

# MODULE FOUR

## **This module addresses functions**

SC Academic Elementary Algebra Standards:

- EA-3.1 Classify a relationship as being either a function or not a function when given data as a table, set of ordered pairs, or graph.
- EA-3.2 Use function notation to represent functional relationships.
- EA-3.3 Carry out a procedure to evaluate a function for a given element in the domain.
- EA-3.4 Analyze the graph of a continuous function to determine the domain and range of the function.
- EA-3.5 Carry out a procedure to graph parent functions  
(including  $y = x$ ,  $y = x^2$ ,  $y = \sqrt{x}$ ,  $y = |x|$ , and  $y = \frac{1}{x}$ ).

**The resources provided in this module are not all inclusive. They are provided to begin to build the conceptual foundation students need. Additional resources will be required to develop the concepts.**

<b>Lesson # 1</b>
<b>Topic: Classifying a relation as a function or not a function</b>
<b>Standard (s): EA – 3.1</b>

## ***I. Planning the Lesson***

*The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.*

- **Continuum of Knowledge**

- In 8<sup>th</sup> grade, students translate among verbal, graphic, tabular, and algebraic representations of linear functions (8-3.1). They also classify relationships between two variables in graphs, tables, and/or equations as either linear or nonlinear (8-3.5).
- In Elementary Algebra, students classify a relationship as being either a function or not a function when given data as a table, set of ordered pairs, or graph (EA-3.1).
- In Intermediate Algebra, students carry out procedures to perform operations on polynomial functions (including  $f(x) + g(x)$ ,  $f(x) - g(x)$ ,  $f(x) \cdot g(x)$ , and  $f(x)/g(x)$ ) (IA-2.5). They also apply a procedure to write the equation of a composition of given functions (IA-2.6). In addition, students carry out a procedure to graph discontinuous functions (including piecewise and step functions) (IA-2.9) and determine the domain and range of discontinuous functions (including piecewise and step functions) (IA-2.10).

- **Taxonomy Level**

Cognitive Process Dimension: Understand

Knowledge Dimension: Conceptual Knowledge

- **Key Concepts**

Function

Relationship

## ***II. Teaching the Lesson***

*In this lesson, students classify relations as either a function or not a function. This can seem to be a very trivial topic to students because they may not see the application of the concept in the real world. To illustrate the real world connection, present two input/output machines: an ATM machine and email accounts. With an ATM machine, the input from your card is your account number and the output is your money. So the ordered pair is (your account number, your money). Create sample ordered pairs using your student's name. For example, (Cindy's acct #, Cindy's \$), (Jack's acct #, Jack's \$), etc... Then include the ordered pair (Cindy's acct #, Sandra's \$)*

and ask would that be OK? Well it might be good for Cindy but not for Sandra. So we have one input with two output and that isn't good because banks would be out of business if they could not guarantee a functional relationship – one input paired with one output.

Go through the same process with the email accounts example where the input is your acct identifier (user name/password) and the output is your emails. The order pair is (your acct identifier, your emails).

Then ask students to come up with other real world situations where a functional relationship is important. Some examples are car keys, phone numbers, house keys, social security numbers.

- **Essential Learning and Understanding**

It is essential for students to do the following for the attainment of this indicator:

- Use the definition of a function to classify a set of ordered pairs in a table or list as a function or not a function.
- Use the definition of a function to classify a set of ordered pairs in a mapping as a function or not a function.
- Use the vertical line test and the graph of a continuous relationship to classify the relationship as a function or not a function.

- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- Classify the relationship shown in the table below as a function or not a function.

X	Y
1	6
2	8
2	10
3	12
4	16

Answer: not a function

- Classify the relationship shown in the table below as a function or not a function.

X	Y
-3	-4
-1	0
1	4
3	8
5	12

Answer: function

- Classify the relationship shown in the table below as a function or not a function.

X	Y
-6	3
-3	-2
0	5
3	-8
6	12

Answer: function

- Classify the relationships shown below a function or not a function.

