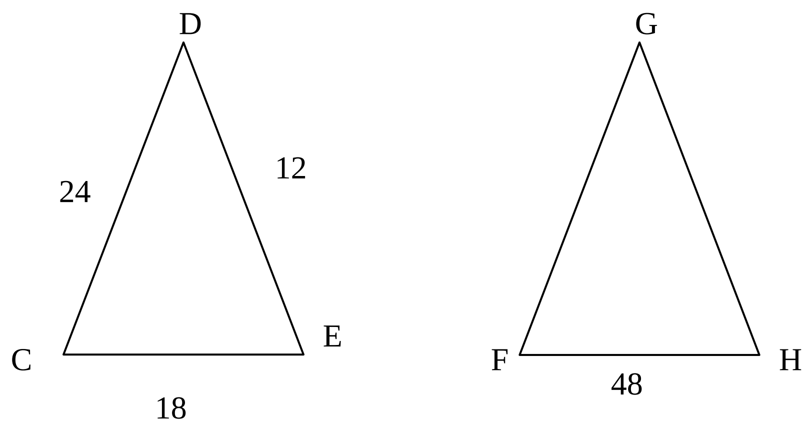


$3/8 = 12/x$

$x = 32$

2. $\triangle$CDE ~ $\triangle$ FGH, and CE = 18 in, CD = 24 in, DE = 12 in, FH = 48 in. Find FG and GH. Draw each triangle and label the vertices.

Answer or possible solution below:



FG = 64
GH = 32

MODULE

3-4

Collection and Representation

This module addresses the following indicators:

8-6.2 Organize data in matrices or scatterplots as appropriate.

This module contains 2 lessons. These lessons are INTRODUCTORY ONLY. Lessons in S3 begin to build the conceptual foundation student need. ADDITIONAL LESSONS will be required to fully develop the concepts.

I. ***Planning* the Module**

Continuum of Knowledge

Indicator 8-6.2 Organize data in matrices or scatterplots as appropriate.

In fourth grade, students organized data in tables, line graphs, and bar graphs whose scale increments are greater than or equal to 1 (4-6.3). In sixth grade, they organized data in frequency tables, histograms, and stem-and-leaf plots (6-6.2). In seventh grade, they organized data in box plots and circle graphs (7-6.2).

In eighth grade, students organize data in matrices or scatterplots as appropriate. Students will also learn about matrices (8-6.2).

In Elementary Algebra, students carry out a procedure to write an equation of a trend line from a given scatterplot (EA-4.4) and they analyze a scattterplot to make predications (EA-4.5).

Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

*Row
*Column
*Matrix (matrices)
*Data set
*Element
*Population
*Scale
*Range
*Rectangular arrangement
*Data
*Scatterplot
*Coordinates

II. *Teaching the Lesson(s)*

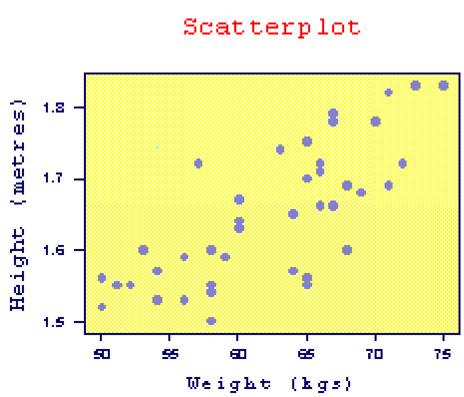
A matrix is a rectangular arrangement of values. Another way to describe a matrix is that it is a two-dimensional table in which the rows (horizontal groups of entries) and the columns (vertical groups of entries) are labeled to indicate the meaning of the data. The plural form of matrix is matrices. The size of the matrix is expressed by giving its dimensions (number of rows by the number of columns). For example, a matrix with 4 rows and 5 columns is a 4 by 3 or 4 x 3 matrix. An example of a matrix giving information about group of isosceles right triangles might look like this:

$$\begin{array}{c} \begin{array}{ccc} \textbf{Hypotenuse} & \textbf{Leg} & \textbf{Area} \end{array} \\ \begin{bmatrix} \sqrt{2} & 1 & \frac{1}{2} \\ \sqrt{8} & 2 & 2 \\ \sqrt{18} & 3 & 9/2 \\ \sqrt{50} & 5 & 25/2 \end{bmatrix} \end{array}$$

Students should be given information about the components of a matrix in order to use a matrix to organize data.

