SOUTH CAROLINA SUPPORT SYSTEMS INSTRUCTIONAL GUIDE

Content Area	**Eighth Grade Math**

Fourth Nine Weeks

Standard/Indicators Addressed:

Standard 8-6: The student will demonstrate through the mathematical processes an understanding of the relationships between two variables within one population or sample.

8-6.3* Use theoretical and experimental probability to make inferences and convincing arguments about an event or events. (B3)
8-6.4* Apply procedures to calculate the probability of two dependent events. (C3)
8-6.5* Interpret the probability for two dependent events. (B2)
8-6.6* Apply procedures to compute the odds of a given event. (C3)
8-6.7* Analyze probability using area models. (B4)

- **These indicators are covered in the following Module for this Nine Weeks Period.**

Module 4-1 Probability and Odds

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines
Module 4-1 Lesson A: **Using Theoretical and Experimental Probability** 8-6.3 Use theoretical and experimental probability to make inferences and convincing arguments about an event or events. (B3)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries	See Instructional Planning Guide Module 4-1 "Introductory Lesson A"	See Instructional Planning Guide Module 4-1 "Lesson A 'Assessing the Lesson'"

	NCTM's Principals and Standards for School Mathematics (PSSM) **Textbook Correlations Appendix A**		

Indicator	Recommended Resources	Suggested Instructional Strategies	Assessment Guidelines
Module 4-1 Lesson B: Dependent Events **8-6.4** Apply procedures to calculate the probability of two dependent events. (C3) **8-6.5** Interpret the probability for two dependent events. (B2)	NCTM's Online Illuminations http://illuminations.nctm.org/ NCTM's Navigations Series SC Mathematics Support Document Teaching Student-Centered Mathematics Grades 5-8 and Teaching Elementary and Middle School Mathematics Developmentally 6th Edition, John Van de Walle www.ablongman.com/vandewalleseries NCTM's Principals and Standards for School Mathematics (PSSM) **Textbook Correlations Appendix A**	See Instructional Planning Guide Module 4-1 "Introductory Lesson B"	See Instructional Planning Guide Module 4-1 "Lesson B 'Assessing the Lesson'"
Module 4-1 Lesson C: Odds **8-6.6** Apply procedures to compute the odds of a given event. (C3)		See Instructional Planning Guide Module 4-1 "Introductory Lesson C"	See Instructional Planning Guide Module 4-1 "Lesson C 'Assessing the Lesson'"
Module 4-1 Lesson D: Probability Using Area Models **8-6.7** Analyze probability using area models. (B4)		See Instructional Planning Guide Module 4-1 "Introductory Lesson D"	See Instructional Planning Guide Module 4-1 "Lesson D 'Assessing the Lesson'"

MODULE

4-1

Probability and Odds

This module addresses the following indicators:

8-6.3 Use theoretical and experimental probability to make inferences and convincing arguments about an event or events.
8-6.4 Apply procedures to calculate the probability of two dependent events.
8-6.5 Interpret the probability for two dependent events.
8-6.6 Apply procedures to compute the odds of a given event.
8-6.7 Analyze probability using area models.

This module contains 4 lessons. These lessons are **INTRODUCTORY ONLY**. Lessons in S^3 begin to build the conceptual foundation student need. **ADDITIONAL LESSONS will be required** to fully develop the concepts.

I. Planning the Module

Continuum of Knowledge

8-6.3 Use theoretical and experimental probability to make inferences and convincing arguments about an event or events.

In sixth grade, students used theoretical probability to determine the sample space and probability for one- and two-stage events using tree diagrams, models, lists, charts, and pictures (6-6.4). In seventh grade students will find and differentiate between the theoretical and experimental probability of the same events. (7-6.7)

In eighth grade students will use theoretical and experimental probability to make inferences and convincing arguments about an event or events (8-6.3)

High school Data Analysis and Probability asks students to compare theoretical and experimental probability [DA-5.9].

8-6.4 Apply procedures to calculate the probability of two dependent events.

