## Lesson: Coding Coordinate Plane Transformations

## Lesson Overview

In this lesson students will explore transformations of figures on a coordinate plane. Transformations will include rotations, reflections, and translations. Students will discover the rules of transformations on ( $x, y$ ). Students will use block coding, through Scratch, to explore the transformations

## Standards Addressed

8.GM. 2 Apply the properties of rigid transformations (rotations, reflections, translations).
a. Rotate geometric figures 90, 180, and 270 degrees, both clockwise and counterclockwise, about the origin.
b. Reflect geometric figures with respect to the $x$-axis and/or $y$-axis.
c. Translate geometric figures vertically and/or horizontally.
d. Recognize that two-dimensional figures are only congruent if a series of rigid transformations can be performed to map the pre-image to the image.
e. Given two congruent figures, describe the series of rigid transformations that justifies this congruence.

Disciplinary Literacy Strategies
Partner Dialogue, Making Thinking Visible, Exit Ticket
Computational Thinking
Tools:
Coding
Cornerstone(s) Addressed:

- Decomposition - students will need to break down the steps to making and labeling ordered pairs in a transformation.
- Pattern Recognition - students from the break-down of the steps of transformations will use patterns of the steps to determine a rule for each transformation.
- Algorithmic thinking - student will follow rules/procedures when finding the ordered pair names ( $x, y$ ) for transformed points.


## Lesson Plan

Time required: three (55-minute) class periods
Focus Question: How can coding help me explore what happens to ordered pairs when transformations
(reflection, rotation, or translation) occur?
Disciplinary Vocabulary: reflection, rotation, translation, transformation, origin, $x$-axis, $y$-axis, counterclockwise, clockwise, ordered pair, coordinate plane, quadrant
Materials needed:

- Graph paper
- Shapes templates (with defined sides)
- Markers or colored pencils
- Devices with scratch
- Internet access (optional but easier)


## Engage

1. Discuss vocabulary around "transformations". A transformation is a change. When applied to ordered pairs, it is a change in location of the ordered pair. Transformations can occur as translations (slide or
shift), reflections (flips), or rotations (turns). Translations can shift or slide up/down or left/right. Reflections can flip over the x-axis or $y$-axis. And, rotations can turn clockwise or counterclockwise.
2. Review Quadrants I, II, III, and IV locations on the coordinate plane. Review the origin and $x$ and $y$ axes.
3. Tell students the goal of this lesson is to explore transformations using coding and discover the rules for transformations on ordered pairs. In the end, they will be writing code for transformations.
4. To introduce transformations, have students plot the ordered pair $(3,7)$ on the coordinate plane (physically on graph paper). Ask them to translate the point up 4 units. What is the new ordered pair? $(3,11)$. What if we wanted to reflect the new ordered pair over the $y$-axis? What would the new ordered pair become? $(-3,11)$. What if we wanted to reflect that new ordered pair over the $x$-axis? What would the new ordered pair become? ( $-3,-11$ ). If we were to rotate the new ordered pair 90 degrees clockwise, where would the new ordered pair be located? ( $-11,3$ ).
5. Repeat this process with a new ordered pair. Have students work with Elbow Partners to start at $(4,5)$. Ask them to translate the ordered pair left 3 units, then down 2 units, then reflect over the $y$-axis, and then rotate 180 degrees clockwise. Ask pairs to determine where the ordered pair would lie after all transformations were complete. (1,-3)

## Explore

1. Still working in pairs students should create a scratch account if at least one on the pair does not already have one. (NOTE: This will be need later to create but some of them may want to save this program to work from)
2. Play the scratch program.
a. Found at this Link: https://scratch.mit.edu/projects/392727983
b. Read the Instructions then press the go full screen.
c. Input the information requested.
d. Once you have completed all 3 transformations you can "look inside" to view the coding.
e. Record the input and answer the following question about each transformation:
i. What did you notice about the movement of the sprite? (NOTE: students may discover the sprite's center is what moves the designated spaces during the translation but that doesn't happen with rotation and reflection - BUT the concept is presented well - they will need to choose a more appropriate sprite to demonstrate them later in the lesson.)
ii. What did you notice about the sprite's position on the graph? (NOTE: same as above)
iii. What did you notice about the change in $x$ and $y$ ?
3. Look inside the program for the following:
a. Flow - how the blocks move the sprite
b. Sequence - the order of the blocks
c. Inputs - (what questions are asked for the user to enter) and Outputs (what happens to the sprite as a result of the answers)