Students should begin looking at a matrix as a table. Students need to become comfortable with matrices and see why they are useful. The students should begin to see the connection between a matrix and a spreadsheet.

A scatterplot is a useful summary of a set of bivariate, numerical data (two variables). It gives a good visual picture of the relationship between the two variables, and aids the interpretation of the correlation coefficient or regression model. Each unit contributes one point to the scatterplot, on which points are plotted but not joined. The resulting pattern indicates the type and strength of the relationship between the two variables. Below is an example of a scatterplot:



Students should encounter many scatterplots that have a nearly linear shape since linearity is an important idea in the middle grades. But, they should also encounter plots that represent nonlinear relationships.

8-6.2 Organize data in matrices or scatterplots as appropriate.
For this indicator, it is **essential** for students to:

- Understand that a matrix is an array or another form of a table
- Understand the purpose of creating a matrix
- Identify an element in by row and column
- Identify the size of a matrix
- Understand the components of a matrix: rows, columns, and size of matrix.
- Understand the purpose of a scatterplot
- Set up the axes of the scatterplot appropriately
- Plot the points

For this indicator, it is **not essential** for students to:

- Perform mathematical operations on the matrices.

1. Teaching Lesson A: Matrices

a.Indicators with Taxonomy

8-6.2 → Organize data in matrices or scatterplots as appropriate. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson A: Matrices

Let students know that one of the most common ways of organizing data is in a matrix. Discuss with the students the components of a matrix: rows, columns, and size of matrix. Model for students how to create a matrix from a given set of data, for example: construct a matrix from information about the dimensions, area, and perimeter of six rectangles (rows – rectangle names, and columns – dimensions). After the matrix is completed, discuss how the size of the matrix is determined – dimensions of matrix which is number of rows by the number of columns. The example above would be a 6 by 4 or 6 x 4 matrix. Also, point out how to identify individual elements (entries) in a matrix. Each element is identified by their row and column numbers. Subscripts are used to identify elements ($e_{2,3}$ – refers to element in row 2, column 3).

Additional Practice:
Give students time to organize data in a matrix.
Have students create a matrix using the following data:
Give three squares with lengths of sides 2, 5, and 9 find the area and perimeter for each square.

Have students create a matrix that displays their weekly expenses. Label the rows the days of the week. For the columns choose labels like food, clothes, and entertainment. The students can select the column labels.

Enrichment:
Have students create a 4x4 matrix in which;
The only entries are a, b, c, and d.
Every column contains a, b, c, and d.
Every row contains a, b, c, and d.

c. Misconceptions/Common Errors

The most common error in matrices is the interchanging of row and column. They need to remember that the elements are identified by their row and column numbers.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- illuminations.nctm.org (Lessons, Activities and Related WebLinks)
- nlvm.usu.edu (National Library of Virtual Manipulatives)
- Oneplacesc.org (ETV Streamline and more!)

- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

f. Assessing the Lesson

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS

Consider the matrix $\begin{bmatrix} 2 & 5 & 1 \\ 4 & 10 & 7 \\ 3 & 6 & 22 \end{bmatrix}$

What are the dimensions of the matrix?
What is entry $e_{3,1}$?
In what locations are the entries two-digit?
In what locations are the entries odd numbers?

