Kitten Population

Lesson Overview

In this lesson students will experiment with exponential growth and connect learning to algebraic and graphical representations of exponential growth. Students will do this through experiments and models based on a problem.

SC Standards Addressed

FLQE.1* Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval.

a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another

Disciplinary Literacy

Strategy Used: Exit Ticket

Computational Thinking

Tools:
Problem Based Learning (PBL)

Cornerstone(s) Addressed:

- Decomposition: Students will break down the problem into necessary steps to find an answer to the problem given.
- Pattern Recognition: Students will try to find patterns in the experiments and data collection to identify and find equations for exponential growth.
- Algorithmic Design: Student will formulate/discover and work with algorithms demonstrating exponential growth.
- Abstraction: Students will need to determine which information is needed to solve the problem and decide the best way to use the data.

Lesson Plan

Time required: Two 60-minute class periods

Focus Question(s): In just 18 months a female cat can have over 2,000 descendants; is that even possible?

Disciplinary Vocabulary: exponential growth, equation, linear, exponential, table representation, graphical representation, percentage, average (mean), best fit

Materials needed:

- Desmos (graphing program) with Internet access – or some other graphing program or device (i.e. graphing calculators)
- Having Kittens - Data Sheet (1 per group)
- Bag of plain M&M's per pair/trio/quad – teacher should group as desired (suggestion is groups of 3)
- M&M's Exponential Growth Lab Sheet
- 11x17 paper (1 sheet per group) or chart paper
- Post-it notes (1 per student)
- Charts or space for the "yes" and "no" post-it notes (from the Engage section)
- Having Kittens - Growth Calculation Sheet (1 per group)

Engage

Share the situation or question to answer with students: “Sarah just got a female kitten. She is trying to decide if she should have the kitten spayed. Research says that in just 18 months, this female cat can have 2,000 descendants. Is that even possible? If you think “yes”, then place a sticky note on the “YES” poster or area and state how many descendants you think the kitten could have in 18 months. If you think “no”, then place a sticky note on the “NO” poster or area and state how many descendants you think the kitten *could* have in 18 months.” The teacher may choose to post the question portion of the above. Designate a chart or space for the “yes” and “no” post-its.

Explore

Provide students with the handout, “Having Kittens - Data Sheet”, to work the solution. Students should be given the opportunity to determine whether this number of descendants is realistic. The students/student groups should record their information and thinking on an 11x17 sheet of paper. (NOTE: Solutions may vary, but students should be able to justify their solutions during the “Explain” section of the lesson. By the way, it is possible.)

Explain

Students should share their solutions and justifications. Students may use tables, charts, tree diagrams, etc. to justify their solutions. Based on the discussion and the group’s own thinking and exploration, students should be asked to update or change their sticky note location and data if desired.

TIME TO SWITCH GEARS TO M&M’s

Engage Part 2

Show the video on cancer and cell division to introduce the idea of exponential changes based on data.
http://www.youtube.com/watch?time_continue=230&v=BmFEoCFDi-w

Explore Part 2

Student groups should be given the “M&M Lab (Exponential Growth)” handout. Students should work through the Math Lab experiment and complete the Lab Sheet.

Explain Part 2

Student groups should discuss the following prompt: “Describe the relationships among the three representations (table, equation, graph)”. After groups have had time to discuss, ask for each group to share out their thinking.

TIME TO SWITCH GEARS BACK TO KITTENS

Elaborate

Students should work to complete the “Having Kittens Growth Calculation Sheet”. After students have had time to work and dialogue, ask each group to share aloud their discoveries and thinking. The teacher should provide additional equations to discuss whole-group about variables within the equations and what they each represent in a real-world scenario.

Evaluate

1- Student Groups should submit the two completed handouts/lab sheets to the teacher for grading.
2- Students should complete an “exit ticket” answering the following questions and using as much of the disciplinary vocabulary as needed (exponential growth, equation, linear, exponential, table representation, graphical representation, percentage, average (mean), best fit): Remind students they may choose to provide any type of representation: written explanation(s), symbols or other representations.
    - What connections did you make as you worked through both the m&m experiment and the kittens’ problem?
    - How do linear and exponential functions and situations compare?

- What might be some other situations where you could apply your learning about exponential functions?

IMPORTANT NOTE: This exit ticket is complex with many expectations. Students should have at least 15 minutes to provide their thinking and explanations.

Assessment Notes: Review the completed handouts for accuracy of work and thinking. Keep in mind, answers will vary based on data collected.

References/Resources:

- www.desmos.com
- Problem adapted from Modeling Population Growth: Having Kittens from the Mathematics Assessment Project. http://map.mathshell.org/lessons.php?unit=9100&collection=8
- http://www.youtube.com/watch?time_continue=230&v=BmFEoCFDi-w

Teacher Biographical Information

Margaret Lorimer
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M.Ed. Administration and Supervision
After teaching mathematics for many years, Margaret is currently an Education Specialist with S^2TEM Centers SC since 2003.
&
Dana Thompson
B. S. Mathematics
M.Ed. Administration and Supervision
After teaching mathematics for many years, Dana is currently a Regional Coordinator with S^2TEM Centers SC/SCCMS since 2002.

Having Kittens - Data Sheet

This is a poster published by an organization that looks after stray cats.

Cats can't add but they do multiply!

In just 18 months, this female cat can have 2,000 descendants.
Have your pets spayed or neutered!

Your job is to figure out whether this number of descendants is realistic.
Here are some facts you will need:

Length of pregnancy
About 2 months

Number of kittens in a litter
Usually 4 to 6

Age at which a female cat can first get pregnant
About 4 months

Average number of litters a female cat can have in one year
3

Age at which a female cat no longer has kittens
About 10 years

Group Members: ____

M&M Lab (Exponential Growth)

Part I: Modeling Exponential Growth M&M Activity

The purpose of this lab is to provide a simple model to illustrate exponential growth of cancerous cells.