In sixth grade, students applied procedures to calculate the probability of complementary events (6-6.5). They also used theoretical probability to determine the sample space and probability for one- and two-stage events such as tree diagrams, models, lists, charts, and pictures (6-6.4) and applied procedures tocalculate the probability of complementary events (6-6.5). In seventh grade, students calculated (7-6.5) and interpreted (7-6.6) the probability of mutually exclusive simple or compound events. They also differentiated between experimental and theoretical probability of the same event (7-6.7) and used the fundamental counting principle to determine the number of possible outcomes for a multistage event (7-6.8)

In the eighth grade, students will calculate the probability of two dependent events (8-6.4).

In high school data analysis and probability, students classify events as either dependent or independent (DA-5.3) and find their probabilities.

8-6.5 Interpret the probability for two dependent events.

In sixth grade, students applied procedures to calculate the probability of complementary events (6-6.5). They also used theoretical probability to determine the sample space and probability for one- and two-stage events such as tree diagrams, models, lists, charts, and pictures (6-6.4) and applied procedures to calculate the probability of complementary events (6-6.5). In seventh grade, students calculated (7-6.5) and interpreted (7-6.6) the probability of mutually exclusive simple or compound events. They also differentiated between experimental and theoretical probability of the same event (7-6.7) and used the fundamental counting principle to determine the number of possible outcomes for a multistage event (7-6.8)

In the eighth grade, students will calculate the probability of two dependent events (8-6.4).

In high school data analysis and probability, students classify events as either or independent (DA-5.3) and find their probabilities.

8-6.6 Apply procedures to compute the odds of a given event.

In fifth grade, students represented the probability of a single-state event in words and fractions (5-6.5). In sixth grade, students applied procedures to calculate the probability of complementary events (6-6.5). They also used theoretical probability to determine the sample space and probability for one- and two-stage events such as tree diagrams, models, lists, charts, and pictures (6-6.4) and applied procedures to calculate the probability of complementary events (6-6.5). In seventh grade, students calculated (7-6.5) and interpreted (7-6.6) the probability of mutually exclusive simple or compound events. They also differentiated between experimental and theoretical probability of the same event (7-6.7) and used the fundamental counting principle to determine the number of possible outcomes for a multistage event (7-6.8)

In eighth grade, students learn what odds are and how to compute the odds of a event (8-6.6).

8-6.7 Analyze probability using area models.

In fifth grade, students applied formulas to determine the perimeters and areas of triangles, rectangles, and parallelograms (5-5.4). In the sixth grade, they applied strategies and procedures to estimate the perimeters and areas of irregular shapes (6-5.4). Seventh graders used ratio and proportion to solve problems involving scale factors and rates (7-5.1). As it relates to probability, seventh grade students calculated (7-6.5) and interpreted (7-6.6) the probability of mutually exclusive simple or compound events. They differentiated between experimental and theoretical probability of the same event (7-6.7) and used the fundamental counting principle to determine the number of possible outcomes for a multistage event (7-6.8)

In eighth grade, students calculate (8-6.4) and interpret (8-6.5) probability for two dependent events. They also analyze probability using area models (8-6.7).

In high school Geometry, student use geometric probability to solve problems [G-2.7].

2. Key Concepts/Key Terms

*These are vocabulary terms that are reasonable for students to know and be able to use. Terms without the * are additional terms for teacher awareness, knowledge and use in conversation with students.

Area model
*Experimental probability
Dependent events
Odds
*Theoretical probability
*Event
Sample Space
Independent Events
*Probability
∗Dependent events
∗Independent events
∗Event
∗Sample Space
∗Favorable
∗Unfavorable
*Outcome
*Ratio
Simulation

II. Teaching the Lessons

1. Teaching Lesson A: Using Theoretical and Experimental Probability

Probability in the eighth grade engages students in inferential thinking skills and persuasive arguments based upon theoretical or experimental probability. Students not only calculate the probability of <u>two dependent</u> events but also analyze the results. (With regard to dependent events, the results are more complex since before multiplying probabilities, students must consider the change in the number of outcomes for each event.) Students need to know or understand that when the outcome of one event affects the outcome of another event, the events are dependent. To find the probability for two dependent events, students can use the formula, P(A and B) = P(A)·P(B given A), where P(B given A) means the likelihood that B will happen when A happens.