$\{(0,2), (3,7), (4,10), (6,15), (8,10)\}$

Answer: function

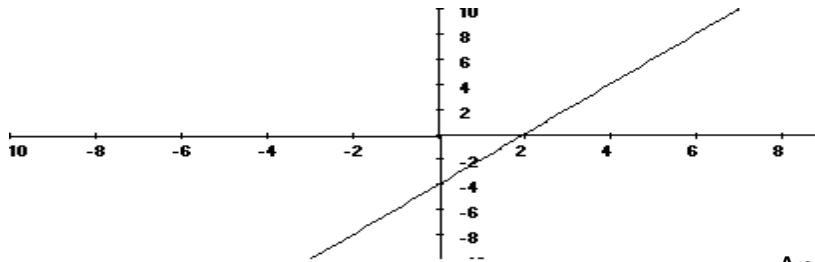
$\{(0,6), (3,6), (4,6), (6,6), (8,6)\}$

Answer: function

$\{(5,2), (5,7), (5,10), (5,15), (5,10)\}$

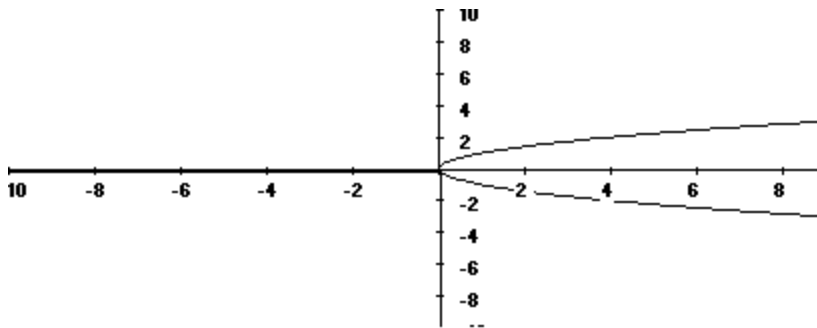
Answer: not a function

- Classify the relationship shown on the graph below as a function or not a function.



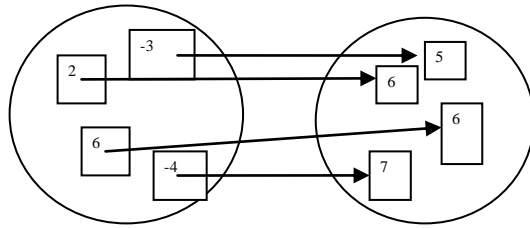
Answer: function

Classify the relationship shown on the graph below as a function or not a function.



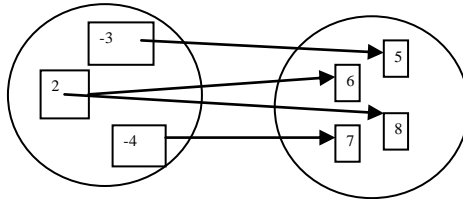
Answer: not a function

- Classify the relationship shown on the mapping below as a function or not a function.



Answer: function

- Classify the relationship shown on the mapping below as a function or not a function.



Answer: not a Function

- **Non-Essential Learning and Understanding**

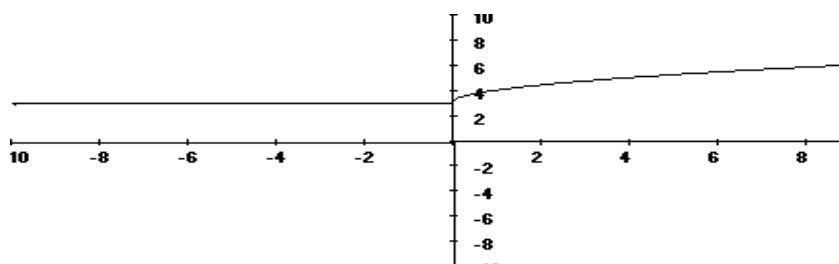
It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

- Classify piecewise relationships as a function or not a function.

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

- Classify the relationship shown on the graph below as a function or not a function.



- **Misconceptions/Common Errors**

Students may not understand that two ordered pairs with the same y-coordinate but different x-coordinates are possible ordered pairs for a function. Students may confuse discrete and continuous data. Tables, lists, and mappings represent discrete data, unless the information provided with the table indicates that the given points are points on a continuous graph. When given a graph, students differentiate between discrete and continuous data more easily.

- **Technology Note**

Use technology where appropriate.

### III. Assessing the Lesson

The objective of this indicator is for the student to understand the definition of a function so that the student can classify a relationship as being either a function or not a function when given data as a table, set of ordered pairs, or graph. In addition to classifying relationships as functions or not functions, students should be able to exemplify, compare, or explain relationships that are functions or not functions.

