Changing Velocity at Constant Acceleration

Lesson Overview

This lesson is "**Lesson 3**" in a 3-lesson series exploring motion. In this lesson, students will observe a tennis ball being dropped using photo burst pictures and analyze data using appropriate software. The observations and data collected are for 1 D Kinematics only where acceleration is constant is nonexistent (values of 0 or greater). Students will first, describe the motion qualitatively, then quantitatively with graphs and finally with equations. They will compare calculations from their data and graphs with those from the equations. They will create a story that will serve as a model of 1D Kinematics. The1 D Kinematics formulas align with the general mathematics formulas for linear and quadratic rates of change:

- Linear equations in slope-intercept form: y = mx + b; 1D Kinematics: $v = at + v_0$ *&* $d = ½ vt + d_0$ *(this variable is usually = to zero and is not represented in general physics 1 D Kinematic equations – it is show here to be representative of the linear equation).*
- Quadratic equations in standard form: $y = ax^2 + bx + c$; 1D kinematics: $d = at^2 + vt + d_0$ *(again usually = 0 so its omitted – shown here to be representative of the quadratic equation)*

Where **d** is final position, **d_0** is initial position (arbitrary, usually zero), **t** is time final, **t_0** is initial time, **v** is velocity final, **v_0** is initial velocity, and **a** is acceleration (for this investigation constant). Recommended The Physics Classroom 1 D Kinematics for Teacher background - http://www.physicsclassroom.com/Physics-Tutorial/1-D-Kinematics

NOTE: This lesson has been written for use without precision equipment. However, if you possess PASCO, Vernier, Pocket Lab, or other equipment – i.e. photogates, light sensors, radar, spark-timer, tickertape-timer, etc... It is recommended that you use them to produce precision data following the procedure for data collection. The rest of the lesson plan can be used with the data to develop the skills knowledge and characteristics presented in Profile of a SC Graduate.

Prerequisite knowledge

Students should be able to *(for 1 dimensional motion)*:

- Identify and accurately graph independent and dependent variables
- Identify and contrast vector and scalar quantities.
- Identify and contrast distance, displacement, and position
- Identify frames (points) of reference.
- Identify changes to distance, displacement and position when a frame of reference changes
- Add and subtract vectors
- Use a line of best fit on a graph of plotted data

Alignment

Science Standards

H.P.2A.3 Use mathematical and computational thinking to apply formulas related to an object's displacement, constant velocity, average velocity and constant acceleration. Interpret the meaning of the sign of displacement, velocity, and acceleration.

H.P.2A.4 Develop and use models to represent an object's displacement, velocity, and acceleration (including vector diagrams, data tables, motion graphs, dot motion diagrams, and mathematical formulas).

H.P.2A.5 Construct explanations for what is meant by "constant" velocity and "constant" acceleration (including writing descriptions of the object's motion and calculating the sign and magnitude of the slope of the line on a position-time and velocity-time graph).

Science and Engineering Practices

H.P.A.2 Develop and use models to (1) understand or represent phenomena, processes, and relationships, (2) test devices or solutions, or (3) communicate ideas to others.

H.P.1A.5 Use mathematical and computational thinking to (1) use and manipulate appropriate metric units, (2) express relationships between variables for models and investigations, and (3) use grade-level appropriate statistics to analyze data.

H.P.A.6 Construct explanations of phenomena using (1) primary or secondary scientific evidence and models, (2) conclusions from scientific investigations, (3) predictions based on observations and measurements, or (4) data communicated in graphs, tables, or diagrams.

Crosscutting Concepts (from the SDE instructional unit resources document)

3. **Scale, proportion, and quantity**: The National Research Council (2012) states that "in considering phenomena, it is critical to recognize what is relevant at different measures of size, time, and energy and to recognize how changes in scale, proportion, or quantity affect a system's structure or performance" (p. 84). The ideas of ratio and proportionality are important here along with being able to predict the effect of a change in one variable on another. For example, how will the speed of an object change if the time traveled is increased but the distance remains the same?

Math Standards

ACE.2 Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.

FIF.5 Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.

FIF.6 Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.

FIF.8 Translate between different but equivalent forms of a function equations to reveal and explain different properties of the function.

Standards for Mathematical Practice

SMP.2 Reason abstractly and quantitatively.

SMP.3 Construct viable arguments and critique the reasoning of others.

SMP.4 Model with mathematics.

ELA Writing

Standard 6: Write independently, legibly, and routinely for a variety of tasks, purposes, and audiences over short and extended time frames.

6.1 Write routinely and persevere in writing tasks over short and extended time frames, for a range of domain-specific tasks, and for a variety of purposes and audiences.