There are more than 100 types of diseases known collectively as cancer. What they all have in common is the overgrowth of cells, tiny units that make up all living things. Cancer (also known as malignancy, pronounced: muh-LIG-nun-see) occurs when cells begin to grow and multiply in an uncontrolled way.

- In our experiment, an M&M represents a cancerous cell. If the M&M lands "M" up, the cell divides into the "parent" cell and "daughter" cell. The cancerous cells divide like this uncontrollably - without end.
- We will conduct 15 trials and record the number of "cancerous cells" on the plate.

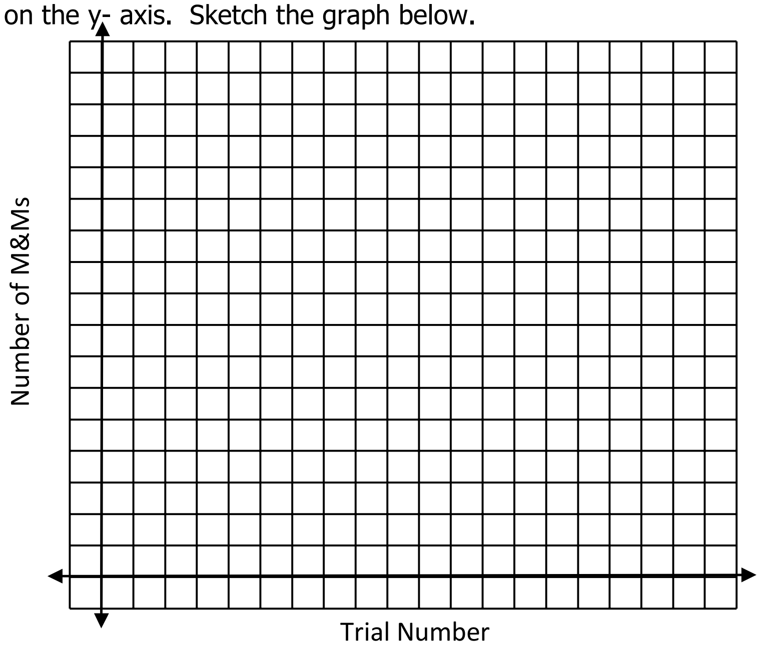
DO NOT EAT THE M&M's!

Exponential Growth Procedure

1) **Place 2 M&M's in a cup/plate. This is trial number 0.**
2) Shake the cup and dump out the M&Ms. For every M&M that shows an "M", add another M&M and then record the new population. (Ex. If 2 M&Ms land face up (M showing), then you add 2 more M&Ms). M&'Ms are never removed from the cup.
3) Repeat step number 2 until you are done with 15 trials OR you run out of M&Ms.

Trial #	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
# of M&M's *(# of cells)*	2															

4) Graph your data (scatterplot) (using a graphing program or device) with the trial number on the x-axis and the number of M&M's on the y- axis. Sketch the graph below.



Exponential Growth

5) Should your graph touch the x-axis? Why or why not?

6) After each time you shook the cup, *approximate* the percentage of M&M's that landed with the imprint of "M" face up by looking at the data in your table. __________

To *calculate* the percentage, we will calculate the percent change for *each* trial using the formula below.

$$\frac{\text{new amount of M\&Ms} - \text{old amount of M\&Ms}}{\text{old amount of M\&Ms}}$$

Complete the table below.

phase #	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Percent *(write as decimal)*	X															

Calculate the average of *ALL* the percentages:__________________

7) We can write an exponential growth function that models the data above using the formula **$y = c(1+r)^t$**

c = Initial amount of M&M's (# of M&Ms you started with)

r = Rate of growth (calculated average from #6 - written as a decimal)

t = # of repetitions (Time - this represents a specific phase number)

Fill in the variables to write your own exponential growth equation:

Use your exponential growth model that you created in #7 to predict the number of "cancerous cells" there would be in:

Trial 25______________________ Trial 50 _________________________

Part II: Lab Discussion

Look at the exponential equations from your calculator … These questions will help you to determine how well your exponential equation fits your actual data

1. **In Part I, what was the "a" value?___________** What does the "a" value represent in the equation

$y = a * b^x$? BE SPECIFIC._______________

____

2. **In Part I, what was the "b" value?___________**

What does the "b" value represent in the equation

$y = a * b^x$? BE SPECIFIC._______________

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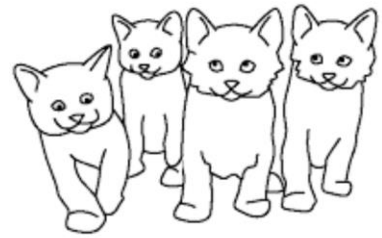
Having Kittens - Growth Calculation Sheet



Model → table → equation

Put the data you modeled into the table below.

Litter #	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
# of cats/ kittens	1															

Calculate the percentage change from litter to litter.

$$\frac{\text{new number of kittens} - \text{old number of kittens}}{\text{old number of kittens}}$$

phase #	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Percent *(write as decimal)*	X															

Calculate the average of ALL the percentages: ____________

We can write an exponential growth function that models the data above using the formula

$y = c\,(1+r)^{t}$

c = initial number of cats / kittens

r = rate of growth (calculated average of percentages – written as a decimal)

t = # of repetitions (litter number)

Fill in the variables to write your own exponential growth equation:

Use your exponential growth model to predict the number of kittens there would be in:

25 kittens ___________________________

50 kittens ___________________________