- **Assessment Item Examples**

- Which equation represents the function in the table?

Input x	Output y
0	3
1	4
2	7

A.  $y = x + 5$

B.  $y = 2x + 3$

C.  $y = x^2 + 3$

D.  $y = x^2 + 5$

E.  $y = 3x + 3$

<b>Lesson # 2</b>
<b>Topic: Writing equations using function notation</b>
<b>Standard (s): EA – 3.2</b>

## ***I. Planning the Lesson***

*The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.*

- **Continuum of Knowledge**

- In 8<sup>th</sup> grade, students translate among verbal, graphic, tabular, and algebraic representations of linear functions (8-3.1). They also classify relationships between two variables in graphs, tables, and/or equations as either linear or nonlinear (8-3.5).
- In Elementary Algebra, students will use function notation to represent functional relationships (EA-3.2).
- In Intermediate Algebra, students carry out procedures to perform operations on polynomial functions (including  $f(x) + g(x)$ ,  $f(x) - g(x)$ ,  $f(x) \cdot g(x)$ , and  $f(x)/g(x)$ ) (IA-2.5).

- **Taxonomy Level**

Cognitive Process Dimension: Apply

Knowledge Dimension: Conceptual Knowledge

- **Key Concepts**

Function Notation

## ***II. Teaching the Lesson***

*In this lesson, students write relationship functions using function notation. The indicator for this lesson goes beyond applying the procedure of simply replacing  $y$  with  $f(x)$ . The indicator asks students to apply conceptual knowledge; therefore, students need to understand the relationship between independent and dependent variable (input and output). The next lesson on EA – 3.2 asks students to evaluate functions. You may want to integrate that lesson here or teach it as a separate lesson.*

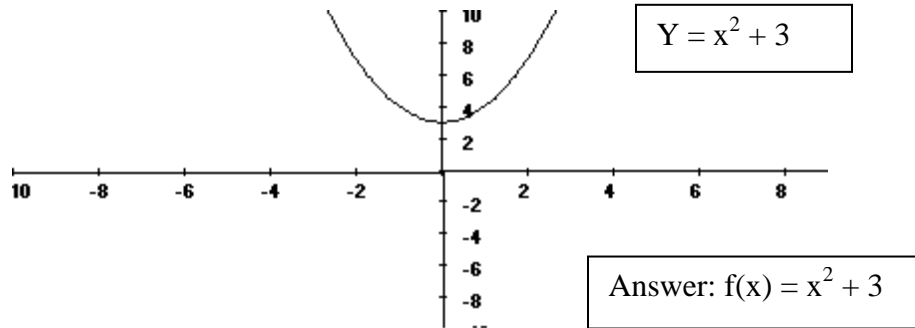
- **Essential Learning and Understanding**

It is essential for students to do the following for the attainment of this indicator:

- Use function notation to represent functions given as an equation, graph, or described in words.

- **Examples of Essential Tasks**

- These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.
- Use function notation to represent the function shown in the graph below.



- Use function notation to represent the function described below. The temperature of an object on the Celsius scale is  $\frac{5}{9}$  the difference between the Fahrenheit temperature ( $t$ ) and  $32^\circ$  F.  
Answer:  $F(t) = \frac{5}{9}(t-32)$
- Given  $y = \frac{1}{x}$ . Express this equation in function notation.  
Answer:  $f(x) = \frac{1}{x}$

- **Non-Essential Learning and Understanding**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

- Derive the functional relationship.

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra

- Use function notation to represent the linear function shown in the table below.

X	Y or $f(x)$
1	6
2	8
3	10
4	12
5	14

Answer:  $f(x) = 2x + 4$



- Use function notation to represent the linear functional relationship represented below containing the set of ordered pairs  $(x, y)$  or  $(x, f(x))$ .  
 $\{(0, 0), (3, -9), (4, -12), (-6, 18), (-8, 24)\}$       Answer:  $f(x) = -3x$

- **Misconceptions/Common Errors**

Students may not understand that function notation defines a function in terms of an independent variable, but does not specify the symbol used to represent the dependent variable.

- **Technology Note**

When graphing a function that is defined in function notation using technology, “ $y =$ ” is used even though the dependent variable may not be “ $y$ ”.

### **III. Assessing the Lesson**

**Assessment Guidelines:** *The objective of this indicator is for the student to apply conceptual knowledge to use function notation to represent functional relationships.*

- **Assessment Item Example**

Which of the following expresses the relationship described below in function notation?