ELA Communication

Standard 1 Interact with others to explore ideas and concepts, communicate meaning, and develop logical interpretation through collaborative conversations; build upon the ideas of others to clearly express one's own views while respecting diverse perspectives.

1.2 Initiate and participate effectively in a range of collaborative discussions with diverse partners; build on the ideas of others and express own ideas clearly and persuasively.

1.4 Engage in dialogue with peers and adults to explore meaning and interaction of ideas, concepts, and elements of text, reflecting, constructing, and articulating new understandings.

1.5 Synthesize areas of agreement and disagreement including justification for personal perspective; revise conclusions based on new evidence.

Connections

Content Area (2 or more) Connections

- Algebra II / Precalculus
- Physics

Content Connections

The use and graphing of quadratic and linear equations in mathematics is the cornerstone skill of this lesson without that context the content for 1 D Kinematics is incomplete.

Active Learning Strategies (for Purposeful Reading, Meaningful Writing, and Productive Dialogue) – These will be hyperlinked in the lesson.

- Quick Write
- Line of Learning
- Frayer Model
- Strategy Harvest

Computational Thinking

The following computational thinking (CT) skills are used during this lesson:

- Logically organizing and analyzing data
- Representing data through abstractions such as models and simulations
- Automating solutions through algorithmic thinking (a series of ordered steps)
- Confidence in dealing with complexity
- Persistence in working with difficult problems
- The ability to deal with open ended problems
- The ability to communicate and work with others to achieve a common goal or solution

Lesson Plan

Time Required – Eight 55 min class periods

Disciplinary Vocabulary –

average velocity, kinematics, frame or point of reference, constant velocity, instantaneous velocity, displacement, velocity, acceleration, constant acceleration, position, acceleration due to gravity, graphing, x-axis, y-axis, variable, average, constant, independent and dependent variables, slope, line of best fit, linear, quadratic, and rate (of change).

Materials Needed:

- Tennis balls (1 per team) (florescent ones work best)
- Smooth flat surfaces (5-7meters for each group)
- Meter sticks (2 per group)
- Painter's tape (1 roll per group) *(NOTE: "blue painters tape from a home improvement store – doesn't leave a residue like masking tape)*
- Stop watches (1 per group) (accuracy to at least 1/10 of a second) – *Smart phones work exceptionally well*
- Graph Paper
- Copies of "Frayer Models" that students started in Lesson 1
- Copies of "Motion of Motorized Cart Activity" (1 per group)

- Constant velocity cart (1per group of 3-4 students) (Carts can be purchased for around $10 For example Arbor Scientific $8.50 https://www.arborsci.com/constant-velocity-car.html)
- 10 Colored pencils (if you have groups of 4 you will need 2 identical sets), 4 markers and a pair scissors per group
- Pack 1 two-meter piece of butcher paper cut into equal length strips, one per group (e.g. cut the butcher paper lengthwise so that each group gets a 2m long strip of paper)
- 1 tablet or smartphone per group of 2-3 students. Each device with either the Fast Burst Camera Lite App for android (Chromebooks and smartphones) or iPhone Burst (comes with most iPhones and iPads) but can also be downloaded. **FYI iPhones and iPads with burst CAN NOT be adjusted they are set at 10fps therefore android devices are preferred b/c you can 20fps and therefore more data points.**
- Computers, laptops, tablets or other devices with photo viewing software of app (Paint is fine) and screens large enough to take measurements.
- *OPTIONAL – Grid lined chart paper*

Formative Assessment Strategies:

- Frayer Model http://www.s2temsc.org/uploads/1/8/8/7/18873120/frayer_model_strategy.pdf
- Line of Learning http://www.s2temsc.org/uploads/1/8/8/7/18873120/line_of_learning_strategy.pdf
- Computer/Tablet/Smart Phone Simulations & Interactives: **The Physics Classroom**: http://www.physicsclassroom.com/Physics-Interactives/1-D-Kinematics & **pHet Interactive Simulations**: https://phet.colorado.edu/en/simulation/legacy/moving-man

Misconceptions:

Students have difficulty connecting motion graphs to physical concepts:

- Differentiating between slope and height
- Relating one variables graph with another (i.e. position, velocity (rate of change in position) and acceleration (rate of change in velocity))
- Matching narrative information with related features on a graph
- The meaning of the area under the graph
- Representing continuous motion with a line or curve
- Differentiating between the shape of the graph and the path of motion
- Negative velocity and acceleration & **constant acceleration (free fall – biggest misconception in 1D Kinematics)**

Safety Note(s): Students should wear appropriate safety equipment. *See MSDS and operating/safety instructions for all equipment and tools used.*

The is a 5E multiday lesson, daily opening and closing is up to the instructor –where time might be a factor we have included “re-engagement” and Exit Tickets. There is a suggested number of periods beside each "E" in the lesson plan – However, it is not advisable to move onto the next part until students have had enough time to process the current information.

