Squares and Cubes

Lesson Overview

In this lesson, students will explore numbers that fall between perfect square and cube roots to begin laying a foundation for further study of rational and irrational numbers.

Standards Addressed

CCSS 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

CCSS 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

NOTE: This lesson is an *introduction* to 8.NS.2; thus, students will *require further instruction* in order to master the content. 8.EE.2 supports 8.NS.2.

Disciplinary Literacy Best Practices

Think Ink Pair Share (TIPS) (Pair with Elbow Partner)
Give Me Five
Pairs Squared
Making Thinking Visible (MTV)
Gallery Walk
Most Important Point (MIP)

Lesson Plan

Time Required – Two 60 minute class periods.

Disciplinary Vocabulary – rational numbers, irrational numbers, square roots, cube roots, (a number) squared (x^2), (a number) cubed (x^3); symbols: $\sqrt{}$ and $\sqrt[3]{}$

Materials Needed:

Both days:

- (1) Pencil per student
- (1) Calculator for each pair of students
- Chart paper or butcher paper (for MTV)
    - Tape to hang butcher paper
- (1) Notecard or post it for each student to use as an exit ticket
- Student math notebooks
- Timer

Day One

- (1) Squares Bell Ringer handout for each student
- "Square Strips"
    - There are 4 different strips included in the materials for this lesson. Students will work with partners. Place Squares Strips A & B in envelopes for half of the pairs. Place Squares Strips C & D in envelopes for half of the pairs.
- Squares Strips instructions (one set for each envelope)

Day Two

- (1) Cubes Bell Ringer handout for each student
- "Cube Strips" – 2 for each pair of students
    - There are 4 different strips included in the materials for this lesson. Students will work with partners. Place Cubes Strips A & B in envelopes for half of the pairs. Place Cubes Strips C & D in envelopes for half of the pairs.
- Cube Strips instructions (one set for each envelope)

NOTE: Bell Ringer handouts, Squares Cards and Cubes Cards may be found after the "Lesson Author" section of this document.

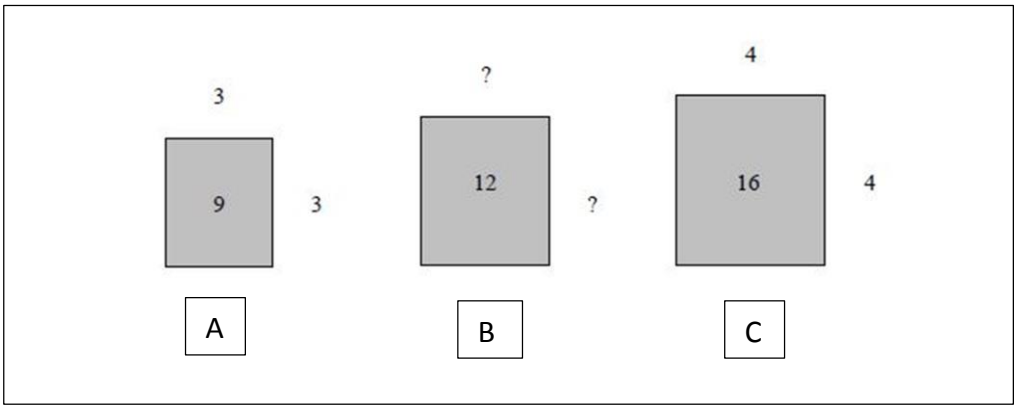
Assessment:

- MTV charts
- Exit Ticket - Most Important Point from the lesson written on a note card or post it and turned in at the end of class.

Day One

Engage

- Bell Ringer – Squares Handout. Distributing it gives students a "notes page" to keep for future reference. You may also display it on the white board or some other device.



  - Students THINK and INK (record) answers to the following:
    - Look at squares A and C. Describe the relationship between the edges and area of each of the squares.
    - Since 12 is between 9 and 16, what can you infer about the edges of the middle square (square B)?
    - Describe how you can use the calculator to find the mystery number WITHOUT using the square root key.
  - Students PAIR with their elbow partner and SHARE their responses. They also try one or both of the strategies for finding the mystery number. Ask them to record their expressions.
  - Use the "give me five" strategy to call on five students to share their responses whole group. Record student guesses as they give them to you.