“ $q$  is a function of  $r$  and 2 times  $r$  equals  $q$ ”

- A.  $q(r) = 2q$
- B.  $q(r) = 2r$
- C.  $r(q) = 2q$
- D.  $r(q) = 2r$

<b>Lesson # 3</b>
<b>Topic: Evaluating a function given a value</b>
<b>Standard (s): EA – 3.3</b>

## ***I. Planning the Lesson***

*The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.*

- **Continuum of Knowledge**

- In 8<sup>th</sup> grade, students apply formulas to determine the exact ( $\pi$ ) circumference and area of a circle (8-5.4) and apply formulas to determine the perimeters and areas of trapezoids (8-5.5). To determine these measurements, students substitute values for variables in appropriate formulas.
- In Elementary Algebra, students carry out a procedure to evaluate a function for a given element in the domain (EA-3.3)..
- In Precalculus, students apply a procedure to evaluate trigonometric expressions. (PC-5.6). Though not specifically stated as an indicator, this essential skill is used in all core areas.

- **Taxonomy Level**

Cognitive Process Dimension: Apply

Knowledge Dimension: Procedural Knowledge

- **Key Concepts**

Value of a Function

Domain Value

Range Value

Substitution

Evaluation

## ***II. Teaching the Lesson***

*In this lesson, students evaluate a function for a given value. Students evaluated expressions in Module 1 and are now applying the procedure to expressions written in function notation. This lesson may be integrated into the previous lesson where students are representing relationships using function notation.*

- **Essential Learning and Understanding**

It is essential for students to do the following for the attainment of this indicator:

- Substitute a given value for the independent variable and use the order of operations to evaluate the function for the given element.

- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- Find the value of the function  $f(x) = 4x + 6$  for  $x = -2$ .
- Given  $g(t) = 2t - 6$ , evaluate  $g(2)$ .
- For  $y = x + 2$ , what is the value of  $y$  when  $x = -4$ ?
- Find the value of  $y$  for  $x = \frac{1}{2}$ , if  $y = \frac{1}{x}$ .

- **Non-Essential Learning and Understanding**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

- Evaluate a function for elements in the domain with variable values.

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

- Evaluate  $g(t) = 2t - 6$  for  $t = a$ .

- **Misconceptions/Common Errors**

Students may think that for  $g(t) = 2t - 6$  that  $g(t)$  means  $g$  multiplied by  $t$ .

- **Technology Note**

- Use technology where appropriate.

### **III. Assessing the Lesson**

**Assessment Guidelines:** The objective of this indicator is for the student to carry out a procedure to evaluate a function for a given element in the domain.

- **Assessment Item Example**

See examples of essential tasks

<b>Lesson # 4</b>
<b>Topic: Analyze the graph to determine the domain and range</b>
<b>Standard (s): EA – 3.4</b>

## ***I. Planning the Lesson***

*The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.*

- **Continuum of Knowledge**

- In 8<sup>th</sup> grade, students use ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate plane (8-4.2).
- In Elementary Algebra, students analyze the graph of a continuous function to determine the domain and range of the function (EA-3.4).
- In Intermediate Algebra, students carry out a procedure to determine the domain and range of discontinuous functions (including piecewise and step functions) (IA-2.10).

- **Taxonomy Level**

Cognitive Process Dimension: Analyze

Knowledge Dimension: Procedural Knowledge

- **Key Concepts**

Domain

Range

## ***II. Teaching the Lesson***

*In this lesson, students analyze continuous graphs to determine the domain and range. Students may not have prior knowledge with this topic; therefore this lesson may need to be revisited throughout the course in order to ensure student proficiency.*

- **Essential Learning and Understanding**

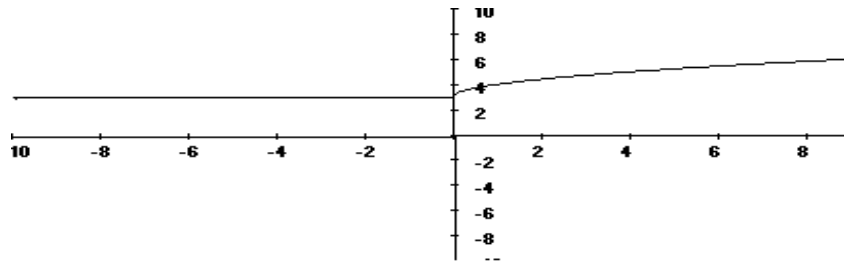
It is essential for students to do the following for the attainment of this indicator:

- Determine the set of x-coordinates of points on the graph.
- Describe the set of x-coordinates using words or mathematical expressions to specify the domain of a continuous function.
- Determine the set of y-coordinates of points on the graph.
- Describe the set of y-coordinates using words or mathematical expressions to specify the range of a continuous function.

