Comparing Function Families

Lesson Overview

In this lesson, students will develop definitions of quadratic functions by comparing quadratic functions with linear and exponential functions. Students will summarize differences between quadratic functions, linear functions, and exponential functions. Prior to this lesson, students should understand linear functions and exponential functions (general equation, graph, rate of change). In this lesson, students build on their understanding of functions by adding "quadratic functions" to their tool box. This two-day lesson is the second mathematics lesson in the unit One Dimensional Kinematics—Modeling Motion.

Alignment

Science and Engineering Practices

H.P.1A.5 Use mathematical and computational thinking to (1) use and manipulate appropriate metric units, (2) express relationships between variables for models and investigations, and (3) use grade-level appropriate statistics to analyze data.

Crosscutting Concepts (from the SDE instructional unit resources document)

3. **Scale, proportion, and quantity**: The National Research Council (2012) states that "in considering phenomena, it is critical to recognize what is relevant at different measures of size, time, and energy and to recognize how changes in scale, proportion, or quantity affect a system's structure or performance" (p. 84). The ideas of ratio and proportionality are important here along with being able to predict the effect of a change in one variable on another. For example, how will the speed of an object change if the time traveled is increased but the distance remains the same?

Math Standards

ACE.2 Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.

FIF.8 Translate between different but equivalent forms of a function equations to reveal and explain different properties of the function.

FIF.9 Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal.

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Standards for Mathematical Practice

SMP.2 Reason abstractly and quantitatively.

SMP.3 Construct viable arguments and critique the reasoning of others.

SMP.7 Look for and make use of structure.

SMP.8 Look for and express regularity in repeated reasoning.

ELA Writing

Standard 6: Write independently, legibly, and routinely for a variety of tasks, purposes, and audiences over short and extended time frames.

6.1 Write routinely and persevere in writing tasks over short and extended time frames, for a range of domain-specific tasks, and for a variety of purposes and audiences.

ELA Communication

Standard 1 Interact with others to explore ideas and concepts, communicate meaning, and develop logical interpretation through collaborative conversations; build upon the ideas of others to clearly express one's own views while respecting diverse perspectives.

1.2 Initiate and participate effectively in a range of collaborative discussions with diverse partners; build on the ideas of others and express own ideas clearly and persuasively.

1.4 Engage in dialogue with peers and adults to explore meaning and interaction of ideas, concepts, and elements of text, reflecting, constructing, and articulating new understandings.

1.5 Synthesize areas of agreement and disagreement including justification for personal perspective; revise conclusions based on new evidence.

Connections

Content Area (2 or more) Connections

- Science (Physics)
- Mathematics (Algebra 2)

Content Connections

The understanding of multiple representations of quadratic functions, as well as the modeling of quadratic relationships graphically and algebraically is a cornerstone skill used in physics to analyze one dimensional kinematics. Motion data collected for objects with a changing velocity at constant acceleration is represented with a quadratic model.

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Active Learning Strategies (for Purposeful Reading, Meaningful Writing, and Productive Dialogue)

Verbal and Visual Word Association

Computational Thinking

In this lesson, students will be developing computational thinking by logically organizing and analyzing data and representing data through abstractions such as models and simulations (during the group activity). In addition, the dispositions of "ability to deal with open ended problems" and "ability to communicate and work with others to achieve a common goal or solution" will be necessary for successful completion of the lesson tasks.

Lesson Plan: Introduction to Quadratic Functions

Time Required – Two 55-minute classes

Disciplinary Vocabulary – rate of change, quadratic function, exponential function, linear function, equation, graph, first differences, second differences

Materials Needed:

- ABCD Cards, 1 set per student
- Group Activity: Comparing Families of Functions, 1 per student
- Compare/Contrast Matrix, 1 per student
- Quadratic Functions Definition Card, 1 per student
- Pattern Blocks, or access to online pattern blocks

Formative Assessment Strategies: ABCD Cards, Visual and Verbal Word Association, Teacher Questioning during group work and class dialogue

Misconceptions:

- Students may think that all relationships are linear, exponential, or quadratic. Let students know that many other functional relationships exist, but these are the ones they currently understand.
- Students may confuse a portion of a graph on a limited domain with the behavior of the function on the entire domain.
- Because the examples are all in the first quadrant, students may have difficulty differentiating between which are quadratic and which are exponential. Encourage students to consider the numerical and algebraic forms of the relationship as well as the graph in making their conjectures.

