Probability: Fair and Unfair Games – Grade 7, Level 3

Lesson Overview

In this lesson, students will use probabilities and models (tree diagrams) to determine if games are "fair" (equal probabilities of winning) or "unfair" (unequal probabilities of winning).

Alignment

Math (7.DSP.8d) Extend the concepts of simple events to investigate compound events.

d. Design and use simulations to collect data and determine probabilities.

Standards for Mathematical Practice

4. Connect mathematical ideas and real-world situations through modeling.
   a. Identify relevant quantities and develop a model to describe their relationships.
   b. Interpret mathematical models in the context of the situation.
   c. Make assumptions and estimates to simplify complicated situations.
   d. Evaluate the reasonableness of a model and refine if necessary.

Connections

Disciplinary Literacy Strategies (for Purposeful Reading, Meaningful Writing, and Productive Dialogue)

Thumbs Up, Thumbs Down

I Think, We Think

Gallery Walk

Exit Ticket

Computational Thinking

*Computational thinking (CT) is a problem-solving process that includes (but is not limited to) the following **characteristics**:*

- Logically organizing and analyzing data
- Representing data through abstractions such as models and simulations

*These skills are supported and enhanced by a number of dispositions or attitudes that are essential dimensions of CT. These **dispositions or attitudes** include:*

- Tolerance for ambiguity
- The ability to communicate and work with others to achieve a common goal or solution

Lesson Plan

Time Required – (One 60 minute Class Period)

Disciplinary Vocabulary – probability, chance, tree diagrams (revisited), experimental probability, theoretical probability, simple event, compound event, simulation

Materials Needed:

- Blue and white counters
- Tree diagram with I Think, We Think Table – handout (1 per student)
- Chart Paper
- Markers

Formative Assessment Strategies: Student dialogue, Exit Ticket

Computational Thinking: This lesson addresses 'tolerance for ambiguity' as it engages the students to reflect on what makes a game fair or unfair. The 'ability to communicate and work with others to achieve a common goal or solution' is demonstrated as students work together to create simulations, record results (data) and dialogue. 'Logically organizing and analyzing data' occurs when students use the tree-diagrams. 'Representing data through abstractions such as models and simulations' happens throughout the lesson as students complete games and record data.

Misconceptions:

- When a probability experiment has very few attempts or outcomes, the result can be deceptive. Computer simulations may help students avoid or overcome erroneous probabilistic thinking. Simulations afford students access to relatively large samples that can be generated quickly and modified easily. (NCTM 2000, p254) Using large samples, the distribution is more likely to be close to the actual distribution. When simulations are used, you will need to help students understand what the simulation data represent and how they relate to the problem situation.
- Students may confuse what has happened with what *may* happen.
- Students may believe that because an event has recently happened it has a high probability of reoccurring.
- Students may make the assumption that outcomes are equally likely (in order to calculate theoretical probabilities) when they are not.
- Students may assume that the theoretical probability and observed relative frequency of an event will be the same.
- Students may allot a probability greater than one to an event.

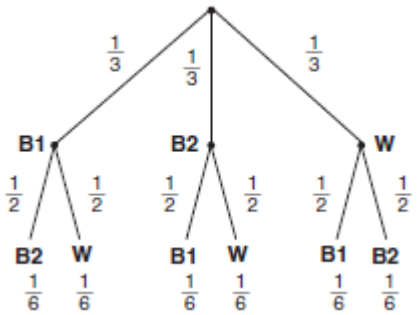
Engage

- Ask students to reflect on what makes a game fair or unfair. The conversation should reach consensus that a fair game is where all players have the same chance of winning and an unfair game is when one or more players have more or less chance of winning than other players. If there is only one player in a game, then the player should have the same chance of winning or losing to be a fair game.
- Pose several game simulations to students based on compound events. Have them use a thumbs up to signify a fair game, a thumbs down to signify an unfair game, and a sideways thumb if they are uncertain. http://www.s2temsc.org/uploads/1/8/8/7/18873120/thumbs_up_thumbs_down_strategy.pdf
- Tally the class "thumb" votes. Make a chart on the board, illustrating the number of students selecting fair, unfair, or uncertain. Then play 6 games with a partner recording wins and losses. Total the class wins and losses for the game.
    - Game Simulation 1: 2 blue counters and 1 white counter are mixed up in a bag. Without looking, draw 1 counter. Then draw a second counter without putting the first counter back in. If the two counters match (are the same color) you win. Otherwise, you lose.
        - Thumbs Up, Down, Sideways Tally
        - Play 5 games with a partner
        - Tally the class wins and losses
        - Discuss whether the game appears to be fair or unfair. The purpose for this is to begin dialogue about how calculating probabilities will help in determining fair and unfair games and if any player has an advantage over another. CT:
    - Game Simulation 2: Now, use 2 blue counters and 2 white counters. Same rules. Fair or Unfair?
        - Thumbs Up, Down, Sideways Tally
        - Play 5 games with a partner
        - Tally the class wins and losses
        - Discuss whether the game appears to be fair or unfair. This is just to begin dialogue about how calculating probabilities will help in determining fair and unfair games and if any player has an advantage over another.
    - Game Simulation 3: Now use 3 blue counters and 1 white counter. Same rules. Fair or Unfair?
        - Thumbs Up, Down, Sideways Tally
        - Play 5 games with a partner