After determining probability of both independent and dependent events, students should be able to use this information to make inferences, draw conclusions, and develop arguments to support their interpretation of the data. Probability thus develops into an applied science at this point.

In eighth grade, the study of probability also includes geometric area models for the first time. Investigations with these matrix models help students make predictions in the same way that the Punnett Square is used in biology to make predictions about genetic crosses. With an area model, students can determine the possible combinations when outcomes of different events are considered. An area model is drawn using a square to represent a whole with partitions drawn to represent the data for first event and then partition the square in the other directions to represent the data for the second event. The area model is easy for students to use and understand.
With appropriate charts, students should be able to calculate ratios and percentages.

Finally, students should be able to calculate the odds of an event (ratio of favorable outcomes to unfavorable outcomes for an event).

Students should have enough experience with this concept to distinguish between the odds of an event and its probability (favorable outcomes to total outcomes).

For this indicator, it is **essential** for students to:
- Understand the characteristics of theoretical probability
- Understand the characteristics of experimental probability
- Understand the purpose of a simulation
- Understand that experimental probability is the background for examining theoretical probability
- Understand that the experimental approach has its limitations because it is impossible to perform an infinite number of trials
- Understand the relationship between a simulation and experimental probability
- Understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of the outcomes in which the event occurs
- Write a probability using appropriate notation
- Interpret the probability

For this indicator, it is **not essential** for students to:
- Conduct the experiment to arrive at the experimental probability.

a. Indicators with Taxonomy

8-6.3 → *Use theoretical and experimental probability to make inferences and convincing arguments about an event or events. (B3)*

Cognitive Process Dimension: Understand
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson A: Using Theoretical and Experimental Probability

Materials Needed: Two color counters, number cubes

These findings below will be used throughout the lessons. Students should be able to write the probability as a fraction or in decimal form.

Review the definitions of theoretical and experimental probability.
theoretical probability - the likelihood that an event will happen, based on mathematical calculation

experimental probability - the frequency with which an event happened, based on an experiment

Compare theoretical probability and experimental results for an event. For example, let students work in small groups to calculate the probability of a coin landing heads up when it is flipped. Then have students flip the coin 20 times, recording the number of times it lands heads up. Have students compare the theoretical and experimental probabilities.

Use theoretical probability and experimental results to make predictions and decisions. Students should analyze the factors that cause experimental data to be poor predictors. For example, have students design games, try out the games, and decide whether they are fair or not using experimental results and theoretical probabilities.

Game 1: Toss 3 two-color counters. You get a point if there are at least two red counters showing. Otherwise, your partner gets a point. Is the game fair? Why or why not?

Game 2: Roll two number cubes. If the product of the two numbers is even, you get a point. If it is odd, your partner gets a point. Is the game fair? Why or why not?

Students then use this information to predict outcomes. For example, if you played the game 100 times, who would win?

Additional questions that can be asked:

If we continued the experiment how could we predict the probability of winning? For 100 games? 200 games?

Mathematically, what makes a game fair?
What was the experimental probability of each event happening?
What is the theoretical probability of each event happening?

How do these compare?
Graph your cumulative experimental probability for 5 games, 10 games, etc. How do these experimental probabilities compare as the number of games increases?
Could you use a graphing calculator to help? How would you use it?
How will the experimental probability compare to the theoretical probability as the number of games played increases?
What are the strategies you have used to find theoretical probabilities?

Let students create a simulation with computer technology to extend the experimental results to 1000 trials and compare the results to theoretical probability.

Have students find the probability of tossing a fair die and getting an odd number. Have the students make convincing arguments about their results.