## Students should be given the Transformation Tables Handout (located at the end of this lesson plan).

## Translations

- Create a rectangle $A B C D$ on the coordinate plane with coordinates $A(1,-1), B(3,-1), C(1,-4), D(3,-4)$. Translate each coordinate (ordered pair) left 2 units and up 3 units.
- Write the new ordered pairs in the chart below under Translation 1.
- Return to the original rectangle ABCD.
- Translate each coordinate (ordered pair) right 4 units and down 2 units.
- Write the new ordered pairs in the chart below under Translation 2.
- Return to the original rectangle ABCD.
- Translate each coordinate (ordered pair) right 1 unit and up 2 units.
- Write the new ordered pairs in the chart below under Translation 3.

Use the Partner Dialogue Strategy to have students look for the rule of what happens to $x$ and $y$ in ( $x, y$ ) when you shift or translate a unit left or right, then what happens when you translate a unit up or down. Which variable is changing? How is it changing? How would you describe the change on $x$ and the change on y algebraically?

Help students put their explanations into an algebraic rule. This is what will go in the last row under each column for the translations found.

| Original Coordinates | Translation 1: <br> Left 2, Up 3 | Translation 2: <br> Right 4, Down 2 | Translation 3: <br> Right 1, Up 2 |
| :--- | :--- | :--- | :--- |
| $(1,-1)$ |  |  |  |
| $(3,-1)$ |  |  |  |
| $(1,-4)$ |  |  |  |
| $(3,-4)$ |  |  |  |
| $(x, y)$ RULES $\rightarrow$ |  |  |  |

Answer Key

| Original Coordinates | Translation 1: <br> Left 2, Up 3 | Translation 2: <br> Right 4, Down 2 | Translation 3: <br> Right 1, Up 2 |
| :--- | :--- | :--- | :--- |
| $(1,-1)$ | $(-1,2)$ | $(5,-3)$ | $(2,1)$ |
| $(3,-1)$ | $(1,2)$ | $(7,-3)$ | $(4,1)$ |
| $(1,-4)$ | $(-1,-1)$ | $(5,-6)$ | $(2,-2)$ |
| $(3,-4)$ | $(1,-1)$ | $(7,-6)$ | $(4,-2)$ |
| $(x, y)$ RULES $\rightarrow$ | $(x-2, y+3)$ | $(x+4, y-2)$ | $(x+1, y+2)$ |

## Reflections

- Create a triangle EFG on the coordinate plane with coordinates $\mathrm{E}(2,2), \mathrm{F}(4,5), \mathrm{G}(2,5)$.
- Reflect each coordinate (ordered pair) over the x-axis.
- Write the new ordered pairs in the chart below under Reflection over the x-axis.
- Return to the original triangle EFG.
- Reflect each coordinate (ordered pair) over the $y$-axis.
- Write the new ordered pairs in the chart below under Reflection over the $y$-axis.

Use the Partner Dialogue Strategy to have students look for the rule of what happens to $x$ and $y$ in ( $x, y$ ) when you reflect an ordered pair over the $x$-axis and the $y$-axis. What happens when you make that reflection? Which variable is changing? How is it changing? How would you describe the change on $x$ and the change on $y$ algebraically?
Help students put their explanations into an algebraic rule. This is what will go in the last row under each column for the reflections over $x$-axis or $y$-axis.

| Original Coordinates | Reflection over the $x$-axis | Reflection over the $y$-axis |
| :--- | :--- | :--- |
| $(2,2)$ |  |  |


| $(4,5)$ |  |  |
| :--- | :--- | :--- |
| $(2,5)$ |  |  |
| $(x, y)$ RULES $\rightarrow$ |  |  |

Answer Key

| Original Coordinates | Reflection over the $x$-axis | Reflection over the $y$-axis |
| :--- | :--- | :--- |
| $(2,2)$ | $(2,-2)$ | $(-2,2)$ |
| $(4,5)$ | $(4,-5)$ | $(-4,5)$ |
| $(2,5)$ | $(2,-5)$ | $(-2,5)$ |
| $(x, y)$ RULES $\rightarrow$ | $(x,-y)$ | $(-x, y)$ |

