Modeling Quadratic Relationships

Lesson Overview

In this lesson, students will examine a video of a basketball shot. They will use mathematics to prove whether the ball will make it into the hoop. Then, they will collect data to find the relationship between the distance a photographer is from his subject and the size of the viewing field through the lens. They will use technology to determine the quadratic regression equation for their data and use the equation to solve problems. This three to five-day lesson is the fifth mathematics lesson in the unit One Dimensional Kinematics—Modeling Motion.

Alignment

Science and Engineering Practices

H.P.1A.5 Use mathematical and computational thinking to (1) use and manipulate appropriate metric units, (2) express relationships between variables for models and investigations, and (3) use grade-level appropriate statistics to analyze data.

Crosscutting Concepts (from the SDE instructional unit resources document)

3. **Scale, proportion, and quantity**: The National Research Council (2012) states that "in considering phenomena, it is critical to recognize what is relevant at different measures of size, time, and energy and to recognize how changes in scale, proportion, or quantity affect a system's structure or performance" (p. 84). The ideas of ratio and proportionality are important here along with being able to predict the effect of a change in one variable on another. For example, how will the speed of an object change if the time traveled is increased but the distance remains the same?

Math Standards

ACE.2 Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.

FIF.5 Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.

SPID.6 Using technology, create scatterplots and analyze those plots to compare the fit of linear, quadratic, or exponential models to a given data set. Select the appropriate model, fit a function to the data set, and use the function to solve problems in the context of the data.

Standards for Mathematical Practice

SMP.1 Make sense of problems and persevere in solving them.

SMP.2 Reason abstractly and quantitatively.

SMP.3 Construct viable arguments and critique the reasoning of others.

SMP.4 Model with mathematics.

SMP.5 Use appropriate tools strategically.

ELA Writing

Standard 6: Write independently, legibly, and routinely for a variety of tasks, purposes, and audiences over short and extended time frames.

6.1 Write routinely and persevere in writing tasks over short and extended time frames, for a range of domain-specific tasks, and for a variety of purposes and audiences.

ELA Communication

Standard 1 Interact with others to explore ideas and concepts, communicate meaning, and develop logical interpretation through collaborative conversations; build upon the ideas of others to clearly express one's own views while respecting diverse perspectives.

1.2 Initiate and participate effectively in a range of collaborative discussions with diverse partners; build on the ideas of others and express own ideas clearly and persuasively.

1.4 Engage in dialogue with peers and adults to explore meaning and interaction of ideas, concepts, and elements of text, reflecting, constructing, and articulating new understandings.

1.5 Synthesize areas of agreement and disagreement including justification for personal perspective; revise conclusions based on new evidence.

Connections

Content Area (2 or more) Connections

- Science (Physics)
- Mathematics (Algebra 2)

Content Connections

The understanding of multiple representations of quadratic functions, as well as the modeling of quadratic relationships graphically and algebraically is a cornerstone skill used in physics to analyze one dimensional kinematics. Motion data collected for objects with a changing velocity at constant acceleration is represented with a quadratic model.

Active Learning Strategies (for Purposeful Reading, Meaningful Writing, and Productive Dialogue)

Think-Ink-Pair-Share

Proof Paragraph

Computational Thinking

In this lesson, students will be developing computational thinking by logically organizing and analyzing data and representing data through abstractions such as models and simulations. In addition, the dispositions of “ability to deal with open ended problems” and “ability to communicate and work with others to achieve a common goal or solution” will be necessary for successful completion of the lesson tasks.