Have students determine whether or not a probability can be greater than 1 or less than zero. Have them explain their thinking. Use the results from the activities above.

c. *Misconceptions/Common Errors*

- Students may attempt to substitute theoretical probability for experimental results because they feel their answer is wrong.
- Students may omit a resultant experimental event because it is an evident outlier and would not fit in with the other data.
- Students may not understand the impact of having very few attempts can cause the results to be deceptive.

d. *Additional Instructional Strategies*

While additional learning opportunities are needed, no suggestions are included at this time.

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:
- http://argyll.epsb.ca/jreed/math7/strand4/4201.htm

- http://illuminations.nctm.org/LessonDetail.aspx?ID=L448
- http://argyll.epsb.ca/jreed/math9/strand4/cards2a.htm (playing cards)

f. Assessing the Lesson

Assessment Note adapted from Teaching Student-Centered Mathematics Grades 5-8 by John A. Van de Walle: it is useful to have a list of conceptual ideas in learning about probability so that as you assess your students, you will not simply focus on the procedural skills. Assessments should be centered on a problem-based task that involves the analysis of an experiment. Have students write explanations to questions you include with the activity.

Formative Assessment Samples of Effective Questions
Sarah has a bag filled with 5 blue chips and 7 red chips.

- What is the probability of drawing a red chip?
- What is the probability if the chip was placed back in the bag, and then a second chip is picked?

2. Teaching Lesson B: Dependent Events

8-6.4 → Apply procedures to calculate the probability of two dependent events. (C3)

For this indicator, it is **essential** for students to:

- Understand the characteristics of dependent events
- Understand the difference between a dependent and independent event
- Be able to multiply fractions to find the probability of a two-stage event.
- Use appropriate strategies such as tree diagrams, area model, list, etc..
- Represent probabilities in fractional form
- Understand probability notation. Example: P(blue, green)
- Determine the reasonableness of their answers

For this indicator, it is **not essential** for students to:

- Calculate complementary, mutually exclusive or compound events.

8-6.5 → Interpret the probability for two dependent events. (B2)

For this indicator, it is **essential** for students to:

- Understand the characteristics of a dependent event
- Understand the difference between an independent and dependent event
- Interpret the meaning of a fractional probability
- Understand probability notation

For this indicator, it is **not essential** for students to:
· Calculate the probability
· Calculate complementary, mutually exclusive or compound events.

a. Indicators with Taxonomy

8-6.4 → Apply procedures to calculate the probability of two dependent events. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

8-6.5 → Interpret the probability for two dependent events. (B2)
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson B: Dependent Events

Materials Needed:
- Four coins for each team – two different denominations
- Small brown paper bags
- Calculators (optional)
- SMART board OR Overhead, transparencies, and markers

Students will calculate the probability of two events. Place all four coins (two different denominations) in a brown paper bag. Have students draw one coin from the bag, replace it, and then draw a second coin. Point out that this experiment is an example of independent events because the coin chosen the second time was not affected by the outcome of the first selection because the first coin was returned to the bag. Then, have students discuss how not returning the first coin could have changed the results of the second coin draw. With this in mind, have the teams calculate the probability that both coins chosen will be the same. In their discussion, have students describe how the ratio of pennies to total number of coins changes if the first coin selected is a penny and is not returned to the bag before the second coin is chosen. Explain that is why this probability is referred to as having ***dependent events***.

c. Misconceptions/Common Errors

- Students sometimes have a difficult time understanding the meaning of the term "dependent" in both probability and in algebra.
- Sample space is a term that is confused by many students as they think it is related to a sample.

d. Additional Instructional Strategies

Have the students define the word dependent. (Definition – when something is dependent, it relies on something other than itself. It is the opposite of independent.) In probability, a dependent event is one that is affected by another event that happens before it.