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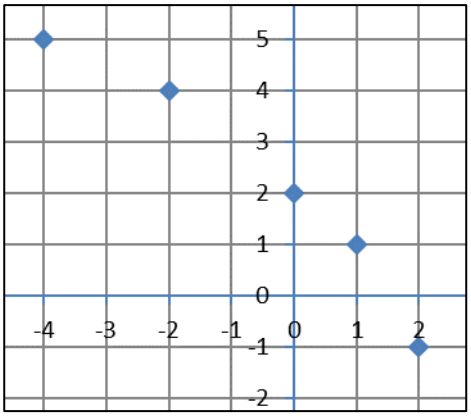
Engage

- **Strategy: ABCD Cards**-Each student will need a set of ABCD cards. For this activity, only cards A and B will be used. (Students can make sets to keep in their notebooks and use throughout the year.) Display one function at a time for students to analyze. Students indicate whether the function is linear (A) or non-linear (B). Allow students to explain their thinking to justify their choices.
    - **Function 1**: A cell phone company charges a monthly fee of $25 for 5 GB data. Each additional GB data is $15.
    - **Function 2**: $y = 2x^2 + 2x - 14$
    - **Function 3:** $y = -4 + 3x$
    - **Function 4:**



    - **Function 5:** The population of rabbits increases 1.5 times a month. The current population of rabbits on the farm is 100.
- **Say**: "Today we are going to further examine the differences between linear and non-linear functions. Specifically, we will explore the rate of change for linear, exponential, and quadratic functions. Tomorrow, we will identify defining characteristics of quadratic functions."

Explore

- Student groups (3-4 students per group) complete the activity: **Comparing Function Families.**
- As students work on the activity, the teacher observes student work and makes notes about each group's progress. These notes will be used to select which student groups share their conclusions on the second day.

Explain

- When groups complete the exploration of the five patterns, provide them with the Compare/Contrast Matrix to complete for the three function families.

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- Teacher selects two or three student groups to share their conclusions with the group. Select groups whose conjectures will promote dialogue about the families of functions.

  Key Concepts to Highlight:

  - The **rate of change** between the variables determines which function family best represents a relationship. Variables with a constant rate of change ($\Delta y/\Delta x$) are represented by linear models. Variables with a rate of change that increases by a common factor are represented by exponential models. Variables with a rate of change that increases by a common value are represented by quadratic models.
  - In relationships that are linear, the **first differences** are equal. In relationships that are quadratic, the **second differences** are equal.
  - The focus of discussion should be on the comparison of quadratic functions with linear and exponential functions.

- **Strategy: Verbal Visual Word Association:** Students use their understanding of quadratic functions from the activity and discussion to develop a definition card for "Quadratic Functions."