2. Teaching Lesson B: Scatterplots

a. Indicators with Taxonomy

8-6.2 → Organize data in matrices or scatterplots as appropriate. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

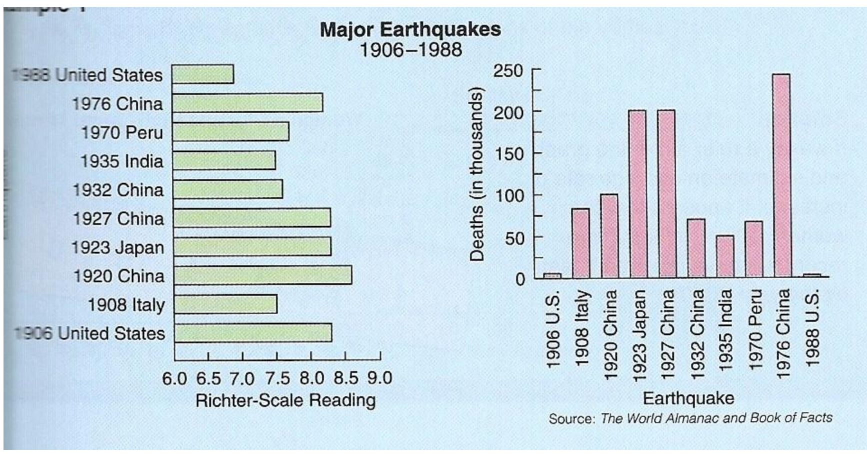
b. Introductory Lesson B: Scatterplots

Gather data in a table or chart on the length of time that students studied for a math test and the resulting test scores. Ask the students to create a class hypothesis from the data. (Use 15 minute intervals for the time.)

Divide the class into groups to discuss other ways this information could be displayed graphically for analysis. Have them also consider if some types of graphs would be inappropriate for this data, leading to incorrect interpretations. After a few minutes, have the groups share their ideas, telling why they selected a particular type of graph and why some graphs would be inappropriate. Next, ask them to tell if their graph would help them decide if the class hypothesis is true and to defend their response. Introduce ***scatterplots*** by using the minutes studied as the independent variable *x* and the grade as the dependent variable *y* on a large wall coordinate grid. Using the index cards as coordinate pairs, have the students tape the cards on the grid. This graph can then be made smaller using graphing calculators, graphing paper, or Smartboard. The teacher will show the same ***scatterplot*** using a transparency or Smartboard display.

Discuss with the students that a scatterplot is appropriate when one variable will yield several different outcomes or when several variables will yield the same outcome. It is also useful when looking for a cause and effect or correlation between two different things.

Additional Practice:



Using the data from the two graphs above, construct a scatterplot. Show the number of casualties on the y-axis and the Richter-scale ratings on the x-axis.

c. Misconceptions/Common Errors

- The most common error in scatterplots is interchanging the axes components when plotted.
- Students may try to connect the data points. They need understand that these points cannot be connected.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

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- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

f. Assessing the <u>Lesson</u>

<u>Formative Assessment Notes</u>
Observe the students as they construct their scatterplots in the "Additional Instruction Strategies" section. Be sure they are using the correct data for each axis, labeling the axes, naming the scatterplot, etc.

III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

Indicator 8-6.2 Organize data in matrices or scatterplots as appropriate.

The objective of this indicator is to <u>organize</u>, which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To organize is to determine how elements (data points) fit or function together within a structure (scatterplot or matrix). The learning progression to **organize** requires students to <u>understand</u> the purpose of matrices and scatterplots. Students <u>recall</u> the components of each and understand how those components are used to organize the data. They <u>analyze</u> the given data and <u>generalize</u> connections (8-1.7) between the data and the components of each representation to determine how to organize their data. After the data has been organized, students explain and justify their answer using correct and clearly written or spoken word (8-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Create a matrix with entries $e_{1,1} = 8$, $e_{1,2} = 9$, $e_{1,3} = 11$, $e_{2,1} = 7$, $e_{2,2} = 15$, and $e_{2,3} = 0$. What is the size of the matrix you created?

2. Using the information below, create a scatterplot.

Age of Trees in Years	**Diameter in Inches**
4	0.8
5	0.8
8	1
8	2
8	3
10	2
10	3.5
12	4.9

Original data source: Chapman, Herman H., and Dwight B. Demeritt, Elements of forest Mensuration (Albany, NY: J. B. Lyon Co., 1932)

MODULE

3-5

Data Analysis

This module addresses the following indicators:

8-6.1 Generalize the relationship between two sets of data by using scatterplots and lines of best fit.