Explore

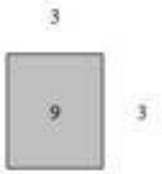
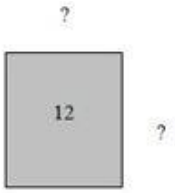
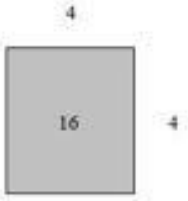
- Give each pair of students a calculator and an envelope with two Squares Cards enclosed. Give half of the groups Cards A & B and the other half Cards C & D. Instruct pairs to find the mystery numbers (dimensions of the middle square in each strip). Remind them that they may not use the square root or x^2 key on the calculator. Also tell them to record the expressions they use while trying to find the dimensions. Tell students they will have 7 minutes to work and set a timer for them.

- Facilitate group dialogue and assist as needed while students work. Possible guiding questions include:
  - Describe the relationship between the edges of the square and the area of the square.
  - How might you get closer to the actual area of the square? (In reference to a specific expression that students have recorded as they work.)
- Have Pairs form Squares so that all four cards (A – D) are in each group. Each pair takes a turn explaining how they found the dimensions of their middle squares. Tell students they will have 7 minutes to work and set a timer for them.
- Visit groups to facilitate dialogue as you did earlier.

Explain

- At the end of 7 minutes, tell students they need to share their thinking using Making Thinking Visible (MTV). Post the following statement: *Write a short explanation for how to find the square root of a number that is not a perfect square.* Give each group a piece of chart or butcher paper to record their explanation as well as the expressions they generated for the mystery dimensions. Tell groups they have 7 minutes to create their MTV and 3 minutes to post and be ready to Gallery Walk.
- Students Gallery Walk to study the MTV posters created by the groups. They should take their notebooks with them so that they can write questions or comments about what they see.
  - Questions to guide their thinking as they study each group's poster:
    - How is this group's explanation for finding the square root of a number that isn't a perfect square the same as or different from our group's explanation? Is there anything we want to add to our explanation?
    - Which of their expressions are the same as ours? Which are different? How are they different?
- Also, engage in whole-class dialogue to establish the following points:
  - x^2 and $\sqrt{x}$ have an inverse relationship
  - x^2 is a number multiplied by itself
  - A perfect square is the result of a whole number being multiplied by itself – call attention to the models on the cards.
  - A number that is not a perfect square still has a square root – call attention to the models on the cards.

- The table below may be helpful during whole-class dialogue and as an anchor chart posted for student reference. Complete the first row while thinking aloud. Ask students to provide the cells that are outlined.

Visual	Multiplication expression	Exponent expression	Radical expression
	$3 \cdot 3 = 9$	$3^2 = 9$	$\sqrt{9} = 3$
	$3.464 \cdot 3.464 \approx 12$	$3.464^2 \approx 12$	$\sqrt{12} \approx 3.464$
	$4 \cdot 4 = 16$	$4^2 = 16$	$\sqrt{16} = 4$

Extend

One natural extension is to have students place the values on the cards on a number line to order and compare them. This should be part of the further instruction that will be required for students to master the content of 8.NS.2.

Lesson Assessment:

- The MTV charts/posters provide valuable formative information.
- As an exit ticket, students write the Most Important Point of what they took away from the lesson that day. They should write this on a notecard or post-it and turn it in at the end of class.

Day Two

Engage

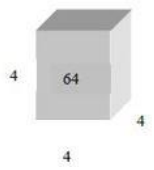
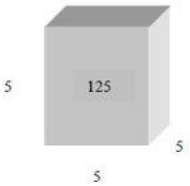
- Cubes Bell Ringer – Follow the same procedure as Day One.

Explore

- Follow the same procedure as Day One.
- Use the Cubes Cards instead of the Squares Cards.

