Spread the Data

Lesson Overview

In this lesson students will study the arm span lengths of the boys and girls within a class. (Students may prefer to use data collected from their own class.) To analyze the data students will compute measures of center (mean, median), quartiles (first, second, and third quartiles) and spread (range, Interquartile Range (IQR), mean absolute deviation (MAD)). Conclusions will be drawn based upon analysis of the data and examinations of graphs, in the context of questions asked about lengths of arm spans.

Alignment

Math Standards

SCCCR-M 7.DSP.3 Visually compare the centers, spreads, and overlap of two displays of data (i.e., dot plots, histograms, box plots) that are graphed on the same scale and draw inferences about this data.

SCCCR-M 7.DSP.4* Compare the numerical measures of center (mean, median, mode) and variability (range, interquartile range, mean absolute deviation) from two random samples to draw inferences about the populations.

Mathematical Process Standards

2. Reason both contextually and abstractly.

a. Make sense of quantities and their relationships in mathematical and real-world situations.
b. Describe a given situation using multiple mathematical representations.
c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation.
d. Connect the meaning of mathematical operations to the context of a given situation.

3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others.

a. Construct and justify a solution to a problem.
b. Compare and discuss the validity of various reasoning strategies.
c. Make conjectures and explore their validity.
d. Reflect on and provide thoughtful responses to the reasoning of others.

4. Connect mathematical ideas and real-world situations through modeling.

a. Identify relevant quantities and develop a model to describe their relationships.
b. Interpret mathematical models in the context of the situation.
c. Make assumptions and estimates to simplify complicated situations.

d. Evaluate the reasonableness of a model and refine if necessary.

Connections

Science Standards

7.EC.5A.3 Analyze and interpret data to predict changes in the number of organisms within a population when certain changes occur to the physical environment (such as changes due to natural hazards or limiting factors).

Science and Engineering Practices

7.S.1A.4. Analyze and interpret data from informational texts, observations, measurements, or investigations using a range of methods (such as tabulation, graphing, or statistical analysis) to (1) reveal patterns and construct meaning or (2) support hypotheses, explanations, claims, or designs.
7.S.1A.5 Use mathematical and computational thinking to (1) use and manipulate appropriate metric units, (2) collect and analyze data, (3) express relationships between variables for models and investigations, or (4) use grade-level appropriate statistics to analyze data.

ELA Inquiry Standards

Standard 3: Construct knowledge, applying disciplinary concepts and tools, to build deeper understanding of the world through exploration, collaboration, and analysis.
3.1 Develop a plan of action by using appropriate discipline-specific strategies.
3.4 Organize and categorize important information, revise ideas, and report relevant findings.
Standard 5: Reflect throughout the inquiry process to assess metacognition, broaden understanding, and guide actions, both individually and collaboratively.
5.1 Acknowledge and value individual and collective thinking; use feedback from peers and adults to guide the inquiry process.
5.2 Employ past and present learning in order to monitor and guide inquiry.

ELA Communication

Standard 1: Interact with others to explore ideas and concepts, communicate meaning, and develop logical interpretations through collaborative conversations; build upon the ideas of others to clearly express one's own views while respecting diverse perspectives.
1.2 Participate in discussions; ask probing questions and share evidence that supports and maintains the focus of the discussion.
1.5 Consider new ideas and diverse perspectives of others when forming opinions regarding a topic, text, or issue.

Active Learning Strategies (for Purposeful Reading, Meaningful Writing, and Productive Dialogue)

Mix-Pair-Share, Partner Dialogue

Computational Thinking

*Computational thinking (CT) is a problem-solving process that includes (but is not limited to) the following **characteristics**:*

- Logically organizing and analyzing data- Students will collect data and use to find measures of centers and spread
- Representing data through abstractions such as models and simulations- Students will used data to create box and line plots
- Automating solutions through algorithmic thinking (a series of ordered steps)- Students will find mean, median, range and mean absolute deviation (MAD) of a set of data

*These skills are supported and enhanced by several dispositions or attitudes that are essential dimensions of CT. These **dispositions or attitudes** include:*

- Confidence in dealing with complexity- Students will transfer prior knowledge/ skills to assist with new learning (MAD) and application
- Persistence in working with difficult problems- Students will apply new learning to increasingly larger data sets
- Tolerance for ambiguity- Students visually determine characteristics of a set of data
- The ability to communicate and work with others to achieve a common goal or solution- Students will work with partners and small groups to solve problems

Lesson Plan

Adapted from: Armspans, Debra L. Hydorn, University of Mary Washington, August 2012
Time Required: One 60-minute class)
Prior Knowledge: Students should have prior learning experiences collecting data and calculating measures of center and spread, as well as constructing box and line plots.