## Rotations

- Create a triangle HIJ on the coordinate plane with coordinates $\mathrm{H}(1,6), \mathrm{I}(1,2), \mathrm{J}(8,2)$.
- Rotate each coordinate (ordered pair) 90 degrees clockwise (or 270 degrees counterclockwise) about the origin.
- Write the new ordered pairs in the chart below under Rotation $90^{\circ}$ clockwise $/ 270^{\circ}$ counterclockwise.
- Return to the original triangle HIJ.
- Rotate each coordinate (ordered pair) 180 degrees clockwise (or 180 degrees counterclockwise) about the origin.
- Write the new ordered pairs in the chart below under Rotation $180^{\circ}$ clockwise $/ 180^{\circ}$ counterclockwise.
- Return to the original triangle HIJ.
- Rotate each coordinate (ordered pair) 270 degrees clockwise (or 90 degrees counterclockwise) about the origin.
- Write the new ordered pairs in the chart below under Rotation $270^{\circ}$ clockwise $/ 90^{\circ}$ counterclockwise.
- Return to the original triangle HIJ.
- Rotate each coordinate (ordered pair) 360 degrees clockwise (or 360 degrees counterclockwise) about the origin.
- Write the new ordered pairs in the chart below under Rotation $360^{\circ}$ clockwise/360 ${ }^{\circ}$ counterclockwise.

Use the Partner Dialogue Strategy to have students look for the rule of what happens to $x$ and $y$ in ( $x, y$ ) when you rotate an ordered pair around the origin at different degrees. Which variable(s) is/are changing? How is/are it/they changing? How would you describe the change on $x$ and the change on $y$ algebraically?
Help students put their explanations into an algebraic rule. This is what will go in the last row under each column for the rotations (at varying degrees respectfully).

| Original Coordinates | Rotation $90^{\circ}$ clockwise or $270^{\circ}$ counterclockwise | Rotation $180^{\circ}$ clockwise or $180^{\circ}$ counterclockwise | Rotation $270^{\circ}$ clockwise or $90^{\circ}$ <br> counterclockwise | Rotation $360^{\circ}$ clockwise or $360^{\circ}$ counterclockwise |
| :---: | :---: | :---: | :---: | :---: |
| $(1,6)$ |  |  |  |  |
| $(1,2)$ |  |  |  |  |
| $(8,2)$ |  |  |  |  |


| $(x, y)$ RULES $\rightarrow$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |

Answer Key

| Original Coordinates | Rotation $90^{\circ}$ clockwise or $270^{\circ}$ <br> counterclockwise | Rotation $180^{\circ}$ clockwise or $180^{\circ}$ counterclockwise | Rotation $270^{\circ}$ <br> clockwise or $90^{\circ}$ <br> counterclockwise | Rotation $360^{\circ}$ clockwise or $360^{\circ}$ <br> counterclockwise |
| :---: | :---: | :---: | :---: | :---: |
| $(1,6)$ | (6, -1) | (-1, -6) | $(-6,1)$ | $(1,6)$ |
| $(1,2)$ | $(2,-1)$ | $(-1,-2)$ | $(-2,1)$ | $(1,2)$ |
| $(8,2)$ | $(2,-8)$ | $(-8,-2)$ | $(-2,8)$ | $(8,2)$ |
| $(\mathrm{x}, \mathrm{y})$ RULES $\rightarrow$ | ( $\mathrm{y},-\mathrm{x}$ ) | (-x, -y) | $(-y, x)$ | ( $\mathrm{x}, \mathrm{y}$ ) |



## Explain

Assign each group/pair a transformation. One of the pair will need an account (they are free and easy to set up just a username (school email) and password (school).

1. Create a scratch program that has one of the following transformations:
a. A translation along both the $x$ and $y$.
b. A rotation at least 90 degrees but not more than 270 degrees
c. A reflection
2. The program should use the following:
a. A sprite (or any shape), other than the cat, in the scratch database (If you want to allow students to use their own images have them get permission first).
b. The program should get input from the player and the output should be based on the player's input.
3. Create a Making Thinking Visible (MTV) to explain their rules/processes to the group. Possibilities include:
a. Make a screen capture of their scratch program with narration.
b. Use another device and do the same with Flipgrid (you will need an account with this assignment in it, so they know where to send the video).
c. Create chart with a drawing or another graphic.

## Elaborate

What happens when a figure undergoes multiple transformations?