Lesson Plan: Modeling Quadratic Relationships

Time Required – Three to Five 55-minute classes

Disciplinary Vocabulary – parabola, intercepts, regression, quadratic, vertex

Materials Needed:

- Video: halftake1.mov (http://blog.mrmeyer.com/2010/wcydwt-will-it-hit-the-hoop/)
- Copies of Picture (halftake1.png), 1 per group
- Video: fulltake1.move (http://blog.mrmeyer.com/2010/wcydwt-will-it-hit-the-hoop/)
- Video: “Photography Tips: Depth of Field” (https://youtu.be/IGIQ-C_S5d8)
- Lab Activity: View Tubes, 1 per student
- Chart paper or butcher paper, 1 large sheet per group
- One tube (empty toilet paper roll, paper towel roll, PVC tube, etc.), 1 per group
- Ruler, 1 per group
- Tape Measure, 1 per group
- 2 sheets of colored paper per group
- Computer with Internet access, 1 per group

Set Up Prior to Lab Activity: View Tubes

- Set up a data collection station for each group of four students. For each station, attach a piece of chart paper or butcher paper to the wall.

- Each station will need one tube, one ruler, one tape measure, and 2 sheets of colored paper.

Formative Assessment Strategies: Think-Ink-Pair-Share,

Misconceptions:

- If students are not familiar with the Desmos, additional instructional time will need to be dedicated to learning to use the online graphing tool.
- Students may fit a linear or exponential equation to their data instead of a quadratic function.

Day One

Engage

- Show students the video for Mr. Meyer's first basketball attempt (**halftake1.mov**). The video stops when the ball is at the vertex of the parabolic path of the ball. Ask: "Will the ball make it into the hoop? How do you know?" Video is available for download at http://blog.mrmeyer.com/2010/wcydwt-will-it-hit-the-hoop/ listed as "Take 1" under attachments.
- **Strategy: Think-Ink-Pair-Share**—Students use the information from the video to Think-Ink the answer to the question individually. Students pair with an elbow partner and share their initial thinking about the path of the ball and whether he makes the shot.
- Teacher leads class dialogue about initial thoughts on whether Mr. Meyer makes the shot or not. Ask: "How might we use mathematics to prove our conjectures about the path of the ball?"

Explore

- Provide pairs of students copies of the photo of the first half of the path of the ball (halfstack1.png, available for download at http://blog.mrmeyer.com/2010/wcydwt-will-it-hit-the-hoop/ listed as "Take 1" under attachments.)
- **Strategy: Proof Paragraph**—Student groups develop a claim (The ball will make it into the hoop. Or The ball will not make it into the hoop.) and write a proof paragraph justifying their claim with mathematics.

Explain

- Each group presents their paragraph and justification with the class for feedback.
- As groups share, the teacher should offer vocabulary to formalize their mathematical language about the path of the ball. If no group offers a method of fitting a quadratic function to the path of the ball, the teacher should model this strategy. First, overlay the picture with a coordinate grid to determine coordinates for the vertex and the

intercepts. These coordinate points can be used to determine an equation of a quadratic function that models the path of the ball.

- After discussion, students return to their groups to develop a list of "best practices" or procedures to follow when modeling data with a quadratic function. Students should record these "best practices" or procedures in their notebooks.
- Show the video of the full path of the basketball so students can determine if their argument was correct. (fulltake1.mov, available for download at http://blog.mrmeyer.com/2010/wcydwt-will-it-hit-the-hoop/ listed as "Take 1" under attachments.)

Day Two and beyond

Extend

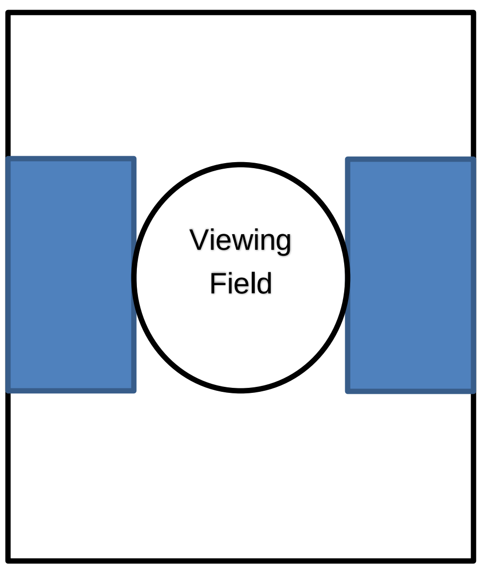
- Say: "Yesterday, we considered how to use mathematics to model the path of a basketball and use that model to make predictions. Today and tomorrow, we are going to extend that understanding by collecting data and using Desmos to calculate a quadratic function that best fits the data. "
- Show video "Photography Tips: Depth of Field" (https://youtu.be/IGIQ-C_S5d8) to set up the lab activity. Say: "Today we are going to further explore the idea of depth of field to determine exactly what effect the distance from the subject has on the viewing area."
- Students read the directions for the **Lab Activity: View Tubes**. Partners paraphrase the directions for the activity.
- Student groups of 4 complete the **Lab Activity: View Tubes.** (Note: This activity is designed using Desmos as graphing technology. It may be adjusted for other available graphing technology.)
- If scheduling permits, students may use the data collected in Physics class during the **Photoburst Drop Zone** activity instead of (or in addition to) **Lab Activity: View Tubes**.