Disciplinary Vocabulary: Mean absolute deviation (MAD)

Materials Needed:

- Tape measure with centimeters (per pair of students)
- Video used to explain and model Mean Absolute Deviation (MAD) or https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-data-statistics/cc-6-mad/v/mean-absolute-deviation
- Copies of the SPREAD the DATA Lesson practice sheet for each student
- Copies of LINE PLOTs! for each student
- Copies of Exit Ticket for each student
- Online Boxplot Grapher or http://www.imathas.com/stattools/boxplot.html

Formative Assessment: Exit Ticket (handout pg 3)

Misconceptions: Comparing two data sets and expressing the difference between centers as a multiple of a measure of variability has several steps. Some students become overwhelmed and need the teacher to break down the process steps to help them make sense. These students may need access to the calculator and/or online tools for smaller data steps.

Measures of center and measures of variability are easily confused. Spend time concentrating on the difference between these two concepts. A foldable may help students separate the two concepts

Engage

- Ask: Did you know that when you stretch your arms (model) to create your arm span, the distance is approximately equal to your height?
- Say: In some situations, an arm span greater than height is thought to be advantageous. Consider some sports, such as swimming: as longer arms may give greater propulsion. A shorter arm span might be considered useful for weight lifters, as they don't have to lift the weight as high.
- Activate: Mix-Pair-Share Prompt: *In what other situations might arm span lengths matter?* Give students a minimum of ten to fifteen seconds to THINK about the prompt and formulate their responses.
    - Round 1: Students silently mix and mingle around the room with no talking. When the teacher says "Pair," the students stop and form a pair with the person closest to them. They shake hands and stand together. The teacher provides the prompt and gives students time to think. Students take turns sharing their responses with their partner. Teachers may have the students thank their partner and repeat several times with a new prompt each time. Teachers may opt to play music while students are mixing and mingling around the classroom and have it stop when it's time to form a pair, think, and share.
    - Round 2: Repeat with this prompt: *Do you think that boys and girls have the same arm spans? Why?* After round 2 is completed, students return to desks.
- Partner Dialogue Have students dialogue. Use some of the listed questions and/or ask students to write some questions that they would be interested in investigating, related to student's arm spans. Move about the groups and observe/monitor conversations.

    - *How is arm span length defined?*
    - *Can someone measure their own arm span length?*
    - *How might the consistency of measurement be an issue with this data? What ideas do you have to help make the measurements consistent?*
    - *Are the arm span lengths of boys and girls the same or different?*

- *What methods might we use to compare the arm span lengths of boy and girls?*

- Use these questions to check <u>understanding</u> of common mathematical vocabulary.
  - *What is the "mean" (representative) arm span length of the class? What is the "median" (typical) arm span length of the class? What is the shortest arm span in the class? What is the longest arm span in the class?*
  - *What might the distribution of arm spans might look like for the whole class. (ex: Would it be the same for boys and girls?)*

Explore

Have students design and implement a process for data collection. Make sure they talk about how to make measurements accurately with precision. After they (whole class) have determined an appropriate protocol for measuring arm span lengths, divide them into pairs to measure the arm span length of their partner. Students should measure and record arm span lengths to the nearest centimeter for their partner.

NOTE* Decide beforehand, how/ where the class data will be compiled and/or displayed.

There are various ways to analyze the data. (a) The class can calculate the measures of center and spread for the boys and girls in their class. (b) They can then construct box plots based on the data to compare boys and girls.

Item 1. Example Class Data

Boy's Arm Spans (cm)	Boy's Arm Spans (cm)		Girl's Arm Spans (cm)	Girl's Arm Spans (cm)
149	155		163	162
140	158		161	158
164	149		148	164
155	158		149	161
150	150		142	163
137	146		144	145
143	152		139	162
155			175	

The sorted values for this data are:

Boys: 137, 140, 143, 146, 149, 149, 150, 150, 152, 155, 155, 155, 158, 158, 164

Girls: 139, 142, 144, 145, 148, 149, 158, 161, 161, 162, 162, 163, 163, 164, 175

Some descriptive statistics for this data are provided in Item 2.

Item 2. Descriptive statistics for Item 1

	Min	Max	Mean	Median	Range
Boys	137	164	150.7	150	27
Girls	139	175	155.7	161	36

What can you infer from the data? (possible) Girls have higher measures of center and greater measures of spread.

Use questions as needed:

- What is the mean arm span for boys? What is the median arm span for boys? What are the shortest and longest arm spans for boys? Give two numbers that cover the middle 50% of the distribution of the arm spans of the boys. What is the range of the most common arm spans for boys?
- What is the mean arm span for girls? What is the median arm span for girls? What are the shortest and longest arm spans for girls? Give two numbers that cover the middle 50% of the distribution of the arm spans of the girls. What is the range of the most common arm spans for girls?
- [*OPT* (Can be used as formative assessment) *What is the mean arm span of the class? What is the median arm span of the class? What is the shortest arm span in the class? What is the longest arm span in the class? Give two numbers that cover the middle 50% of the distribution of the arm spans. What is the range of the most common arm spans for the class?*]

What generalizations, if any, can we make about boys and girls arm lengths, using the results from this data?