Part 1: Start with triangle $A B C$ where $A(2,3), B(5,5), C(4,2)$.
Have students work with their elbow partner or table group to complete the chart with the algebraic rules for each given transformation in the chart. They may also choose to create the figure on a coordinate plane.

| Original Coordinates | Reflection over the $y$-axis | Rotation $90^{\circ}$ clockwise | Translation 2 units left and <br> 1 unit down |
| :--- | :--- | :--- | :--- |
| $(x, y)$ | Rule: |  | Rule: |
| $(2,3)$ |  |  | Rule: |
| $(5,5)$ |  |  |  |
| $(4,2)$ |  |  |  |

Answer Key

| Original Coordinates | Reflection over the $y$-axis | Rotation $90^{\circ}$ clockwise | Translation 2 units left and <br> 1 unit down |
| :--- | :--- | :--- | :--- |
| $(x, y)$ | Rule: $(-x, y)$ | Rule: $(y,-x)$ | Rule: $(x-2, y-1)$ |
| $(2,3)$ | $(-2,3)$ | $(3,2)$ | $(1,1)$ |
| $(5,5)$ | $(-5,5)$ | $(5,5)$ | $(3,4)$ |
| $(4,2)$ | $(-4,2)$ | $(2,4)$ | $(0,3)$ |

Part 2: Start with rectangle EFGH where $\mathrm{E}(-1,3), F(-4,3), G(--4,1), H(-1,1)$
Have students work with their elbow partner or table group to complete the chart with the algebraic rules for each given transformation in the chart. They may also choose to create the figure on a coordinate plane.

| Original Coordinates | Reflection over the $x$-axis | Rotation $90^{\circ}$ counterclockwise | Translation 2 units right and 3 units up |
| :---: | :---: | :---: | :---: |
| ( $x, y$ ) | Rule: | Rule: | Rule: |
| $(-1,3)$ |  |  |  |
| $(-4,3)$ |  |  |  |
| $(-4,1)$ |  |  |  |
| $(-1,1)$ |  |  |  |

Answer Key

| Original Coordinates | Reflection over the $x$-axis | Rotation $90^{\circ}$ <br> counterclockwise | Translation 2 units right <br> and 3 units up |
| :--- | :--- | :--- | :--- |
| $(x, y)$ | Rule: $(x,-y)$ | Rule: $(-y, x)$ | Rule: $(x+2), y+3)$ |
| $(-1,3)$ | $(-1,-3)$ | $(3,-1)$ | $(5,2)$ |
| $(-4,3)$ | $(-4,-3)$ | $(3,-4)$ | $(5,-1)$ |
| $(-4,1)$ | $(-4,-1)$ | $(1,-4)$ | $(3,-1)$ |
| $(-1,1)$ | $(-1,-1)$ | $(1,-1)$ | $(3,2)$ |

## Evaluate

## Exit Ticket

Consider the following graphical representation. Pay attention to the color key as well to determine which figure is based on which transformation. What would the next set of ordered pairs become if the figure was next translated into Quadrant IV? What type of transformation would need to occur? What is the rule for that transformation?


Assessment Notes
MTV plus exit ticket - formative assessment purposes
Teacher Biographical Information
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## Transformation Tables Handout

## Translations:

| Original Coordinates | Translation 1: <br> Left 2, Up 3 | Translation 2: <br> Right 4, Down 2 | Translation 3: <br> Right 1, Up 2 |
| :--- | :--- | :--- | :--- |
| $(1,-1)$ |  |  |  |
| $(3,-1)$ |  |  |  |
| $(1,-4)$ |  |  |  |
| $(3,-4)$ |  |  |  |
| $(x, y)$ RULES $\rightarrow$ |  |  |  |

## Reflections:

| Original Coordinates | Reflection over the $x$-axis | Reflection over the $y$-axis |
| :--- | :--- | :--- |
| $(2,2)$ |  |  |
| $(4,5)$ |  |  |
| $(2,5)$ |  |  |
| $(x, y)$ RULES $\rightarrow$ |  |  |

## Rotations:

| Original | Rotation $90^{\circ}$ <br> clockwise or <br> $270^{\circ}$ <br> Coordinates <br> counterclock- | Rotation $180^{\circ}$ <br> clockwise or <br> $180^{\circ}$ <br> counterclock- <br> wise | Rotation $270^{\circ}$ <br> clockwise or <br> $90^{\circ}$ <br> counterclock- <br> wise | Rotation $360^{0}$ <br> clockwise or |
| :--- | :--- | :--- | :--- | :--- |
| $(1,6)$ |  |  |  | $360^{\circ}$ <br> counterclock- <br> wise |
| $(1,2)$ |  |  |  |  |
| $(8,2)$ |  |  |  |  |
| $(x, y)$ RULES $\rightarrow$ |  |  |  |  |