Independent Events

Give students an activity where they determine the possible outcomes for independent event in order to help them distinguish between dependent and independent events. For example, have students draw a tree diagram and list all possible outcomes for a situation in which a marble is picked from a bag containing 2 blue and 2 white marbles, placed back in the bag, and then a second marble is picked. After completing their diagrams and lists, have the students find the probability that both the marbles are white. Put a diagram on the overhead or SmartBoard to show that there are 4 favorable outcomes out of 16 possible outcomes for picking 2 white marbles when the marbles are replace after the first pick. Have the students discuss how to represent the probability of picking 2 white marbles with replacement. (Ans. – 4/16 = ¼)

Dependent Events (Use the same bag as above)
Have students draw a tree diagram and list all possible outcomes representing 2 picks from the same bag, but **not** replacing the marble after the first pick. Have students identify the probability for picking 2 white marbles. Put a diagram on the overhead or Smartboard. Since the marble was not replaced, ask the students if there would be the same number of possible outcomes as in the first activity. They should observe that there are 4 fewer outcomes. Therefore, the number of possible outcomes is 12 instead of 16. Using the diagram, have them determine the number of favorable outcomes for both marbles being white. Have the students determine the probability for picking 2 whites without replacement. (Ans., 2/12 or 1/6)
(It is important to note that by not replacing the marble reduced the number of marbles available for the second pick.)

These two activities show the students that finding the probability with replacement represents an example of independent events; but **without** replacement, the situation is different. This is an example of dependent event where the outcome of the second is depended on outcome of the first event.

To find the probability of the dependent event have the students use multiplication. Explain that for getting 2 white marbles without replacement, you have to assume that the first event you are looking for occurred because that is the situation you are interested in. If a white marble was removed, there will be only 3 marbles left and only one of the 3 will be white. The sentence for 2 white marbles without replacement should be $\frac{1}{4} \times \frac{1}{3} = 1/12$.

Give student several problems to compute the probability of two dependent events.

If two events, A and B, are dependent, then the probability of both events occurring is the product of the probability of A and the probability of B after A occurs.

P(A and B) = P(A) x P(B following A)

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:

Conduct probability exercises using marbles in a bag, candy or fake lotto tickets with student names. Have students use the software Inspiration, the drawing tools in MS Office or another graphic organizer to create tree diagrams to represent probability results visually.

f. Assessing the Lesson

Assessment Note adapted from Teaching Student-Centered Mathematics Grades 5-8 by John A. Van de Walle: it is useful to have a list of conceptual ideas in learning about probability so that as you assess your students, you will not simply focus on the procedural skills. Assessments should be centered on a problem-based task that involves the analysis of an experiment. Have students write explanations to questions you include with the activity.

Have each team prepare a brief argument explaining how they would determine the experimental probability of the dependent event described in the lesson. Require the teams to develop a convincing argument to support their prediction based upon their analysis of the theoretical probability and the knowledge that they have acquired in this lesson about experimental probability.

3. ***Teaching Lesson C: Odds***

The discussion of odds should include that odds are another way to express probability; ratio involving favorable outcomes and unfavorable outcomes or Chances for versus Chances against.

$$\text{Probability} = \frac{\#\text{ of favorable outcomes}}{\#\text{ of possible outcomes}}$$

$$\text{Odds} = \frac{\#\text{ of favorable outcomes}}{\#\text{ of unfavorable outcomes}}$$

To go from probability to odds:
1. Students should note that the numerator is the same in both probability and odds, so the numerator of the answer will be the same.
2. To find the denominator (or the number of undesirable outcomes), we subtract the numerator from the denominator.

Example: If the probability of winning is 7/15, then the odds of winning is ___?
- The numerator is the same which is 7.
- The denominator is: 15-7 = 8 (possible – favorable = unfavorable).
- The answer is 7/8.

Students may also explore going from odds to probability.

For this indicator, it is **essential** for students to:
- Understand the meaning of odds
- Understand the difference between odds and probability
- Understand the relationship between odds and probability
- Understand the difference between favorable outcomes and unfavorable outcomes (other phrases like desirable/undesirable, chances for/chances against, etc…)
- Write the odd in fractional form (3/2) or as a ratio 3:2
- Interpret the meaning of odds. For example, 3 to 2 means 3 favorable outcomes to every 2 unfavorable outcomes, and we write 3:2.