Let's examine the data in a box plot.

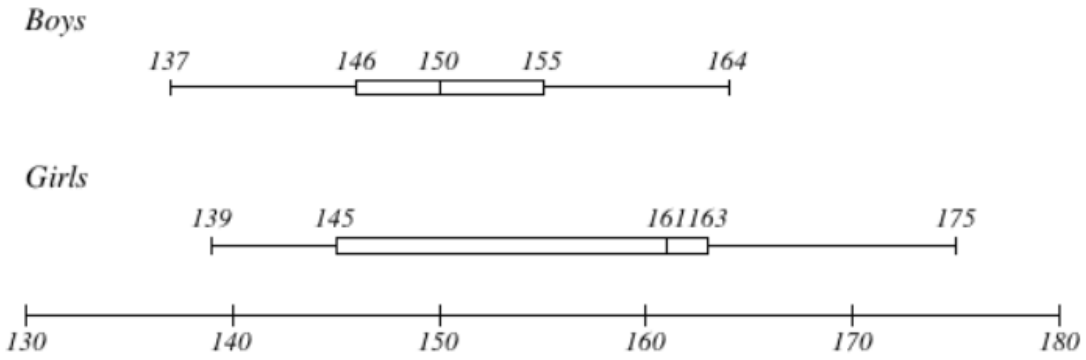
NOTE*: To create a boxplot, students should use the 5-number summary:

minimum, first quartile (25th percentile, Q1), second quartile (median, 50th percentile, Q2), third quartile (75th percentile, Q3), and maximum. [These five numbers are plotted on a line extended from the minimum to the maximum and then a box is created around Q1 and Q3 with lines drawn at the first quartile, 0 is called the interquartile range (IQR). That is, IQR = Q3 - Q1.]

Use the Online Boxplot Grapher. Have partners create box plots for the data sets.

(Box plots for the two groups are shown in Item 3.)

Item 3.

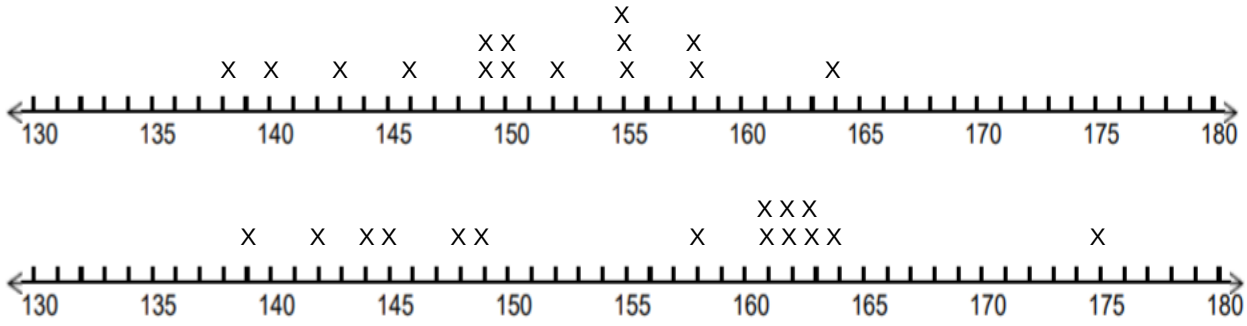


What are some things you are noticing about the data? (possible) The distributions have roughly the same shapes; both are somewhat symmetric (balanced). The distribution for the girls has a larger range and a larger median. The middle 50% of the boy's distribution is contained within the middle 50% of the girl's data, etc.

Question to ponder: How might a line plot of the data sets, visually compare to a box plot?

- Create a line plot of the data sets. Compare with the box plots. What do you notice visually about the data? (Handout, pg 2)

Boys: 137, 140, 143, 146, 149, 149, 150, 150, 152, 155, 155, 155, 158, 158, 164



Girls: 139, 142, 144, 145, 148, 149, 158, 161, 161, 162, 162, 163, 163, 164, 175

What generalizations, if any, can we make about boys and girls arm lengths, using the results from this data?

Explain

- Show the video clip, Mean Absolute Deviation (MAD). Debrief. If using one-to-one technology, students may use the online interactive practice found on the site.
- Provide examples of small data sets for students to practice. (Handout, pg 1- Spread the DATA! Mad practice)
- Next, assign pairs to find the MAD of one of the data sets (boys or girls).

Mean absolute deviation (MAD) of a data set is the average distance between each data value and the mean. Mean absolute deviation is a way to describe variation in a data set. Mean absolute deviation helps us get a sense of how "spread out" the values in a data set are.

