NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Discussion

Research Brief

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What Does Research Say the Benefits of Discussion in Mathematics Class Are?

Introducing new material in mathematics class in the United States has typically been done through teacher presentations of a few sample problems followed by demonstrations of how to solve them. The step-by-step demonstrations are often carried out by asking short-answer questions of students along the way (Stigler & Hiebert, 1999). Over the last 20 years, however, mathematics educators have observed and analyzed alternatives to recitation, the questioning pattern described above. In particular, a growing body of literature supports the use of discussion in mathematics class. In this brief, after describing and providing examples of recitation and discussion, some benefits of discussion in mathematics class will be presented. These recommendations are based on published studies that suggest that discussion is a productive alternative to other more passive talk formats. In short, discussion can:

- Increase student learning
- Motivate students
- Support teachers in understanding and assessing student thinking
- Shift the mathematical authority from teacher (or textbook) to community

Recitation versus Discussion

In *Classroom Discourse*, Cazden (2001) made the following observation: The three-part sequence of teacher **I**nitiation, student **R**esponse, and teacher **E**valuation (IRE) is the most common pattern of classroom discourse at all grade levels. The IRE interaction pattern repeats itself throughout a recitation-type lesson. In their succinct summary of implicit rules, Edwards and Mercer (1987) noted: (a) It is the teacher who asks the questions; (b) The teacher knows the answers; and (c) Repeated questions imply wrong answers (p. 45). Below is an example of a recitation sequence contained in a lesson in which Ms. R is working with students on the problem of finding the points of intersection of a line and a parabola:

Ms. R: Let's look at the third problem. How many points of intersection did you come up with? Chris?

Chris: One.

Ms. R: One? Jamie?

Jamie: Uh, two.

Ms. R: Good, two. And what were those two points?

Jamie: One, six and, um, six, eleven.

Ms. R: Good. The intersection points are one, six and six, eleven. Let's look at another one.

In this recitation sequence, Ms. R seemed to be looking for correct answers. She did not appear to be focused on understanding her students' thinking or providing opportunities to discuss strategies using mathematical language. One of the most striking features of a typical recitation sequence is that the teacher tends to be the only one asking questions, as seen above. Thus, recitation could foster the impression that students must participate in accordance with the pattern established by the teacher—namely, students speak only when invited to respond to their teachers' questions.

Discussion provides an alternative to recitation. Within discussions, assessing students' subject-matter knowledge is not necessarily the primary and sole objective. In addition, teachers are interested in helping their students to develop understandings. In the example below, Ms. D works on the same problem as Ms. R, but this time through a discussion rather than a recitation.

Ms. D: Okay, let's talk about the next problem. You were asked to figure out something about the points of intersection of the parabola and the line. What did people come up with?

Jen: We said there was one point.

Juan: My group got two.

Maria: Yeah, we got two too.

Ms. D: All right then, let's take a look at this. I'm hearing that some groups found that there was one point of intersection and others thought that there were two. Let's hear from Maria's group first. Maria, can you describe your strategy?

Maria: Well, we just graphed the parabola and the line, and then we found that they intersected at one, six and at six, eleven.

Ms. D: You graphed it how?

Maria: We used our graphing calculator. At first we thought that there was one point too, and then we had to change the screen and we found the second point.

Ms. D: Does anyone have questions for Maria?

Jen: What do you mean you changed your screen? Because we graphed ours too.

Maria: We had to change the numbers so we could see the graph bigger. Then we saw the two points when we changed to bigger numbers.

Ms. D: Does anyone understand what Maria is saying about seeing the graph bigger and changing to bigger numbers? Can anyone else restate what Maria said using some of the terminology that we discussed yesterday? Grady?

Grady: Yeah, I think she's saying that she changed her viewing window. She probably had to change the y-values so she could see the graph higher. That's what we did because if you just use the normal window then you can only see one point. But we knew there had to be two points because we talked about how if there's only one point, it goes along the side of the graph.