Lab Activity: View Tubes

Group Members

Question: In photography, how does the distance from the subject effect the size of the viewing field?

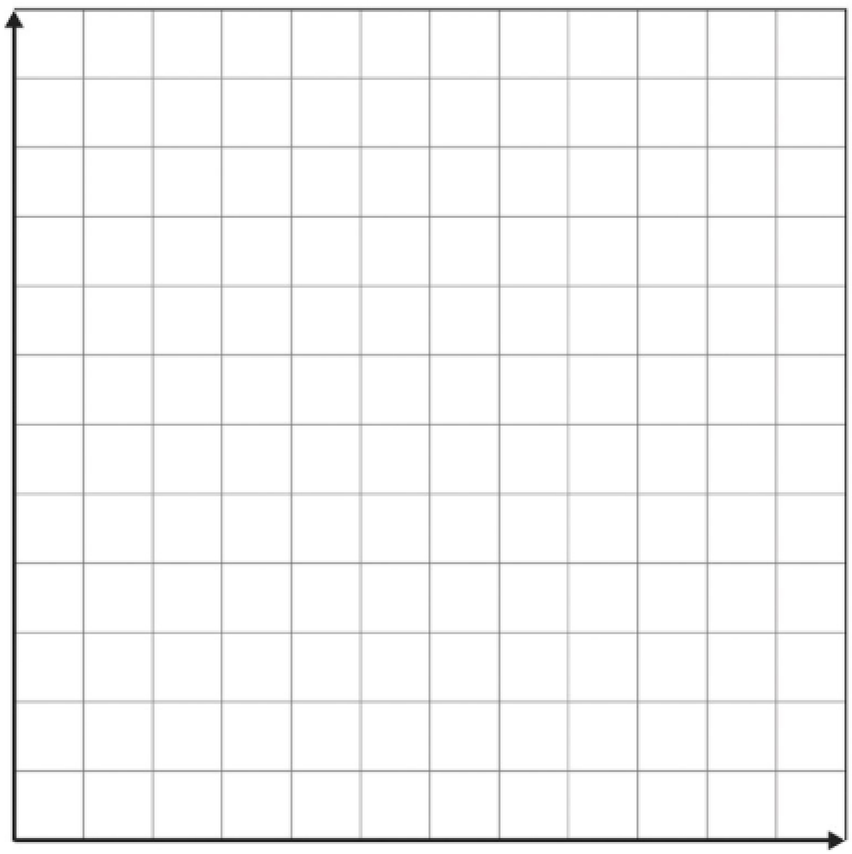
1. Assign group roles:
   - **a.** Photographer/Viewer--Will look through the tube. The Photographer must hold the tube parallel to the floor and must hold the tube steady while measurements are taken.
   - **b.** Data Collectors (2 people)—Will communicate with the photographer to determine the location and diameter of the target circle.
   - **c.** Enforcer—Measures the distance of the photographer from the wall, makes sure the tube is parallel to the floor and steady, and records the data.
2. You will collect 10 sets of data. To determine the size of the viewing field, the data collectors will use the colored pieces of paper against the chart paper to determine the amount of the chart paper the viewer can see. Measure the distance between the two pieces of colored paper to find the diameter of the viewing area.