8-6.8 Interpret graphic and tabular data representations by using range and the measures of central tendency (mean, median, and mode).

This module contains 2 lessons. These lessons are INTRODUCTORY ONLY. Lessons in S3 begin to build the conceptual foundation student need. ADDITIONAL LESSONS will be required to fully develop the concepts.

I. ***Planning the Module***

Continuum of Knowledge

Indicator 8-6.1 Generalize the relationship between two sets of data by using scatterplots and lines of best fit.

In seventh grade, students were introduced to slope as a constant rate of change. [7-3.3] Seventh grade students predicted the characteristics of two populations.

In eighth grade, students identify the slope of a linear equation from a graph, equation, and/or table [8-3.7]. They make scatterplots for two sets of data, find the line of best fit for each and compare these lines of best fit.

In Elementary Algebra, students carry out a procedure to write an equation of a trend line from a given scatterplot (EA-4.4) and they analyze a scattterplot to make predications (EA-4.5).

Indicator 8-6.8 Interpret graphic and tabular data representations by using range and the measures of central tendency (mean, median, and mode).

In third grade, students applied a procedure to find the range of a data set (3-6.1). In fifth grader, students calculated (5-6.3) and interpreted (5-6.4) the measures of central tendency (mean, median, and mode). In the sixth grade, students analyzed which measure of central tendency was the most appropriate for a given purpose (6-6.3) and predicted the characteristics of one population based on the analysis of sample data (6-6.1).

In eighth grade, students interpret graphic and tabular data representations by using range and the measures of central tendency (mean, median, and mode) (8-6.8).

High school students will apply and interpret data via these concepts in Data Analysis and Probability.

Key Concepts/Key Terms

* These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

*Inference
*Data sets
*Line of best fit

*Trend line
*Range
*Scale
*Outlier
*Range
*Measures of Central Tendency (mean, median, mode)
*Scatterplot
*Correlation (positive, negative or no)

II. Teaching the Lessons

With the use of scatterplots, students learn to analyze the relationship between variables to determine if there is a positive or negative correlation, or if there is no correlation at all. Students should estimate a line of fit to determine these relationships and make predictions. This line of fit prepares students for linear functions taught in Algebra 1.

Some generalizations to consider when determining the relationship between two sets of data:

a) The more the points tend to cluster around a straight line, the stronger the linear relationship between the two variables (the higher the correlation).
b) If the line around which the points tends to cluster runs from lower left to upper right, the relationship between the two variables is positive (direct).
c) If the line around which the points tends to cluster runs from upper left to lower right, the relationship between the two variables is negative (inverse).
d) If there exists a random scatter of points, there is no relationship between the two variables (very low or zero correlation).
e) Very low or zero correlation could result from a non-linear relationship between the variables. If the relationship is in fact non-linear (points clustering around a curve, not a straight line), the correlation coefficient will not be a good measure of the strength.

A scatterplot will also show up a non-linear relationship between the two variables and whether or not there exist any outliers in the data. Scatterplots give students the opportunity to interpolate and predict values not present in a display by estimating where the values will fall. Students need to realize that predictions from data have limitations. Also it is important for students to be able to distinguish between variables that covary "by accident" and variables that illustrate a causal relationship.

Students need to develop skills in identifying trends in the data represented in a scatterplot.

In *Navigating through Data Analysis in Grades 6 – 8,* mean, median, and mode are considered "numerical summaries" of a data set. These values reveal something different about the data. Numerical summaries should be used along with descriptive language. The range should be related to the numerical summaries. Students need to realize that a great deal of information is ignored when computing the range since only the largest and the smallest data values are considered while the remaining data are ignored. Also the range value of a data set is greatly influenced by the presence of just one unusually large or small value in the sample (outlier).

Students need to understand what each value is saying about the data. For example:

Mean - a measure of location, commonly called the average

value depends equally on all of the data which may include outliers and may not appear representative of the central region for skewed data sets.

useful as being representative of the whole sample for use in subsequent calculations.