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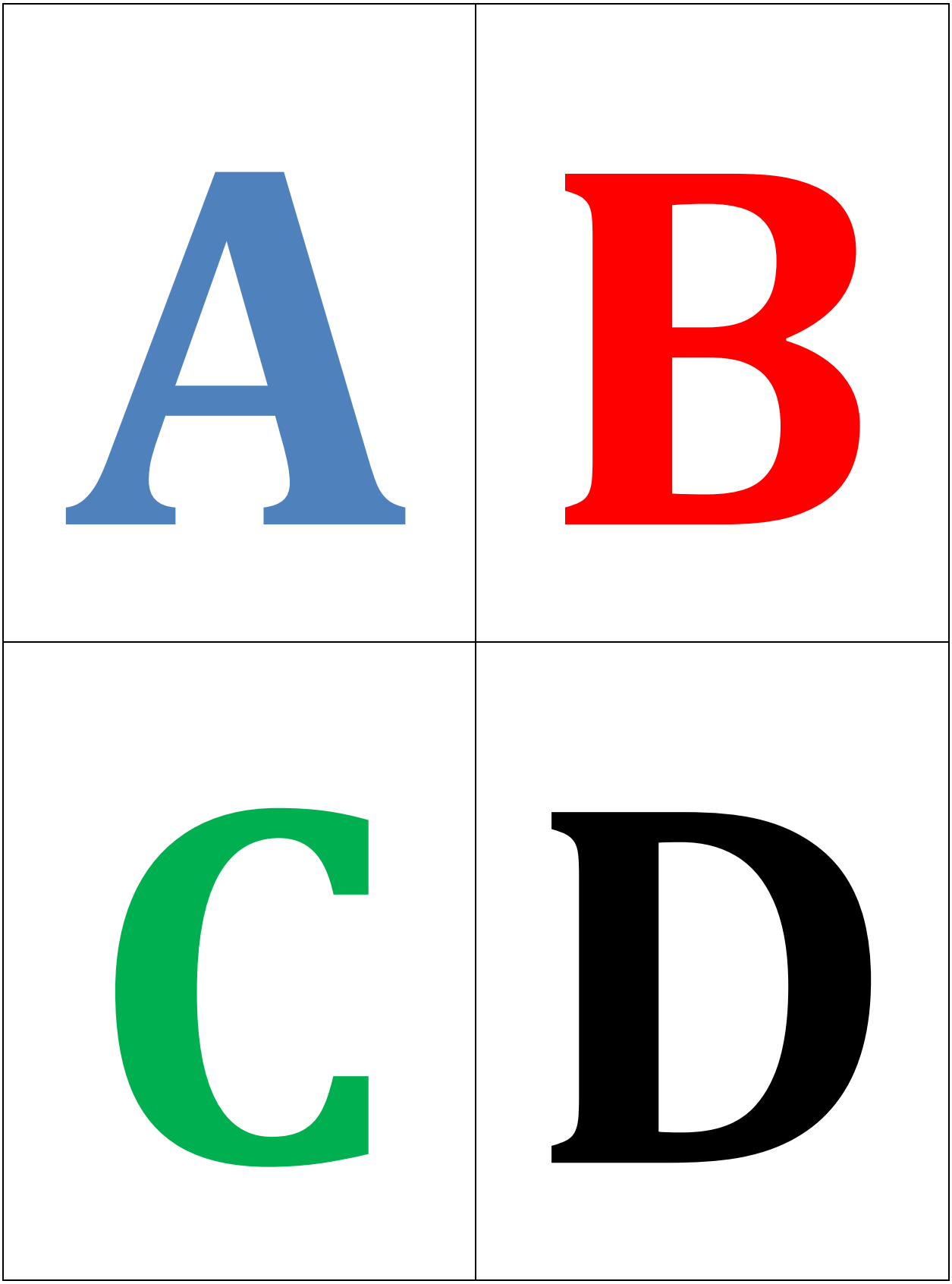

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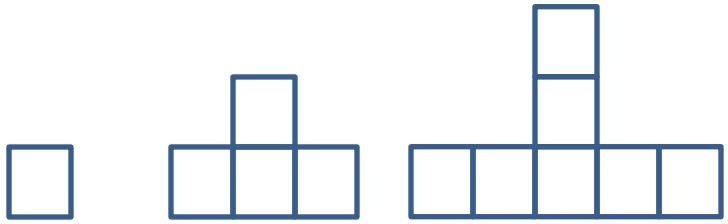
Group Activity

Group Members:

Comparing Function Families

Use tiles to recreate and extend each pattern as many steps as needed to determine a function rule for the number of tiles in each step. Represent the function as a table, graph, and equation and identify the function as linear, exponential, or quadratic.

1.

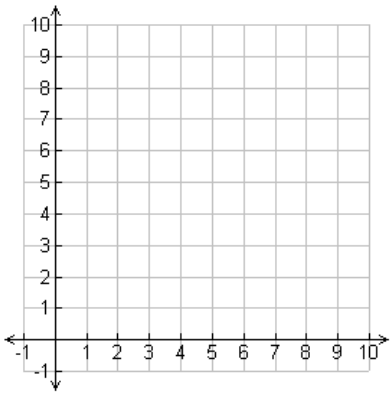


Step 1 Step 2 Step 3

Table:

Step #	# Tiles

Graph:



Equation:

Linear, Exponential, or Quadratic? How do you know?

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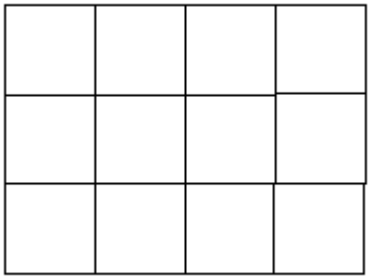
2.