Day One

Engage:

- Show Armageddon or Deep Impact Trailer (you can find these on YouTube) and tell the students that they are about to embark on an exploration of an objects motion that is accelerated by the same force that accelerates asteroids.
- Revisit the Frayer models created in Lessons 1 and 2

Explore:

- Students will use their graphs from constant velocity and compare their data and graphical analysis with the following kinematic equations:

  $v = at + v_0$ & $d = \frac{1}{2} vt + d_0$

  - Students should use both graphs and data tables to pull quantities for each of the formulas. Make sure students know that the formulas may need to be manipulated to solve for the ungiven variable.

Day Two

Explain:

- Formatively assess what students have learned about constant velocity and linear graphs AS well as providing an opportunity for students to explore other types of motion.
  Use the following simulations (NOTE: This part of the lesson is best if each student has access to their own device – these sims run on almost any platform including smart phones and tablets.):
  - To assess or refresh graphing skills: http://www.physicsclassroom.com/Concept-Builders/Relationships-and-Graphs (Both sections are equally effective – let students choose)
  - To assess student comprehension of the relationship between position vs time and velocity & To provide an opportunity for students to explore changing velocity at a constant acceleration and explore the vector (directional) component of motion in more detail at their own pace: http://www.physicsclassroom.com/Physics-Interactives/1-D-Kinematics & https://phet.colorado.edu/en/simulation/legacy/moving-man (*There are several pHet approved lessons for this simulation under the teacher section*).
- Prior to the Photo Burst Drop Zone Activity:
  - Ensure that you have enough charged devices (BYOD is great here b/c they can practice at home).
  - Set the camera quality (picture quality) to HD.
  - If the devices are being reused in multiple classes have appropriate changers available.

    - Make sure you have practiced using the burst camera if you have never used one.

Day Three

Extend:

- Students use both graphs and the data they have collected and compare and contrast it with the 1 D Kinematic Equations:
- Give the following 3 problems to each group and have the students solve them individually and then compare the strategies they used to get the answer (Provide the answer to each problem with the problem – the goal is for them to figure out how to get that answer)
    - A ball is dropped from rest, off a cliff of height 100m. Assuming gravity accelerates masses uniformly on Earth's surface at 9.8m/s^2, how fast is the ball going when it hits the ground? (44.3 m/s) How long does it take to hit the ground? (4.5s)
    - A soccer ball is kicked from rest at the penalty spot into the net 11m away. It takes .4s for the ball to hit the net. If the soccer ball does not accelerate after being kicked, how fast was it traveling immediately after being kicked? (27.5 m/s)
    - A continuously accelerating car starts from rest as it zooms over a span of 100m. If the final velocity of the car is 30m/s, what is the acceleration of the car? (4.5m/s^2)

Days Four—Eight

Evaluate:

- Use the [Strategy Harvest] for the strategies individuals in each group used to solve 1 D Kinematic problems. The strategies each student uses should include one of the following general approaches:
    - Graphical Analysis
    - Utilizing Equations
    - Dimensional Analysis (This lesson has not directly addressed DA)
- Provide students with the following problems and have them solve these for the answer including a description or procedure for how they got their answer.
    - A basketball is dropped from a height of 10m above the surface of the moon, accelerating downwards at 1.6m/s^2. How long does it take to hit the surface, in seconds to the nearest tenth? (DO NOT SHARE THE ANSWER until group is finished working them t = 3.5s. Use $v^2 = v_0 + 2ad$ where $v_0 = 0$)
    - A train traveling at 40m/s is heading towards a station 400m away. If the train must slow down with constant deceleration into the station, how long does it take to come to a complete stop, in seconds? (t = 20s, Use $v^2 = v_0^2 + 2ad$, then use $v = at + v_0$)
    - A pitcher throws a baseball towards home plate, a distance of 18m away, at 40m/s. Suppose the batter takes .2s to react before swinging. In swinging, the batter accelerates the end of the bat from rest through 2m at some constant

acceleration *a*. Assuming that the end of the bat hits the ball if it crosses the plate within .05s of the ball crossing the plate, what is the minimum required *a* in m/s^2 to the nearest tenth for the batter to hit the ball? (*The ball takes t = d/v = .45s to reach the plate, but the batter has until .45s + .5s = .5s to make contact due to the buffer. Since the batter takes .2s to react, we have a maximum time of t=.03s to swing the bat. The distance the end of the bat has traveled as a function of its acceleration and time is* $d = \frac{1}{2} at^2$*, so to reach home plate in t requires an acceleration of* $a = 2d/t^2 = 44.4 m/s^2$)