Explain

- Follow the same procedure as Day One.
- Also, engage in whole-class dialogue to establish the following points:
  - x^3 and $\sqrt[3]{x}$ have an inverse relationship
  - x^3 is a number used as a factor 3 times → $x * x * x$
  - A perfect cube is the result of a whole number being used as a factor 3 times – call attention to the models on the cards.
  - A number that is not a perfect cube still has a cube root – call attention to the models on the cards.
- The table below may be helpful during whole-class dialogue and as an anchor chart posted for student reference. Complete the first row while thinking aloud. Ask students to provide the cells that are outlined.

Visual	Multiplication expression	Exponent expression	Radical expression
	$4 \cdot 4 \cdot 4 = 64$	$4^3 = 64$	$\sqrt[3]{64} = 4$
	$4.38 \cdot 4.38 \cdot 4.38 \approx 84$	$4.38^3 \approx 84$	$\sqrt[3]{84} \approx 4.38$
	$5 \cdot 5 \cdot 5 = 125$	$5^3 = 125$	$\sqrt[3]{125} = 5$

Extend

One natural extension is to have students place the values on the cards on a number line to compare them. This should be part of the further instruction that will be required for students to master the content of 8.NS.2.

Lesson Assessment:

- The MTV charts/posters provide valuable formative information.
- As an exit ticket, students write the Most Important Point of what they took away from the lesson that day. They should write this on a notecard or post-it and turn it in at the end of class.

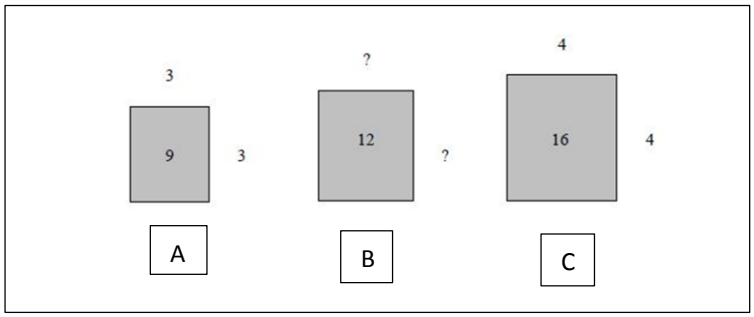
Teacher Reflections and Biographical Information

Students often mistakenly compute 3^2 as if the exponent were a factor, so they come up with 6 instead of 9. They do the same when the exponent is 3, thinking that 4^3 is 12 rather than 64. By using the visual representation of squares and cubes, that common mistake may be avoided. If students gain an understanding of the relationships between multiplication, exponent, and radical, they will be much less likely to make this common error. It will also lay a good foundation for working with other roots.

Lesson Author:

Adapted from the S^3 Curriculum Framework by Margaret Lorimer, S^2TEM Centers Education Specialist. Mrs. Lorimer was a classroom teacher of mainly middle school math for 14 years before serving as a Middle School Math coach for three years. She has been an Education Specialist since 2007 and was part of the team that wrote the original S^3 document.

Squares Bell Ringer – Think Ink Pair Share



THINK and INK

- Look at squares A and C. Describe the relationship between the edges and area of each of the squares.

- Since 12 is between 9 and 16, what can you infer about the edges of the middle square (square B)?

- Describe how you can use the calculator to find the mystery number WITHOUT using the square root ($\sqrt{x}$) or x^2 key. Don't use the calculator yet!

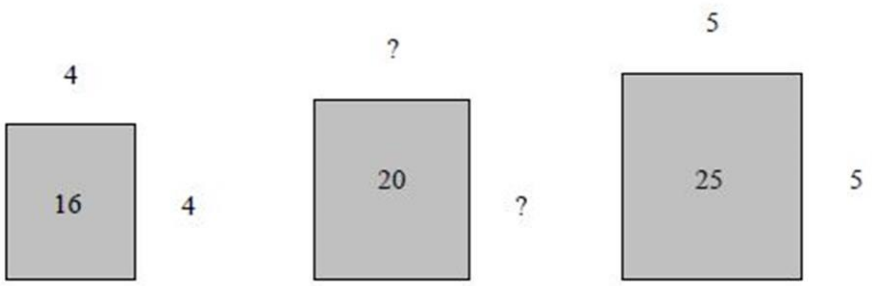
PAIR and SHARE

Compare your THINK and INK answers with your partner.