For this indicator, it is **not essential** for students to:
- None noted

a. Indicators with Taxonomy

8-6.6 → Apply procedures to compute the odds of a given event. (C3)
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge

b. Introductory Lesson C: Odds

Materials Needed: coins

Discuss what is meant by odds. The discussion should have the students saying that odds are another way to express probability; ratio involving favorable outcomes and unfavorable outcomes or Chances for : Chances against

We say the odds are "3 to 2," which means 3 favorable outcomes to every 2 unfavorable outcomes, and we write 3 : 2. For example, the odds of rolling a 5 or greater are 2 : 4, which reduces to 1 : 2.

Have the students toss a coin two times, what are the odds for it landing heads at least once?
(Ans., Favorable outcomes: 3 -- HH, HT, TH. Unfavorable outcomes: 1 -- TT. Thus, the odds for it landing heads at least once are 3 to 1, or 3 : 1.)

Ask, "If the odds for an event are 3: 2, what is the probability of the event happening?" Discuss with the students how to solve this problem.
(Ans., Favorable outcomes = 3.
Possible outcomes = favorable outcomes + unfavorable outcomes = 3 + 2 = 5.
Thus, the probability of the event happening is $\frac{3}{5}$.)

Give groups of students bags filled with 5 blue chips and 7 red chips. Have students draw a table that lists the number of favorable outcomes in the first column and the number of non-favorable outcomes in the second column. Explain that they will only need to take the results of the table to determine the odds: favorable to unfavorable.

Favorable Outcomes	Unfavorable outcomes

If they are asked the number of blue chips to red chips, blue chips would be considered favorable because it was mentioned first, so the table would be completed as follows:

Favorable Outcomes	Unfavorable outcomes
5	7

The odds would be 5 to 7 in this situation.

If they are determining the odds of red chips to blue chips, the table would be completed as follows:

Favorable Outcomes	Unfavorable Outcomes
7	5

The odds are reported as 7 to 5.

c. Misconceptions/Common Errors

Students may confuse finding probability with determining the odds.

d. *Additional Instructional Strategies*

While additional learning opportunities are needed, no suggestions are included at this time.

e.Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:
http://illuminations.nctm.org/ActivityDetail.aspx?ID=205
The Dynamic Paper application allows you to create images of nets for 3-D shapes, tessellations of polygons, number grids with any number of rows, columns, and integers, spinners with various numbers of sectors, and more! You can then create a PDF worksheet with the images you choose, or you can export the images to JPEG format for use in other applications or on the web.

f. Assessing the Lesson

Assessment Note adapted from Teaching Student-Centered Mathematics Grades 5-8 by John A. Van de Walle: it is useful to have a list of conceptual ideas in learning about probability so that as you assess your students, you will not simply focus on the procedural skills. Assessments should be centered on a problem-based task that involves the analysis of an experiment. Have students write explanations to questions you include with the activity.

Formative Assessment Notes

Observe the students as they complete the table and determine the odds for the chips in the activity above.

Formative Assessment Samples of Effective Questions

What do we mean when we say the odds are 5:3?

4. Teaching Lesson D : Analyzing Probability Using Area Models

The area model will not solve all probability problems; however, it fits well into a developmental approach to the subject because it is conceptual, it is based on existing knowledge of fractions, and more symbolic approaches can be derived from it.

For this indicator, it is **essential** for students to:
- Compute the area of polygons
- Interpret probability notation
- Understand how to represent probability using a rectangular area model
- Understand that the probability is the ratio between the area of the shapes; regardless of the shapes
- Understand how to set up a ratio
- Interpret area models

For this indicator, it is **not essential** for students to:
- None noted

a. Indicators with Taxonomy

8-6.7 → Analyze probability using area models. (B4)
Cognitive Process Dimension: Analyze
Knowledge Dimension: Conceptual Knowledge

b. Introductory Lesson D: Analyzing Probability Using Area Models

Materials Needed: area models, coins

An area model is drawn to represent a whole with partitions drawn to represent the data for first event and then partition the square in the other directions to represent the data for the second event. The area model is easy for students to use and understand.
Have student toss a coin. Correlate the possible combinations with the number patterns in the model and analyze the probability for each probability.

