Transformations of Quadratic Functions

Lesson Overview

In this lesson, students will explore the effect of changes on the equation on the graph of a quadratic function. Students will examine quadratic functions in standard form, vertex form, and intercept form and make conjectures about the impact of changing the constants in each form on the resulting parabola. On day 2, students will practice their knowledge of the connection between the equation and graph by playing a game in pairs. Last, students will demonstrate their understanding of the three forms using an online card sort. This three-day lesson is the fourth mathematics lesson in the unit One Dimensional Kinematics—Modeling Motion.

Alignment

Science and Engineering Practices

H.P.1A.5 Use mathematical and computational thinking to (1) use and manipulate appropriate metric units, (2) express relationships between variables for models and investigations, and (3) use grade-level appropriate statistics to analyze data.

Crosscutting Concepts (from the SDE instructional unit resources document)

3. **Scale, proportion, and quantity**: The National Research Council (2012) states that "in considering phenomena, it is critical to recognize what is relevant at different measures of size, time, and energy and to recognize how changes in scale, proportion, or quantity affect a system's structure or performance" (p. 84). The ideas of ratio and proportionality are important here along with being able to predict the effect of a change in one variable on another. For example, how will the speed of an object change if the time traveled is increased but the distance remains the same?

Math Standards

FBF.3 Describe the effect of the transformations $kf(x), f(x)+k, f(x+k)$, and combinations of such transformations on the graph of $y=f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.

FIF.8 Translate between different but equivalent forms of a function equations to reveal and explain different properties of the function.

Standards for Mathematical Practice

SMP.2 Reason abstractly and quantitatively.

SMP.3 Construct viable arguments and critique the reasoning of others.

SMP.7 Look for and make use of structure.

SMP.8 Look for and express regularity in repeated reasoning.

ELA Writing

Standard 6: Write independently, legibly, and routinely for a variety of tasks, purposes, and audiences over short and extended time frames.

6.1 Write routinely and persevere in writing tasks over short and extended time frames, for a range of domain-specific tasks, and for a variety of purposes and audiences.

ELA Communication

Standard 1 Interact with others to explore ideas and concepts, communicate meaning, and develop logical interpretation through collaborative conversations; build upon the ideas of others to clearly express one's own views while respecting diverse perspectives.

1.2 Initiate and participate effectively in a range of collaborative discussions with diverse partners; build on the ideas of others and express own ideas clearly and persuasively.

1.4 Engage in dialogue with peers and adults to explore meaning and interaction of ideas, concepts, and elements of text, reflecting, constructing, and articulating new understandings.

1.5 Synthesize areas of agreement and disagreement including justification for personal perspective; revise conclusions based on new evidence.

Connections

Content Area (2 or more) Connections

- Science (Physics)
- Mathematics (Algebra 2)

Content Connections

The understanding of multiple representations of quadratic functions, as well as the modeling of quadratic relationships graphically and algebraically is a cornerstone skill used in physics to analyze one dimensional kinematics. Motion data collected for objects with a changing velocity at constant acceleration is represented with a quadratic model.

Active Learning Strategies (for Purposeful Reading, Meaningful Writing, and Productive Dialogue)

Chum Check
Partner Dialogue
Graphic Organizer
Collaborative Comments

Computational Thinking

In this lesson, students will be developing computational thinking by logically organizing and analyzing data and representing data through abstractions such as models and simulations (during the Desmos activity). In addition, the dispositions of "ability to deal with open ended problems" and "ability to communicate and work with others to achieve a common goal or solution" will be necessary for successful completion of the lesson tasks.

Lesson Plan: Transformations of Quadratic Functions

Time Required – Three 55-minute classes

Disciplinary Vocabulary – parabola, standard form, vertex form, intercept form, axis of symmetry, vertex, x-intercept, y-intercept, quadratic function

Materials Needed:

- Completed VVWA from Lesson 2, 1 per student
- Computer or Tablet with Internet Access, 1 per student
- Graphing Quadratic Functions Graphic Organizer, 1 per student

Formative Assessment Strategies: Chum Check, Graphic Organizer, Card Sort

Misconceptions:

- Students may confuse the attributes of different forms. For example, they identify the zeroes in intercept form as the vertex of the parabola.
- When graphing parabolas, students may connect coordinates using straight lines instead of a parabolic curve.