Median - good descriptive measure of the location which works well for skewed data, or data with outliers.

Mode – most frequent data

more than one mode in a data set is possible

use to describe either categorical or quantitative data

Range - is a measure of the spread or the dispersion of the observations

difference between the largest and the smallest observed value of some quantitative characteristic

Students should use descriptive statistics, including mean, median, mode, and range, to summarize and compare data sets. They should organize and display data to pose and answer questions. Students need to understand that a measure of center alone does not thoroughly describe a data set because very different data sets can share the same measure of center. (*Focal Points*, NCTM, p20)

8-6.1 Generalize the relationship between two sets of data by using scatterplots and lines of best fit.

For this indicator, it is **essential** for students to:

- Understand the relationship between correlation and slope
- Recognize data that is increasing (positive correlation – positive slope)
- Recognize data that is decreasing (negative correlation – negative slope)
- Recognize data that is varying or no change (no correlation)
- Understand the structure and purpose of a scatterplot
- Understand the purpose of a line of best fit
- Interpret a trend in data
- Understand the limitations of a trend line when interpreting data
- Determine if a given line drawn with the scatterplot is a line of best fit based on the relationship between the points and the line

For this indicator, it is **not essential** for students:

- Understand how to determine slope as a numerical value.
- Find the line of best fit.

8-6.8 Interpret graphic and tabular data representations by using range and the measures of central tendency (mean, median, and mode).

For this indicator, it is **essential** for students to:

- Calculate range
- Calculate the measures of central tendency
- Understand the meaning of range
- Understanding what each measure of central tendency says about the data
- Describe the data using appropriate mathematics language

For this indicator, it is **not essential** for students to:

- None noted

1. Teaching Lesson A: Generalizing Relationships Using Scatterplots and Lines of Best Fit

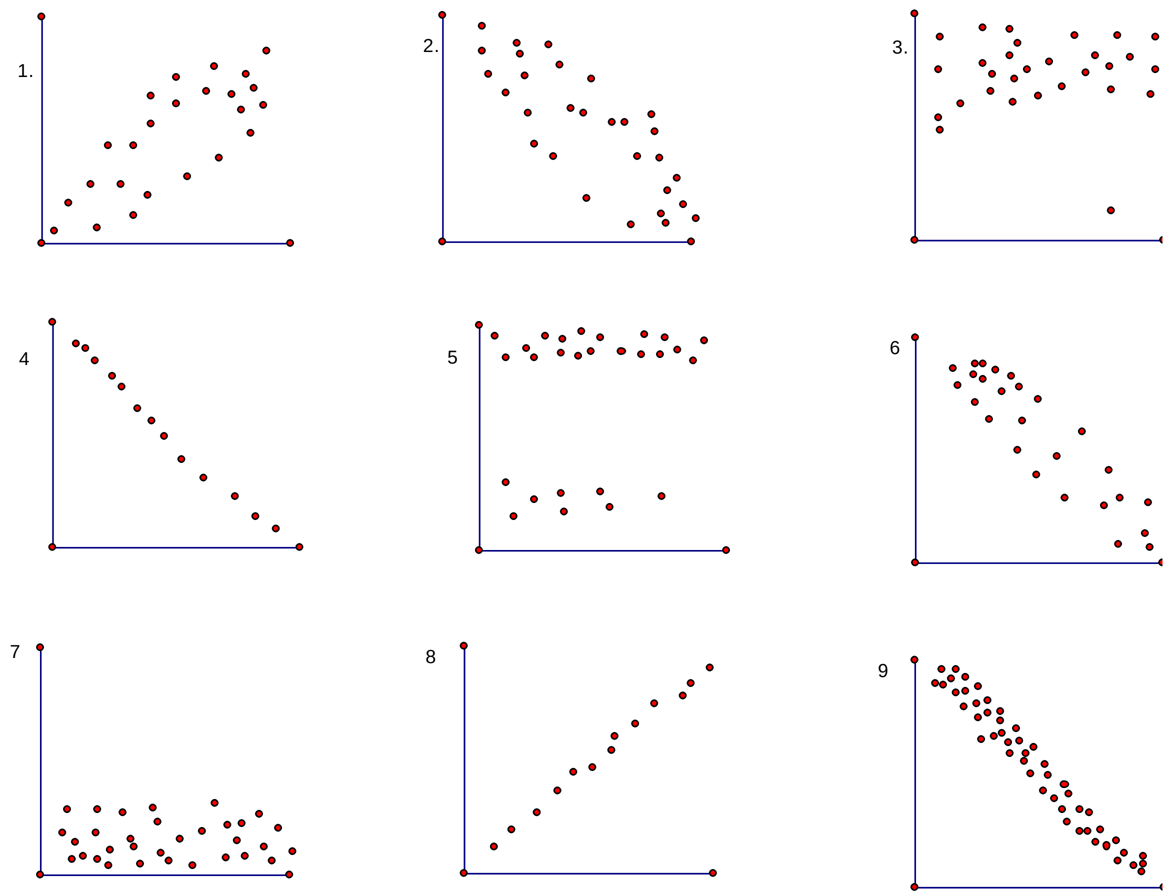
a. Indicators with Taxonomy

8-6.1 → Generalize the relationship between two sets of data by using scatterplots and lines of best fit. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson A: Generalizing Relationships Using Scatterplots and Lines of Best Fit

Part 1
Since students have created scatterplots in 8-6.2, they can now begin to discuss the relationships and draw conclusions from the scatterplots.

Have students work in groups. Use example 1 to model your thinking about how to interpret the scatterplot by referring the generalizations, a – e, in the teacher notes. (for example: trends in the data – positive, draw line of best fit, etc.)
Have the groups discuss the remaining graphs and record the findings. Have each group report their findings. Be sure to discuss each graph for their relationships.



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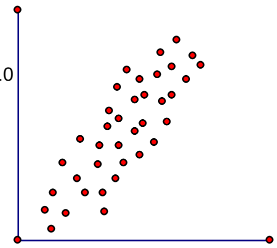

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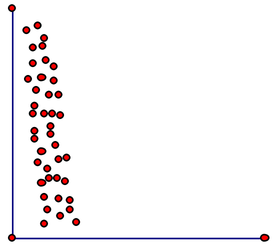

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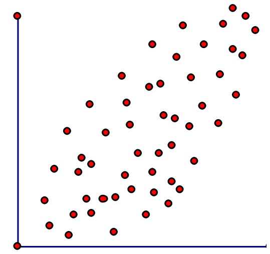


Part 2

Adapted from: Van de Walle, John A. & Lovin, LouAnn H., 2006. Teaching Student Centered Mathematics: Grades 5-8.

Investigate your own data or search around the Internet to collect data (see technology resource section). After you have your data (use a set that indicates a relationship), have students graph the data, using graph paper. Once the data is graphed, have students interpret the graph based on the generalizations listed in teacher notes and used in Part 1.

Provide students with a piece of spaghetti to use as a line. Have students place or tape the spaghetti on the scatterplot so that it is the "best" line to represent the relationship. Have students develop a rationale for why they placed their lines as they did. Compare the various lines and discuss their rationales. Let this lead into a discussion about "lines of best fit" and how "lines of best fit" can be used to predict.

c. Misconceptions/Common Errors

- In the process of finding the line of best fit many students want to make sure that their line goes through the origin even though this is not in line with the data points.
- Students may try to connect all of the data points.
- Some students may include an outlier in their estimate of the line of best fit and have their line be far from the expected slope.

d. Additional Instructional Strategies/Differentiation

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- Specific Resources:
  - Illuminations: Exploring Linear Data; http://illuminations.nctm.org/LessonDetail.aspx?id=L298
  - www.mste.uiuc.edu and use the search engine for scatter plots
  - Shows scatterplots, how they are set up, positive and negative correlation, and example problems: http://www.bbc.co.uk/schools/gcsebitesize/maths/data/scatterd iagramsrev1.shtml
- illuminations.nctm.org (Lessons, Activities and Related WebLinks)
- nlvm.usu.edu (National Library of Virtual Manipulatives)
- Oneplacesc.org (ETV Streamline and more!)
- http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

f. Assessing the Lesson

Formative Assessment Suggestion:
Use an exit slip: What types of generalizations can be made concerning the relationship between two data sets on a scatterplot?