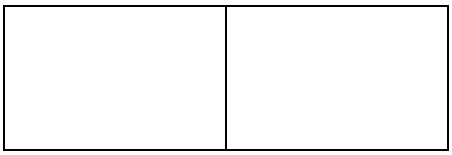
- Tally the class wins and losses
- Discuss whether the game appears to be fair or unfair. This is just to begin dialogue about how calculating probabilities will help in determining fair and unfair games and if any player has an advantage over another.

Explore

- Tell students, "Calculating probabilities help us determine whether games are fair or unfair. Playing the games yields experimental probabilities and not the theoretical probabilities. Theoretical probability is what determines true fairness of a game."
- Tell students, "In the first game, there were 2 blue counters and 1 white counter. Although the two blue counters were the same color, they are not the same counter. Refer to the blue counters as "blue 1 (B1)" and "blue 2 (B2)".
- Review tree diagrams with students using the flipping-two-coins Simulation where the outcomes will be HH, HT, TH, and TT.
- Tell students, "Tree diagrams can help us determine the probabilities of winning Game 1" to determine fairness.
- Provide students with a photocopy of the tree diagram where they can write on it and express their thinking. At the bottom of the tree diagram, students should be given or should create, a table for I Think, We Think. http://www.s2temsc.org/uploads/1/8/8/7/18873120/i_think_we_think_strategy.pdf
- Individually first, students should write (in the I Think column) some connections they think are evident in the tree diagram. Then, they should discuss with their table mates and complete the We Think Column with any ideas they all agree upon.



I Think	We Think



- Ask student groups to share their thinking with the class.
- Discussion Points should include (for example):
    - Each counter in the first draw has an equal chance (1/3 chance) of being drawn.
    - If one of the blue counters is drawn first, then the other blue counter and the white counter have equal chances (1/2) of being selected next.
    - Ultimately, there are 6 ways the selections could turn out as demonstrated when following the lines down the tree diagram. Each of the 6 ways have an equal chance of happening (1/6).
    - However, drawing two blue counters in a row only happens in 2 of the 6 results, or 2/6 of the time which is the same as 1/3.
    - So, selecting matching colors from two draws is not a fair game. There are 2/6 ways to win this game and 4/6 ways to lose it.
- Have students refer to game Simulations 2 and 3. They should complete tree diagrams for each of these games in their table groups. Record the tree diagrams on chart paper and post them in the room when completed.
- Have students do a gallery walk of the tree diagrams as they look for similarities and differences.
  http://www.s2temsc.org/uploads/1/8/8/7/18873120/gallery_walk_strategy.pdf

Explain

- Facilitate a class discussion to reach consensus on whether the game Simulations were fair or unfair focusing on the explanations and diagrams.
- Exit Ticket:
  http://www.s2temsc.org/uploads/1/8/8/7/18873120/exit_slips_or_exit_tickets_strategy.pdf
  Pose the following game Simulation: "Suppose the game was played with 4 blue counters. You will win if you select 2 blue counters in a row without replacing the first one when drawn. Is this game fair or unfair? Explain your thinking with words and/or an illustration. Be thorough."

Other information on this indicator(s) can be found in the support documents/resources on the SC State Department website.

www.ed.sc.gov (Instruction → Standards and Learning → Mathematics or Science → Support Documents and Resources)

Content Area (Disciplinary) Literacy strategies and descriptions can be found on the S2TEM Centers SC website:

s2temsc.org (Resources → Disciplinary Literacy Virtual Library → Strategy Warehouse)

Computational Thinking Reference:

https://csta.acm.org/Curriculum/sub/CurrFiles/CompThinkingFlyer.pdf
https://csta.acm.org/Curriculum/sub/CompThinking.html

Additional Information

Level 1 lessons contain a realignment to the 2014 Science and/or the 2015 Mathematics Standards.

Level 2 lessons contain Level 1 information and Content Area Literacy and Disciplinary Literacy Strategies.

Level 3 lessons contain Level 1 and 2 information and Computational Thinking Connections.

Level 4 lessons contain Level 1, 2, and 3 and integration of at least 2 content areas