Ms. D: Okay, so I think what you're saying at the end is that if there was only one point of intersection, it would have to be a tangent line, tangent to the parabola. [Ms. D draws a diagram on the board.] Is that what you're saying, Grady?

Grady: Yeah.

Ms. D: Jen's group—did what Maria and Grady said make sense?

Jen's group: Yeah.

Here, Ms. D seemed genuinely curious about how students were making sense of the problem. She was interested in learning about the range of solutions, and she allowed misconceptions to surface. Although the teacher was orchestrating the discourse, students were encouraged to speak with one another in the discussion. Ms. D also provided students with opportunities to use mathematically precise language and to engage with the reasoning of their classmates. *Attending to precision* by communicating precisely to others and *constructing viable arguments and critiquing the reasoning of others* are two practices that are promoted in the Common Core State Standards. These practices are based on the belief that students learn and come to understand mathematics by working to justify why a mathematical statement is true or where a rule comes from (NGA Center and CCSSO, 2010). In the sections that follow, some benefits of discussion are described.

Discussion Can Increase Student Learning

The classroom culture, the ways in which students and teachers interact, the kinds of learning experiences students have, and the tasks that students are asked to engage with all greatly influence the opportunities that students have to learn mathematics in any given classroom (Hiebert et al., 1997). We learn through social interaction (Lave & Wenger, 1991; Vygotsky, 1978). A Vygotskian viewpoint, as articulated by Gibbons (2006), suggests that language use is at the root of learning. More specifically, this view of language calls for any examination of teaching and learning to treat interactions between teacher and learner as crucial. These interactions not only shape students' talk, but they help to construct understanding (Gibbons, 2006). Discussions can take place in small groups or as a whole class. When viewing a classroom as a community of learners, it must be remembered that interacting is not optional, but rather it is essential because communication is necessary for building understanding (Hiebert et al., 1996). In the remainder of this section, three studies which support the idea that discussion-based classrooms can increase student learning are summarized.

The results of Project Challenge offer compelling evidence that shifting to a discussion-based teaching format positively impacts student learning. In their work with Project Challenge, Chapin, O'Connor, and Anderson (2003) put a great deal of emphasis on students talking with one another and with the teacher in particular ways that have been found to be academically productive. The work of Project Challenge took place over four years in a low-income Boston school district and involved about 400 students and 18 teachers in grades 4–7. The majority of these students (65%) were English Language Learners, and most students (78%) qualified for free and reduced lunch. Using *Standards*-based curricula, daily logic-problem warm-ups, and weekly quizzes, these classrooms "emphasized communication by supporting discussions, both lengthy and brief, and by maintaining a constant focus on explanations for students' reasoning" (Chapin & O'Connor, 2007, p. 114). Results on the California Achievement Test (CAT) were used as a measure of student learning. After about three years of the study, the class mean of the Project Challenge students reached the 90th percentile. Project Challenge students also scored better as a whole than students in one of the most highly ranked cities in the state of Massachusetts (see Chapin & O'Connor, 2004; 2007 for more details). These results provide strong evidence that student learning is greatly supported by engagement in academically productive talk (Chapin & O'Connor, 2007).

The case of Railside also suggests that students learn more in classrooms that provide them with opportunities to learn mathematics through discussion. At Railside High School, an urban school in California, the focus of the approach to teaching mathematics was "communicative," meaning that "the students learned about the different ways that mathematics could be communicated through words, diagrams, tables, symbols, objects, and graphs" (Boaler, 2008, p. 59). As they worked on algebra and geometry tasks in heterogeneous classes, the students would frequently be asked to explain their work to each other. In fact, teachers lectured only about 4% of the time. Approximately 72% of the time, students worked in groups while the teachers circulated around their rooms showing methods to students, helping students, and asking them questions about their work. Students presented their work about 9% of the time, and they were questioned by the teacher in a whole-class format about 9% of the time (Boaler & Staples, 2008). As part of the research project, the achievement of Railside students was compared to that of similar-size groups of students being taught through more traditional approaches in two other high schools. In these classes, students did not typically discuss mathematics, but rather watched the teacher demonstrate procedures and then worked through textbook exercises. At the beginning of the year, the two suburban schools using the more traditional approach started with higher mathematics achievement levels than the students at Railside, but by the end of the first year of the study the students at Railside were achieving at the same level in algebra as the students in the suburban schools. By the end of the second year, the Railside students were outperforming the other students on algebra and geometry tests (Boaler, 2008).