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS:

- When two quantities increase or decrease together on a scatterplot, the correlation between quantities is said to be ________________.

- When one quantity increases as the other decreases on a scatterplot, the correlation between quantities is said to be ________________.

- Explain when a line of best fit can be drawn.

2. Teaching Lesson B: Interpreting Data Using Range and the Measures of Central Tendency

a. Indicators with Taxonomy

8-6.8 → Interpret graphic and tabular data representations by using range and the measures of central tendency (mean, median, and mode). (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson B: Interpreting Data Using Range and the Measures of Central Tendency

Your students while in the sixth grade have predicted the characteristics of one population based on the analysis of sample data and analyzed which measure of central tendency (mean, median, or mode) was the most appropriate for a given purpose.

For this lesson, the students will interpret graphic and/or tabular data representations by using range and measures of central tendency. Discuss with the students how to determine which measure to choose for a given set of data using the information from "Content Overview" section.

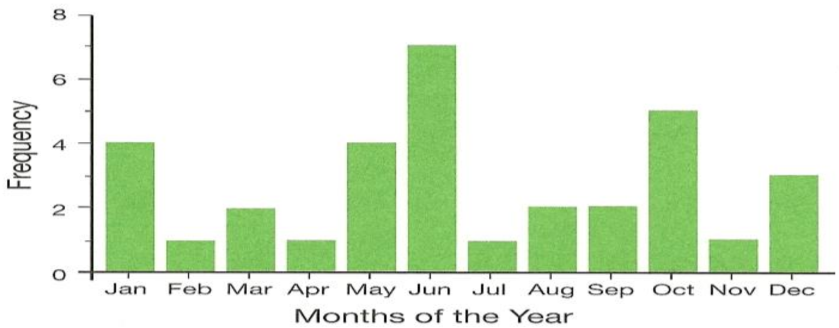
In the table is a sample of the students' last math test scores in Mrs. Smith's second period class.

Student	1	2	3	4	5	6	7	8	9	10	11	12	13
Score	38	43	98	96	55	95	95	56	62	63	89	87	65

Have students, working in groups of 3 or 4, find the measures of central tendency as well as the range. A spreadsheet can used to find the mean and the median. Have the groups discuss which average is a good measure of central tendency for this set of data. Each group will share their selection and explain their choice.
Be sure to include a discussion about the range for this set of data. Discuss usefulness of the range for this set of data.

<u>Additional Practice:</u>
Using the graph below, determine range and the measures of central tendency. Decide which result would be best to describe the data set and explain your thinking.

Rainfall for Year



c. Misconceptions/Common Errors

- The word average has been misinterpreted by students. The belief is that average refers to only the mean. Students need to realize that it can refer to the median or mode as well.

d. Additional Instructional Strategies/Differentiation

- While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

- <u>Specific Resources</u>:
  - Illuminations: Playing Games
    illuminations.nctm.org/LessonDetail.aspx?id=U77

    - Illuminations: Building Height illuminations.nctm.org/LessonDetails.aspx?id=L764
  - illuminations.nctm.org (Lessons, Activities and Related WebLinks)
  - nlvm.usu.edu (National Library of Virtual Manipulatives)
  - Oneplacesc.org (ETV Streamline and more!)
  - http://www.shodor.org/interactivate/ (Interactive Lessons and Activities based on our 2007 SC Mathematics Standards and Indicators)

f.Assessing the <u>Lesson</u>

<u>FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS</u>

- Explain when the mean is a good measure in summarizing data.

- When might the median be used to summarize a given data set?

- Give an example of when is it better to use mean to interpret data.

- Why is it misleading to use the term "average" when interpreting data?