- Provide answers once all students in a group have solved the problem.
- Have students revisit their "Quick Write" from Lesson 1 and add a "Line of Learning" "What you know NOW about motion?"
- Have student share their work with the group for feedback, at this time students should also reflect on the strategies they used to solve the problems.
  - Have each group assign the following roles:
    - facilitator – this person will track the participation. Share the following as examples: *Indicate that they may choose different categories*
      - Time spent on sharing strategies
      - Time spent on comparing results
      - Time spent asking questions
      - Time spent making suggestions
      - Time spent explaining (point of view or how something worked)
    - strategy hound – this person will track the category of each strategy. Share this example: 3 used graphical analysis to solve the problem and 1 used it as a check, 3 used the formulas as a check and 1 used it to solve, dimensional analysis was used by 2 to check that the formula they were using was the correct one
    - recorders – 2 or more folks that record the groups ideas, positions and questions for the whole class. Suggest that these folks alternate recording information – For example, the group likes something A says so recorder 1 records it but before recorder 1finishes with the idea that A had then B asks a question that no one can answer at this time the second recorder should record the questions et…
- Finally – have students submit 2 -3 problems that meet the criteria below for the summative assessment, one that is graphical and two that are verbal – use the rubric and then use the best problems to create your test. Checklist:
  - Motion must be in 1 dimension
  - Must include an explanation of how to use one of the strategies to solve the problem and demonstrate specifically how it solves the problem (i.e. show the work and explain each step in solving the problem).
  - Information that must be available via graphic representation (dot diagrams, data tables, graphs, etc…)
    - Constant velocity
    - Constant acceleration

    - Instantaneous velocity
    - Average velocity
    - Displacement
    - Position
  - Must include a velocity time graph
  - Verbal problem must include an opportunity to solve a problem involving constant acceleration and velocity (changing or constant)

Photo Burst
Drop Zone

Purpose:

- To use graphing as a model representing an object's motion (position, displacement, velocity and acceleration).
- To develop an operational definition for constant acceleration.
- To compare results from graphical analysis to the 1 D Kinematic Equation:

$$d = at^2 + vt + d_0$$

Materials:

- 1 Device with a photo burst app (*instructions for android and iOS setting are separated in the procedure be sure to follow the set-up instructions for your app*).
- 1 tennis ball
- 2 meter sticks
- Blank wall that contrasts with tennis ball OR 2 m long piece of butcher paper
- Painters or Masking Tape
- Movable table/desk that can be used to hold camera steady

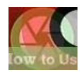
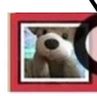
Description:
In this lab, you will be observing the motion of a tennis ball dropped from a specific height using a burst camera. Your goal will be to determine the changes in motion by measuring and analyzing the series of photographs. You will be looking at changes in position (displacement) as a function of time – very similar to the procedure used in the motorized cart exercise with the butcher paper. In addition, you will also be examining velocity as a function of time. You will create 2 graphs one position vs time and one velocity vs time. You will again draw a best fit curve for the points on your graph. In addition, you will calculate the instantons velocity (the velocity at a specific instance in time) for each data point.

Part I Procedure:

Set Up:

1. Assign the following roles:
   a. Camera operator – this person is responsible for taking the burst shots with the app. This person should have very steady hands.
   b. Photographer – this person is responsible for sequencing the groups photographs for each of the 3 trials and ensuring photos are properly labeled.
   c. The Ball Handler – this person is responsible for dropping the ball so that the measurement from the meter sticks can be read easily.
2. Set Up the investigation:
   a. Select an area of wall that is free from distractors (posters, books, shelving, etc…) and tape your two metersticks vertically from the floor up

2 meters of the wall. Be sure the metersticks are facing the same direction. If the wall is too distracting or the tennis ball will not show up on it then tape up the butcher paper first and then the meter sticks.