H	T

Analysis: Row 1: 1 coin, 1 toss – 2 combinations (1+1=2) – probability of heads is 1 out of 2 or ½ , tails: 1 out of 2 or ½

HH	HT
TH	TT

Analysis: Row 2: 1 coin, 2 tosses, 4 possible outcomes (1 + 2 + 1=4) 2 heads: ¼ or 1 out of 4; 1 head and 1 tail 2/4 or 2 out of 4 chances; 2 tails ¼ or 1 out of 4

Have students complete 20 flips and analyze probability and share with the class.

c. *Misconceptions/Common Errors*

- When constructing the rectangular area models, students may not understand that the answer is the overlapping region.
- Students may struggle with the conceptual basis of constructing area models.

d. *Additional Instructional Strategies*

- An area model is drawn to represent a whole with partitions drawn to represent the data for first event and then partition the square in the other directions to represent the data for the second event. The area model is easy for students to use and understand. Students can make connections with pictorial models used to multiply fractions.
- Another aspect is to actually work with area itself. The dart board analogy usually interests students. Find the probability of hitting the center.

$$P(\text{hitting the center}) = \frac{\text{area of center circle}}{\text{area of dart board circle}}$$

- Students' understanding of the area model should extend beyond circle inside of circle. They should transfer their understanding to circle inside of rectangles, rectangles inside of circles, etc… (any combination of polygons their know).

e. Technology

Virtual manipulatives should NOT take the place of concrete manipulation of objects/materials. Once conceptual understanding has been reached, you may move to pictorial representations and then virtual manipulatives. Concrete manipulatives should be the focus of learning to build conceptual understanding. Real life situations/representations are critical for conceptual understanding.

These are suggestions for resources:
http://illuminations.nctm.org/ActivityDetail.aspx?ID=205
The Dynamic Paper application allows you to create images of nets for 3-D shapes, tessellations of polygons, number grids with any number of rows, columns, and integers, spinners with various numbers of sectors, and more! You can then create a PDF worksheet with the images you choose, or you can export the images to JPEG format for use in other applications or on the web.

f. Assessing the Lesson

Assessment Note adapted from Teaching Student-Centered Mathematics Grades 5-8 by John A. Van de Walle: it is useful to have a list of conceptual ideas in learning about probability so that as you assess your students, you will not simply focus on the procedural skills. Assessments should be centered on a problem-based task that involves the analysis of an experiment. Have students write explanations to questions you include with the activity.

FORMATIVE ASSESSMENT SAMPLES OF EFFECTIVE QUESTIONS

A spinner has 7 equal sized sectors of green, red, yellow, purple, pink, orange, and blue. Describe the area model you would use to display the outcomes.

III. Assessing the Module

At the end of this module summative assessment is necessary to determine student understanding of the connections among and between the indicators addressed in this module.

8-6.3 Use theoretical and experimental probability to make inferences and convincing arguments about an event or events.

The objective of the indicator is to use, which is in the "apply conceptual" knowledge cell of the Revised Taxonomy. To apply conceptual knowledge means to use your understanding of interrelationships among concepts to solve problems as opposed to an algorithm. The learning progression to **use** requires students to recall and understand the meaning of theoretical and experimental probability. They use that understand to make observations about the results of an event or events and explain and justify their observations to their classmate and teacher. Based on their discussions, students generalize mathematical statements (8-1.5) about the data and use those statements to make inferences about the event or events. They use correct and clearly written or spoken words to communicate their understanding (8-1.6).

8-6.4 Apply procedures to calculate the probability of two dependent events.