- **Examples of Essential Tasks**

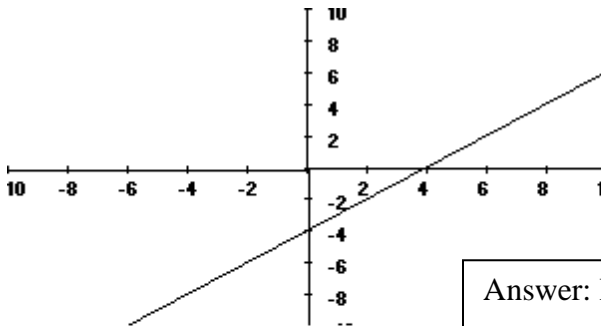
These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- Analyze the graph below to determine the domain and range of the function. Arrows are on the ends.



Answer: Domain: all real numbers  
Range:  $y \geq 3$

- Analyze the graph below to determine the domain and range of the function. Arrows are on the ends.



Answer: Domain: all real numbers  
Range: all real numbers

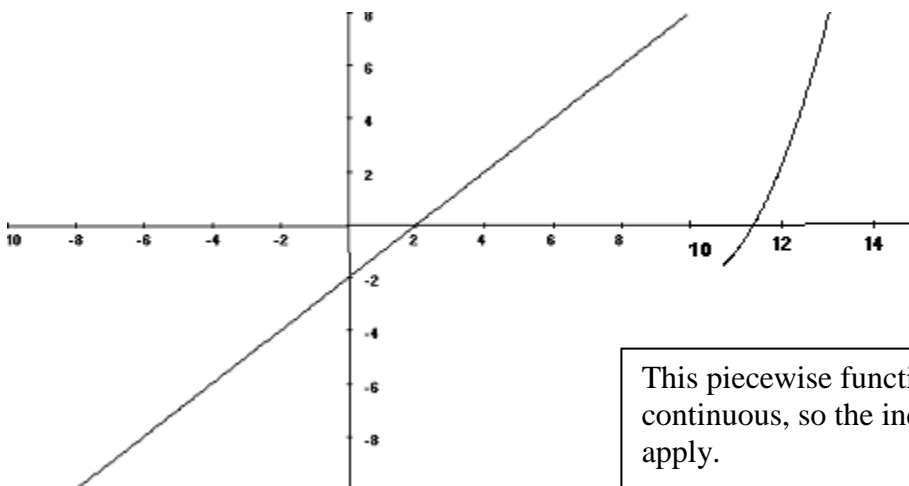
- **Non-Essential Learning and Understanding**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

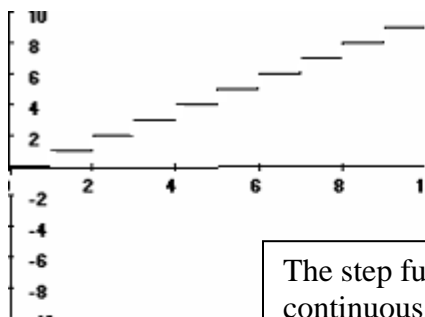
- Analyze the graph of non-continuous functions that are piecewise, including step functions.

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.



This piecewise function is not continuous, so the indicator does not apply.



The step function ( $y = [x]$ ) shown at the left is not continuous, so the indicator does not apply. The greatest integer function is not continuous.

- **Misconceptions/Common Errors**

Students sometimes reverse the domain and range of functions.

- **Technology Note**

Students may enter a function, look a graph, and use the trace feature.