3. Set up your device for data collection & Practice using the burst camera use “a” for android smart phones and tablets as well as Chromebooks, use “b” for all iOS devices (iPhone, iPad, iPod):
   a. Android (Chrome) –
      i. Open the App: 
      ii. Select the icon  at the bottom right of the screen
      iii. Once it is open use the slider to select 1/20 second – so that 20 pictures are taken
      iv. TO practice point the camera at the meter stick and have then ball handler drop the tennis ball a second after you press and hold the shutter icon 
      v. BE SURE TO HOLD IT – Let go when the tennis ball hits the ground
      vi. Wait until the saving icon  stops – i.e. the white doughnut has completely disappeared, and a small photo icon  appears in its place.

   b. iOS – The time between bursts can’t be changed it runs at 10 frames per second or 10fps or 1/10 sec for each picture – so 10 pictures are taken in 1 second.
      i. Burst mode is a “hidden” feature in the native camera app that allows you to continuously capture ten photos every second.
      ii. To activate burst mode, simply tap and hold the shutter button within the iPhone’s camera app.
   c. Practice using the burst camera to capture the ball drop:
      i. Handler holds the ball at the top of the 2 meter sticks
      ii. Camera Operator presses and holds the shutter button and tells the handler to release the ball
      iii. The Photographer tells the Camera Operator to stop pressing the shutter when the ball hits the ground
      iv. All group members review the pictures AFTER they have saved and adjust so that the ball can be seen in reference to the meter stick (i.e. you can zoom in and see the number on the meter stick)
      v. If its blurry use a desk or table to hold the camera/device on while pressing the shutter button.

Data Collection:

4. Create a data table in which you can record the position at each time interval (burst photograph was taken), velocity at each time interval and acceleration at each time interval. For iOS this will be in increments of .10 sec for android it will be in increments of .05 sec The number of photographs (position entries) for each trial will depend upon the time interval.
5. The camera operator starts and tells the handler to release the ball AFTER the shutter button had been pressed (the shutter button is held until the photographer tells the operator to stop)
6. The Handler Drops the ball and holds very still.
7. The photographer tells the operator to stop pressing the shutter button when the ball hits the ground.
8. The operator gives the device/camera to the photographer who opens the pictures AFTER they have been saved.
9. The group then views the photos deleting all unneeded.
   a. The first picture should be of the handler holding the ball
   b. The rest should be of the ball in motion
   c. The Last one should be of the ball hitting the floor or near the floor. Do not keep pictures of the ball bouncing – i.e. if the camera does not capture the ball hitting the floor your last picture should be the one right before the first ball bounce.
   d. The total number of pictures is the number of position/time entries you will have for each trial (The number of pictures per trial should ONLY vary by 1 or 2 pictures)
10. The photographer then uploads or prints the photos for the trial and ensures that each photo for that trial is identified with that trial number.
11. Repeat steps 5 - 10 until you have 3 trial sets of satisfactory of photographs.

Analysis:

4. Measure each position of the ball from the first picture through the last.
   a. Use the zoom feature either on the device or on the computer (device) where you saved the images, to determine where the on the meter sticker the midpoint of the ball is in the picture.
      1. Use a ruler or other straight edge if there is too much space between where the ball is and where to the read the meter stick.
      2. If the ball is blurry just use the midpoint of the blur.
   b. Repeat "a" recording the position of the ball at each time interval.
   c. For example, if you have 21 pictures and your burst was set for 1/20 than you will have 21 positions - the first position starting at time interval .00s and the next at .05s and the third at .10s until you reach the 21st at time interval 1.05s.
5. Record your position data for each trial next to its corresponding time interval.
6. Calculate the average position at each time interval.
7. Plot the average of each position/time data pair on a position vs. time graph.

   a. Remember to label the graph and both axis.
   b. Include the proper units for each axis.
   c. You may ask your group members questions to assist you, BUT you must draw and label your own graph and plot the (time/position) ordered pair for each time interval
9. Draw a best fit for the data points on your graph. See the examples from the Motorized Cart activity – Do your points fit best to a line or a curve?
10. Calculate the velocity at each time interval using either graphical analysis (tangent line) or an equation (1 D Kinematic).
11. Plot velocity vs time on a graph – remember proper units and that this is also a rate.
12. Calculate the slope using the y-intercept formula (y=mx + b). BE SURE TO KEEP THE UNITS!!!
13. Calculate the slope using the graph (rise/run OR $(y_2 – y_1)/(x_2 - x_1)$)
14. Calculate the area under the curve – look at the shape it makes
15. Substitute the independent and dependent variables into the quadratic equation. DO the same for the 1 D Kinematic equation.

Results and Conclusions:

1. How does the displacement compare to the position? Why?
2. What does a curved velocity graph tell us about the motion of an object? How do the dot motion diagrams compare to what you are seeing in this graph – THINK about what you observed the ball doing.
3. What does the slope of the velocity vs time graph tell us? What does the area under the line tell us?
4. How do the quadratic equation and this 1 D Kinematic equation compare?