Engage

- **Strategy: Chum Check**—Students pair with another student to explain and refine their definition of "quadratic functions" represented on their VVWA chart from the previous lessons. Student partners provide feedback to one another using the following sentence stems:
  - Some characteristics you noted that I did not are _____
  - Your definition helped me better understand _____about quadratic functions.
  - I'm not sure I understand this part of your VVWA: _____. Can you explain that in more detail?
  - I'm going to add _____to my VVWA based on what you've shared with me.
- Say: "Today we are going to continue our study of quadratic functions by examining the relationship between the equation and the graph more closely. Your goal is to learn how each of the constants in the general quadratic equation ($y = a(x-h)^2 + k$ *or* $y = a(x-p)(x-q)$ *or* $y = ax^2 + bx + c$) effects the graph." Display the chart with the three forms of the quadratic equation.

Explore

- Students complete the **Desmos** activity "**Match My Parabola**" (https://teacher.desmos.com/activitybuilder/custom/5605bb6200701ed10fb0931a)
- **Desmos** is a free online graphing calculator. From the activity page on the **Desmos** website: "*In this activity, students work through a series of scaffolded quadratic graphing challenges to develop their proficiency with standard, vertex, factored, and other quadratic function forms."* Teachers should download and read the Teacher Guide for the activity prior to use.

Explain

- **Strategy: Graphic Organizer**--Students summarize their learning from the Desmos activity using the **Quadratic Functions Graphic Organizer.** They should be able to complete the sections for Vertex Form and Intercept Form from the Desmos activity.
- Partners practice their learning about quadratic functions in the **Desmos** activity "Marbleslides: Parabolas" (https://teacher.desmos.com/marbleslides-parabolas#)
- Students revisit their **Quadratic Functions Graphic Organizer** to add new learning from their practice during "Marbleslides: Parabolas", focusing on the sections for Vertex Form and Intercept Form. Teacher should examine student work and ask questions to support student conclusions written on the graphic organizer.

Extend

- Say: "In the Desmos activity, we experimented with changing the values of the constants for quadratic functions written in Vertex Form and Intercept Form. What about quadratic functions that aren't in one of these forms? How might we graph a quadratic function in the form $y = 2x^2 + 3x - 1$? This form is called Standard Form."
- Say: "The following three equations represent the same quadratic function. Use what you know about Vertex Form and Intercept Form to make conjectures about the vertex, intercepts, and axis of symmetry for a quadratic function written in Standard Form."
    - Set 1: $y = x^2 - 10x + 16, y = (x-5)^2 - 9, y = (x-8)(x-2)$
    - Set 2: $y = 2x^2 - 10x - 28, y = 2(x - \frac{5}{2})^2 - \frac{81}{2}, y = 2(x-7)(x+2)$
- Groups of 3-4 students use the two sets of equations to develop conjectures about the relationship between the Standard Form of the Quadratic Function and the vertex, axis of symmetry, and intercepts of the related parabola.
- **Strategy: Collaborative Comments**—Teacher leads class discussion using Collaborative Comments as a guide for dialogue. Class develops consensus on the relationship between the Standard Form of the Quadratic Function and the vertex, axis of symmetry, and intercepts of the related parabola.
- Students add learning about the Standard Form of the Quadratic Function to their **Quadratic Functions Graphic Organizer.**

Evaluate

- Students complete the Desmos activity "Card Sort: Parabolas" to demonstrate their understanding of the characteristics and forms of a parabola. (https://teacher.desmos.com/activitybuilder/custom/579bda013037419e171c207e)
- **Desmos** is a free online graphing calculator. From the activity page on the **Desmos** website: "*There are many strategies for determining the shape of a graph given its equation. In this activity, students will find the shape of a parabola by using its form to reveal its characteristics. The activity begins with a review of both the characteristics and forms of a parabola. Later, students will determine characteristics of the graph of a parabola given either in standard form, vertex form, or intercept form."* Teachers should download and read the Teacher Guide for the activity prior to use.

Graphing Quadratic Functions Graphic Organizer

Forms of Quadratic Functions

Standard Form: $y = ax^2 + bx + c$

What can I learn from this form?

Example

Vertex Form: $y = a(x - h)^2 + k$

What can I learn from this form?

Example

Intercept Form: $y = a(x - p)(x - q)$

What can I learn from this form?

Example