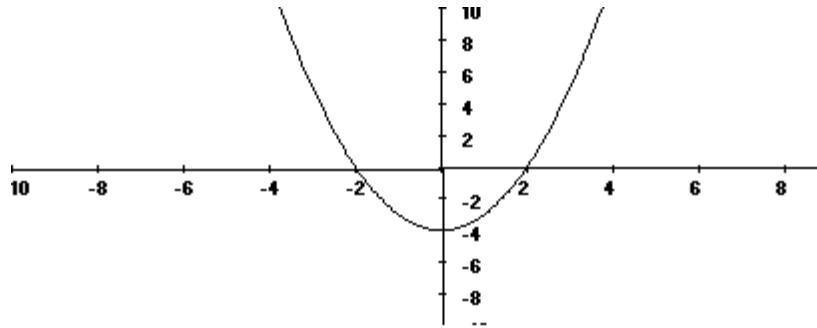
Students may use the table feature.

### III. Assessing the Lesson

**Assessment Guidelines:** The objective of this indicator is for the student to analyze the graph of a continuous function to determine the domain and range of the function.

- Assessment Item Example

Analyze the graph below to determine the domain and range of the function.



<b>Lesson # 5</b>
<b>Topic: Graphing parent functions</b>
<b>Standard (s): EA – 3.5</b>

## ***I. Planning the Lesson***

*The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.*

- **Continuum of Knowledge**

- In 8<sup>th</sup> grade, students use ordered pairs, equations, intercepts, and intersections to locate points and lines in a coordinate plane (8-4.2).
- In Elementary Algebra, students carry out a procedure to graph parent functions (including  $y = x$ ,  $y = x^2$ ,  $y = \sqrt{x}$ ,  $y = |x|$ , and  $y = \frac{1}{x}$ ) (EA-3.5).
- In Intermediate Algebra, students carry out a procedure to graph translations of parent functions (including  $y = x$ ,  $y = x^2$ ,  $y = \sqrt{x}$ ,  $y = |x|$ , and  $y = \frac{1}{x}$ ) (IA-2.7).

- **Taxonomy Level**

Cognitive Process Dimension: Apply

Knowledge Dimension: Procedural Knowledge

- **Key Concepts**

x-coordinate

y-coordinate

Ordered pair

Coordinate plane

## ***II. Teaching the Lesson***

*In this lesson, students are introduced to the parent functions. Student's prior knowledge may not include functions other than linear functions. This lesson may be integrated into other lesson later in the course or it may be addressed in its entirety here. Students explored the graphs of linear functions in Module Three and inverse relationships in Module Two. They will explore quadratic functions in Module 8. The absolute value and square functions are not addressed in other modules and may be addressed separately or integrated into other modules.*

- **Essential Learning and Understanding**

It is essential for students to do the following for the attainment of this indicator:

- Use the equation of a parent function to determine a set of ordered pairs that lie on the graph of the equation, locate those



points in a coordinate plane, and sketch a continuous graph by analyzing the pattern of the points plotted.

- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- Sketch a graph of  $y = x$  in a coordinate plane.
- Sketch a graph of  $y = x^2$  in a coordinate plane.
- Sketch a graph of  $y = \sqrt{x}$  in a coordinate plane.
- Sketch a graph of  $y = |x|$  in a coordinate plane.
- Sketch a graph of  $y = \frac{1}{x}$  in a coordinate plane.

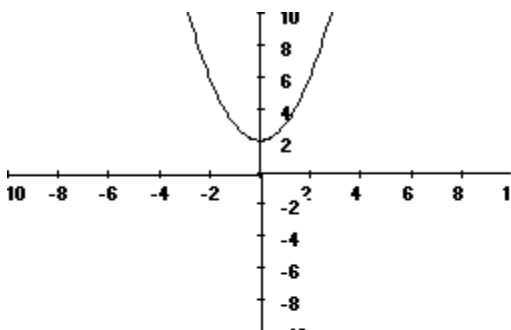
- **Non-Essential Learning and Understanding**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

- Graph transformations of parent functions

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.



This is a translation of  $y = x^2$  ( $y = x^2 + 2$ ). It is not essential for students to graph translations of parent functions.

- **Misconceptions/Common Errors**

None noted

- **Technology Note**

If students graph  $y = 1/x$  in connected mode, then the calculator will incorrectly connect the parts of the graph in quadrant I and quadrant III. A more accurate representation is produced when students graph in the unconnected mode.

### **III. Assessing the Lesson**

**Assessment Guidelines:** The objective of this indicator is for the student to carry out a procedure to graph parent functions

(including  $y = x$ ,  $y = x^2$ ,  $y = \sqrt{x}$ ,  $y = |x|$ , and  $y = \frac{1}{x}$ ).