The objective of this indicator is to interpret which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To interpret is to change from one form (numerical) to another form (verbal). The learning progression to **interpret** requires students to understand the meaning of dependent events. Given a real world problem and a probability, students use inductive and deductive reasoning (8-1.3) and summarize observations. They use their understanding of the characteristics of dependent events to generalize mathematical statements (8-1.5) about their observations. Students use correct and clearly written or spoken words to communicate their understanding.

8-6.5 Interpret the probability for two dependent events.

The objective of this indicator is to interpret which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To interpret is to change from one form (numerical) to another form (verbal). The learning progression to **interpret** requires students to understand the meaning of dependent events. Given a real world problem and a probability, students use inductive and deductive reasoning (8-1.3) and summarize observations. They use their understanding of the characteristics of dependent events to generalize mathematical statements (8-1.5) about their observations. Students use correct and clearly written or spoken words to communicate their understanding.

8-6.6 Apply procedures to compute the odds of a given event.

The objective of this indicator is to analyze which is in the "analyze conceptual" knowledge cell of the Revised Taxonomy. To analyze means break material (probability) into its constituent parts (area of shapes) and determine how the parts relate to one another and to the overall structure. The learning progression to **analyze** requires students to understand the meaning of probability. Students recall their knowledge of multiplying fractions using rectangular models and how to compute the area of polygons. They explore probability using these models and generalize the connections (8-1.7) between these concepts. They generalize mathematical statements (8-1.5) about these connections and use that understanding to solve problems. Students explore other types of area models and generalize the connection between (8-1.5) part/whole ratios and area of shape/areas of whole shape ratios. They solve a variety of problems and explain and justify their answers using correct and clearly written or spoken words (8-1.6).

8-6.7 Analyze probability using area models.

The objective of this indicator is to interpret, which is in the "understand conceptual" knowledge cell of the Revised Taxonomy. To interpret involves changing from one form of representation to another (e.g. from graphic or tabular to verbal). The learning progression to **interpret** requires students to recall and understand the meaning of range, mean, median and mode. Students use their understanding of these statistics to analyze data in graphical and tabular form. They use inductive and deductive reasoning (8-1.5) to reach a conclusion and explain how each statistic impacts the data. Students explain and justify their answers using correct and clearly written or spoken words (8-1.6).

The following examples of possible assessment strategies may be modified as necessary to meet student/teacher needs. These examples are not derived from nor associated with any standardized testing.

1. A spinner has three equal sections labeled 1, 2, and 3. A second spinner has five equal sections labeled A, B, C, D, and E. What is the probability that you would get 1-B after spinner both spinners? Support your conclusion.

2. A box contains chips of different colors. For a single draw, the probability of getting a red chip is 3/8, the probability of getting a green chip is 1/8, and the probability of getting a white chip is ½. What is the probability of drawing a blue chip?

3. The table shows the results of an experiment in which one cube was selected from a bag, its color recorded, and the cube was returned to the bag and then another cube was selected. Find each experimental probability.

Color	# of times

	selected
Yellow	20
Red	39
Blue	26
Green	15

P (yellow) = P (red) = P (blue) =
P (green) = P (yellow or blue) =

In the next 300 picks, how many times would you expect to get a blue cube?

4. Two names are picked from a basket to see who will have to bring refreshments for the next meeting. Jean, Robert, Camille, Mary Ann, and Rodney have their names in the basket. What is the probability that both Camille and Rodney will have their names picked?

Answer: 1/5 x ¼ = 1/20

5. At the school carnival, an average of 4 students in 10 win a prize at the bean bag toss booth. Give the odds against winning.

6. Tom has 2 nickels, 3 dimes, 4 pennies and 5 quarters in his pocket. What are the odds that he will draw a dime out of the pocket on the first try?

7. Sam has 3 nickels, 4 dimes, 5 pennies and 6 quarters in his pocket. What are the odds that he will draw a penny out of his pocket on the first try?

8. Based on the data in the table;

P (red) =
P (blue) =
P (yellow) =

What is the theoretical probability for landing on each color?

1	Red
2	Red
3	Blue
4	Red
5	Yellow
6	Blue
7	Yellow
8	red