III. Assessing the <u>Module</u>

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

Indicator 8-6.1 Generalize the relationship between two sets of data by using scatterplots and lines of best fit.

The objective of this indicator is to <u>generalize,</u> which is the "understand conceptual" knowledge cell of the Revised Bloom's Taxonomy. To generalize to abstract a general theme or major points; therefore, students are summarizing general themes as it related to the relationship between two set of data. The learning progression to **generalize** requires students to recall and understand the meaning of a scatterplot and a line of best fit. Students <u>analyze</u> graphical data to determine trends and the relationship between the points and the line of best fit. They <u>use inductive reasoning</u> (8-1.3) to formulate mathematical arguments about their observations. Students <u>explain</u> and <u>justify</u> their answers to their classmates and teacher using correct and clearly written or spoken words (8-1.6). If necessary, they <u>revise</u> their thinking based on the classroom discussion then <u>generalize</u>

mathematical statements (8-1.5) about the relationships between two sets of data.

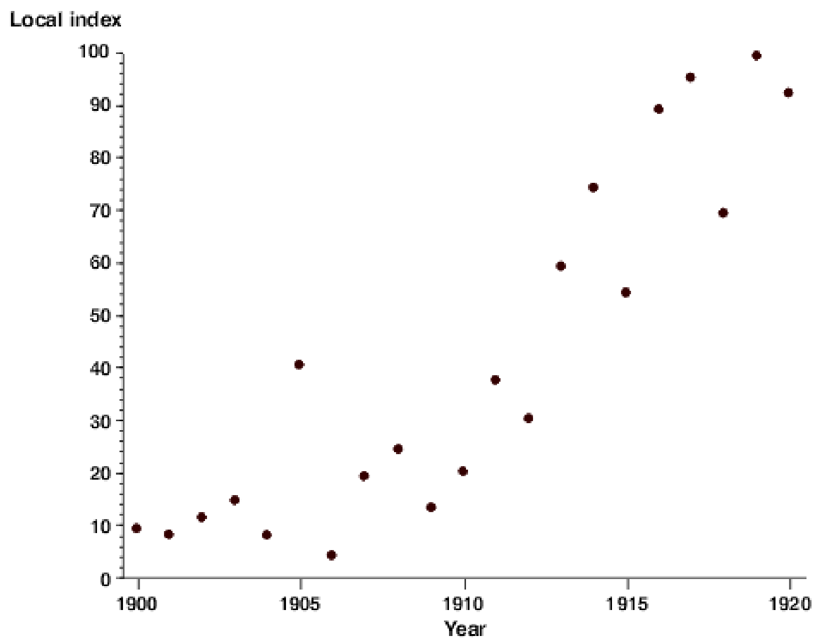
Indicator 8-6.8 Interpret graphic and tabular data representations by using range and the measures of central tendency (mean, median, and mode). (B2)

The objective of this indicator is to interpret, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To interpret involves changing from one form of representation to another (e.g. from graphic or tabular to verbal). The learning progression to **interpret** requires students to recall and understand the meaning of range, mean, median and mode. Students use their understanding of these statistics to analyze data in graphical and tabular form. They use inductive and deductive reasoning (8-1.5) to reach a conclusion and explain how each statistic impacts the data. Students explain and justify their answers using correct and clearly written or spoken words (8-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. Use the scatterplot to answer the following:

   Does the graph support a line of best fit? If your answer is yes, then draw the line of best fit for the data. If, your answer is no, then explain why there is no line of best fit. Describe the relationship between the two quantities.



2. Explain when a line of best fit can be drawn.

3. According to the *New York* Times on May 4, 2008, the top 16 nonfiction bestsellers had been on the bestseller list these numbers of weeks: 8, 3, 4, 2, 4, 2, 1, 3, 16, 4, 2, 1, 2, 8, 5, 1. Determine the range and the measures of central tendency . Decide which result would be best to describe the data set and explain your thinking.

4. Find the mean, median, and mode for the given set of data. Explain which measure would best summarize this set of data.

8.6, 9.1, 7.3, 8.4, 5.9