One more piece of evidence to support the idea that discussing mathematics can lead to increased student learning comes from a study focused on students' perspectives. In *Listening to My Students' Thoughts on Mathematics Education*, mathematics teacher Joseph Obrycki (2009) described the results of his action research project in which he analyzed six interviews of students in his high school geometry course. The interviews were conducted for Obrycki by a university researcher after he participated in three years of professional development focused on classroom discourse. Obrycki's students noted again and again that his teaching style was different from their past mathematics instructors (who *told* them about mathematical ideas) because he expected them to *think* and "figure stuff out" themselves. Some students noted an initial frustration with this approach, but eventually all students interviewed concluded that working in groups to prove theorems and solve problems was in their best interest in terms of their learning. All six students agreed that it was possible to generate mathematical knowledge on their own, with many noting that this was the best way to learn. When asked at the conclusion of the interview if there was anything she would like to share with other mathematics educators, one student noted: "I don't know if the answer should be withheld all the time, but letting students get to the answer and not just presenting it to them is definitely worthwhile, even if it takes longer" (Obrycki, 2009, p. 201). When students begin to recognize that participating in mathematics discussions helps them to learn mathematics, their motivation to participate may be increased.

Discussion Can Motivate Students

In *Motivation Matters and Interest Counts*, Middleton and Jansen (2011) suggested that teachers should make efforts to involve their students in class by convincing them that many types of contributions will help advance the class's knowledge (e.g., questions, alternative solutions, false starts, conjectures). When teachers do this, they argued, more students feel comfortable and courageous enough to contribute to classroom discussions. Active participation in a collaborative mathematics classroom, therefore, can have a positive impact on student motivation: "Knowledge is built. Understanding grows. Relationships with mathematics and with classroom community members develop" (Middleton & Jansen, 2011, p. 164).

The case of Railside High School also offers evidence that students' motivation to learn mathematics can be positively impacted by participating in discussion-focused classrooms. The results of questionnaires given to students showed that each year the Railside students were significantly more positive about their mathematics experiences than their peers in more traditional classes. For example, 71% of Railside students in Year 2 classes (n = 198), reported "enjoying math class," while only 46% of students in the more traditional classes (n = 318) agreed to this statement (Boaler & Staples, 2008). By their senior year, 41% of Railside students were in advanced classes of precalculus and calculus, compared to only 23% of students coming from the more traditional classes (Boaler, 2008).

Discussion Can Support Teachers in Understanding and Assessing Student Thinking

Some classroom interaction patterns promote deeper mathematical thinking than others (Herbel-Eisenmann & Breyfogle, 2005; Martens, 1999), and skillful questioning of student thinking can provide the teacher with valuable knowledge about students' developing mathematical ideas (Martino & Maher, 1999). NCTM's (2000) Teaching Principle begins with the following claim: "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (p. 16). Discussion is a strategy that can support teachers in understanding what students already know and in determining what they still need to learn. In this sense, listening to students' ideas in discussions can serve as formative assessment that helps teachers make decisions about instruction. To maximize the instructional value of discussion using formative assessment, "teachers need to move beyond a superficial 'right or wrong' analysis of tasks to a focus on how students are thinking about the tasks" (NCTM, 2000, p. 24). Rather than concentrating solely on misconceptions or errors, teachers should make efforts to identify valuable student insights on which further progress can be based (NCTM, 2000). Emphasizing tasks that focus on reasoning and sense-making and providing students with opportunities to discuss mathematics serves to afford teachers with ongoing assessment information. Teachers must then guide the students toward new understandings and support their development as they work to communicate mathematically.