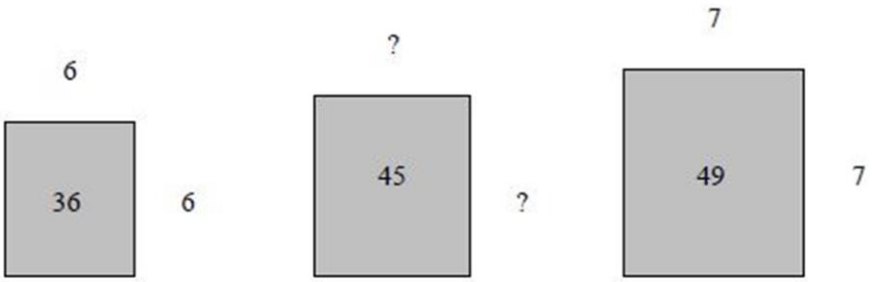
Use the calculator to try the strategies you and your partner described. DO NOT use the SQUARE ROOT ($\sqrt{x}$) or x^2 key. Record your work.

Squares Cards

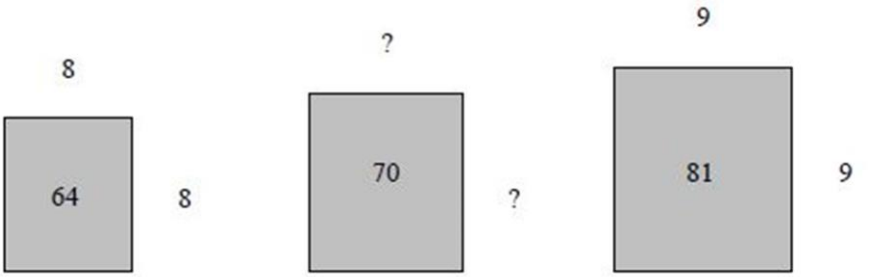
Card A



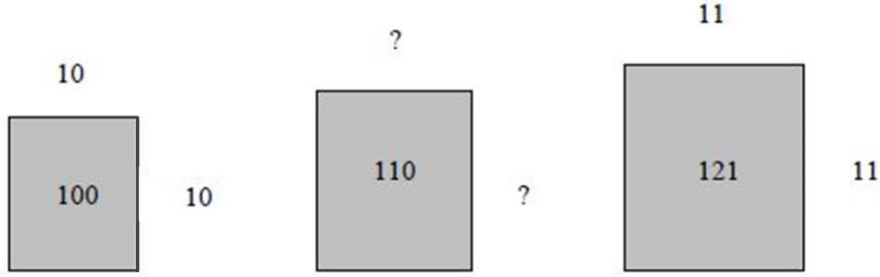
Card B



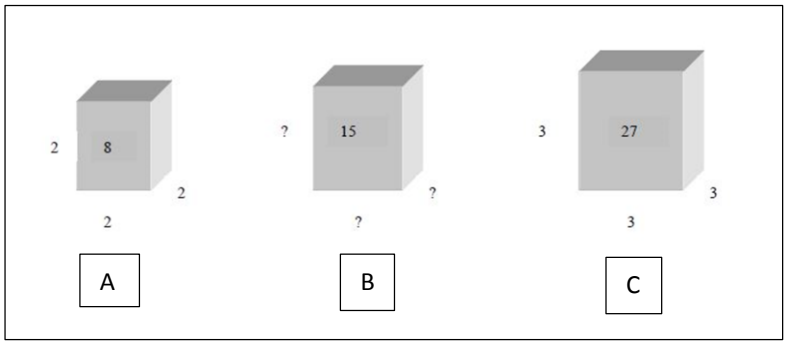
Card C



Card D



Cubes Bell Ringer – Think Ink Pair Share



THINK and INK

- Look at cubes A and C. Describe the relationship between the edges and volume of each of the cubes.