With your partner, calculate the Mean Absolute Deviation or MAD, of one of the two data sets.

Boys: 137 (13.7), 140 (10.7), 143 (7.7), 146 (4.7), 149 (1.7), 149 (1.7), 150 (0.7), 150 (0.7), 152 (1.3), 155 (4.3), 155 (4.3), 155 (4.3,) 158 (7.3), 158 (7.3), 164 (13.3)

Girls: 139(16.7), 142(13.7), 144(11.7), 145(10.7), 148(7.7), 149(6.7), 158(2.3), 161(5.3), 161(5.3), 162(6.3), 162(6.3), 163(7.3), 163(7.3), 164(8.3), 175(19.3)

MAD= average distance the values in a data set are from their mean

	Min	Max	Mean	Median	Range	MAD
Boys	137	164	150.7	150	27	83.7/15 = 5.58
Girls	139	175	155.7	161	36	107.9/15 = 7.19

Monitor student progress to offer support as needed. Bring together as whole group to check accuracy, to dialogue, and make inferences about the data.

Interpret the Results

What information about the data can we get from the MAD? (possible + other)

- The boys' data has less spread than the girls' data.
- The girls' data points are about 7 away from the mean.
- The boys' data points are about 6 away from the mean.

Other resources

A resource for teacher learning and/or enrichment/extended learning, or differentiation
https://www.learner.org/courses/learningmath/data/support/lmd7.pdf

Online number line generator
https://www.helpingwithmath.com/printables/others/NumberLineGenerator01.htm

Spread The Data!

Lesson Practice sheet

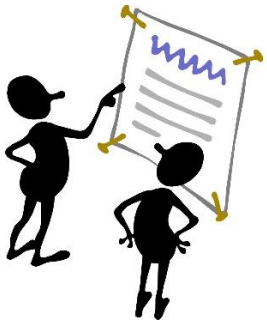
MAD is a tricky concept so let's use easy numbers to get started.

To find the mean absolute deviation (MAD) of the data, start by finding the mean of the data set. DATA SET: 1,2,4,6,8,9

1. Add up all the values 1+2+4+6+8+9 = 30
2. Divide by the number of values to get the mean: 30 divided by 6 values is 5(mean)
3. Find the absolute value of the difference between each data value and the mean: |data value – mean|.
4. Find the sum of the absolute values of the differences. (4+3+1+1+3+4) = 16
5. Divide the sum of the absolute values of the differences by the number of data values. 16 / 6 = 2.67

Calculate the MAD of the data set.

Data set	Mean	|Data value - mean|	Sum of |Differences|	Sum / # of data	MAD
1+2+4+6+8+9 = 30 / 6 = 5	5	|1-5|, |2-5|, |4-5|, |6-5|, |8-5|, |9-5|,	4+3+1+1+3+4 = 16	16 / 6	2.67
3,3,4,6,6,8					
3,4,5,6,7,8,9					
0,3,7,10,15					



LINE PLOTs!

How might a line plot of the data sets, visually compare to a box plot?

Create a line plot of the boys and girls data sets to answer the question.

Boys: 137, 140, 143, 146, 149, 149, 150, 150, 152, 155, 155, 155, 158, 158, 164
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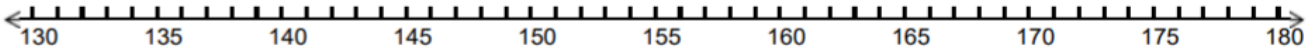
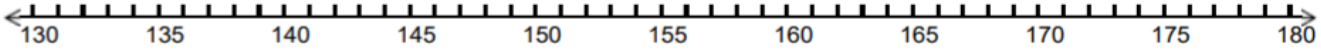
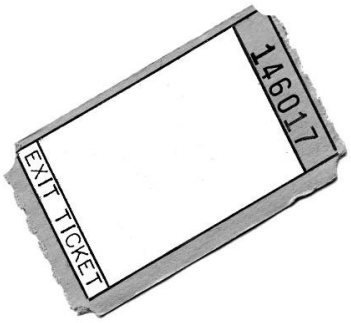




What do you notice about the line plot data?

What generalizations, if any, can we make using the results from this data?



After comparing the box plot and line plot graphs from today's lesson, which presentation of the data (graph) do you think tells the best visual story? Why?

OPT* What questions do you still have?

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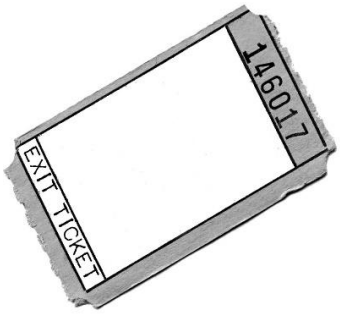

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