Data Collection: View Tubes

Distance from Wall										
Viewing Diameter										
Area of Viewing Field										

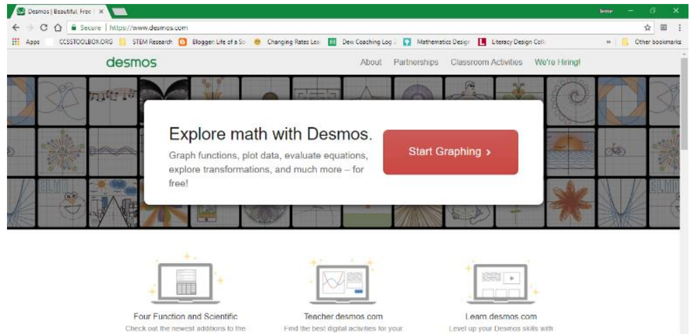
1. Graph a scatter plot of your data on the grid below.



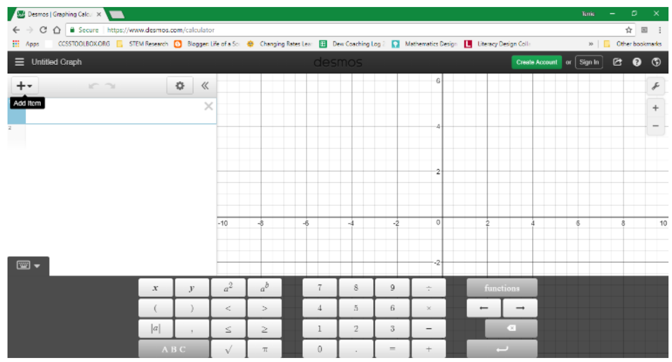
2. Use your knowledge of quadratic functions to create an equation to model the data of distance from the subject vs. viewing field.

3. In statistics, a process called "regression" is used to determine the curve that best fits a given set of data. For data whose graph is a parabola, this is called "Quadratic Regression." We can use technology to find the quadratic regression equation for a given data set.

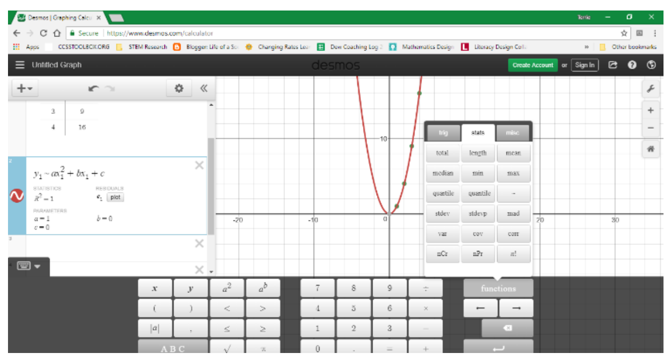
   a. Open www.desmos.com on your computer. Select "Start Graphing".



   b. Select the "+" at the top left of the screen to add an item. Add a table, then enter your data into Desmos.



   c. After you enter your data, select the "+" again to add a regression equation. Since the general form of a quadratic function is $y = ax^2 + bx + c$, enter $y_1 \sim a{x_1}^2 + bx_1 + c$. The "~" symbol is under the function menu on the lower right side of the keyboard. Select the "stats" tab to access the "~" symbol.



   d. Adjust the graph to get a quadratic regression equation that best fits your data. Record your quadratic regression equation below:

4. How does the equation you found using Desmos compare with the equation you created in #2?

Use your quadratic regression equation to answer the following questions:

5. Suppose a photographer wanted to take a photo of the entire senior class sitting on the bleachers in the football stadium. When seated, the distance from end to end of the whole group is 50 meters. If the photographer is using a lens the size of your tube, how far away will they need to stand to get the entire group in the photo?

6. A photographer is working in a room with limited space. The distance from corner to corner in the room is only 5 meters. Assuming the size of her lens is the same as your tube, what is the largest viewing size she will be able to capture inside this room?