- Since 15 is between 8 and 27, what can you infer about the edges of the middle square (square B)?

- Describe how you can use the calculator to find the mystery number WITHOUT using the CARROT (^) or $\sqrt[3]{x}$ key. Don't use the calculator yet!

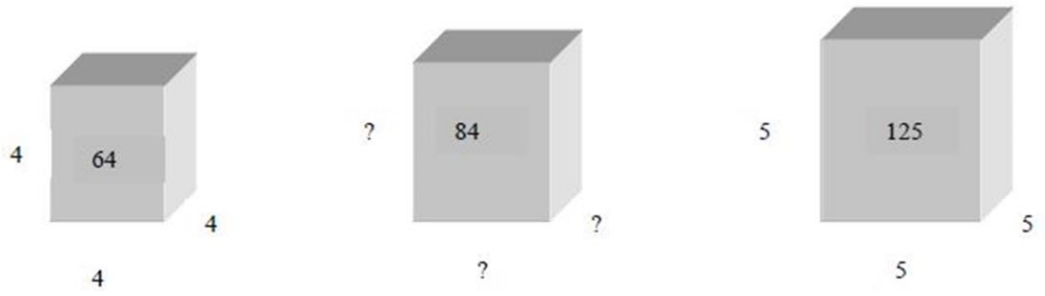
PAIR and SHARE

Compare your THINK and INK answers with your partner.

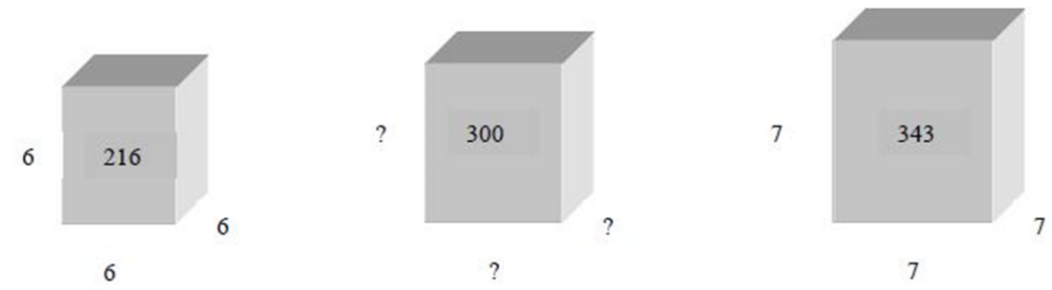
Use the calculator to try the strategies you and your partner described. DO NOT use the CARROT (^) or CUBE ROOT ($\sqrt{x}$) key. Record your work.

Cubes Strips

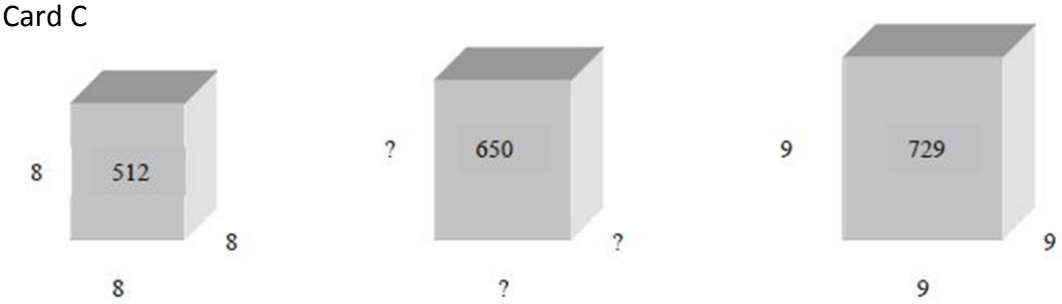
Card A



Card B



Card C



Card D