Step 1

Step 2

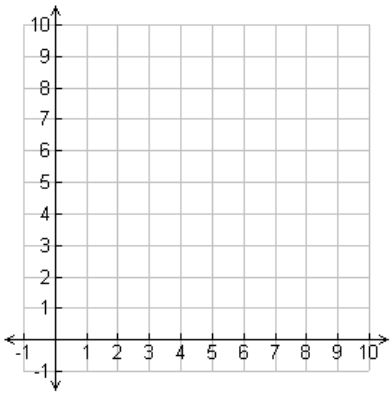


Step 3

Table:

Step #	# Tiles

Graph:



Equation:

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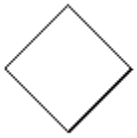
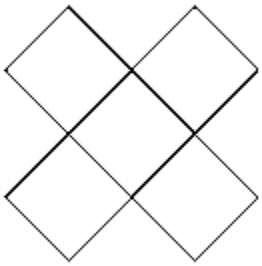
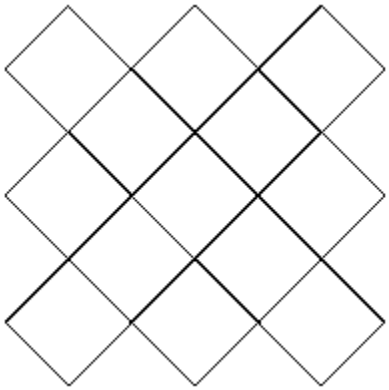
3.



Step 1



Step 2

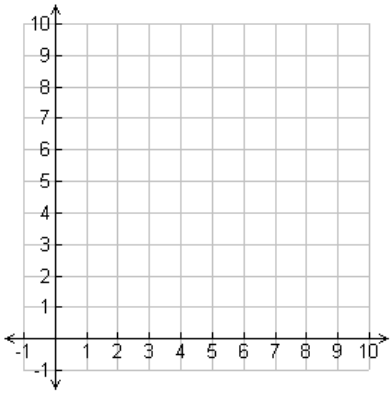


Step 3

Table:

Step #	# Tiles

Graph:



Equation:

Linear, Exponential, or Quadratic? How do you know?

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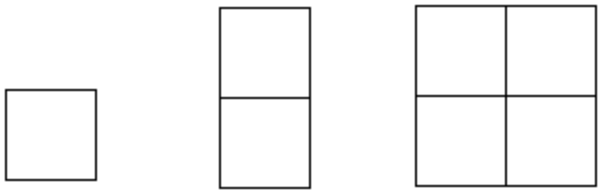
4.

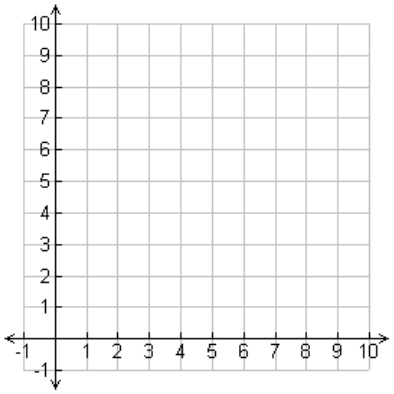


Step 1 Step 2 Step 3

Table:

Step #	# Tiles

Graph:



Equation:

Linear, Exponential, or Quadratic? How do you know?

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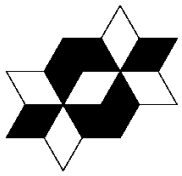
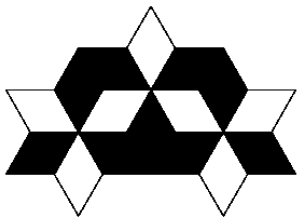
5.

Step 1



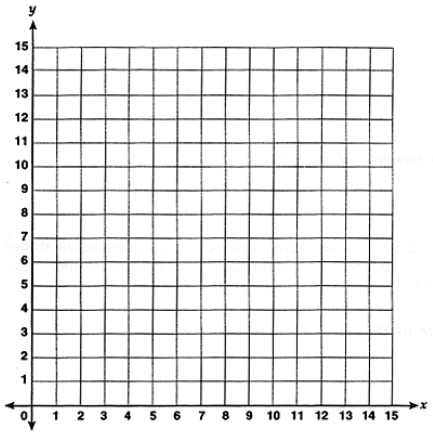
Step 2



Step 3

Table: **Graph:** **Equation:**

Step #	# Tiles



Linear, Exponential, or Quadratic? How do you know?

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Function Families
Compare/Contrast Matrix

Attribute	Linear Functions	Exponential Functions	Quadratic Functions
General Form of Equation			
Graph			
Rate of Change			

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Quadratic Functions	**Visual Representation**
Definition (in your own words)	**Characteristics**

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