# Inverting a Triangular Array

## Lesson Overview

This problem-based lesson promotes strategic, mathematical thinking, encourage student dialogue and reflection. Students are assigned a problem in which their challenge is to find the BEST solution possible. There should be multiple solution strategies occurring within the classroom and many students will challenge each other. The role of the teacher in this lesson is to facilitate and to question student thinking and encourage student dialogue.

## Standards Addressed

CCSS.MP.1 - Make sense of problems and persevere in solving them.
CCSS.MP.3 - Construct viable arguments and critique the reasoning of others.
CCSS.MP.4 - Model with mathematics.
CCSS.MP.5 - Use appropriate tools strategically.

## Disciplinary Literacy Best Practices

Bounce Cards
Turn and Talk
Making Thinking Visible
Two Minute Paper

## Lesson Plan

Time Required – Two 60-minute Class Periods

Disciplinary Vocabulary : invert, inductive reasoning, deductive reasoning, conjecture, solution, reflect, translate, counterexample

Materials Needed:

- Two-side Counters – 36 per pair of students
  - Paper and Pencil – for each student
  - Chart Paper or poster paper – 1 per pair
  - Markers – 1 set per pair

Assessment: Partner Making Thinking Visible charts and Individual Two-Minute Paper

## Engage

- The teacher first captures the students' interest by telling them that they are going to solve a problem today that everyone will find a solution for; however, the challenge for this problem will be to find the best solution possible.
- Explain to students that there will be a reward at the end (if you choose to provide one) for the pair who finds the best solution possible.
- Leader Board: Make a t-chart on the board with one column for student names and another for solution. Explain the leader board after presenting students the challenge problem.

## Explore

- Present the problem:
  - "What is the fewest number of moves it would take to turn a triangle of 36 pennies upside down? Constraint: Only one penny may be slid or otherwise moved per move."
- After presenting the problem, ask students: What questions might you have about this problem? What clarifications do you need at this point?
- Have students **turn and talk** to their partner (Elbow partner or however you have paired them for this lesson) and talk about how they might want to go about solving this problem. "Turn to your partner and talk about strategies, or ways, that you might solve this problem."
- Question for students: As you think about solving this problem, what are some materials you might need? (i.e. counters as pennies or some other object to model with, paper, pencil)
- Before beginning work, explain the "Leader Board" to students, if you are choosing to use this during the lesson. As they PROVE a solution, they may post their names and the fewest number of moves they have proven under the solution column. However, once posting, that means that they now have to work to beat their own solution (while others work to tie or beat theirs as well). Make it a friendly competition.
- Allow students to begin exploring how to invert the triangle with the fewest number of moves possible. Encourage students to use **Bounce cards** as they dialogue. What is the fewest number of moves you have proven so far? What strategy did you use? How might that strategy help you find fewer moves? Will this strategy work all of the time? (Especially if they are trying a pattern or a table.) How do you know? How could you find out? How might you solve this a different way to see if this solution is still the "lowest"? (Finding a counterexample)
- For stuck students…after allowing ample time to grapple with 36 penny triangle but before they reach the point of frustration and giving up, try:
  - Tell me about what you're thinking.
  - What have you tried so far?
  - Have you thought about…(at this point, if students are completely stuck and not sure where to go, suggest starting with a smaller triangle and building up or making a table or looking for patterns with a smaller triangles).

- For students who say "this is the lowest I can go" – challenge them to go further. Have students show their strategy and explain how they found their solution. What might be another way to solve this problem (other than the method they are currently using)?
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## Explain

- Each pair of students has the opportunity to explain their lowest solution and strategy for finding that solution to the class by making a **Making Thinking Visible (MTV).** Students are given chart paper (or poster paper) and a marker. They record their solution and strategy on paper to share out. This must include a "picture" of what they did and a written explanation.

## Extend

- For students who need the extra challenge, have them attempt a 66 penny triangular array.
- Challenge students who are only using direct modeling with counters to use a different solution method, such as a table or looking for patterns. Giving them a larger array causes them to use a different solution strategy because modeling no longer seems to be a reasonable strategy to use with a large number of pennies.

Lesson Assessment: **Two-Minute Paper**--Students complete reflective writing at the end to assess themselves and their partner. Students will write for two minutes, answering the following questions:

- How well did you and your partner use the Bounce Cards? What are your thoughts about the Bounce Cards?
- From this lesson, what are some things you learned about yourself as a problem solver?

## Teacher Reflections and Biographical Information

There are many ways students will try to solve this problem – modeling, patterns, making tables, etc. It is very important to allow students time to think about this problem and grapple with it before offering suggestions for solving it. Wait until they are completely blocked at 36 before suggesting a simpler triangle or tables or patterns. Always disprove solutions considered "lowest" in order to encourage students to keep trying. Many solutions will seem logical to them but they are not reasonable solutions. Explanations are required for every solution and for claim on the leader's board. (Students how tie can put names in a hat and draw.) Reward students for perseverance!

Lesson Authors:

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