A key component of formative assessment is feedback. When students routinely take part in discourse in which meanings are developed and shared, they are provided with feedback that supports them to move their learning forward (Lee, 2006). In particular, feedback allows students to compare how their thinking correlates with that of other students in the class as well as the conventional mathematical ideas. It also allows students opportunities to reconsider and revise their thinking from the early "first draft" stage to a more refined "final" version (Choppin, 2007). A discussion-rich learning environment can provide students with agency over their own learning.

Discussion Can Shift the Mathematical Authority to Community

When teachers shape the discourse by opening it up through discussion, there is real potential to shift the mathematical authority from teacher (or textbook) to community (Webel, 2010). For this shift to truly be realized, however, the students must be aware of and willing to take on roles that differ from their roles in recitation sequences. More specifically, for discussions to be productive, students must "share the responsibility for developing a community of learners in which they participate" (Hiebert et al., 1997, p. 16). Two important aspects of the students' role in discussion-oriented classroom communities are sharing and listening. First, students must take responsibility for sharing the results of their explorations and for explaining and justifying their strategies. Second, students must realize that learning means learning from others, taking advantage of others' ideas, and listening to the results of their classmates' investigations (Hiebert et al., 1997). Thus, to become full participants in a community of peers doing mathematics, students must be willing to share with and actively listen to one another.

Research by Otten et al. (2011) suggested that when students actively listen to one another, mathematical reasoning can be made more explicit and more accessible. As a result, more students can participate in the discussion by articulating mathematical thoughts and developing shared meanings. This type of community knowledge-building can cause students to compare and contrast their own mathematical thinking to that of their peers, change their own thinking, and come to new understandings (Kosko, 2012). The teacher plays an important role in helping students understand what counts as an acceptable explanation and justification in mathematics class (Yackel & Cobb, 1996) so that students' efforts to listen to each other are not hampered by student talk that is unclear or imprecise.

Concluding Thoughts

The fact that the rules of the IRE pattern, the defining characteristic of recitation, so heavily favor the power of the teacher is undoubtedly one reason why it has become such a popular style of teaching (Lemke, 1990). Teachers understandably may find it difficult to deviate from IRE because maintaining it offers many advantages to them, such as setting the topic, controlling the pace, and steering the direction that the topic develops (Lemke, 1990). Thus, navigating a new terrain of teaching can be challenging for teachers at any level, particularly because they may never have experienced, as a learner, an approach to teaching other than lecture or recitation (Marrongelle & Rasmussen, 2008). Some teachers have handled this challenge by believing they need to stop all "telling" (see, e.g., Chazan & Ball, 1999). Yet, the recitation versus discussion interaction patterns need not be dichotomous. Acknowledging that talk formats operate on a continuum, some researchers have pointed out that most classrooms operate somewhere between recitation and discussion (Herbel-Eisenmann, 2001). Cazden (1988) contended that within a matter of moments, a lesson can move from recitation to discussion, and the activity that students are engaged in can determine the form of the lesson. As a general rule, however, any extreme version of the IRE-recitation sequence can be viewed as having the potential for closing down the discourse. In contrast, as teachers move away from recitation toward more purposeful discussions, there is a potential for opening up the discourse and shifting the mathematical authority from teacher to community.

To be clear, it is not just getting students to talk more that matters. The orchestration of the discourse must be purposeful (Smith & Stein, 2011), and it must be academically productive "in that it supports the development of students' reasoning and students' abilities to express their thoughts clearly" (Chapin & O'Connor, 2007, p. 115). The field is just beginning to understand and develop ways to support teachers in facilitating productive discussions in mathematics class. More studies are needed that validate the effectiveness of some of the existing strategies available for orchestrating productive discussions. In addition, the field would benefit from studies that identify features of unproductive discussions that inhibit student learning.

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